AD Patterns of Architecture

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Mark Garcia

Geometry and New and Future Spatial Patterns

Helmut Pottmann

Geometric patterns have fascinated mankind since ancient times. Artists had an excellent understanding of the generation principles of patterns, long before mathematicians devoted deep studies to this subject. A prominent example is furnished by Moorish architecture: In the Alhambra (13^{th} century, Granada, Spain) we find all essentially different types of patterns which can be formed by congruent tiles; the mathematical classification has only been achieved in the 20^{th} century. Contemporary architecture generates a stunning variety of new designs and spatial patterns, but architects do not always have the right tools at their disposal to realize such structures. We consider it a challenging and exciting task for mathematicians to bridge the gap between design and construction and devise new fabrication-aware design tools for the architectural application.

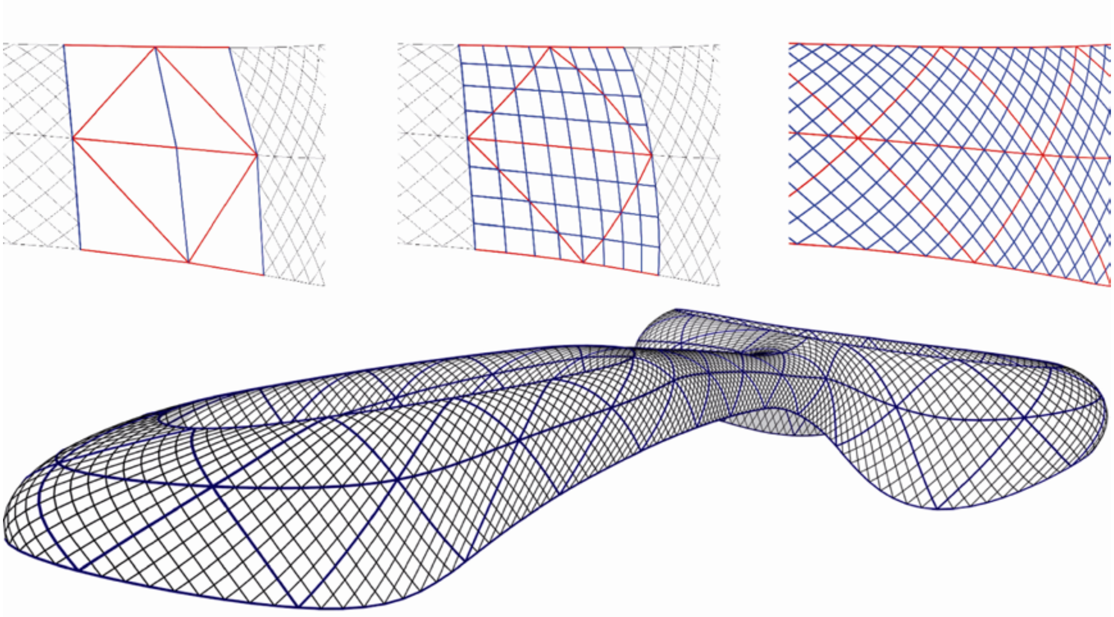
Patterns are a vast subject. Even in architecture, they come in a variety of applications. In the present paper, we mainly focus on recent research which is motivated by the realization of complex architectural freeform shapes. There, patterns arise naturally through the layout of panels and by supporting structures associated with freeform geometry. Especially on large seamless surfaces, the generation of panel patterns is challenging since it is not only a matter of aesthetics, but also heavily influenced by material and manufacturing constraints.

Texture mapping vs. paneling

Computer Graphics has intensively investigated the problem of mapping planar texture onto a double curved surface while minimally distorting it. Texture mapping techniques may be useful in architecture for purposes of decoration and presentation, but for paneling they are hardly sufficient. Based on the choice of material and manufacturing technology, certain geometric shapes of panels will be preferred: Planar panels are always the simplest and cheapest. Curved panels should be cylindrical when working with glass; they can be more general single curve panels when metal is used and they should be ruled surfaces for certain technologies to manufacture curved glass-fibre reinforced concrete panels. These facts are not respected by available CAD software. Hence, patterns of panels and associated structures are an active topic of research and development[1,2]. Companies which provide geometry consulting services in this field include Designtoproduction[3], Evolute[4] and Gehry Technologies[5]; some of them develop specialized software (based on optimization algorithms) which is capable of solving the basic paneling problems. The results shown in the present article have been achieved with software implemented at the author's research group at TU Vienna and at Evolute.

