Case Studies in Cost-Optimized Paneling of Architectural Freeform Surfaces

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Abstract.

Paneling an architectural freeform surface refers to an approximation of the design surface by a set of panels that can be manufactured using a selected technology at a reasonable cost, while respecting the design intent and achieving the desired aesthetic quality of panel layout and surface smoothness. Eigensatz and co-workers [Eigensatz et al. 2010] have recently introduced a computational solution to the paneling problem that allows handling large-scale freeform surfaces involving complex arrangements of thousands of panels. We extend this paneling algorithm to facilitate effective design exploration, in particular for local control of tolerance margins and the handling of sharp crease lines. We focus on the practical aspects relevant for the realization of large-scale freeform designs and evaluate the performance of the paneling algorithm with a number of case studies.

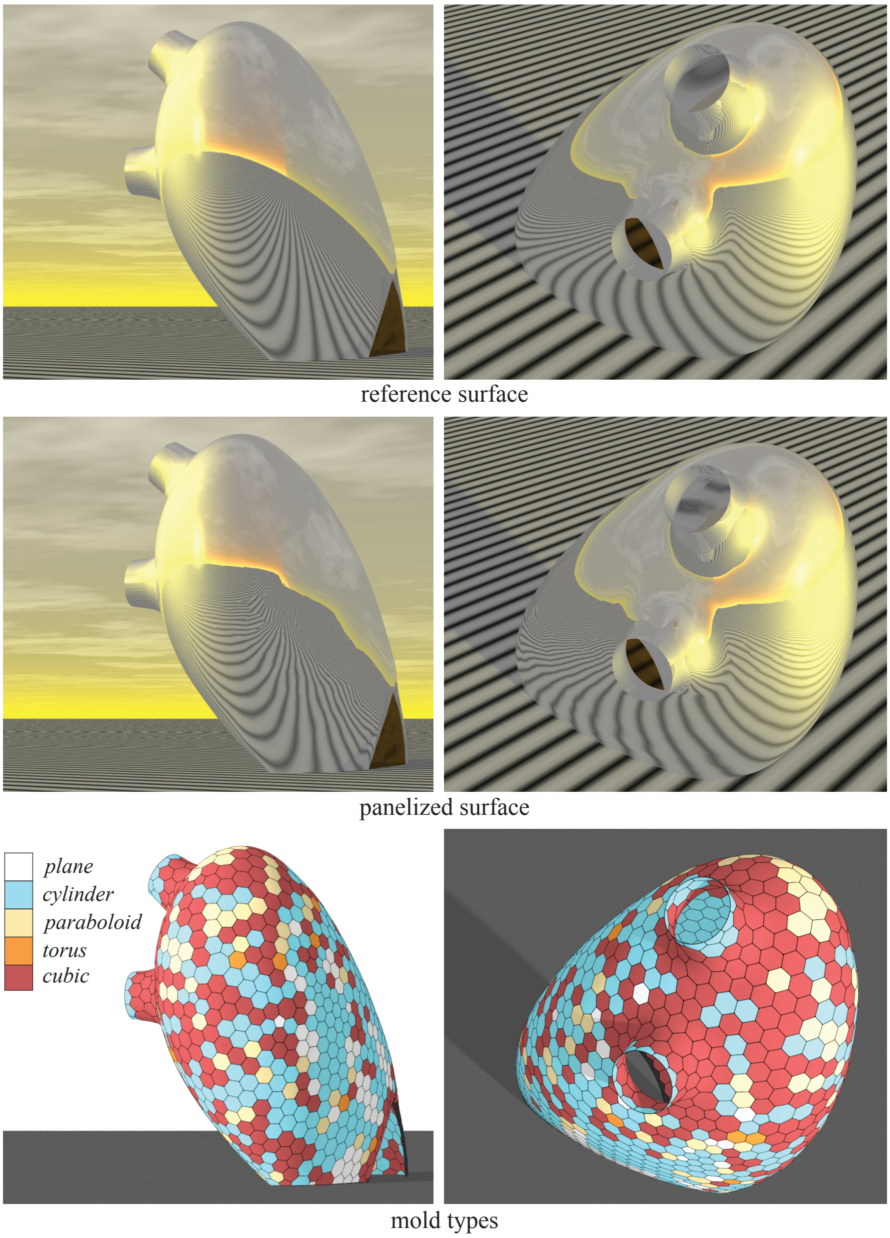


Figure 1: *Given a reference surface (top row), the* paneling algorithm *produces a rationalization of the the input. The paneling solution (middle row) employs a small set of molds that can be reused for cost-effective panel production (bottom row), while preserving surface smoothness and respecting the original design intent. The shown metal paneling solution is* 40% *cheaper than the production alternative of using custom molds for each individual panel. Figure 11 presents a variety of solutions that achieve cost savings of up to* 60%. *Figure 4 lists the metal cost ratios used.*

1 Introduction

Freeform shapes play an increasingly important role in contemporary architecture. Recent technological advances enable the large-scale production of single- and double-curved panels that allow panelizations of architectural freeform surfaces with superior inter-panel continuity compared to planar panels. However, the fabrication of curved panels incurs a higher cost depending on the complexity of the panel shapes, as well as on the employed material and panel manufacturing process (see Table 1). This gives rise to the so-called *paneling* task: The approximation of a design surface by a set of panels that can be manufactured using a selected technology at a reasonable cost, while respecting the design intent and achieving the desired aesthetic quality of panel layout and surface smoothness. The paneling task is a key component of the *rationalization* process for architectural freeform designs.

The challenge in paneling architectural freeform surfaces lies in the complex interplay of different objectives related to geometric, aesthetic, and fabrication constraints that need to be considered simultaneously. In this paper we discuss the paneling solution recently introduced in [Eigensatz et al. 2010], henceforth referred to as the *paneling algorithm*, and focus on the practical aspects relevant for the realization of large-scale freeform designs. We enhance the algorithm to handle spatially adaptive quality thresholds and propose an extension that allows incorporating sharp feature lines. With these new functionalities, the algorithm offers improved control for the architect to adapt the paneling according to the design specifications. We present three case studies to evaluate the performance of the paneling algorithm and provide insights into how the different parameter tradeoffs affect the quality of the results.

The rest of the paper is organized as follows: After discussing related work in the area of surface rationalization, we first classify different available panel types and fabrication processes (Table 1). We then formalize the paneling problem as stated in [Eigensatz et al. 2010] and review the main algorithmic contributions of their paneling solution. Section 4 presents our extensions to the existing formulation that allow processing freeform surfaces with sharp feature curves and enable local control of the paneling quality. In Section 5, we present three case studies to evaluate the performance of the algorithm, before concluding with a discussion of future research directions to address current limitations in Section 6.

Related Work

A forward approach to surface rationalization is to use parametric design. An example for this was proposed by Glymph and coworkers [Glymph et al. 2002], where certain classes of surfaces are rationalized using planar quadrilateral panels. Parametric design is also available in many standard CAD tools nowadays. Such an approach introduces a logic into a geometric model by means of a generative sequence and relations between geometric objects. This logic helps in enabling simultaneous

Figure 2: *Projects involving double-curved panels where a separate mold has been built for each panel. These examples illustrate the importance of the curve network and the existing difficulties in producing architectural freeform structures. (Left: Peter Cook and Colin Fournier, Kunsthaus, Graz. Right: Zaha Hadid Architects, Hungerburgbahn, Innsbruck.) Figure taken from [Eigensatz et al. 2010].*

control of the surface shape and the paneling layout. The simple causal chains inherent to parametric modeling, however, are insufficient for the rationalization of complex freeform geometries.

