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[ > restart:with(LinearAlgebra) :
[ >
[ > # General sphere condition
[ > Lambda:=-R^2*e0^2+b^2*e0^2+c^2*e0^2+A^2*e0^2+B^2*e0^2+C^2*e0^2+a
^2*e0^2-R^2*e1^2+b^2*e1^2+c^2*e1^2+A^2*e1^2+B^2*e1^2+C^2*e1^2+a^
2*e1^2-R^2*e2^2+b^2*e2^2+c^2*e2^2+A^2*e2^2+B^2*e2^2+C^2*e2^2+a^2
*e2^2-R^2*e3^2+a^2*e3^2+b^2*e3^2+c^2*e3^2+A^2*e3^2+B^2*e3^2+C^2*
e3^2+4*A*f0*e1+4*C*f1*e2-2*A*a*e0^2+2*A*a*e3^2-4*c*e3*f0-4*b*e3*
f1+4*a*e3*f2+4*a*e0*f1+4*c*e0*f3+4*A*f2*e3-4*A*f1*e0-4*C*f2*e1+4
*b*e0*f2-2*C*c*e0^2+2*C*c*e2^2+4*C*f0*e3-4*b*e2*f0-4*a*e2*f3+2*C
*c*e1^2+4*B*f3*e1-2*B*b*e2^2-4*B*f1*e3+4*B*f0*e2-4*c*e1*f2-2*A*a
*e1^2-4*A*f3*e2+2*B*b*e3^2-4*B*f2*e0-2*B*b*e0^2-4*a*e1*f0-2*C*c*
e3^2-4*C*f3*e0+4*c*e2*f1+2*B*b*e1^2+4*b*e1*f3+2*A*a*e2^2-4*A*c*e
0*e2+4*C*a*e0*e2-4*B*a*e1*e2-4*A*b*e1*e2-4*C*b*e2*e3-4*B*a*e0*e3
+4*A*b*e0*e3-4*A*c*e1*e3-4*C*a*e1*e3-4*B*c*e2*e3+4*B*c*e0*e1-4*C
*b*e0*e1+4*f1^2+4*f0^2+4*f3^2+4*f2^2:
[ >
[ > # Coordinates of base anchor points
[ > A1:=0:B1:=0:C1:=0:
[ > A2:=0:B2:=1:C2:=0:
[ > A3:=0:B3:=-1:C3:=0:
[ > A4:=1:B4:=0:C4:=0:
[ > A5:=1:B5:=B:C5:=0:
[ >
[ > # Coordinates of platform anchor points
[ > a1:=0:b1:=0:c1:=0:
[ > a2:=0:b2:=b2:c2:=0:
[ > a3:=0:b3:=b3:c3:=0:
[ > a4:=a:b4:=b4:c4:=0:
[ > a5:=a:b5:=b5:c5:=0:
[ >
[ > # Set of equations
[ > Lambda1:=factor(subs(a=a1,b=b1,c=c1,A=A1,B=B1,C=C1,R=R1,Lambda))
:
[ > Lambda2:=factor(subs(a=a2,b=b2,c=c2,A=A2,B=B2,C=C2,R=R2,Lambda))
:
[ > Lambda3:=factor(subs(a=a3,b=b3,c=c3,A=A3,B=B3,C=C3,R=R3,Lambda))
:
[ > Lambda4:=factor(subs(a=a4,b=b4,c=c4,A=A4,B=B4,C=C4,R=R4,Lambda))
:
[ > Lambda5:=factor(subs(a=a5,b=b5,c=c5,A=A5,B=B5,C=C5,R=R5,Lambda))
:
[ > N:=e0^2+e1^2+e2^2+e3^2:
[ > Phi:=e0*f0+e1*f1+e2*f2+e3*f3:
[ >
[ > Delta21:=Lambda2-Lambda1:
[

