# Lineare Pentapoden mit einfacher Singularitätsvarietät
Linear pentapods with a simple singularity variety

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Abstract

A *linear pentapod* can be defined as a five degree-of-freedom (DOF) *line-body component* of a Gough-Stewart platform consisting a linear motion platform ℓ with five identical spherical-prismatic-spherical (S$\underline{\text{P}}$S) legs, where the prismatic joints are active and the rest are passive. The pose of ℓ is uniquely determined by a point $(u,v,w,p_x,p_y,p_z) \in \mathbb{R}^6$ located on the singular quadric $\Gamma : u^2 + v^2 + w^2 = 1$, where (u,v,w) determines the orientation of ℓ and (p_x,p_y,p_z) its position.
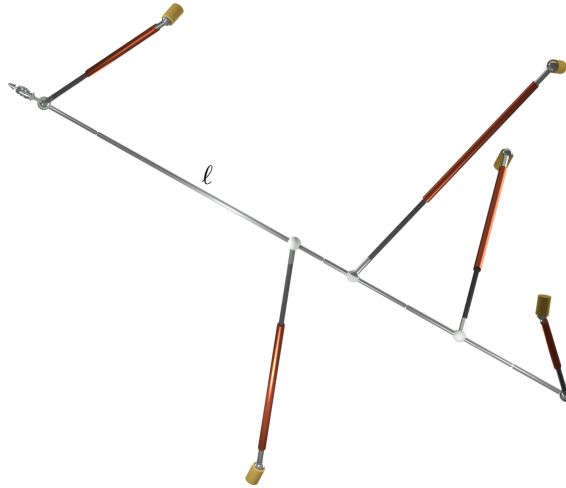
It turns out that this kind of manipulator is an interesting alternative to serial robots handling axis-symmetric tools. Some fundamental industrial tasks such as 5-axis milling, laser engraving and water jet cutting can be counted as its applications in industry.

Singularity analysis

Singularity analysis plays an important role in motion planning of parallel manipulators. Special configurations referred to as *kinematic singularities* have always been central in mechanism theory and robotics. Beside being an intellectually appealing topic, the study of kinematic singularities provides an insight of major practical and theoretical importance for the design, control, and application of robot manipulators. In such singularities, the *kinetostatic* properties of a mechanism undergo sudden and dramatic changes. This motivates the enormous practical value of a careful study and thorough understanding of the phenomenon for the design and use of parallel manipulators.

From the line-geometric point of view a linear pentapod is in a singular configuration if and only if the five carrier lines of the legs belong to a linear line congruence; i.e. the Plücker coordinates of these lines are linearly dependent. From this latter characterization the following algebraic one can be obtained: The set of all singular robot configurations equals the intersection of Γ with a cubic hypersurface Σ of $\mathbb{R}^6$. Moreover, a rational parametrization of this singularity loci $\Gamma \cap \Sigma$ was given by the authors in [1].

Goals and results

It can easily be seen that the equation of the cubic hypersurface Σ is only quadratic in position as well as orientation variables. The intention of our work is to find necessary conditions for Σ to be:

- linear in position variables,

- linear in orientation variables,

- quadratic in total.

Clearly, due to the degree reduction it becomes easier to obtain closed form information about singular poses. But the main motivation for our research is the computational simplification of singularity-free zones, for which the state of the art is as follows:

In [1] it was proven that for a generic linear pentapod, the computation of the maximal singularity-free zone in the position/orientation workspace (with respect to the Euclidean/spherical metric) needs the solution of a polynomial of degree 6 and 8, respectively.

In contrast, the determination of the closest singular pose within the complete configurations space (with respect to an object oriented metric) reduces to the solution of a polynomial of degree 80. Due to this high degree, computation in real time is not possible. Our first idea to cope with this problem was to relax the motion group from the Euclidean one to the group of equiform motions (similarity transformations), which is equivalent to omitting the normalizing condition Γ. Doing so, the degree drops to 28, which was demonstrated in the addendum of [1, arXiv version]. As the obtained distance of the relaxed problem is less or equal the distance of the original problem, it can be used as the radius of a hypersphere, which is guaranteed singularity-free.