Other early contributions to the field of freeform architecture come from research at Gehry Technologies (see, e.g., [Shelden 2002]). These are mostly dedicated to developable or nearly developable surfaces, as a result of the specific design process that is based on digital reconstruction of models made from material that assumes (nearly) developable shapes. This approach is well suited for panels made of materials like sheet metal that may be deformed to developable or nearly developable shapes at reasonable cost. The approach is not sufficient, however, for panels made of materials like glass, for which the production processes limit shapes achievable at reasonable cost to very restricted classes of developable surfaces (see Table 1).

Most previous work on the paneling problem deals with planar panels. For various reasons, planar quadrilateral (quad) panels are preferred over triangular panels. Based on the theory of discrete differential geometry (see also [Bobenko and Suris 2008]), Pottmann and colleagues propose algorithms for covering general freeform surfaces with planar quad panels with new ways of supporting beam layout and for the related computation of multi-layer structures [Liu et al. 2006; Pottmann et al. 2007]. More recently, this approach was extended to the covering of freeform surfaces by single-curved panels arranged along surface strips [Pottmann et al. 2008b]. Figure 3 shows an example freeform surface rationalized using planar quads and developable strips, respectively. Additional results in this direction, e.g., hexago-

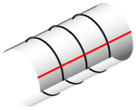
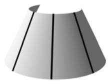
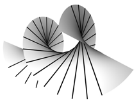
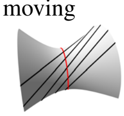
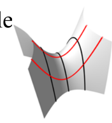
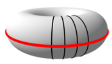
surface types	**manufacturing possibilities**		
	glass	*metal*	*fibre reinforced concrete/plastic*
single curved isometric to the plane, no or little plastic deformation of material			
cylindrical parts of right circular cylinders 	machine for bending and thermal tempering	roll bending machine	configurable mold or custom hot-wire cut foam mold
conical parts of right circular cones 	configurable or custom mold, no thermal tempering	machine or reconfig-urable mold	configurable mold or custom hot-wire cut foam mold
general single curved developable surfaces 	custom mold, no thermal tempering		custom hot-wire cut foam mold
double curved usually plastic deformation of material is involved			
general double curved 	custom molds, no thermal tempering of glass	machine or reconfig-urable mold	custom molds commonly made of EPS foam
general ruled generated by a moving straight line 	straight lines can be exploited	see above	foam molds can be hot-wire cut
translational carries two families of congruent profile 	congruent profiles can be exploited		congruent profiles can be exploited
rotational, cf. Figure 6 carries one family of congr. profiles 			

Table 1: *Classification of panel types and state-of-the-art production processes for common materials in architecture. Although we do not cover all the relevant production processes, this table is for a rough guideline. Planar panels have been left out.*

nal meshes with planar faces, have been presented at "Advances in Architectural Geometry" [Pottmann et al. 2008a].

These approaches, however, focus on one specific type of panels (planar or developable) for rationalizing a given freeform surface, and do not explicitly consider the aesthetic quality of panel layout or surface smoothness. With these rationalization approaches it is difficult to freely choose the paneling seams, since they need to closely follow a so-called conjugate curve network on the given freeform surface, a notion that is defined by the curvature behavior of the surfaces (see [do Carmo 1976] and [Liu et al. 2006]).

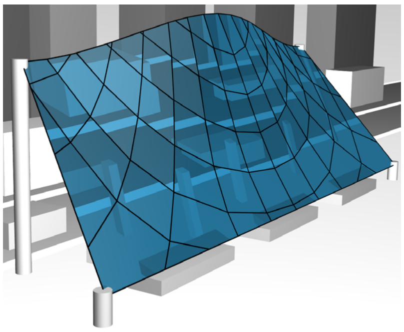
The optimization leading to a paneling solution is obtained by controlled deviation of the reference surface to increase the mold reuse. This is similar in spirit to symmetrization [Mitra et al. 2007; Golovinskiy et al. 2009] proposed to enhance object symmetry, i.e., repetitions, by controlled deformation of the underlying meshing structure.

2 Panels and Fabrication

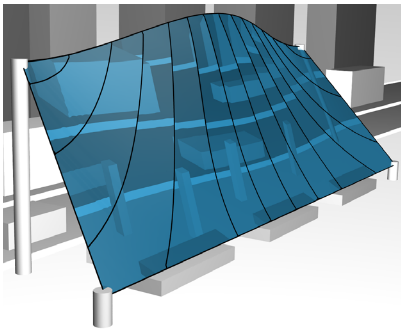
Table 1 gives an overview of the state-of-the-art in architectural panel production. Curved panels are either produced using specially fabricated *molds* with the cost of mold fabrication often dominating the panel cost, or the panels require unique machine configurations, which drive cost by means of machining time. There is thus a strong incentive to reuse the same mold or machine configuration for the production of multiple panels to reduce the overall cost. In the following we use the term *mold* to also refer to machine configuration.

The choice of panel types depends on the desired material and on the available manufacturing technology. The paneling algorithm does not depend on materials: they may be transparent or opaque, include glass, glass-fibre reinforced concrete or gypsum, metal, wood, etc. Currently the algorithm supports five panel types that possess different cost to quality tradeoffs: planes, cylinders, paraboloids, torus patches, and general cubic patches (see Figure 4). If these types cannot approximate a surface segment within the required tolerances, a custom general double curved panel is used.

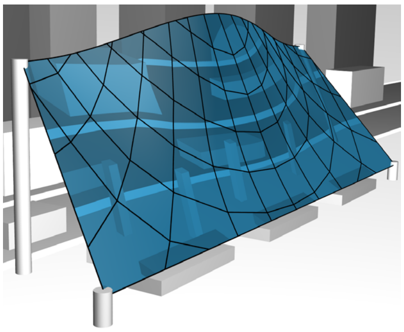
Planar panels are easiest to produce, but result in a faceted appearance when approximating curved freeform surfaces, which may not satisfy the aesthetic criteria of the design. A simple class of curved panels are cylinders, a special case of single-curved (developable) panels. Naturally, such panels can lead to a smooth appearance only if the given reference surface exhibits one low principal curvature. General free-form surfaces often require double-curved panels to achieve desired quality specifications prescribed in terms of tolerances in divergence and kink angles (see Section 3 for details). The paneling algorithm currently supports three instances of such panels: paraboloids, torus patches, and cubic patches. Paraboloids and tori are important



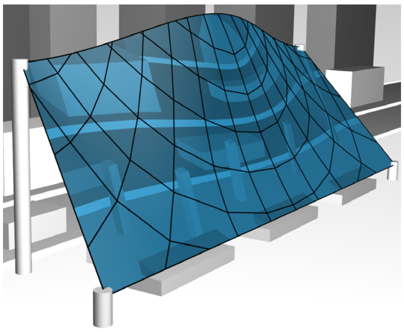
(a) *A conical planar quad mesh according to [Liu et al. 2006] results in a maximum kink angle of* 11°.