Subdivision techniques

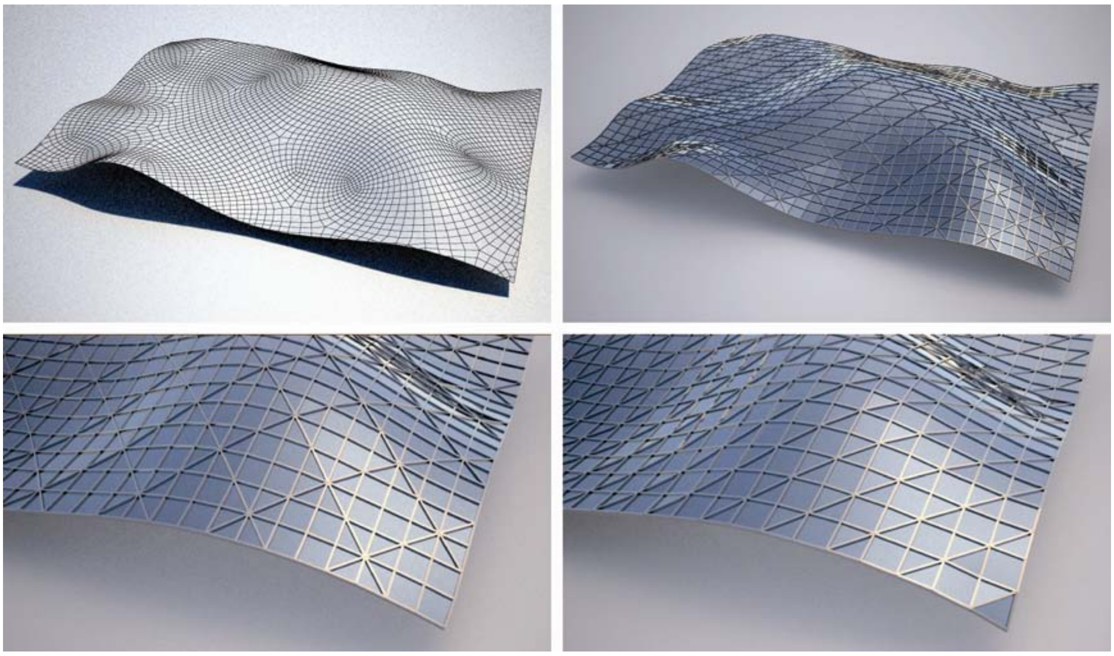
A popular approach to shape modeling, known as *subdivision,* is based on successive refinement of a coarse input mesh[1]. Subdivision algorithms, a first simple variant of which led Buckminster-Fuller to his geodesic domes, tend to produce aesthetically pleasing meshes and are thus of high interest for architecture. They may be used to generate meshes formed by triangles, quads or hexagons, or even more complicated patterns some of which we will address below in connection with circle patterns. Subdivision may also be part of the geometric optimization strategy for an architectural structure.



The quad pattern of supporting beams computed for the grid shell of the Yas Island Marina Hotel (Asymptote Architecture, Abu Dhabi, completion in 2009) receives structural stability through the formation of especially strong beams, arranged in a triangular macro-structure (bold). The geometric computation and optimization of the beam layout (by Evolute, Gehry Technologies and Waagner Biro) proceeds as follows (top row): An initial quad mesh is refined via subdivision; part of the resulting mesh together with its diagonal mesh contain the desired triangular macrostructure and are finally optimized towards high aesthetics while staying close to the design surface and meeting a number of constraints (bottom).

Patterns from planar quads

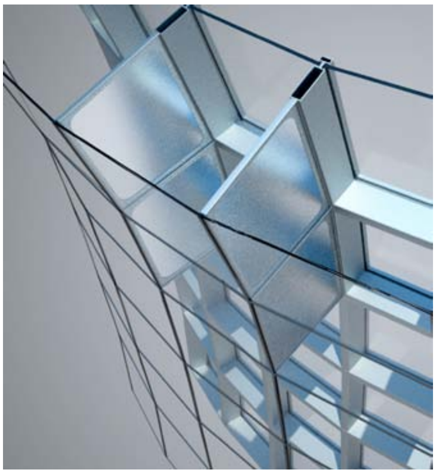
In the YAS Island Marina Hotel project, the panels are mounted with gaps to the structure and thus planarity of the quadrilateral mesh faces has not been an issue. However, for watertight quad structures, planarity of faces may be essential. Recently, important progress has been achieved in the design and computation of freeform quad meshes with planar faces (PQ meshes)[1,6]. The geometric study revealed a close relation between PQ meshes and the curvature behavior of the underlying surface. Especially if the quad panels should be close to rectangles, the layout is guided by the principal curvature lines of the surface; these curves follow the directions of locally extreme normal curvature. The resulting patterns may be intriguing, but also limiting when one has to meet specific requirements on the panel layout for an already designed surface. The design of PQ meshes with help of subdivision, interleaved with optimization, is a lot easier and ready for practical use[1].



