```
> restart:with(FGb):
Path set to c:\Programme\Maple 10\bin.win/libfgbint.so
    FGb/Maple interface package Version 1.26
    JC Faugere (jcf@calfor.lip6.fr)
    Type ?FGb for documentation
>
> # For the given input parameters we get:
> Omega2:=23534*I*f1*e3+40344*I*e1*e3-11286*e3^2+50430*e1*e3+40344
  *e2*e3+23534*f0*e1-23534*f3*e2+23534*f2*e3+15610*e2^2+15610*e1^2
  -50430*I*e2*e3-63608*I*e3^2+17080*I*e2^2+17080*I*e1^2-23534*I*f0
  *e2-23534*I*f3*e1;
```

$$\Omega 2 := 23534\, I\, f1\, e3 + 17080\, I\, e1^2 - 11286\, e3^2 + 50430\, e1\, e3 + 40344\, e2\, e3 + 23534\, f0\, e1 - 23534\, f3\, e2 + 23534\, f2\, e3 + 15610\, e2^2 + 15610\, e1^2 - 50430\, I\, e2\, e3 - 23534\, I\, f0\, e2 + 40344\, I\, e1\, e3 + 17080\, I\, e2^2 - 63608\, I\, e3^2 - 23534\, I\, f3\, e1$$

```
> Omega3:=-23534*I*f1*e3-11286*e3^2+50430*e1*e3+40344*e2*e3+23534*
  f0*e1-23534*f3*e2+23534*f2*e3+15610*e2^2+15610*e1^2-40344*I*e1*e
  3+50430*I*e2*e3+63608*I*e3^2-17080*I*e2^2-17080*I*e1^2+23534*I*f
  0*e2+23534*I*f3*e1;
```

$$\Omega 3 := -23534\, I\, f1\, e3 - 11286\, e3^2 + 50430\, e1\, e3 + 40344\, e2\, e3 + 23534\, f0\, e1 - 23534\, f3\, e2 + 23534\, f2\, e3 + 15610\, e2^2 + 15610\, e1^2 - 17080\, I\, e1^2 + 50430\, I\, e2\, e3 + 23534\, I\, f0\, e2 - 40344\, I\, e1\, e3 - 17080\, I\, e2^2 + 63608\, I\, e3^2 + 23534\, I\, f3\, e1$$

```
> Omega4:=574*f0*e3-574*f2*e1+574*f1*e2-1804*e1*e3-2788*e2*e3-700*
  e1^2-700*e2^2-700*e3^2;
```

$$\Omega 4 := 574\, f0\, e3 - 574\, f2\, e1 + 574\, f1\, e2 - 1804\, e1\, e3 - 2788\, e2\, e3 - 700\, e1^2 - 700\, e2^2 - 700\, e3^2$$

```
> Pi5:=26600*e1^2+141204*f0*e1+70602*f2*e1-47068*f3*e1+26600*e2^2+
  70602*f0*e3+47068*f0*e2+47068*f1*e3+26600*e3^2-70602*f1*e2+14120
  4*f3*e2-141204*f2*e3;
```

$$\pi 5 := 26600\, e1^2 + 141204\, f0\, e1 + 70602\, f2\, e1 - 47068\, f3\, e1 + 26600\, e2^2 + 70602\, f0\, e3 + 47068\, f0\, e2 + 47068\, f1\, e3 + 26600\, e3^2 - 70602\, f1\, e2 + 141204\, f3\, e2 - 141204\, f2\, e3$$

```
> # Lambda is the sphere condition:
> Lambda:=-4*f2*e3*x+2*Y*y*e1^2+2*Y*y*e3^2-4*e1*y*f3+4*e1*z*f2+e1^
  2*X^2+e2^2*X^2+e3^2*X^2+e1^2*Y^2+e2^2*Y^2+e3^2*Y^2+e1^2*x^2+x^2*
  e2^2+x^2*e3^2+e1^2*y^2+e2^2*y^2+y^2*e3^2+e1^2*z^2+z^2*e2^2+4*f0^
  2+4*f3^2+4*f2^2+4*f1^2-4*Y*e1*e2*x-4*X*e1*e2*y-4*X*e1*e3*z-4*Y*e
  2*e3*z-4*Z*e1*e3*x-4*Z*e2*e3*y-4*X*f0*e1+4*X*f3*e2-4*X*f2*e3-4*Y
  *f3*e1-4*Y*f0*e2+4*Y*f1*e3+4*e1*x*f0+4*f3*e2*x-2*X*x*e1^2+2*X*x*
  e2^2+2*X*x*e3^2+4*f0*e2*y+4*f1*e3*y-2*Y*y*e2^2-4*f1*e2*z+4*f0*e3
  *z+2*Z*z*e2^2+4*Z*f2*e1-4*Z*f1*e2-4*Z*f0*e3+2*Z*z*e1^2-2*Z*z*e3^
  2+e3^2*z^2+e1^2*Z^2+e2^2*Z^2+e3^2*Z^2-e1^2*R^2-e2^2*R^2-e3^2*R^2
  ;
```

