EXEMPLARY SELF-MOTIONS OF PLANE-SYMMETRIC CONGRUENT STEWART GOUGH PLATFORMS

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1 Introduction

In the following we study a concrete example of a plane-symmetric congruent Stewart Gough platform regarding its self-motions, whose theoretical aspects were presented in Section 5.2 of:

NAWRATIL, G.: *Congruent Stewart Gough platforms with non-translational self-motions.* Proc. of the 16th Int. Conf. on Geometry and Graphics, Innsbruck, Austria, August 4–8, 2014

All references given and notations used in this discussion are done with respect to this article.

1.1 Fundamentals

The geometry of the plane-symmetric congruent Stewart Gough platform is determined by:

$$\mu = \pi/4, \quad \lambda = -3\pi/4, \quad c_1 = c_2 = c_3 = -1.$$

In this case it is not difficult to verify that all four cylinders of revolutions are real (cf. footnote 6). Three of these axes equal the coordinate axes of the reference frame. The remaining axis s located in the plane ε of symmetry ($= xy$-plane) is given by $y - x = \cos(\mu)$.

The SCHÖNFLIES self-motion with respect to the direction of the i-axis (with $i \in \{x, y, z\}$) possesses the following bond β_i (up to conjugation of coordinates):

$$\beta_x = (0:0:0:0:1:I:0:0), \quad \beta_y = (0:0:0:0:1:0:I:0), \quad \beta_z = (0:0:0:0:1:0:0:I).$$

The bond β_s of the SCHÖNFLIES self-motion with respect to the direction of the axis s is given by:

$$\beta_\mathsf{s} := (0:0:0:0:1:I\sqrt{2}/2:I\sqrt{2}/2:0).$$

Finally, it should be noted that the manipulator's bond-set $\mathscr{B}$ equals $\{\beta_x, \beta_y, \beta_z, \beta_{\mathrm{s}}\}$ up to conjugation of coordinates.

1.2 Visualization

All figures and animations are given in one of the following three projections:

- Axonometric view with projection direction $(1, -1, \sqrt{2})$: The labeled base of the manipulator with respect to this projection is given in Fig. 1(a).
- Top view: The labeled base with respect to this projection is given in Fig. 1(b).
- Side view with projection direction $(0, 1, 1)$:The labeled base with respect to this projection is given in Fig. 1(c).

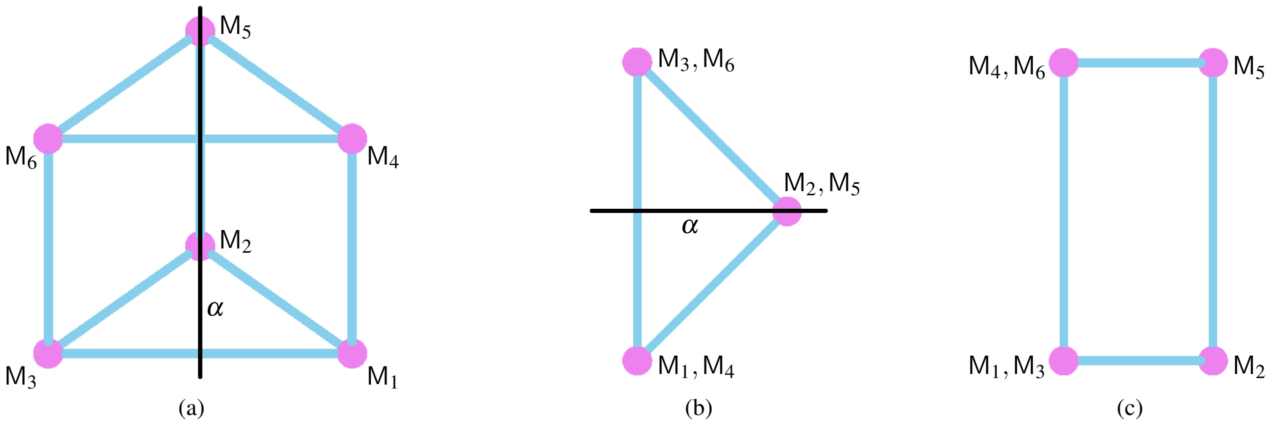

Figure 1: The projection centers of the axonometric view (a) and the top view (b) are located in a second symmetry-plane α of the base (beside ε), which is given by $x + y = 0$. The projection direction of the side view (c) is orthogonal to α and parallel to the axis s.