All resulting designs of our study have the property that the degree of the polynomial associated with the closest singular pose drops from 80 to 10 and for the relaxed problem from 28 to 3, respectively. In conclusion we propose three kinematically redundant linear pentapods (illustrated below), which have a simplified singularity variety in all of their possible configurations.
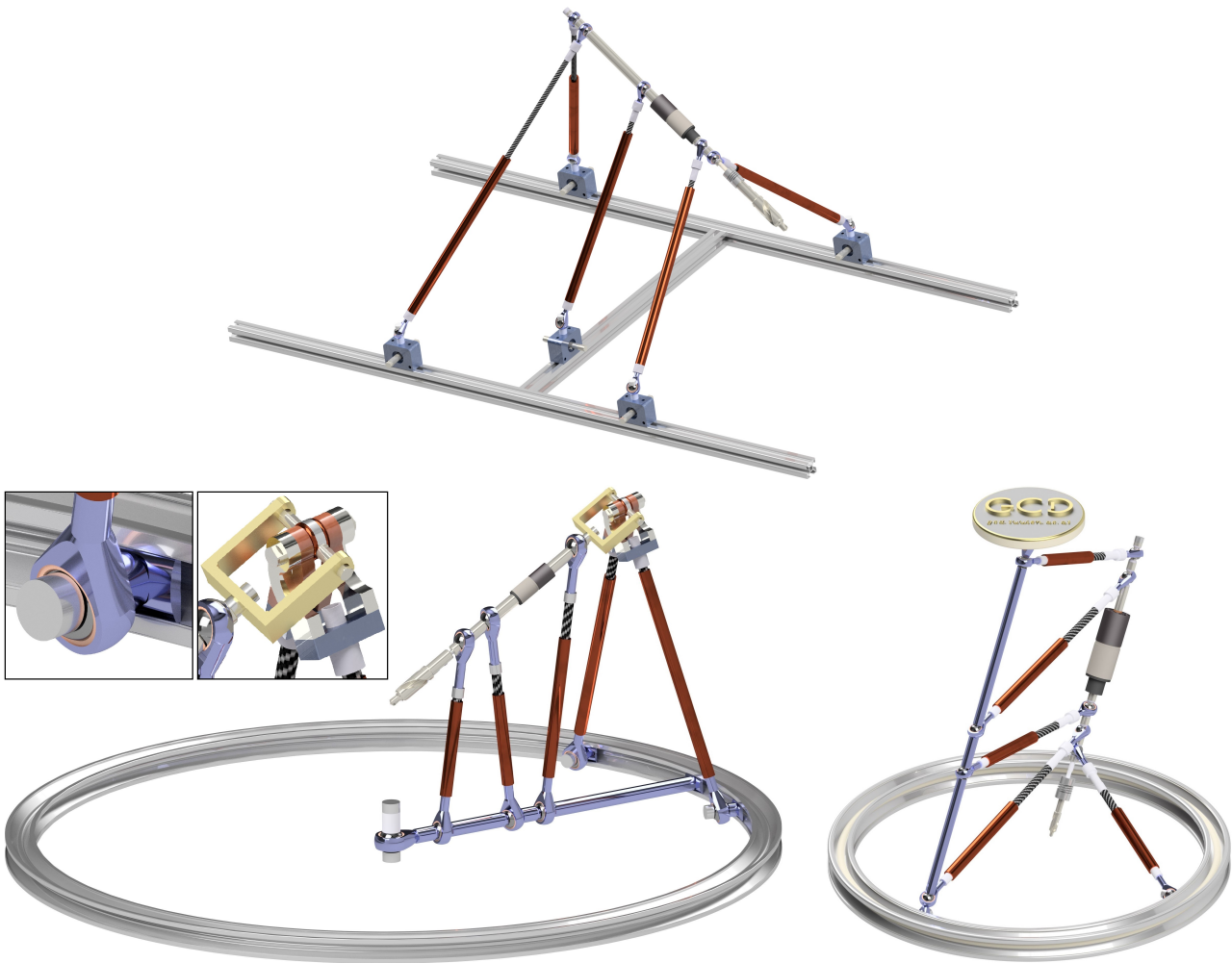
References

[1] Rasoulzadeh, Arvin, and Nawratil, Georg: *Rational parametrization of linear pentapods singularity variety and the distance to it*. Computational Kinematics (S. Zeghloul et al. eds.), pages 516–524, Springer (2017) arXiv:1701.09107 (extended version)

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