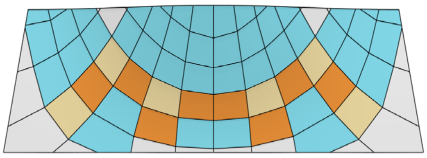
(b) *Developable surface strips according to [Pottmann et al. 2008b] results in a maximum kink angle of* 6° *between strips.*



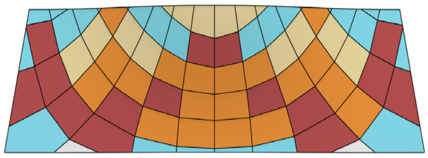
(c) *Paneling solution using* 1° *kink angle threshold (divergence:* 4.7*mm; cost:* 294*).*



(d) *Paneling solution using* $1/4^\circ$ *kink angle threshold (divergence:* 1.6*mm; cost:* 998*).*



(e) *Panels colored by type of corresponding mold.*



(f) *Panels colored by type of corresponding mold.*

Figure 3: *Comparison with state-of-the-art rationalization algorithms on a freeform facade design study. (a, b) Rationalization using a planar quad mesh and developable surface strips, respectively. (c-f) Rationalization using the paneling algorithm with* 1° *and* $1/4^\circ$ *kink angle thresholds, shown along with visualization of respective mold types (using glass cost ratios listed in Figure 4). A detailed overview of mold reuse for (e) is shown in Figure 8.*

Panel types

plane | *torus*
cylinder | *cubic*
paraboloid | *custom*

Costs per mold and per panel

	glass						*metal*					
mold	-	2	18	24	30	-	-	3	6	6	6	-
panel	1	2	5	5	5	35	1	3	6	6	6	12

Figure 4: *The panel types currently supported by our algorithm and two typical cost sets.*

because they are special classes of translational and rotational surfaces and carry families of congruent profiles (parabolae and circles, respectively). This typically simplifies mold production (see Table 1 and Figure 6). Although cubic panels do not have any such advantage for manufacturing, they offer the highest flexibility and approximation power. Thus a small number of cubic or more general double-curved molds are often indispensable to achieve a reasonable quality-cost tradeoff.

Mold reuse is a critical cost saving factor. In order to compute paneling solutions with mold reuse in reasonable time one needs to restrict the search space and parameterize panel types using a few parameters only. The paneling algorithm, therefore, uses the restricted panel types paraboloids, tori and cubics instead of the much more general translational, rotational and general double-curved surfaces. Paraboloid, torus, and cubic are defined by 2, 3 and 6 shape parameters, respectively (please refer to [Eigensatz et al. 2010] for details). In Section 6 we discuss the possibility of adding other panel types.

3 Paneling Architectural Freeform Surfaces

We review both the specification of the paneling problem and the optimization approach presented by Eigensatz and coworkers. For a more detailed description, in particular with respect to mathematical and algorithmic aspects, we refer the reader to [Eigensatz et al. 2010].

3.1 Problem Specification

Let F be a given input freeform surface, called *reference surface*, describing the shape of the design. The goal is to find a collection of *panels*, such that their union approximates the reference surface. Since the quality of the approximation strongly depends on the position and tangent continuity across panel boundaries, Eigensatz and coworkers identify two quality measures (see Figure 5):

- **divergence:** quantifies the spatial gap between adjacent panels and,
- **kink angle:** measures the jump in normal vectors between adjacent panels.

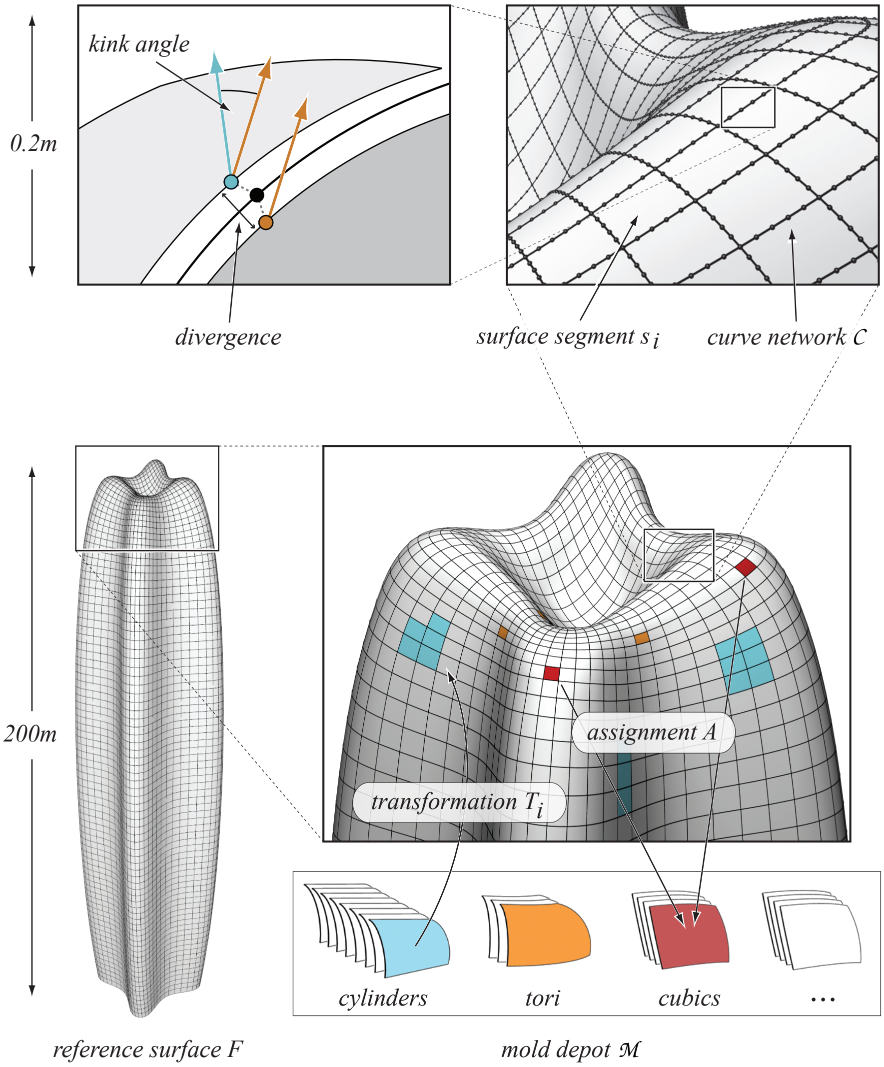


Figure 5: *Terminology and variables used in the paneling algorithm. The reference surface F and the initial curve network $\mathcal{C}$ are given as part of the design specification. The optimization solves for the mold depot $\mathcal{M}$, the panel-mold assignment function A, the shape parameters of the molds, the alignment transformations T_i, and the curve network's normal displacement. Figure taken from [Eigensatz et al. 2010].*

While divergence is strongly related to the viability of a paneling solution, the kink angles influence the visual appearance, since they are related to reflections. Hence one can allow higher kink angles in areas not or only barely visible to an observer. We will elaborate on this possibility in Sections 4.2 and 5.2.

The intersection curves between adjacent panels are essential for the visual appearance of many designs (see Figure 2) and typically reflect the structure of the building, as they often directly relate to the underlying support structure. An initial layout of these curves is usually provided by the architect or engineer as an integral part of the design. While small deviations are typically acceptable in order to improve the paneling quality, the final solution should stay faithful to the initial curve layout and reproduce the given pattern as good as possible by the intersection lines of adjacent panels. The collection of all panel boundary curves (strictly speaking panel intersection curves) forms the *curve network*, which splits the given input freeform surface into *segments*. Each segment, in general polygonal, of the curve network has to be covered by a panel.

The *paneling problem* is formulated as follows: Approximate a given free-form surface F by a collection of panels of selected types such that pre-defined thresholds on divergence and kink angle are respected, the initial curve network is reproduced as good as possible, and the total production cost is minimized. The production cost of a panelization comprises the following terms: the production cost of each employed mold and the cost of producing each panel from its assigned mold (see Figure 4 for two typical cost sets and Figure 8 for an illustration).

3.2 Paneling Algorithm

A paneling solution can be computed using the optimization algorithm described in [Eigensatz et al. 2010]. This algorithm takes as input the reference surface F, the initial curve network, and global thresholds on maximal kink angle and divergence, along with a permitted deviation margin of the final paneled surface from the reference surface. As output, the algorithm computes the parameters that determine the shape of the fabrication molds and the alignment transformations that position the panels in space. These parameters are computed in such a way that the reference surface is approximated as good as possible, while the kink angle and divergence thresholds are satisfied everywhere. At the same time, the cost of fabrication is minimized by favoring panels that are geometrically simple and thus cheaper to manufacture wherever possible, and maximizing the amount of mold reuse.