2 Exemplary self-motions

For the following choice of leg lengths:

$$R_1^2 = 6, \quad R_2^2 = 4, \quad R_3^2 = 6, \quad R_4^2 = 9, \quad R_5^2 = 7, \quad R_6^2 = 9,$$

the planar cubic P_3 in e_0, e_1, e_2 splits into a regular conic section $e_0^2 + e_1^2 + e_2^2 + 4e_1e_2 = 0$ and a straight line $e_1 - e_2 = 0$. Therefore the resulting self-motions can easily be parametrized.[1]

[1] In general the cubic P_3 is elliptic and therefore no parametrization exists.

2.1 Self-motion induced by the quadratic term

This self-motion, which is symmetric with respect to the plane α, has two branches stemming from the solution of $\Lambda_1 = 0$ with respect to f_0. They are visualized in the following animations:

- Animations 1a and 2a: Axonometric view of 1st and 2nd branch,
- Animation 1b and 2b: Top view of 1st and 2nd branch,
- Animation 1c and 2c: Side view of 1st and 2nd branch.

The trajectories of the anchor points are displayed in Fig. 2, 3 and 4, respectively. It can easily be seen on basis of the trajectory of the second (or fifth) anchor point (see Fig. 3) that there is no bifurcation between these two branches of the self-motion. Moreover there cannot be a bifurcation within one of the branches itself, as a regular conic has no self-intersection.

Remark 1: *It can easily be checked that the bonds of the self-motion induced by the quadric term are (up to conjugation of coordinates) β_x and β_y. Finally it should be noted that this self-motion is a line-symmetric motion as $f_3 = 0$ holds.*[2] ◇

2.2 Self-motion induced by the linear term

This self-motion, which is visualized in the following animations:

- Animation 3a: Axonometric view,
- Animation 3b: Top view,
- Animation 3c: Side view,

is a SCHÖNFLIES motion, where the direction of the axis of rotation equals the direction of the cylinder axis s.[3] It belongs to a set $\mathscr{X}_+$ of SCHÖNFLIES self-motions (with respect to the axial direction of s), which is determined by:

$$R_3 = R_1, \quad R_6 = R_4, \quad R_5^2 = R_4^2 - R_1^2 + R_2^2.$$

This set is 3-parametric as the choice of the three leg lengths R_1, R_2 and R_4 remains free. $\mathscr{X}_+$ is a superset of the 2-parametric set $\mathscr{X}$ (with respect to the axial direction of s) for which the additional condition $R_1 = R_2$ holds.[4]

[2] cf. SELIG, J.M., HUSTY, M.: *Half-turns and line symmetric motions.* Mech. Mach. Theory **46**(2) 156–167 (2011) .

[3] Therefore the bond of this self-motion is given by β_s (up to conjugation of coordinates).

[4] Note that this superset $\mathscr{X}_+$ of $\mathscr{X}$ with respect to the axial direction of s only exists due to the special geometry of the chosen example.

The trajectories of the anchor points under the self-motion induced by the linear term are displayed in Fig. 5, which show that this motion is symmetric with respect to α.

Remark 2: *There is no bifurcation between the self-motion induced by the linear term and the one induced by the quadratic term, as the conic* $e_0^2+e_1^2+e_2^2+4e_1e_2=0$ *and the line* $e_1-e_2=0$ *do not have a real intersection point.* $\diamond$

2.3 Self-motion of $\mathscr{X}$

If we modify the leg length of the second and fifth leg slightly by setting $R_2^2=6$ and $R_5^2=9$, we obtain a SCHÖNFLIES self-motion of $\mathscr{X}$ (with respect to the axial direction of s), which is visualized in the following animations:

- Animation 4a: Axonometric view,
- Animation 4b: Top view,
- Animation 4c: Side view.

