

# Geometry in the CAAD Curriculum

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*Modern architecture takes advantage of the greatly increasing design possibilities. Yet, architects are not just a new group of CAD users. Scale and construction technologies pose new challenges to engineering and design. We are convinced that those can be met more effectively with a solid understanding of geometry. Thus, an academic CAAD curriculum has to meet these requirements. We introduce our approach taken at TU Vienna and focus on the first-term mandatory course on geometry for architecture.*

**Keywords:** *Geometry; CAAD curriculum; geometric design.*

## Introduction

We discuss the important role that geometry plays in the CAAD curriculum. Since a few years we are developing a series of modern applied geometry courses for students of architecture at TU Vienna. In their first year of studies students have a mandatory course where they study fundamental geometric concepts for computer aided design. A follow-up course in the second year deals with advanced topics of geometric modeling ranging from subdivision surfaces to shape deformations. Toward the end of their university education students can take a research driven class that brings them in touch with cutting edge results in geometry processing, e.g., 3D scanning, rapid prototyping, and milling with a robot arm.

In this paper we focus on the geometry that is taught in the first year mandatory course. This course is structured in a lecture class covering the theoretical background and an exercise class where students in small groups practice geometric modeling with an appropriate CAD system. By means of 3D modeling of interesting geometric and architectural

objects students are introduced to geometric modeling (figure 1).

## Curriculum overview

### Creating a digital 3D model

We start with teaching the efficient use of different types of coordinate systems, basic shapes and modeling tools. Using parallel and central extrusion we can already generate prisms and pyramids (basic discrete surfaces), cylinders and cones (basic smooth surfaces).

### Fundamental projections, shapes and modeling operations

The course continues with a lecture on projections, light models and basic rendering. While parallel projection is advantageous during modeling, an architectural presentation usually includes a rendered perspective image of the design. Built-in tools in CAD systems automatically produce a correct image. Nevertheless, geometrically correct freehand-drawing is useful for sketching a design. This is often achieved

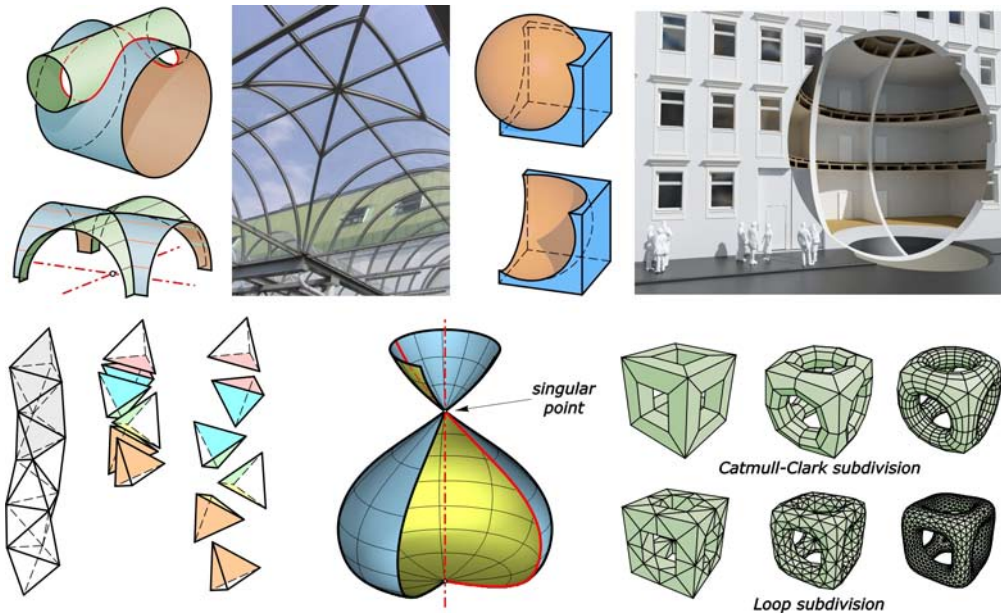
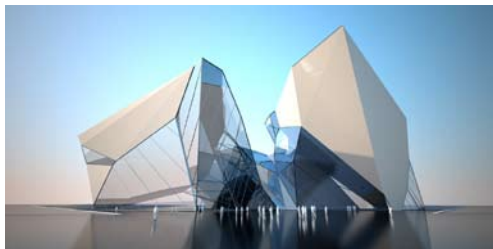


Figure 1  
Some geometric concepts  
which are introduced in the  
first term course

by respecting a few basic geometric facts that we teach our students.