In order to achieve these conflicting goals, the paneling optimization is formulated as a mixed discrete/continuous optimization that simultaneously explores many different paneling solutions (see [Eigensatz et al. 2010] for details). From all these different alternatives, the solution of minimal overall fabrication cost is selected that satisfies the kink angle and divergence thresholds. An essential ingredient in this optimization is controlled deviation of the paneling from the initial design surface. By allowing the curve network to move away from the reference surface, panels can fit

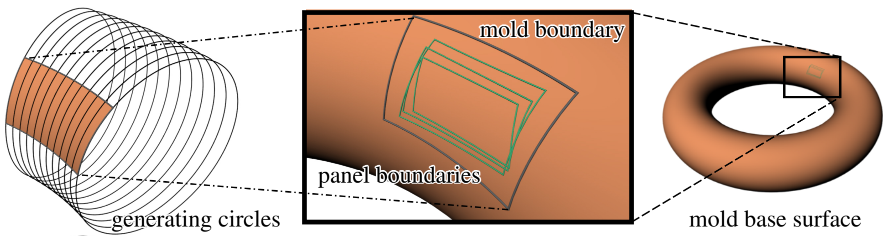


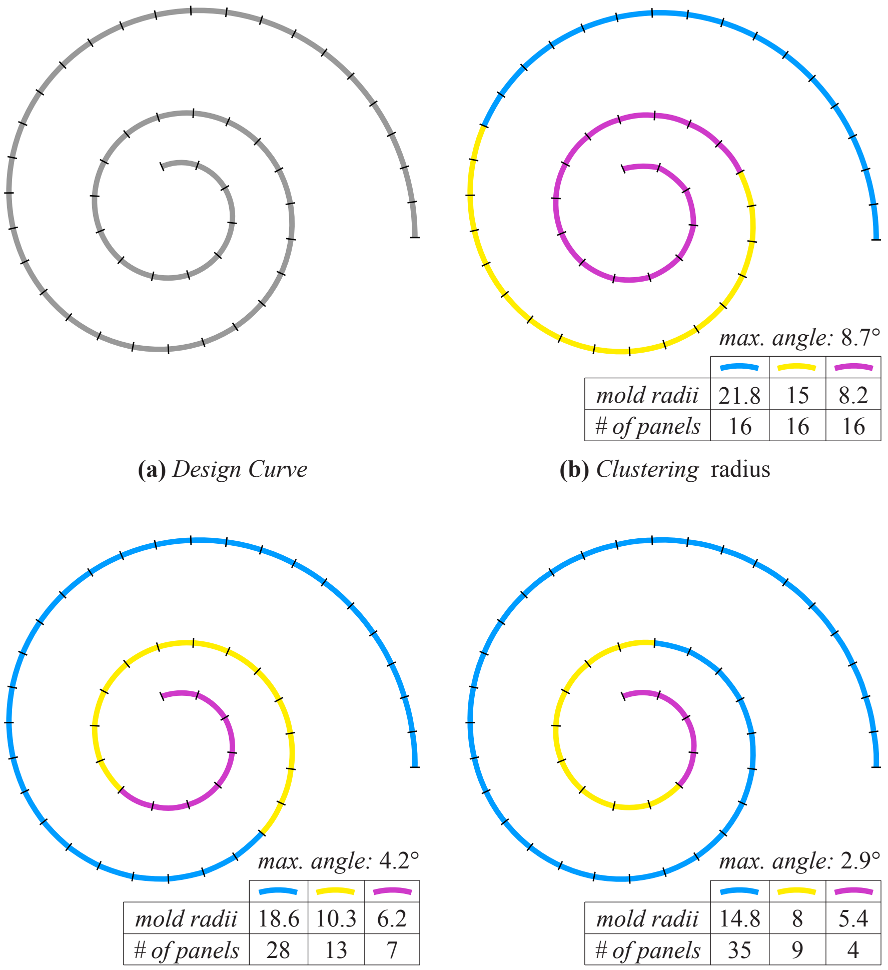
Figure 6: *Example of mold reuse. Panel boundary curves are in general not congruent. However, several panels may be closely grouped together on the same mold base surface. In that case the same mold or machine configuration, which embraces all affected panels, may be used to manufacture the panels. This figure further illustrates how the congruent profiles of a rotational or translational surface, in this case the circles generating a torus, can be exploited for mold fabrication.*

together with smaller kink angles and divergence, simpler and thus cheaper panels can be used in certain regions, and the amount of reuse of molds can be increased. Figure 7 demonstrates the effectiveness of the discrete optimization presented by [Eigensatz et al. 2010] on an illustrative example, comparing different techniques to enable mold reuse.

The results shown in [Eigensatz et al. 2010] include solutions to the paneling problem for large-scale architectural freeform designs that often consist of thousands of panels. Typically, these paneling solutions consist of patches of flat, single and double curved panels as shown in Figure 3, therefore partly generalizing the approaches introduced in [Liu et al. 2006] and [Pottmann et al. 2008b] to include double curved panels. The main innovations of the paneling algorithm can be summarized as follows:

- Given a table of mold and panel production costs, the paneling algorithm computes a panelization with minimal cost while meeting predefined quality requirements.
- The algorithm is adaptable to numerous production processes and materials.
- The possibility to explore diverse quality requirements and cost tables provides valuable information to guide design decisions.
- The rationalized 3D models produced by the algorithm may be used for visual inspection, prototype panel manufacturing, quality control, and the final production of freeform surfaces.
- Interference with the architects design intent is minimized.

The original paneling algorithm provides a general framework and is extensible in various ways. We propose and investigate two specific extensions in Section 4 and discuss further extension possibilities in Section 6.



(a) *Design Curve* **(b)** *Clustering* radius

(c) *Clustering* (1 / radius) **(d)** *Discrete Optimization*

Figure 7: *Illustrative comparison of different techniques for mold reuse. The curve should be approximated with circle arcs of varying radii. This can be understood as a simple paneling with cylinders of varying radii, where the figure shows an orthogonal cross section. The input design curve shown in* ***(a)*** *consists of nicely aligned circle arcs with decreasing radii from* 25 *to* 5. *The method shown in* ***(b)*** *clusters these radii (using k-means clustering) to obtain* 3 *molds and assigns the best mold to each segment. The colors indicate the segments sharing the same mold. The method shown in* ***(c)*** *does the same, but performs a clustering of* $(1/radius)$ *instead of clustering the radius itself, which is a much better distance approximation for cylinders as shown in [Eigensatz et al. 2010] and therefore the maximal kink angle is already much lower compared to* ***(b)***. *The method shown in* ***(d)*** *performs the full discrete optimization presented in [Eigensatz et al. 2010] and leads to an even better mold depot that enables a paneling with only* 3 *molds but very low kink angles. The differences presented on this schematic example become even more prominent if more complex surfaces and/or panel types are involved.*

4 Extensions

In this section we discuss algorithmic extensions to the method of Eigensatz and coworkers [Eigensatz et al. 2010] that broaden its applicability.