The trajectories of the anchor points under this self-motion are displayed in Fig. 6, which show that this motion is again symmetric with respect to α.

2.4 Self-motion of $\mathscr{X}_-$

For the choice

$$R_1^2=R_2^2=R_3^2=R_4^2=R_5^2=R_6^2=6,$$

we obtain an exemplary SCHÖNFLIES self-motion (with respect to the axial direction of s) of $\mathscr{X}_-$. This self-motion with equal leg lengths has two separated branches, which are symmetric with respect to α. Therefore only one of these branches is animated in

- Animation 5a: Axonometric view,
- Animation 5b: Top view,
- Animation 5c: Side view,

which also include the trajectories of all six anchor points.

Remark 3: *As all above given* SCHÖNFLIES *self-motions are parametrized with respect to the rotation angle, the respective animations speed up close to pure translational motions. This property can be seen very well in the animations 3 and 4.* $\diamond$

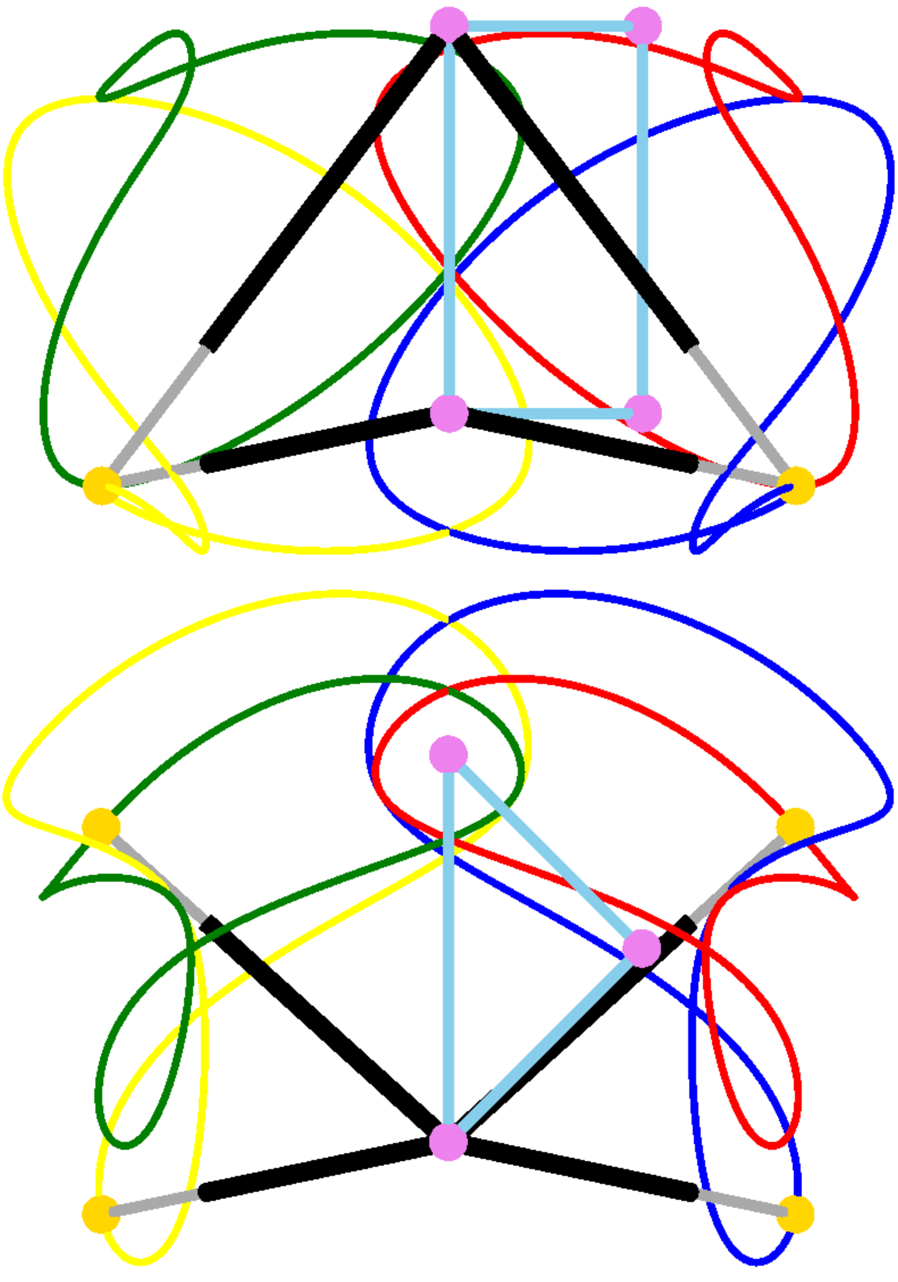

Figure 2: Side view and top view arranged as paired normal projections: The trajectories of the 1st resp. 2nd branch of the first anchor point (red resp. green) and fourth anchor point (blue resp. yellow).