### Polyhedra and polyhedral surfaces

Polyhedra and polyhedral surfaces (figure 2) are used to introduce the important concept of corner cutting for the derivation of new polyhedra from basic shapes. Starting with the geodesic domes of Buckminster Fuller, polyhedral surfaces have gained importance in architecture. Nowadays they are often used in the realization of freeform shapes as e.g. steel-glass structures with planar faces. However, a



realization of a freeform surface as a polyhedral surface (i.e., with planar faces other than triangles, e.g., quadrilaterals) is challenging and a hot-topic of current research.

### Boolean operations

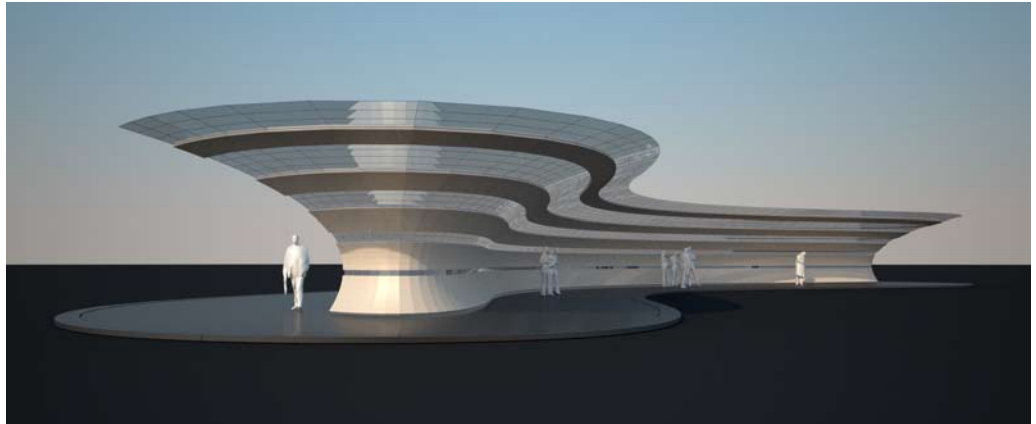
The Boolean operations are basic geometric modeling tools for solid models. The corresponding operations for surface models are the trim and split operations. Using these operations we can derive a rich variety of geometric shapes from a few fundamental objects.

### Planar and spatial transformations

Planar and spatial transformations are fundamental tools employed in geometric modeling and we study some of their important properties. As one application we show how tilings are connected with planar transformations. Spatial transformations are needed to describe spatial motions and thus we introduce some basic techniques how to generate meaningful animations. A good understanding of

Figure 2  
Polyhedra derived from 3D  
Voronoi cells used in an  
architectural design by a  
student

Figure 3  
Kinematic surface used in an architectural design by a student



these operations is later needed for kinematic surface generation (figure 3). Together with other advanced topics these are taught in a follow-up course on CAAD and geometry.

### Curves and surfaces

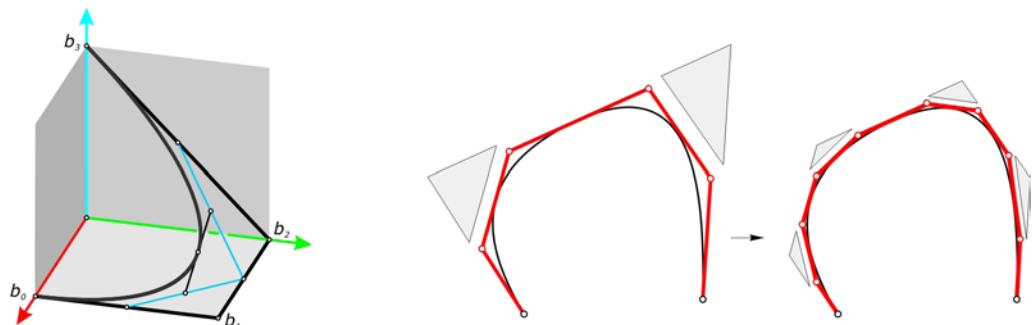
Students learn about basic properties of curves and surfaces. Going beyond the classical conic sections we study wide-spread freeform curves such as Bézier, B-spline and NURBS curves. We introduce algorithms for the generation of smooth and discrete freeform curves with intuitive geometric algorithms because we see an advantage in avoiding the standard mathematical approach employed in textbooks on freeform curves and surfaces. For example, Bézier curves have a beautiful geometric construction via