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> gcd(resultant(G13_, simplify(K_), e3_),
      resultant(G13_, simplify(H3_), e3_));
1427247692705959881058285969449495136382746624
      2
(625 B4 - 2000 B3 + 2400 B2 - 1280 B + 256) B2 e17 e24 (3696 B2 e14 + 9848 B2 e13 e2
- 14425 B2 e12 e22 - 9848 B2 e1 e23 + 3696 B2 e24 + 3792 B e14 - 13168 B e13 e2
- 504 B e12 e22 + 13168 B e1 e23 + 3792 B e24 + 960 e14 - 960 e13 e2 - 1680 e12 e22
+ 960 e1 e23 + 960 e24)
> factor(V);
68501504 B6 e14 + 585515008 B6 e13 e2 + 1346052096 B6 e12 e22 - 585515008 B6 e1 e23
+ 68501504 B6 e24 - 229195776 B5 e14 - 2560827392 B5 e13 e2
- 10657035264 B5 e12 e22 + 2560827392 B5 e1 e23 - 229195776 B5 e24
+ 115998720 B4 e14 - 810762240 B4 e13 e2 + 28947865600 B4 e12 e22
+ 810762240 B4 e1 e23 + 115998720 B4 e24 - 1376583680 B3 e14
+ 14789181440 B3 e13 e2 - 26377912320 B3 e12 e22 - 14789181440 B3 e1 e23
- 1376583680 B3 e24 + 4583915520 B2 e14 - 14115799040 B2 e13 e2
- 5020057600 B2 e12 e22 + 14115799040 B2 e1 e23 + 4583915520 B2 e24
- 2319974400 B e14 - 2359296000 B e13 e2 + 8683520000 B e12 e22
+ 2359296000 B e1 e23 - 2319974400 B e24 + 131072000 e14 + 1205862400 e13 e2
+ 996147200 e12 e22 - 1205862400 e1 e23 + 131072000 e24
> a:='a':b2:='b2':b3:='b3':b4:='b4':b5:='b5':
> R1:='R1':R2:='R2':R3:='R3':R4:='R4':R5:='R5':
>
> # Therefore U=0 has to hold:
>
> U40:=factor(coeff(coeff(U, e1, 4), e2, 0)):
> U31:=factor(coeff(coeff(U, e1, 3), e2, 1)):
> U22:=factor(coeff(coeff(U, e1, 2), e2, 2)):
> U13:=factor(coeff(coeff(U, e1, 1), e2, 3)):
> U04:=factor(coeff(coeff(U, e1, 0), e2, 4)):
>
> simplify(U40-U04);
0
> simplify(U31+U13);
0
> W1:=simplify(U40/((b2+b3)^2)):nops(%);
44
> W2:=simplify(U31/((b2+b3))):nops(%);
81
> W3:=U22:nops(%);

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[ > # Discussing b2=-b3
[ > b3:=-b2:
[ > factor(simplify(W3));
[
[ 
$$-64 b_2^5 (B b_2 + b_4 - b_5)^2$$

[ > b3:='b3':
[ >
[ > # Therefore W1=W2=W3=0 has to hold, which is discussed next:
[ >
[ > E1:=simplify(resultant(W2,W3,b5)):
[ > E2:=simplify(resultant(W1,W3,b5)):
[ > E3:=simplify(resultant(W1,W2,b5)):
[ >
[ > GCD:=factor(gcd(gcd(E1,E2),E3));
[
[ 
$$16 (b_2 - b_3) (b_2 + b_3)^2 (a^2 b_2^2 + a^2 b_3^2 - 2 a b_2^2 b_3 + 2 a b_2 b_3^2 + 2 b_2^2 b_3^2$$

[ 
$$- 2 b_2^2 b_3 b_4 + b_2^2 b_4^2 - 2 b_2 b_3^2 b_4 + b_3^2 b_4^2) B^4 a b_2 b_3$$

[ > T:=a^2*b2^2+a^2*b3^2-2*a*b2^2*b3+2*a*b2*b3^2+2*b2^2*b3^2-2*b2^2*b3*b4+b2^2*b4^2-2*b2*b3^2*b4+b3^2*b4^2;
[
[ 
$$a^2 b_2^2 + a^2 b_3^2 - 2 a b_2^2 b_3 + 2 a b_2 b_3^2 + 2 b_2^2 b_3^2 - 2 b_2^2 b_3 b_4 + b_2^2 b_4^2 - 2 b_2 b_3^2 b_4$$

[ 
$$+ b_3^2 b_4^2$$

[ > # Disussion of the GCD
[ >
[ > # Item 1
[ > b2:=0:
[ > factor(W1);
[
[ 
$$-B b_3^2 (b_4 - b_5) (a + b_5) (B a + a + b_4)$$

[ >
[ > # (a)
[ > b5:=-a:
[ > factor(W2);
[
[ 
$$4 B (a + b_4) a b_3^3 (B a + a + b_4)$$

[ > b4:=solve(B*a+a+b4,b4);
[
[ 
$$-B a - a$$

[ > factor(W3);
[
[ 
$$8 B^2 a^3 b_3^4 (B^2 + 2 B + 2)$$

[ > solve(B^2+2*B+2,B);
[
[ 
$$-1 + I, -1 - I$$

[ > b4:='b4':b5:='b5':
[ >
[ > # (b)
[ > b4:=solve(B*a+a+b4,b4);
[
[ 
$$-B a - a$$

[ > factor(W2);