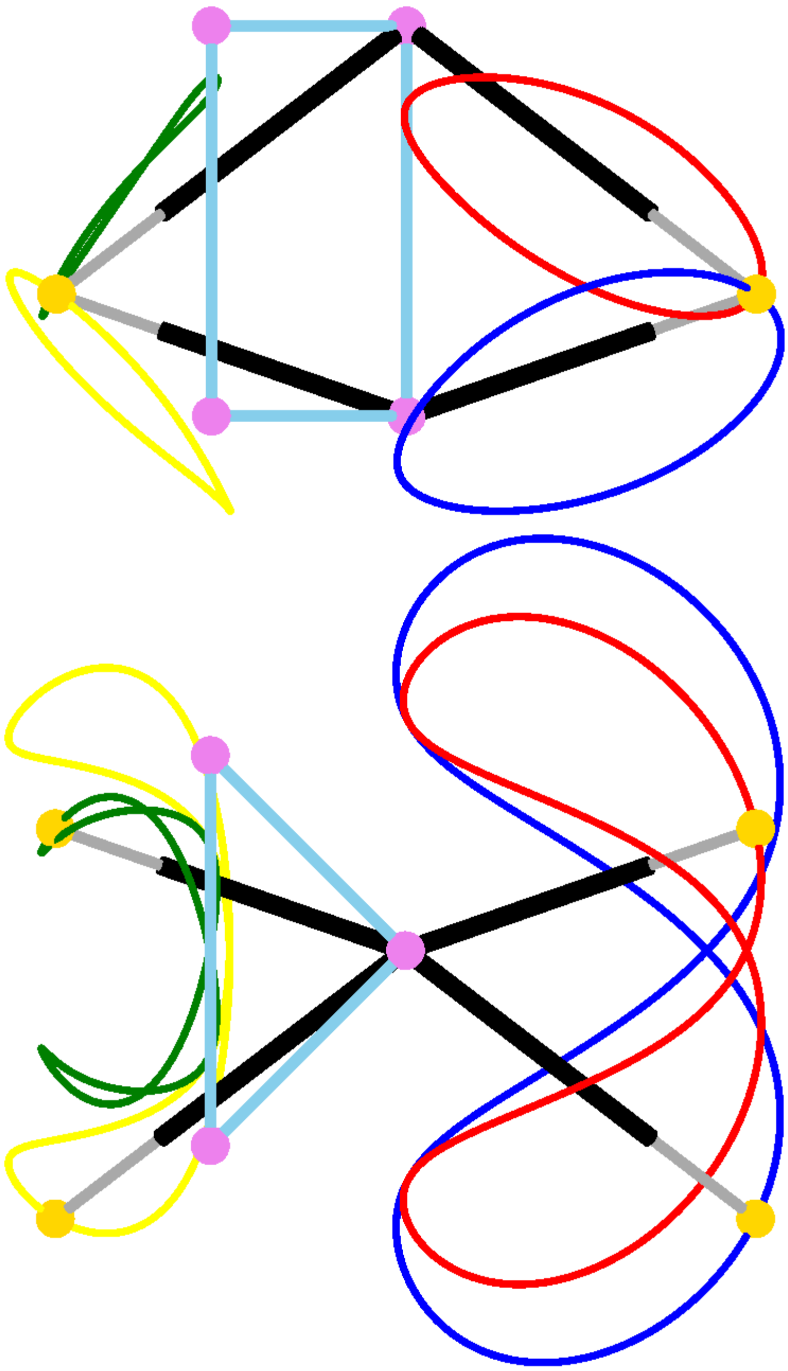

Figure 3: Side view and top view arranged as paired normal projections: The trajectories of the 1st and 2nd branch of the second anchor point (red and green) do not intersect each other. This also holds for the trajectories of the 1st and 2nd branch of the fifth anchor point (blue and yellow).

