On Sets of Lines corresponding to Affine Spaces

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The set $\mathscr{L}$ of lines of a 3-dimensional projective space with commutative underlying field K may be represented in a well-known way in a 5-dimensional projective space by a Graßmann-variety G (the Plücker-quadric). Those subsets of $\mathscr{L}$ which correspond to the intersection of G and k-dimensional subspaces have been discussed in detail (e.g. linear complexes of lines, linear congruences of lines, reguli, ruled planes, stars of lines, pencils of lines,...). However the existence of G dependes on the commutativity of K.

Irrespective of weather K is a commutative field or a proper skew field, there exists a 4-dimensional affine space $\mathscr{A}$ corresponding to all lines of $\mathscr{L}$ which are skew to a fixed line a. By adding a hyperplane at infinity to the affine space $\mathscr{A}$ we get an "absolute regulus" in this hyperplane. Thus we have a kind of "space-time geometry".

If K is commutative then the above mentioned sets of lines yield (all) affine subspaces of $\mathscr{A}$ provided that they contain the fixed line a.

This in turn motivates our investigation of those sets of lines which correspond to the subspaces of $\mathscr{A}$ when K is a proper skew field. The subspaces of $\mathscr{A}$ will be classified by using the intersection of their projective clusure with the absolute regulus.