Patterns of planar quads are strongly reflecting the curvature behavior of a surface (top left; mesh courtesy Konrad Polthier, Berlin). They may exhibit extraordinary points at undesired locations, a strong variation in panel sizes and incompatibility with boundary conditions. A practical solution to meet constraints can be achieved by hybrid meshes of triangles and planar quads (top right and bottom; images courtesy Alexander Schiftner, Evolute, and Heinz Schmiedhofer), since they offer more degrees of freedom and thus are more likely to meet imposed constraints. The data set for this example comes from the design of an Islamic Art Museum in the Louvre, Paris, by Bellini Architects. The mesh on the lower right has been chosen as the solution to be realized (by Waagner Biro Stahlbau, Vienna).

Supporting structures

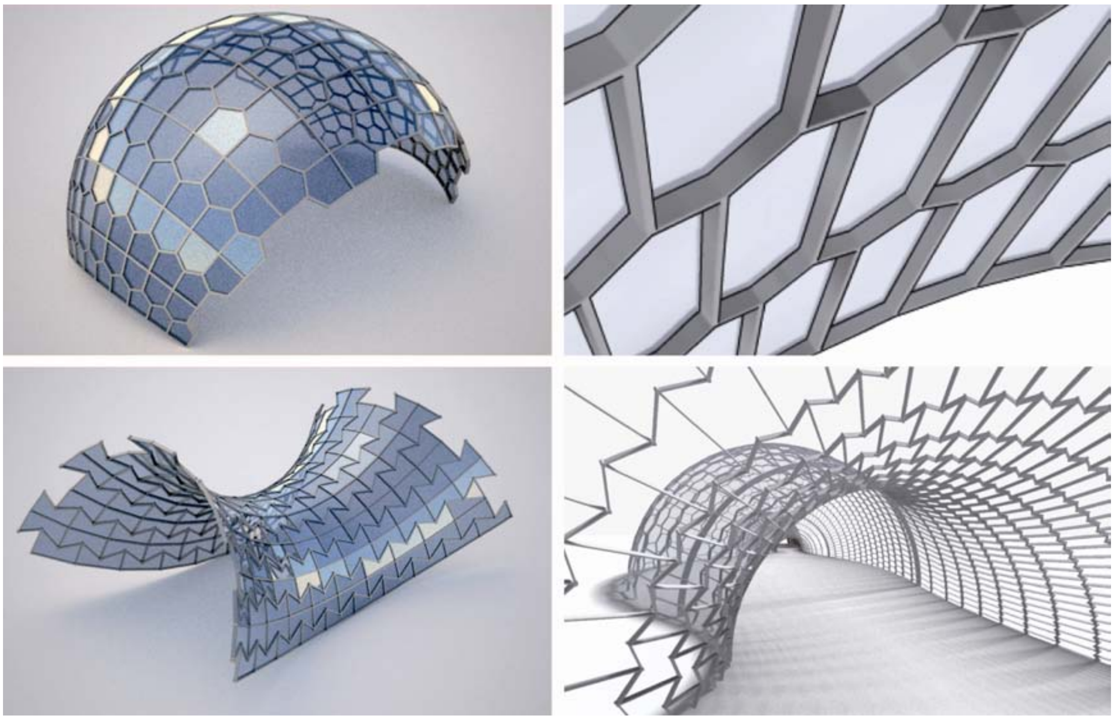
Special PQ meshes can be embedded into a sequence of PQ meshes which lie at constant distance of each other. The distance may be measured in different ways, leading to different mesh types. For example, the so-called conical meshes possess offset meshes where corresponding faces lie at constant distance. Related to that are layouts of supporting beams with torsion-free nodes, i.e., at each vertex the central planes of the beams are coaxial [1,6]. On the other hand, hybrid meshes can be associated with offsets at variable distance, but a torsion-free beam layout can in general be achieved only in an approximate way via an optimization algorithm.