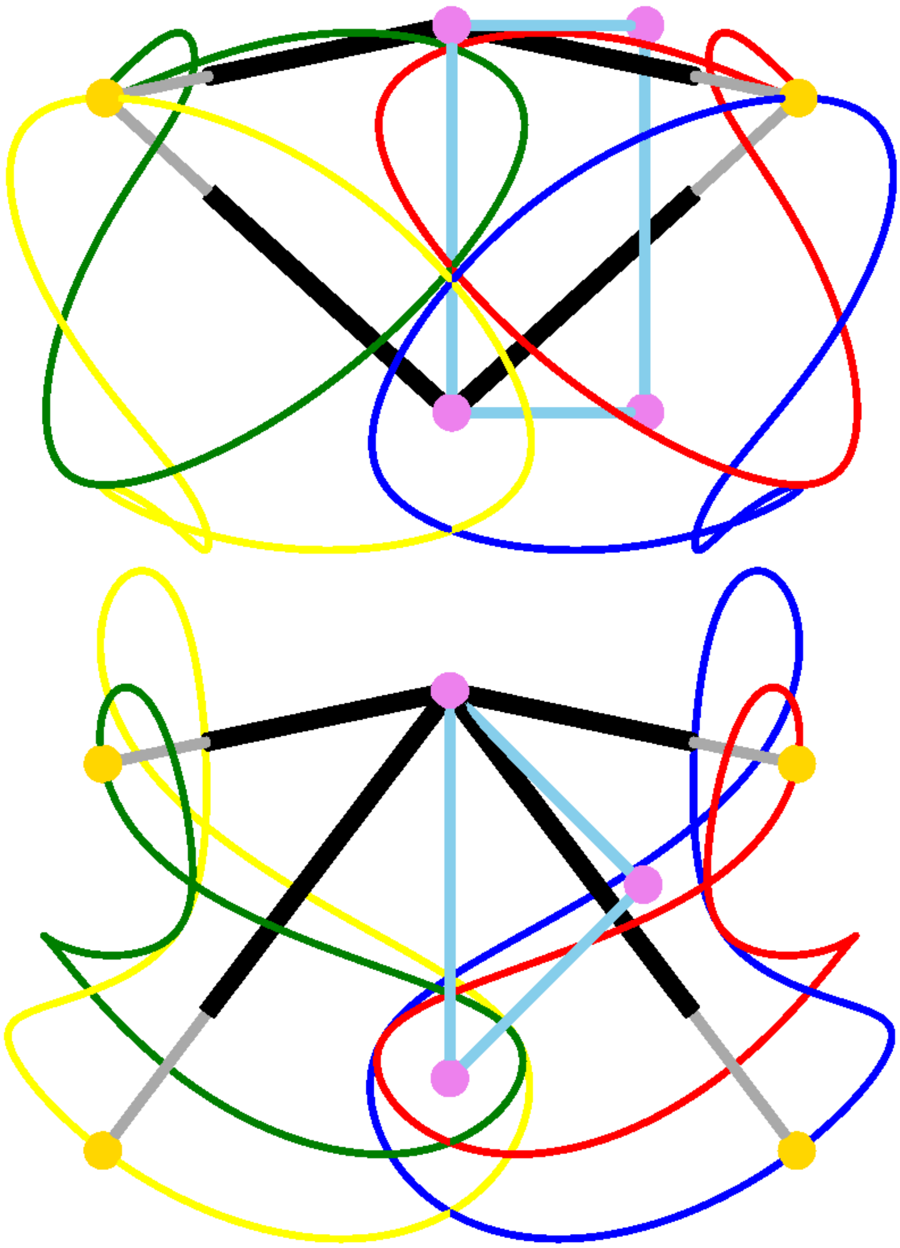

Figure 4: Side view and top view arranged as paired normal projections: The trajectories of the 1st resp. 2nd branch of the third anchor point (red resp. green) and sixth anchor point (blue resp. yellow).