4.1 Sharp Features

The algorithm introduced by Eigensatz and coworkers assumes that the input reference surface is smooth everywhere. Sharp feature lines, however, are used in architectural freeform designs to highlight strong characteristic features and to enhance the visual appeal of a design. We therefore propose an extension of the paneling algorithm to incorporate sharp features.

mold type / *cost*	**plane**	**cylinder**	**parab.**	**torus**
cost per **mold** dependent on type	plane 1	cylinder 1 cylinder 2 cylinder 3 cylinder 4-6 cylinder 7-8	paraboloid 1-3	torus 1 torus 2
cost per **panel** dependent on type	18 panels	16 panels 8 panels 6 panels 4 panels each 1 panel each	2 panels each	6 panels 2 panels

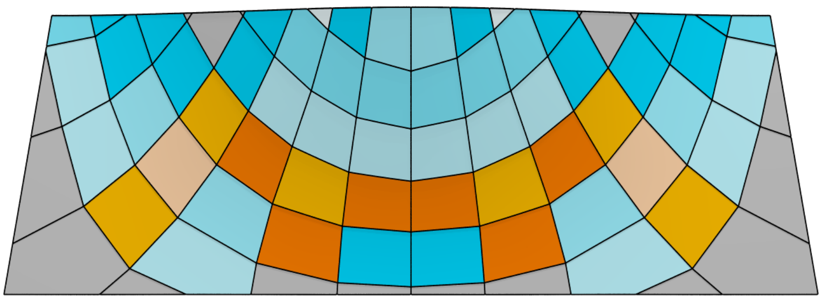


Figure 8: *Illustration of the mold depot and the cost model by means of the example shown in Figure 3(e). The colors of panels are saturated according to mold reuse. Figure 4 lists the glass cost ratios used for this example.*

Sharp feature lines can either be specified by the designer as specially marked lines of the initial curve network, or automatically computed by detecting sharp creases on the design surface. To support sharp features we adapt the original paneling algorithm such that

- kink angle thresholds are not applied along the curves describing sharp features and
- the tangent continuity between two panels on opposite sides of a sharp feature is not optimized.

Figure 14 demonstrates how this extension enables paneling freeform surfaces with sharp features.

4.2 Adaptive Control of Paneling Quality

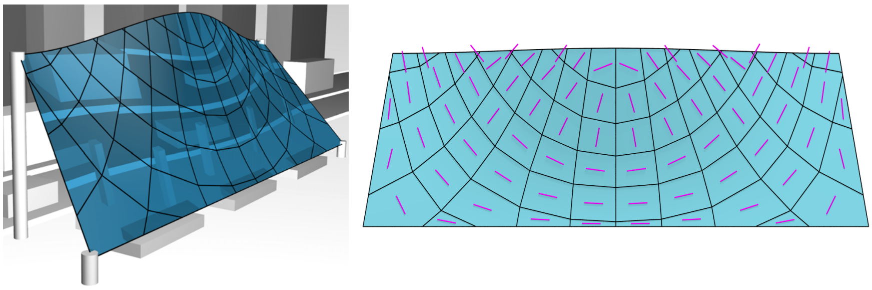
The paneling algorithm introduced in [Eigensatz et al. 2010] guarantees compliance with user-specified tolerance thresholds on divergence and kink angle. These thresholds are specified globally for the entire surface. In practice, however, the quality requirements might vary for different regions of the design. For regions not visible from certain view-points, for example, higher kink angles might be acceptable to reduce manufacturing cost. We therefore extend the original paneling algorithm to optimize the paneling quality with respect to a spatially adaptive importance function on the design surface.

As shown in Figure 10 this importance function can, for example, be computed using a visibility calculation that computes the visibility for every point on the design surface, if the design is viewed from a path or street around the building. This importance function is then an additional input to our extended paneling algorithm to

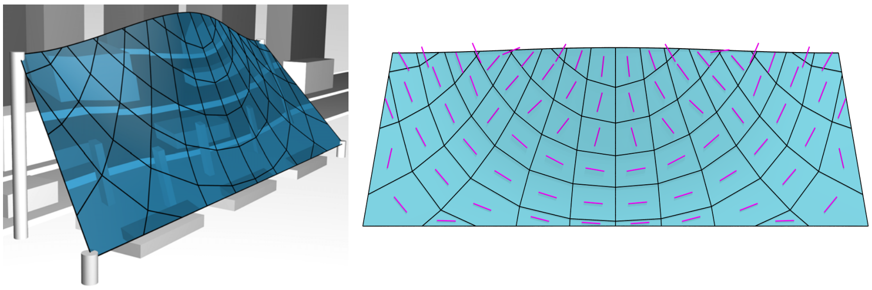
- adaptively specify a separate kink angle threshold for every point on the curve network and
- focus the tangent continuity optimization on important regions.

Figures 10-13 demonstrate how this adaptive quality control directs the use of expensive panels towards regions where they are needed most, leading to an improved paneling quality at similar or lower costs compared to globally specifying thresholds. Achieving the same quality at the important regions with the original paneling algorithm using global thresholds requires a much more expensive paneling.

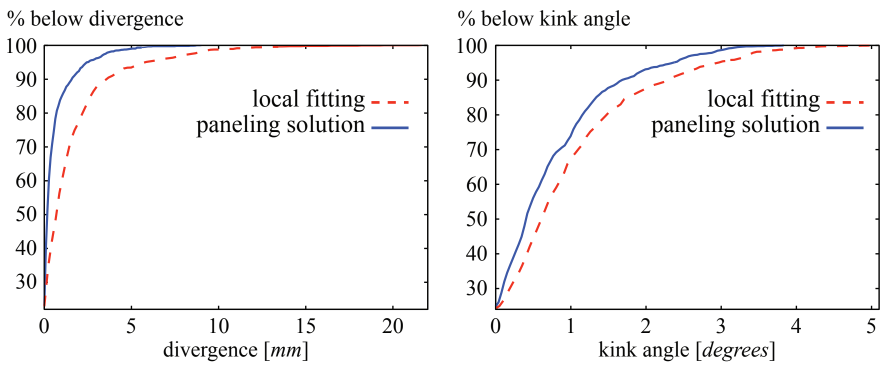
The same technique can be used to adaptively control the divergence or the deviation from the original design surface.



(a) *Local fitting of cylinders.*



(b) *Paneling solution.*



(c) *Cumulative histograms of divergence and kink angles for the above solutions.*

Figure 9: *The paneling algorithm restricted to cylindrical panels. Here we compare a result on the Facade Design Study computed using simple local fitting of cylinders (a) to a paneling solution using only cylinders (b). For both results we show the axis directions of cylinders colored in magenta and the cumulative histograms of resulting divergences and kink angles.*

5 Case Studies

In this section we demonstrate the performance of the paneling algorithm on three case studies. Specifically we compare our solutions with state-of-the-art rationalization alternatives, study the preservation of sharp features, and compare the cost trade-offs for global kink angle specifications versus spatially adapted ones.

5.1 Facade Design Study

We compare several rationalization possibilities for a freeform facade. For this case study we use glass mold cost ratios as listed in Figure 4.

Figure 3a shows a rationalization result using a conical planar quad mesh, which implies very favorable properties for simplifying the substructure, cf. [Liu et al. 2006; Pottmann et al. 2007]. Naturally this approach leads to a facetted result with kink angles up to 11°. A further option makes use of the close relation between planar quad meshes and developable strip models ([Pottmann et al. 2008b]): Refining the planar quad mesh in one direction and keeping the faces planar leads to a rationalization using single-curved strips. Clearly this results in a much smoother representation of the surface as can be seen in Figure 3b (maximum 6° kink angle), while one could still make use of a planar quad mesh for the substructure. The deformation of glass to general single-curved panels, however, requires molds to be built, a possibility that was ruled out because of budgetary issues. Therefore the paneling algorithm was used to proof feasibility for the competition, making use of cylindrical panels only. The superiority of such a restricted paneling solution to results that are achievable using local fitting of cylinders is documented in Figure 9. Figure 3 compares further paneling solutions with respect to cost and paneling quality, making use of the complete set of mold types.

