VU Diskrete Mathematik

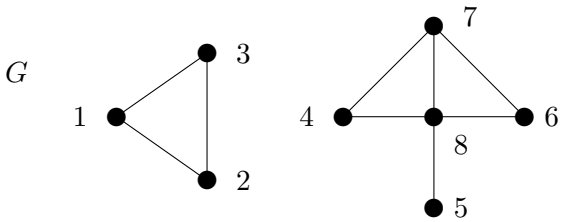
Exercises for Oct 20, 2023

9) If T is a tree having no vertex of degree 2, then T has more leaves than internal nodes. Prove this claim

(a) by induction,

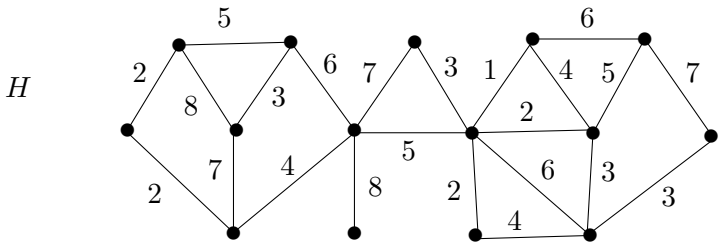
(b) by considering the average degree and using the handshaking lemma.

10) Use the matrix tree theorem to compute the number of spanning forests of the graph below!



11) Use the matrix tree theorem to compute the number of spanning forests of the graph $G = (V, E)$ where $V = \{v_0, v_1, v_2, \ldots, v_{2n}\}$ and $E = \{v_0 v_i \mid i = 1, 2, \ldots, 2n\} \cup \{v_{2i-1} v_{2i} \mid i = 1, 2, \ldots, n\}$

12) Use Kruskal's algorithm to find a minimal and a maximal spanning tree of the following weighted graph.



13) Let $G = (V, E)$ be a connected graph with an even number of vertices. Show that there is a (not necessarily connected) spanning subgraph (i.e. a subgraph with vertex set V) in which all vertices have odd degree. Is this also true for non-connected graphs?

14) List all matroids (E, S) with $E = \{1\}$, $E = \{1, 2\}$ or $E = \{1, 2, 3\}$.

15) Prove: If $M = (E, S)$ is a matroid and A and B are two bases of M, then $|A| = |B|$.

16) Let E be a set, $1 \leq k \leq |E|$ an integer, and let S denote the set of all subsets $X \subseteq E$ with cardinlity at most k. Examine whether (E, S) is a matroid.