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[
      -2 B a b33 (a + b5) (B2 + 2 B + 2) (B a + a + b5)
[ > b4:='b4':b2:='b2':
[ >
[ > # Item 2
[ > # Case b3=0
[ > b3:=0:
[ > factor(W1);
      B b22 (b4 - b5) (a - b5) (B a - a + b4)
[ > # (a)
[ > b5:=a:
[ > factor(W2);
      -4 B (a - b4) a b23 (B a - a + b4)
[ > b4:=solve(B*a-a+b4,b4);
      -B a + a
[ > factor(W3);
      8 B2 a3 b24 (B2 - 2 B + 2)
[ > solve(B^2-2*B+2,B);
      1 + I, 1 - I
[ > b4:='b4':b5:='b5':
[ >
[ > # (b)
[ > b4:=solve(B*a-a+b4,b4);
      -B a + a
[ > factor(W2);
      2 B a b23 (a - b5) (B2 - 2 B + 2) (B a - a + b5)
[ > b4:='b4':b2:='b2':
[ >
[ > # Case b3=b2
[ > b3:=b2:
[ > factor(W1);
      4 B b22 (b4 - b5) (b2 - b5) (B a - b2 + b4)
[ >
[ > # (a)
[ > b5:=b2:
[ > factor(W2);
      -16 B (b2 - b4) a b23 (B a - b2 + b4)
[ > b4:=solve(B*a-b2+b4,b4);
      -B a + b2
[ > factor(W3);
      64 B2 a3 b24 (B2 + 1)
[ > solve(B^2+1,B);
      I, -I
[ > b4:='b4':b5:='b5':
[

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[ > # The two possibilites e1=0 or e2=0 imply the same condition
[ > e1:=0:
[ > factor(K_);

$$\frac{4 e3\_^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 e2^4 b2^4 b3^2}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e1:='e1':
[ > e2:=0:
[ > factor(K_);

$$\frac{4 e1^4 b3^2 b2^4 (e1^2 + e3\_)^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e2:='e2':
[ > solve(b2^2-b2*b3+2*b3^2,b3);

$$\left(\frac{1}{4} + \frac{1}{4} I 7^{(1/2)}\right) b2, \left(\frac{1}{4} - \left(\frac{1}{4} I\right) 7^{(1/2)}\right) b2$$

[ > B:='B':b5:='b5':
[ >
[ > # (ii) Discussion of the second branch of b5
[ > b5:=Sol_b5[2];

$$\frac{(-2 I) B b2^3 + 2 I B b2^2 b3 - (I) B^2 b3^3 - (2 I) B b3^3 + 2 B^2 b2^3 + 2 B^2 b2^2 b3 + 2 B^2 b2 b3^2 + 2 B^2 b3^3 + 2 I B b2 b3^2 + B^2 b2^2 b3 I - (I) B^2 b2 b3^2 + B^2 b2^3 I - 2 B b2^3 + 2 B b2^2 b3 - 2 B b2 b3^2 + 2 B b3^3 + 2 b2^3 + 2 b2^2 b3 + 2 b2 b3^2 + 2 b3^3}{(B^2 b2^2 + 2 B^2 b2 b3 + B^2 b3^2 - 2 B b2^2 + 2 B b3^2 + 2 b2^2 + 2 b3^2) (b2^2 + b3^2)} b3 b2 / ($$

[ > solve(coeff(numer(b5), I, 1), B);

$$0, \frac{2 (b2 - b3)}{b2 + b3}$$

[ > B:=(2*(b2-b3))/(b2+b3):
[ > factor(simplify(H3_));

$$\frac{256 (b2 - b3)^4 e2^2 b2^2 b3^2 e1^2}{(b2^2 + b3^2)^2}$$

[ > e1:=0:
[ > factor(K_);

$$\frac{4 e3\_^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 e2^4 b2^4 b3^2}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e1:='e1':
[ > e2:=0:
[ > factor(K_);

```

$$\frac{4(e1^2 + e3\_)^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 b2^4 b3^2 e1^4}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

```

[ > e2:='e2':
[ > B:='B':b5:='b5':a:='a':b4:='b4':
[ >
[ > # (b) Discussion of the second branch of a
[ > a:=Sol_a[2]:
[ > b4:=solve(coeff( numer(a), I, 1), b4);
[ >
[ >

$$\frac{b2 b3 (b2 + b3)}{b2^2 + b3^2}$$

[ > factor(gcd(gcd( numer(simplify(W1)), numer(simplify(W2))), numer(simplify(W1)))):
[ > J:=simplify(%/b2/b3/(b2-b3)):nops(%);
[ >
[ > Sol_b5:=solve(J, b5):
[ >
[ > # (i) Discussion of the first branch of b5
[ > b5:=Sol_b5[1];
[ >

$$-((-2 I) B b2^3 + 2 I B b2^2 b3 - (I) B^2 b3^3 - (2 I) B b3^3 - 2 B^2 b2^3 - 2 B^2 b2^2 b3 - 2 B^2 b2 b3^2 - 2 B^2 b3^3 + 2 I B b2 b3^2 + B^2 b2^2 b3 I - (I) B^2 b2 b3^2 + B^2 b2^3 I + 2 B b2^3 - 2 B b2^2 b3 + 2 B b2 b3^2 - 2 B b3^3 - 2 b2^3 - 2 b2^2 b3 - 2 b2 b3^2 - 2 b3^3) b3 b2 / ((B^2 b2^2 + 2 B^2 b2 b3 + B^2 b3^2 - 2 B b2^2 + 2 B b3^2 + 2 b2^2 + 2 b3^2) (b2^2 + b3^2))$$