5.2 Skipper Library

Initially issued by Texxus, the skipper library is a feasibility study also picked up by Formtexx for freeform metal cladding. The case study demonstrates our extension of the paneling algorithm allowing adaptive control of the paneling quality, as well as the ability of the paneling algorithm to handle arbitrary panel layouts. The presented panel layout was created using the dual mesh of a circle packing mesh (cf. [Schiftner et al. 2009]), which leads to a panel layout consisting mainly of hexagonal panels combined with a torsion free support structure. Our motivation to adaptively control the paneling quality is given by the following:

Due to various constraints imposed by surrounding buildings, restricted access paths, neighboring trees and foliage, different sections of architectural buildings have different visibility. This can be exploited to reduce the manufacturing cost of such buildings by allowing larger kink angles in less visible regions. As described in

Section 4, we generalize the paneling algorithm proposed in [Eigensatz et al. 2010] to allow spatially variable kink angle specifications as opposed to a global maximum kink angle threshold. Figures 10-13 compare the results on manufacturing cost for a global threshold versus two spatially adapted threshold specifications. The local importance functions are computed based on visibility of the reference surface when moving along the specified access paths (see Figure 10). For this case study we use metal mold cost ratios as listed in Figure 4. The middle row in Figure 1 shows a paneling solution with 1° global kink angle threshold.

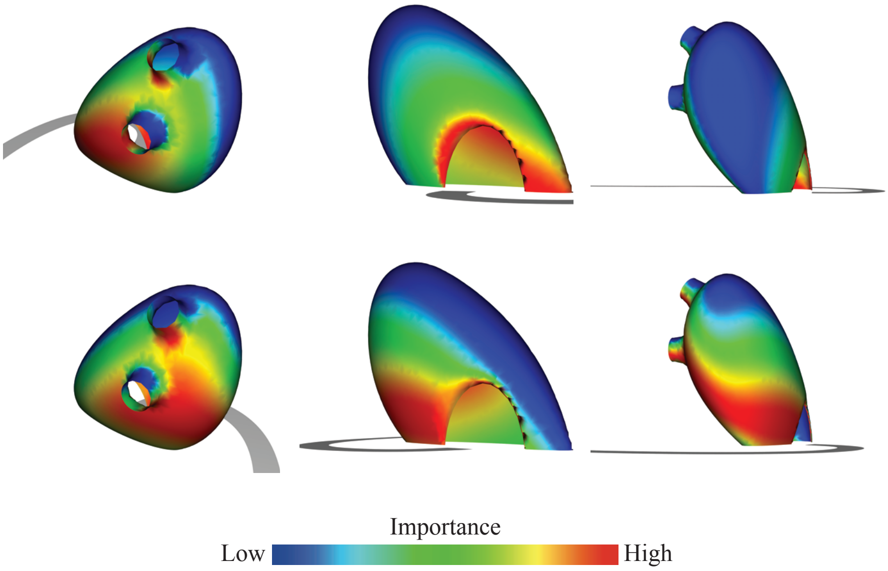
5.3 Lissajous Tower

Lissajous Tower is an example skyscraper specifically created for illustrating our extension to the paneling algorithm for handling sharp features. The surface contains large nearly flat and single-curved parts as well as small highly curved parts, which can not be approximated by cylinders within realistic tolerances. Figure 14 compares two paneling solutions produced by the paneling algorithm with maximum kink angle thresholds of 1° and 3°, respectively. While both solutions preserve the characteristic sharp feature line of the design, the production cost is significantly reduced (by 40%) for a slight relaxation in the maximum kink angle constraint. For this case study we use glass mold cost ratios as listed in Figure 4.

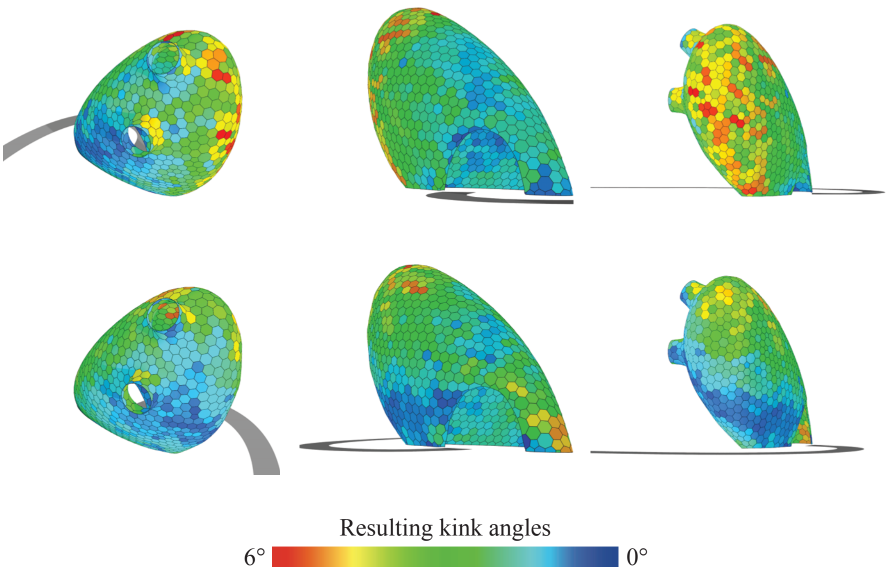
6 Discussion

Limitations. The input to the paneling algorithm is a design surface and a set of curves (panelization seams) that define how the surface is divided into panels. We consider both the surface and the panelization seams as design intent and thus aim to change them as little as possible. This approach leads to the following implications:

- If design surface or seams inherently violate the limits of a certain material or production process, for example with respect to maximum panel sizes, then the paneling algorithm will not eliminate this.
- When computing minimum cost solutions the paneling algorithm cares about cost of panel production only. This is reasonable because it just minimally changes the design surface and panelization seams, and therefore does not influence the cost of further parts like the substructure.

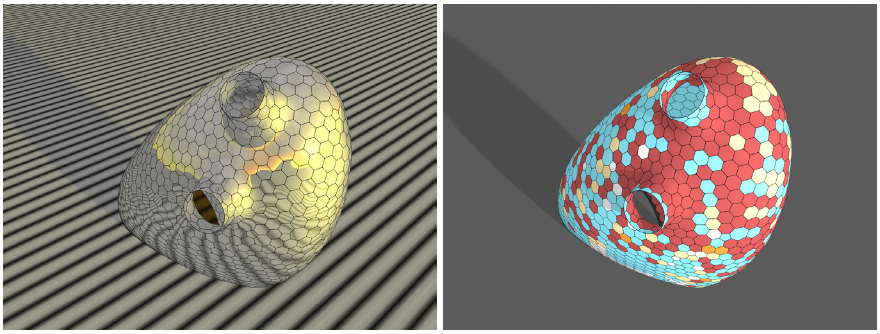


(a) *Spatially adaptive importance functions computed based on visibility from path 1 (top row) and path 2 (bottom row). These importance functions are used for paneling solutions as shown in 10(b) and Figures 11-13 (b) and (c), respectively.*

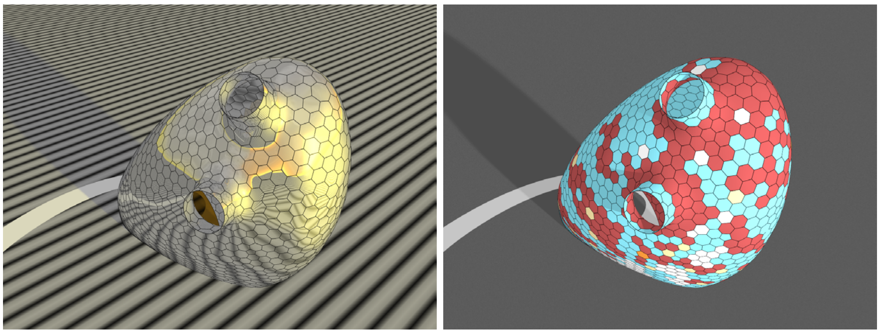


(b) *Kink angles of two paneling solutions (top and bottom rows) using adaptive thresholds based on the two importance functions shown in 10(a). Further renderings of the results are shown in Figures 11-13.*