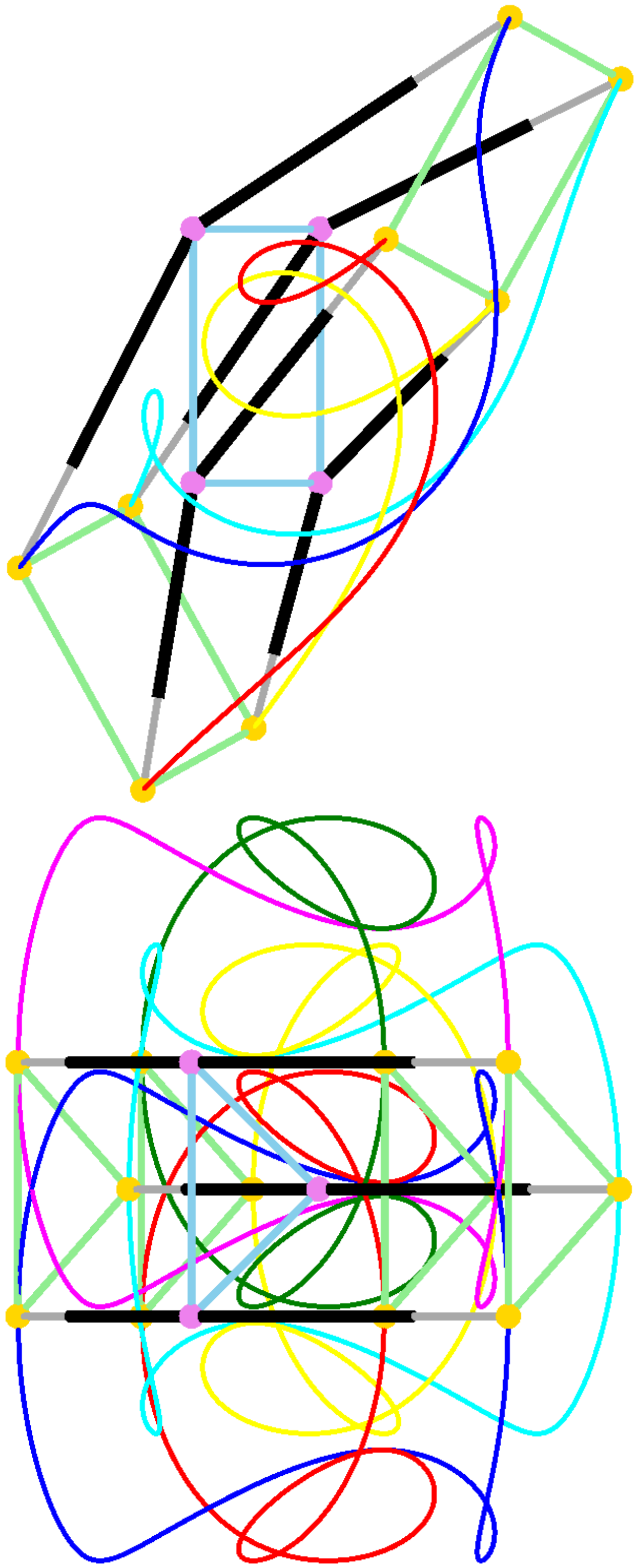

Figure 5: Side view and top view arranged as paired normal projections: The trajectories of the anchor points are colored in ascending order as follows: red, yellow, green, blue, cyan and magenta.