[ > solve(coeff( numer(b5), I, 1), B);
[ >

$$0, \frac{2(b2 - b3)}{b2 + b3}$$

[ > B:=(2*(b2-b3))/(b2+b3):
[ > factor(simplify(H3_));
[ >

$$\frac{256(b2 - b3)^4 e2^2 e1^2 b2^2 b3^2}{(b2^2 + b3^2)^2}$$

[ > e1:=0:
[ > factor(K_);
[ >

$$\frac{4 e3\_^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 e2^4 b2^4 b3^2}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e1:='e1':
[ > e2:=0:
[ > factor(K_);

```



$$\frac{4(e1^2 + e3\_)^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 b2^4 b3^2 e1^4}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e2:='e2':

[ > B:='B':b5:='b5':

[ >

[ > # (ii) Discussion of the second branch of b5

[ > b5:=Sol\_b5[2];

$$\frac{(-(2I) B b2^3 + 2 I B b2^2 b3 - (I) B^2 b3^3 - (2I) B b3^3 + 2 B^2 b2^3 + 2 B^2 b2^2 b3 + 2 B^2 b2 b3^2 + 2 B^2 b3^3 + 2 I B b2 b3^2 + B^2 b2^2 b3 I - (I) B^2 b2 b3^2 + B^2 b2^3 I - 2 B b2^3 + 2 B b2^2 b3 - 2 B b2 b3^2 + 2 B b3^3 + 2 b2^3 + 2 b2^2 b3 + 2 b2 b3^2 + 2 b3^3) b3 b2}{(B^2 b2^2 + 2 B^2 b2 b3 + B^2 b3^2 - 2 B b2^2 + 2 B b3^2 + 2 b2^2 + 2 b3^2) (b2^2 + b3^2)}$$

[ > solve(coeff(numer(b5), I, 1), B);

$$0, \frac{2(b2 - b3)}{b2 + b3}$$

[ > B:=(2\*(b2-b3))/(b2+b3):

[ > factor(simplify(H3\_));

$$\frac{256(b2 - b3)^4 e2^2 e1^2 b2^2 b3^2}{(b2^2 + b3^2)^2}$$

[ > e1:=0:

[ > factor(K\_);

$$\frac{4 e3\_^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 e2^4 b2^4 b3^2}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e1:='e1':

[ > e2:=0:

[ > factor(K\_);

$$\frac{4(e1^2 + e3\_)^2 (b2^2 - b2 b3 + 2 b3^2)^2 (b2 - b3)^4 b2^4 b3^2 e1^4}{(b2 + b3)^2 (b3 I + b2)^4 (b3 I - b2)^4}$$

[ > e2:='e2':

[ > B:='B':b5:='b5':a:='a':b4:='b4':

[ > # This closes Item 3

[ >

[ > # Therefore we can assume GCD different from zero

[ > E1\_star:=numer(factor(simplify(E1/(GCD^1)))):nops(%)

464

[ > E2\_star:=numer(factor(simplify(E2/(GCD^1)))):nops(%)

271

[ > E3\_star:=numer(factor(simplify(E3/(GCD^1)))):nops(%)

```

[
[ >
[ > D1:=factor(resultant(E2_star,E3_star,B)):
[ > D2:=factor(resultant(E1_star,E3_star,B)):
[ > D3:=factor(resultant(E1_star,E2_star,B)):
[ >
[ > gcd(gcd(D1,D2),D3);
32768 a b2 b3 (b2 - b3) (b2 + b3)20 (a2 b22 + a2 b32 - 2 a b22 b3 + 2 a b2 b32 + 2 b22 b32
- 2 b22 b3 b4 + b22 b42 - 2 b2 b32 b4 + b32 b42)
[ > GCD;
16 (b2 - b3) (b2 + b3)2 (a2 b22 + a2 b32 - 2 a b22 b3 + 2 a b2 b32 + 2 b22 b32
- 2 b22 b3 b4 + b22 b42 - 2 b2 b32 b4 + b32 b42) B4 a b2 b3
[ > # This finishes the discussion of the general case.
[ >

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