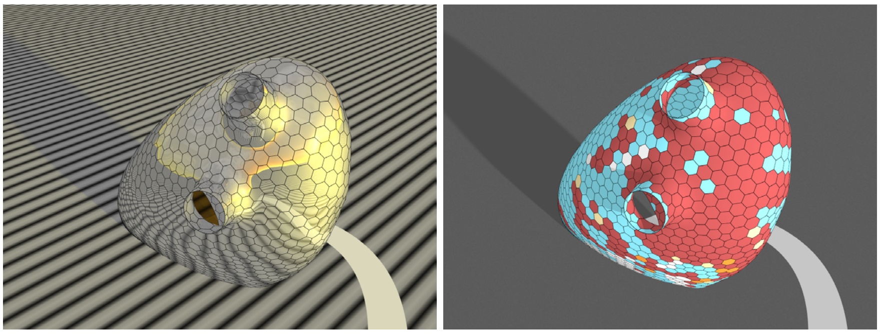
Figure 10: *Adaptive quality control.*



(a) *Paneling solution with kink angle thresholds specified globally over the surface.*

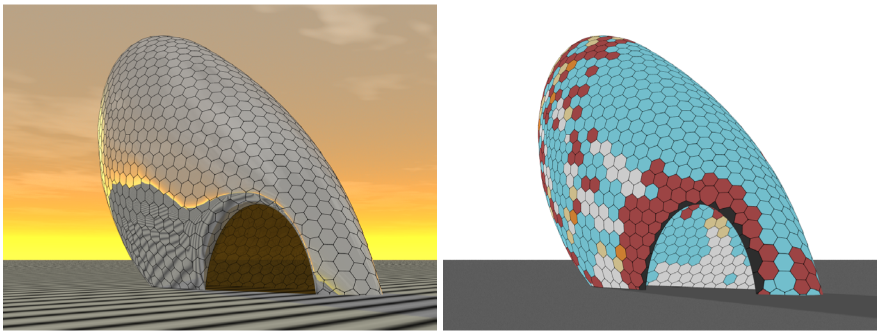


(b) *Paneling solution with spatially adaptive kink angle thresholds.*

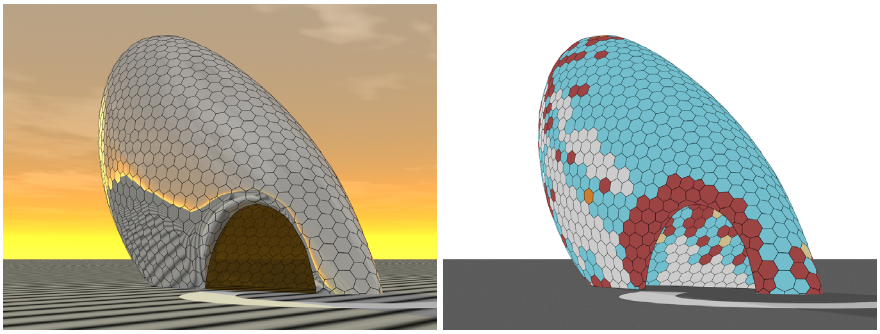


(c) *Paneling solution with another set of spatially adaptive kink angle thresholds.*

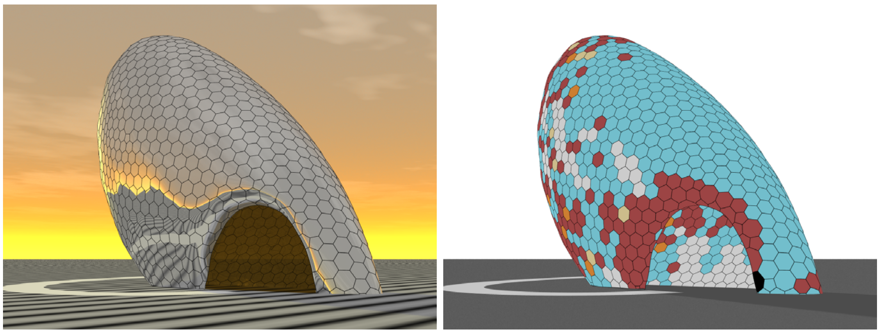
Figure 11: *Effect of global vs spatially varying kink angle specifications on the Skipper Library dataset. Paneling solutions using a global kink angle specification (a) and using adaptive kink angle thresholds computed based on the extent of visibility while moving along the indicated ground paths (b, c). Left column images show the reflection lines on paneled surfaces, while right column images show the mold types for individual panels (color convention same as in Figure 1). Figures 12 and 13 show the same solutions from two other views. Figure 10 shows the spatially varying kink angle thresholds used in (b) and (c).*



(a) *Paneling solution with kink angle thresholds specified globally over the surface.*



(b) *Paneling solution with spatially adaptive kink angle thresholds.*



(c) *Paneling solution with another set of spatially adaptive kink angle thresholds.*

(a) global *cost:* 5946

	(white)	(blue)	(yellow)	(orange)	(red)	(dark grey)
molds	-	38	15	2	119	32
panels	102	622	84	11	349	32

divergence: 6mm
max angle: 3°

(b) path 1 *cost:* 5810

	(white)	(blue)	(yellow)	(orange)	(red)	(dark grey)
molds	-	73	8	1	169	22
panels	152	683	17	5	321	22

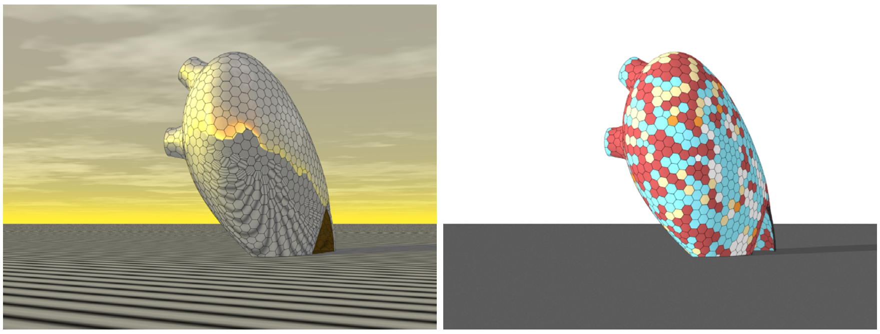
divergence: 6mm
max angle: 1°-6° (adaptive)

(c) path 2 *cost:* 6265

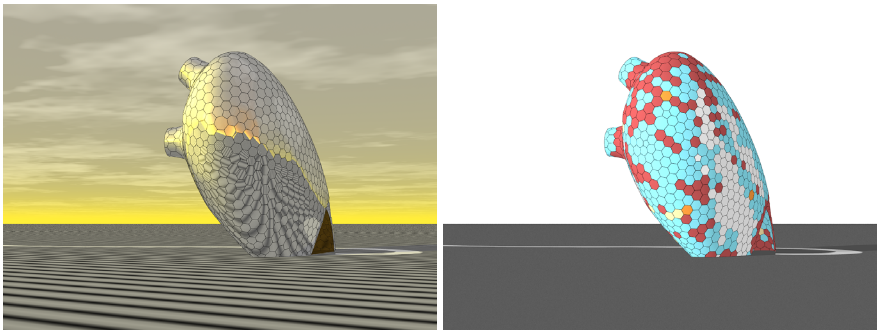
	(white)	(blue)	(yellow)	(orange)	(red)	(dark grey)
molds	-	45	7	5	191	15
panels	97	631	17	13	427	15

divergence: 6mm
max angle: 1°-6° (adaptive)