the algorithm of de Casteljau (figure 4a). Similarly, smooth B-spline curves can be approximated by a sequence of polygons that are refined by an elegant corner cutting algorithm (figure 4b). Thus we have a connection between discrete and smooth curves via so-called subdivision algorithms. Later on these intuitive geometric concepts are generalized from the curve to the surface case.

### Traditional surface classes and freeform surfaces

Traditional surface classes used in architecture include rotational, translational (figure 5a), and ruled surfaces (figure 5b). We study their generation principles and properties next. As a special case of ruled surfaces we study developable surfaces which can be unfolded

Figure 4  
(a) The algorithm of de Casteljau is a geometric method for the generation of Bézier curves. (b) Corner cutting is an important principle to derive new curves and surfaces from basic ones. We illustrate it by means of Chaikin's algorithm that generates a quadratic B-spline curve from a polygon



into a plane. They are of special interest in architecture (see for example the works of Frank O. Gehry) and geometric model building. The course is concluded with the study of freeform surfaces and an outlook towards advanced geometric modeling topics.

### Key assignments in the curriculum

From our fundamental course we have chosen the following two examples that illustrate some teaching concepts that we employ.

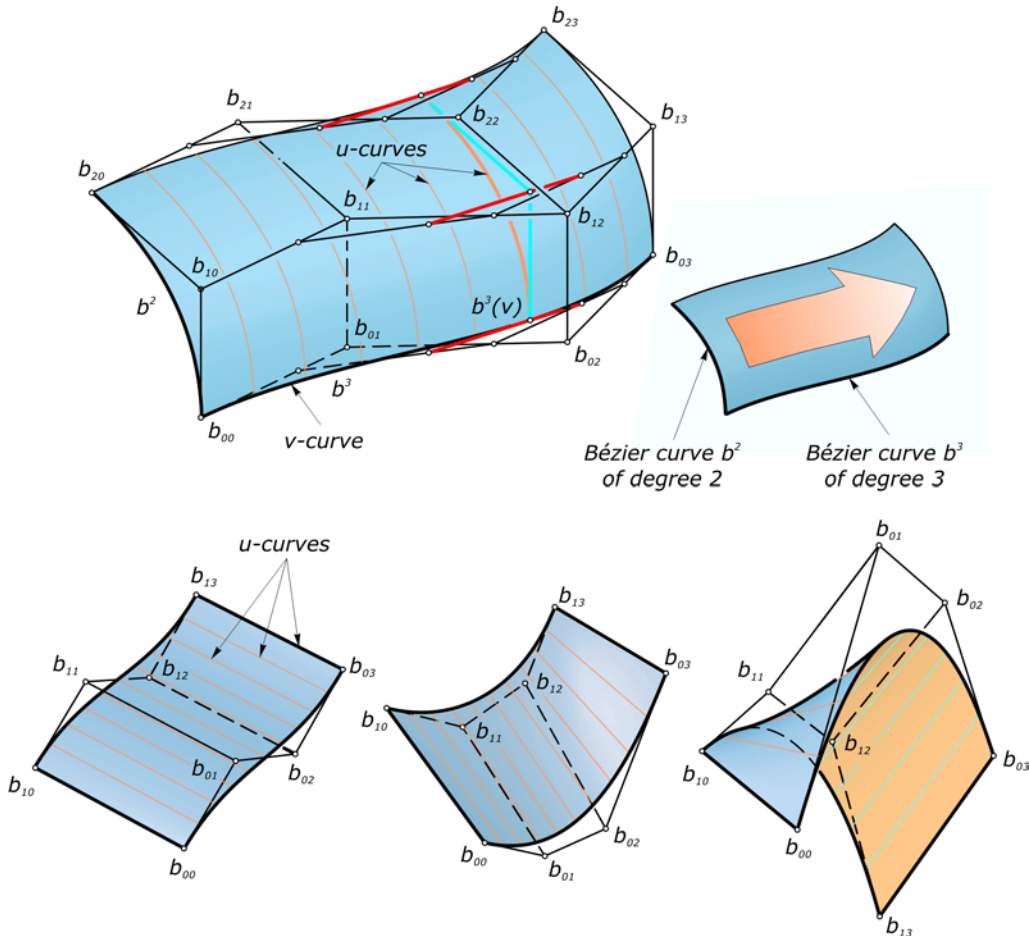


Figure 5  
 (a) A translational surface generated as a Bézier surface of degree (2, 3). (b) Ruled surfaces generated as Bézier surfaces of degree (1, 3)

### Example 1: Introduction to geometric modeling by means of the Winton guest house

It goes without saying that the architect's design work starts before geometric modeling. According to Frank O. Gehry, his inspiration for the Winton guest house in Wayzata, Minnesota, came from the still-life paintings by Giorgio Morandi. When asked to build a guest house in the 1980s, he set a counterpoint to the main house—designed by Philip Johnson in 1952. Gehry conceived the guest house as a large outdoor sculpture in which each room constitutes its own mini-building (figure 6).