Multilayer structures are spatial patterns around a surface-like pattern. This image (courtesy Benjamin Schneider) shows a structure which is based on a conical mesh and formed by planar quads only.

Hexagonal structures

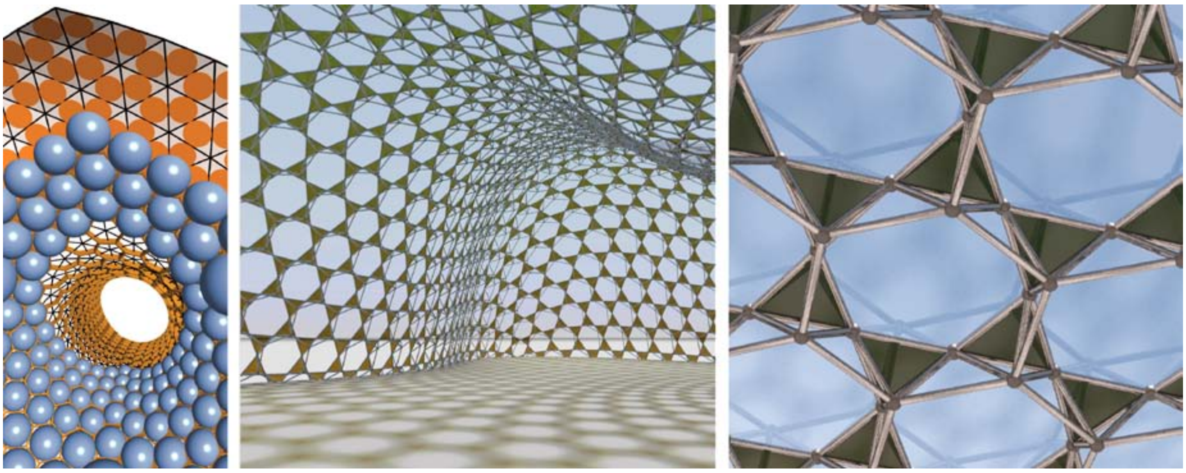
R. Buckminster Fuller, Frei Otto[7], Lars Spuybroek[8] and others have been fascinated by the shapes and structural efficiency of the siliceous micro-skeletons of radiolaria whose shapes are often based on hexagonal meshes. Hexagonal meshes representing freeform shapes are a geometrically fascinating subject. Aiming at planarity of faces yields surprising patterns, since panels in negatively curved (locally saddle shaped) areas will not be convex. To generate a hex mesh with planar faces, one may start with a triangle mesh and intersect tangent planes at mesh vertices; the result will be a mesh formed by mostly hexagonal panels. This is a numerically very sensitive process, accompanied by difficulties in controlling the behavior in areas where Gaussian curvature changes its sign[9]. Thus, achieving high aesthetic quality in a hex mesh with planar faces is a challenging and largely unexplored topic. It should be mentioned that patterns of planar panels with more than four edges per panel are always exhibiting non convex panels in the areas of negative Gaussian curvature.



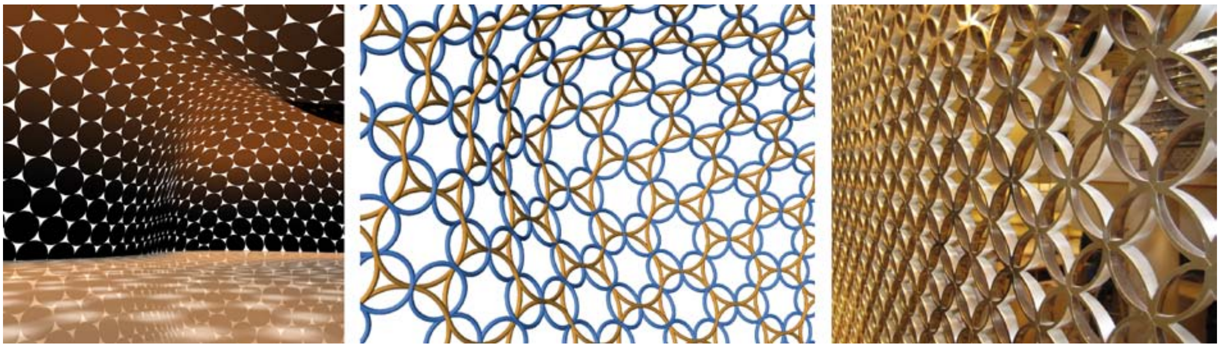
Pentagonal and hexagonal meshes with planar faces feature non-convex panels in negatively curved areas, such that structural feasibility may require additional elements. The shown meshes are so-called edge offset meshes [1,6] which allow for a particularly clean layout of supporting beams. At each vertex, beams of constant height meet perfectly aligned on both sides and without torsion, i.e., their central planes are coaxial (images courtesy Liu Yang, Heinz Schmiedhofer and Johannes Wallner).

