**A STEINER-like approach to some BAER-subplanes**

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Let $K$ be a *non-commutative field* and let $\mathcal{A}(K^2)$ be the *affine plane* on the *right vector space* $K^2$ over $K$. Denote by $Z(M)$ the *centralizer* of $M \subset K$ in $K$. If $a \in K \setminus Z(K)$, then

$$\Gamma^\circ := \{ (x,y) \in K^2 \mid y = xa \} \qquad (1)$$

is the *proper part* of a *degenerate conic* $\Gamma$ ($y = ax$ would be the equation of a line). The *improper part* of $\Gamma$ is given by the line at infinity of the projective closure of $\mathcal{A}(K^2)$. The term *conic* is motivated by the fact that $\Gamma$ can be generated (in the classical way of J. STEINER) by two projectively related pencils of lines. We shall call $a \in K \setminus Z(K)$ a *parameter* of the degenerate conic $\Gamma$.

It was shown in [1] that those degenerate conics (1) with $|a:Z(K)| = 2$ have completeley different geometric properties than degenerate conics with $|a:Z(K)| > 2$. In this talk we are concerned with this phenomenon from a *geometric point of view*, while in [1] and [3] the major tools are *algebraic theorems* on *generalized polynomial identities*. The main result is this

THEOREM. *If $\Gamma^\circ$ is furnished with the structure of a trace space $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ of $\mathcal{A}(K^2)$, then it is isomorphic to the affine space on the right vector space $K$ over $Z(\{a\})$, i.e. the centralizer of $a$ in $K$. Moreover $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ is an affine subplane of $\mathcal{A}(K^2{}_K)$ if , and only if, $|a:Z(K)| = 2$. If $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ is an affine plane, then it is even an affine BAER-subplane of $\mathcal{A}(K^2{}_K)$.*

By virtue of that theorem, the automorphic collineations of $\Gamma$ (cf. [1], [3]) are easily seen to be given by those affinities of the affine space $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ which can be extended to the projective closure of $\mathcal{A}(K^2{}_K)$. If $|a:Z(K)| = 2$, then the conditions for the existence of such an extension are much weaker than in the general case. This in turn explains the different *"sizes"* of the group of automorphic collineations of $\Gamma$ mentioned in [1].

## References


[1] HAVLICEK, H.: Applications of Results on Generalized Polynomial Identities in Desarguesian Projective Spaces. In: R. KAYA, P.PLAUMANN, K. STRAMBACH (eds): *Rings and Geometry*, Dordrecht, D.Reidel, 1985.

[2] HAVLICEK, H.: Degenerate Conics revisited, *J. Geometry*, to appear 1990.

[3] RIESINGER, R.: Entartete Steinerkegelschnitte in nicht-papposschen Desarguesebenen, *Monatsh. Math.* **89**, 243 - 251 (1980).

Cf. [1] and [2] for further references.