Our students learn how to create a digital 3D model of this building using an appropriate CAD system. Furthermore, by means of this key assignment students learn how to efficiently employ basic solid models, different coordinate systems, visualization techniques, and the overall handling of a CAD system (layers, ...). During each step in the construction process a new geometric or CAAD concept is introduced. The Winton guest house consists of various types of basic objects which have interesting mutual positions. Thus, all basic solids (cuboids, prisms, cylinders, ...) and the use of different coordinate systems are introduced in a very intuitive and generic way.

In our experience the students' motivation to learn CAAD is increased by geometric modeling of built architecture. Figure 6 shows one step in the design process where the prismatic garage is constructed. Using a local polar coordinate system we correctly position the garage. While the theoretical background is taught in a lecture, during the exercises students learn self-dependent following an illustrated handout. In groups of 30, students work out geometric concepts by means of carefully designed examples and tasks. They are supported by instructors and teaching assistants. Those help individually when troubles arise.

### Example 2: Smooth Bézier junction between a parabolic cylinder and a plane

The design of a smooth blending surface between a parabolic cylinder  $C$  and a plane  $P$  requires a good geometric understanding. As a blending surface we use a Bézier surface  $B$  of degree (2,3) that connects a boundary parabola  $p_1$  of  $C$  to a parabola  $p_2$  lying in  $P$  (figure 7). The first and the last column of the control mesh of  $B$  coincide with the control polygons of the two parabolas  $p_1$  and  $p_2$ . The second column of the control mesh can be chosen according to a construction via equal ratios. The third column just has to lie in the plane  $P$  since then the tangent ruled

Figure 6  
The Winton Guest House by Frank O. Gehry is a nice example for students of architecture to get started with geometric modeling of fundamental shapes