Circle patterns and derived structures

Motivated by work of Future Systems (Selfridges, Birmingham), we currently investigate patterns associated with packings of circles and spheres on surfaces which are derived from so-called circle-packing (CP) meshes[10]. These are triangle meshes where the in-circles of neighboring triangles have the same contact point at their common edge. CP meshes enjoy a high aesthetical value and lead to various remarkable surface patterns as well as to a number of unsolved mathematical problems. We note here that the in-circles of CP meshes do not form a complete packing. Complete circle packings (where any generic circle has six pairwise tangent neighbour circles) exist in the plane and on the sphere only. On surfaces, one has to move to an approximate solution, for which CP meshes provide various possibilities. From CP meshes, one can compute meshes formed by hexagons and triangles. Here, the presence of triangles allows us to use nearly planar and almost regular hexagons in a structure; we do not have the structural disadvantage of non-convex panels anymore.



Left: Circle-packing (CP) meshes are triangle meshes where the inscribed circles of neighboring triangles are tangent to each other; they give rise to sphere packings on surfaces, to hexagonal patterns and to torsion-free beam layouts. Middle: A tri-hex structure derived from a CP mesh; the hexagons are nearly planar and regular. Right: A supporting framework of a tri-hex mesh which makes use of the triangles in the mesh and has been inspired by the Eden project, Bodelva, Cornwall, by N. Grimshaw (which however represents spherical shapes and where a different layer in the framework plays the role of the roof). Images: Alexander Schiftner.



Left: Approximate circle packing of a freefrom shape, derived from a CP mesh. Middle: CP meshes also serve as basis for the computation of freeform circle patterns; images: Alexander Schiftner. Right: A planar circle pattern in the Louis Vuitton store, Paris (image courtesy RFR).

Conclusion and future research

Design and fabrication of geometrically complex architectural structures is expected to benefit a lot through interaction of architects with mathematicians and engineers. This has been illustrated at hand of a few recent solutions to paneling problems and supporting structures for freeform hulls. Mathematics also offers promising tools for the design of fully 3D spatial patterns. Those include Voronoi diagrams and shape evolution algorithms[1]. Shape evolution received a lot of interest within mathematics and geometric computing in recent years, but in view of applications in Computer Vision and Image Processing rather than Architecture. A related and highly challenging topic for future research is the computation of 4D patterns in form of animated facades and other flexible space structures. Numerous ideas towards dynamic architectural designs have been contributed by Kas Oosterhuis and his coworkers within the Hyperbody research group at Delft University of Technology; it could be a highly inspiring source for further investigations from a mathematical / computational/ engineering perspective so as to reach a state of maturity which would promote flexible architectural structures on a larger scale.

Voronoi diagrams associate nearest neighbor regions (Voronoi cells) to a set of input points. The concept works in the plane (upper right), for input points on a curved surface (lower right; only the part of the diagram close to the surface is shown) and in space (left; image courtesy B. Schneider). Voronoi diagrams and their numerous generalizations are a rich source for the design of spatial structures.

Notes.

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3. see http://www.designtoproduction.com
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11. The author gratefully acknowledges support by the Austrian Science Fund (grants 19214-N18, S92), the Austrian research council (FFG, grant 813391), and Waagner-Biro Stahlbau, Vienna.

Helmut Pottmann - Short biography

Helmut Pottmann received a PhD in mathematics from Vienna University of Technology (TU Wien) in 1983. Since 1992 he is professor at TU Wien and head of the 'Geometric Modeling and Industrial Geometry' research group. On leave from TU Wien, he is currently director of the KAUST Geometric Modeling and Scientific Visualization Research Center, Saudi Arabia. His recent research concentrates on Geometric Computing for Architecture and Manufacturing.