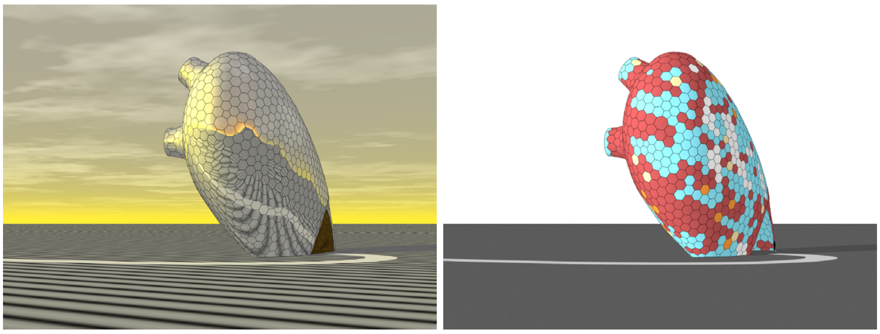
Figure 12: *Effect of global vs spatially varying kink angle specifications on the Skipper Library dataset, along with statistics for corresponding paneling solutions (see also Figure 11).*



(a) *Paneling solution with kink angle thresholds specified globally over the surface.*

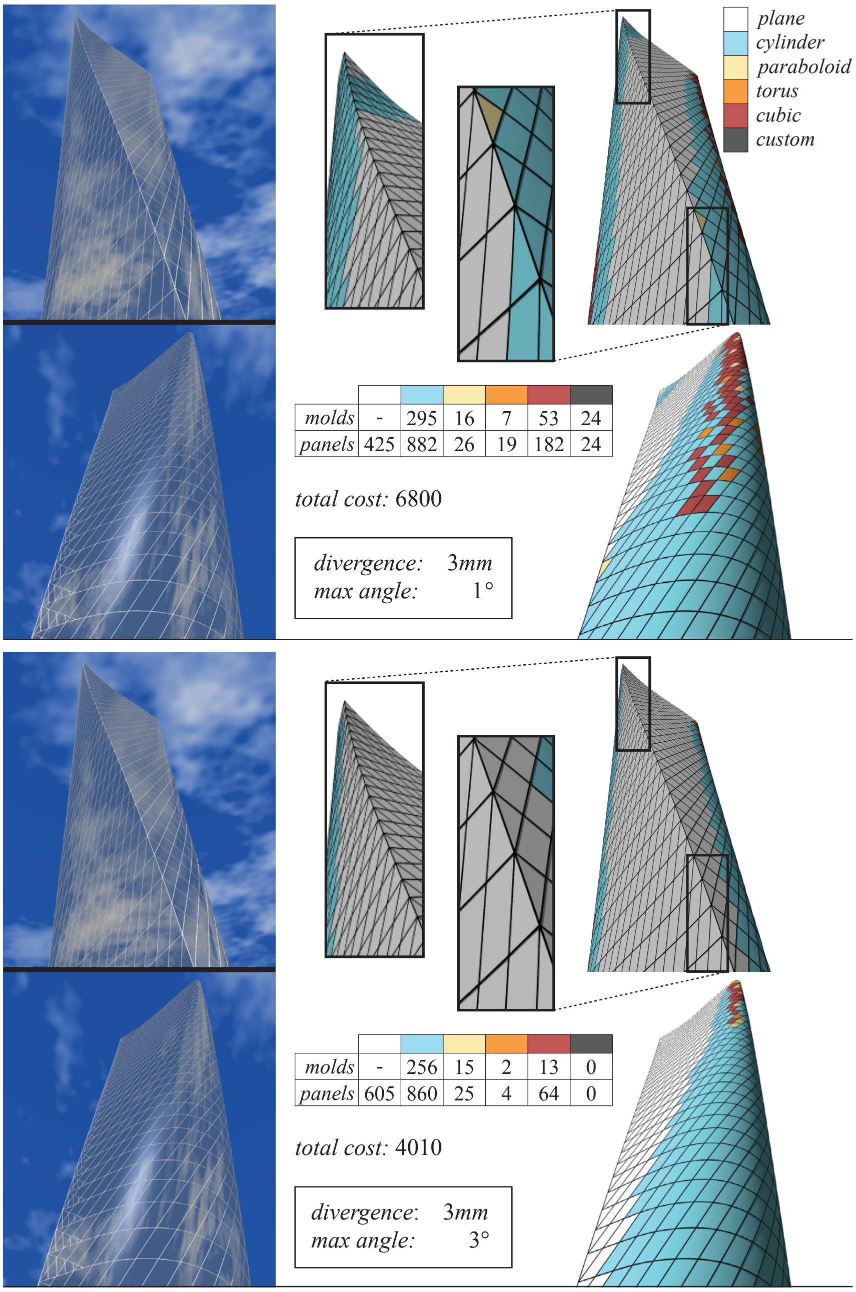


(b) *Paneling solution with spatially adaptive kink angle thresholds.*



(c) *Paneling solution with another set of spatially adaptive kink angle thresholds.*

Figure 13: *Effect of global vs spatially varying kink angle specifications on the Skipper Library dataset. Please refer to Figure 11 for details.*



	plane	cylinder	paraboloid	torus	cubic	custom
molds	-	295	16	7	53	24
panels	425	882	26	19	182	24

	plane	cylinder	paraboloid	torus	cubic	custom
molds	-	256	15	2	13	0
panels	605	860	25	4	64	0

Figure 14: *Paneling solution respecting crease line(s) on the input model. The characteristic sharp feature line of the Lissajous Tower is preserved in our paneling solution.*

Future Work. There are a few desirable extensions to the paneling algorithm leading to challenging problems for future research.

Figure 3 compares the paneling algorithm with rationalization approaches given by planar quad meshes and developable strip models. The latter include favorable geometric properties for the layout of substructure. It is natural to ask for possibilities of combining these approaches with the paneling algorithm. This motivates an adaption of the paneling algorithm towards the incorporation of optimization goals for the curve network, for example with respect to offsets and supporting structures.

For the three presented case studies, the Facade Design Study, the Lissajous Tower, and the Skipper Library, the paneling solutions are obtained in roughly 10 minutes, 1 hour, and 10 hours, respectively. In future, we plan to explore both algorithmic and computational changes to speed up the process in order to enable interactive and simultaneous exploration of reference surface design, curve network layout, and paneling solutions.

An obvious possibility for extending the paneling algorithm concerns the support of further mold types. We plan to include simple additional types like cones, but also more general surface types like general ruled surfaces.

Conclusion. This paper presents improvements of the paneling algorithm introduced by Eigensatz and coworkers [Eigensatz et al. 2010] to enable the preservation of sharp feature lines and the adaptive control of tolerance margins, allowing advanced exploration of cost effective rationalizations of architectural freeform surfaces. In our case studies on cutting edge architectural designs we evaluate the various modes of control enabled by our extended paneling algorithm and demonstrate the effectiveness of the algorithm with new examples, focusing on practical aspects complementary to the ones presented in [Eigensatz et al. 2010].

Acknowledgments

We would like to thank Yves Brise, Peter Kaufmann and Sebastian Martin for their help with the project. Special thanks to Formtexx for providing the architectural datasets and to RFR for fruitful comments. This work was supported by the Swiss National Science Foundation, the Austrian Research Promotion Agency grant number 824197, and the European Community's 7th Framework Programme under grant agreement 230520 (ARC). Niloy Mitra was partially supported by a Microsoft outstanding young faculty fellowship.

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