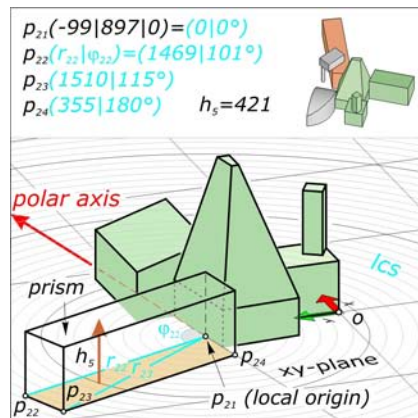
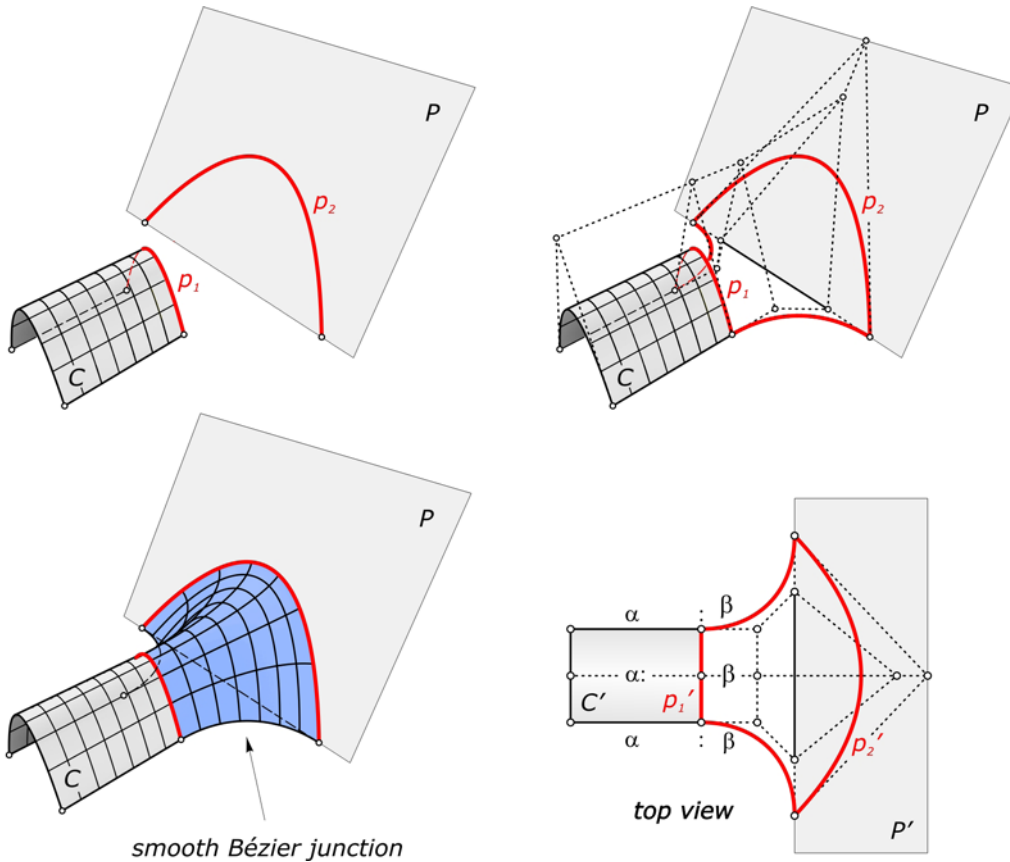


Figure 7  
Smooth Bézier junction between a parabolic cylinder  $C$  and a plane  $P$



*smooth Bézier junction*

surface is a patch in  $P$  and we get a smooth join with  $P$ . This still gives some choice to the designer to vary the shape of the blending surface, however always ensuring a smooth blend between the cylinder and the plane.

We introduce this example toward the end of the course. It is a typical example which encourages students to apply their knowledge gathered during our one-term course. They have to employ properties of conic sections, especially parabolas, and free-form surfaces to a practical design task. Working out

different solutions for the smooth junction, students gain an even deeper insight into the design and application of Bézier curves and surfaces.