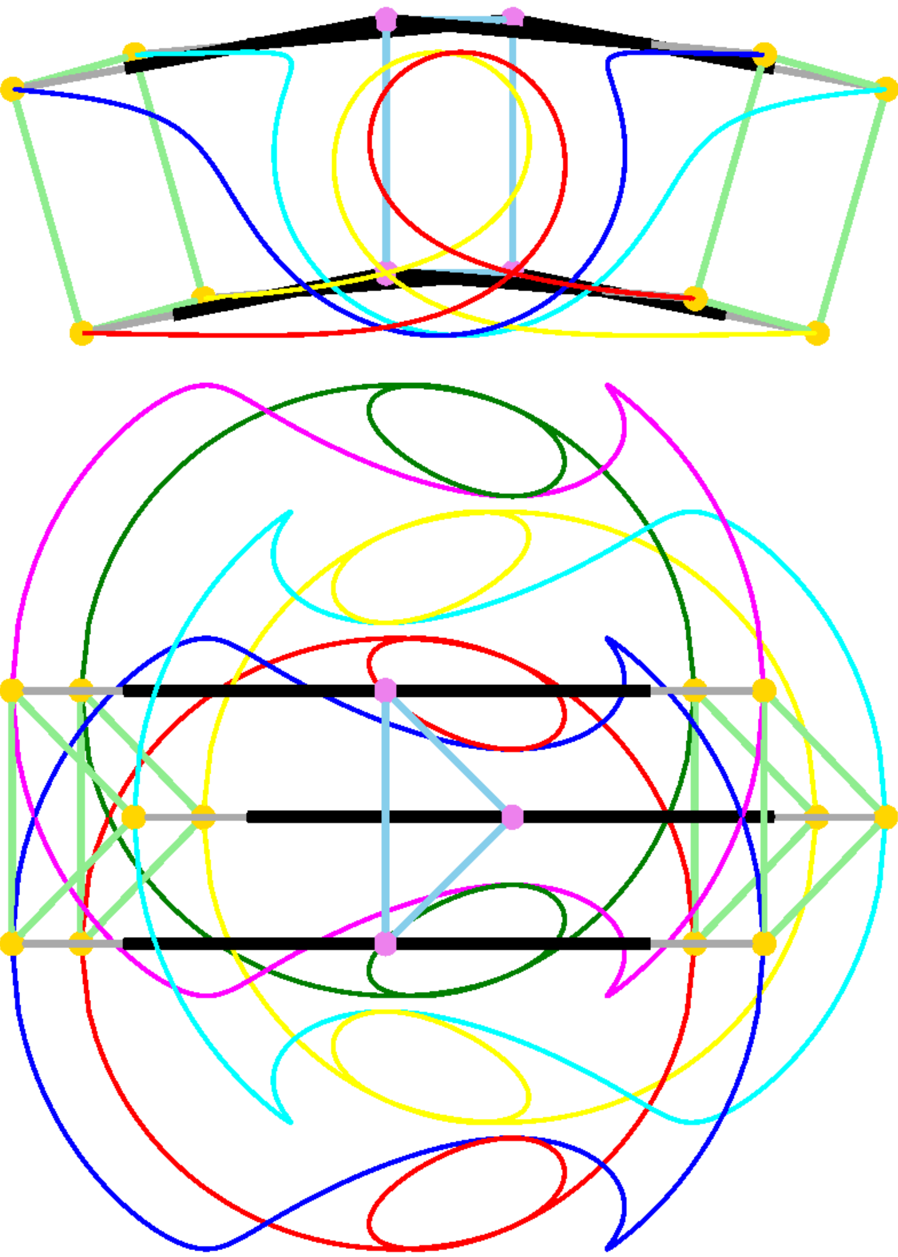

Figure 6: Side view and top view arranged as paired normal projections: The trajectories of the anchor points are colored in ascending order as follows: red, yellow, green, blue, cyan and magenta.