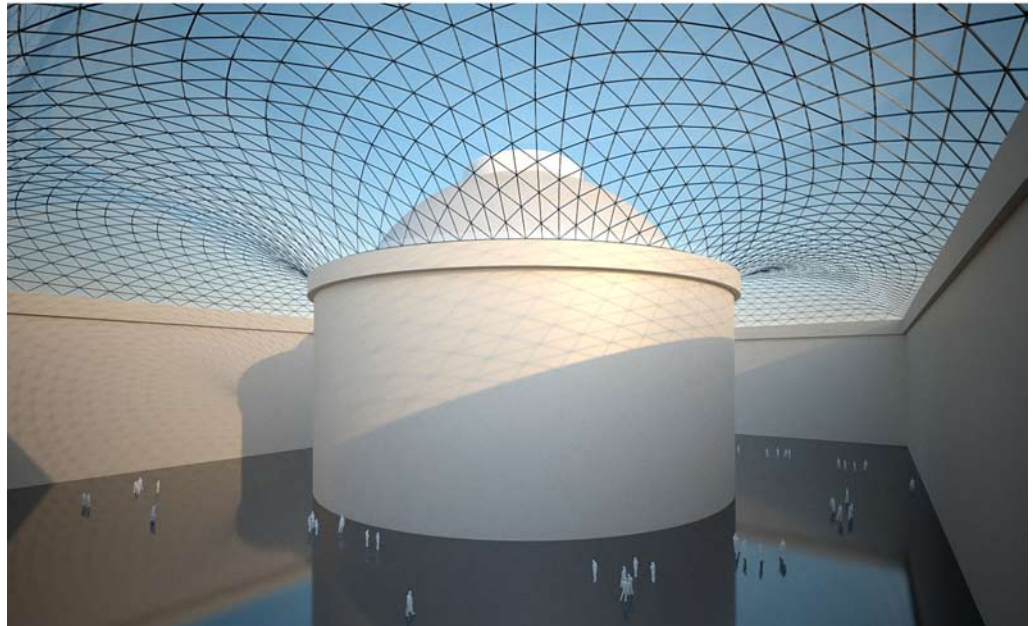
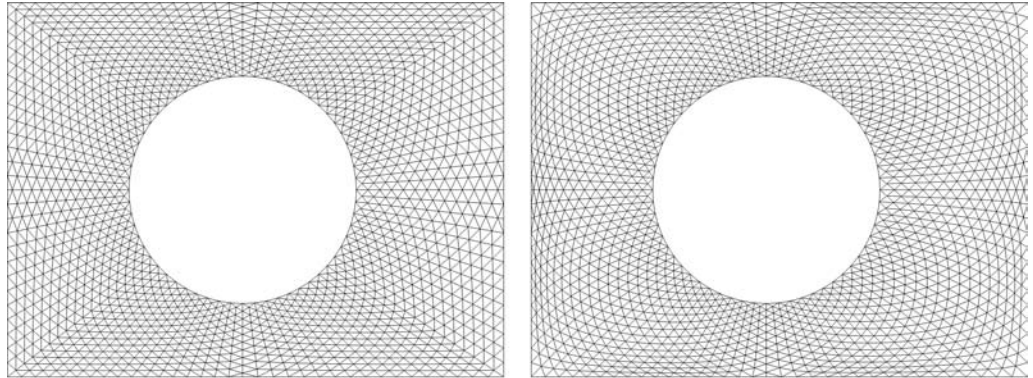
To confront students with some challenging tasks at the end of the first-term course is motivated including the following reasons. They see that with their geometric knowledge they are able to design interesting architecture. Furthermore, these tasks also encourage students to learn more geometry to stand the challenge of contemporary and future architectural design.

## Conclusion

Recent masterpieces in architecture often employ geometric shapes that could only be built by means of sophisticated techniques of geometric description and integration of structural and fabrication principles (figure 8). Our goal is to confront students

in the CAAD curriculum already at the beginning of their studies with the geometry that is fundamental to later thrive with their projects. The first term course adopts new methodologies and algorithms from geometry processing and computer graphics for the purpose of computer aided architectural design.

*Figure 8*  
*The glass roof of the British museum by Lord Norman Foster is a freeform shape realized as a steel-glass structure and is featured in the shown rendering. Relaxation was applied to the initial mesh*



We focus on efficient CAD construction methods and use CAD to support geometry teaching and understanding. Most books on the geometric aspects of CAD require quite some mathematics which most students of architecture will not be familiar with. Therefore we try to explain all geometric objects and methods available in CAD systems including free-form curves and surfaces without much mathematics. We explain the material via geometric considerations and support those by many figures, instructive examples and demanding exercises.

Our experience is summarized in a textbook written especially for an architectural audience (Pottmann et al., 2007). This book closes the gap between technical possibilities and an effective working knowledge of new geometric design methods.

## Acknowledgements

We thank our students for critical comments during our courses, which help us to continuously improve the quality of our CAAD curriculum. They provided images of their sophisticated works made during the courses, especially thanks to Benjamin Schneider. Thanks go to Chris Williams for providing the mesh data used in figure 8. We acknowledge support by FWF grant S9201 of the Austrian Science Fund.

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