$$\Lambda := -4\, Z\, f1\, e2 + e1^2\, X^2 + e2^2\, X^2 + e3^2\, X^2 + e1^2\, Y^2 + e2^2\, Y^2 + e3^2\, Y^2 + e1^2\, x^2 + x^2\, e2^2 + x^2\, e3^2 + e1^2\, y^2 + e2^2\, y^2 + y^2\, e3^2 + e1^2\, z^2 + z^2\, e2^2 + e3^2\, z^2 + e1^2\, Z^2 + e2^2\, Z^2 + e3^2\, Z^2 - e1^2\, R^2 - e2^2\, R^2$$

$$-e3^2 R^2 - 4\,f2\,e3\,x + 2\,Y\,y\,e1^2 + 2\,Y\,y\,e3^2 - 4\,e1\,y\,f3 + 4\,e1\,z\,f2 + 4\,f0^2 + 4\,f3^2 + 4\,f2^2 + 4\,f1^2$$
$$- 4\,Y\,e1\,e2\,x - 4\,X\,e1\,e2\,y - 4\,X\,e1\,e3\,z - 4\,Y\,e2\,e3\,z - 4\,Z\,e1\,e3\,x - 4\,Z\,e2\,e3\,y - 4\,X\,f0\,e1$$
$$+ 4\,X\,f3\,e2 - 4\,X\,f2\,e3 - 4\,Y\,f3\,e1 - 4\,Y\,f0\,e2 + 4\,Y\,f1\,e3 + 4\,e1\,x\,f0 + 4\,f3\,e2\,x - 2\,X\,x\,e1^2$$
$$+ 2\,X\,x\,e2^2 + 2\,X\,x\,e3^2 + 4\,f0\,e2\,y + 4\,f1\,e3\,y - 2\,Y\,y\,e2^2 - 4\,f1\,e2\,z + 4\,f0\,e3\,z + 2\,Z\,z\,e2^2$$
$$+ 4\,Z\,f2\,e1 - 4\,Z\,f0\,e3 + 2\,Z\,z\,e1^2 - 2\,Z\,z\,e3^2$$

```
> # where R denotes the leg length, (X,Y,Z) are the fixed
  coordinates of the base anchor point and (x,y,z) are the
  coordinates of the corresponding platform anchor point with
  respect to the moving system.
>
> # Computation of the f0,...,f3:
> fs:=solve({e1*f1+e2*f2+e3*f3,Omega2,Omega3,Omega4},{f0,f1,f2,f3}
  );
```

$$fs := \{f1 = (48614\,e2^2\,e3 - 8540\,e1^2\,e3 + 39565\,e2\,e3^2 + 31804\,e3^3 - 20172\,e1\,e3^2$$
$$+ 14350\,e2\,e1^2 + 14350\,e2^3 + 36982\,e1\,e2\,e3) \,/\, (11767\,(e2^2 + e3^2 + e1^2)), f2 = ($$
$$-57154\,e1\,e2\,e3 - 14350\,e1^3 - 14350\,e1\,e2^2 - 44787\,e1^2\,e3 - 39565\,e1\,e3^2 + 5643\,e3^3$$
$$- 7805\,e2^2\,e3 - 20172\,e2\,e3^2) \,/\, (11767\,(e2^2 + e3^2 + e1^2)), f0 = 5\,(8525\,e1\,e3^2$$
$$+ 5070\,e2\,e3^2 - 1561\,e1\,e2^2 - 2173\,e2^2\,e3 - 2173\,e1^2\,e3 - 1561\,e1^3 + 1708\,e2^3 + 2870\,e3^3$$
$$+ 1708\,e2\,e1^2) \,/\, (11767\,(e2^2 + e3^2 + e1^2)), f3 = -(-8540\,e1\,e2^2 - 7805\,e2\,e1^2$$
$$+ 5643\,e2\,e3^2 - 7805\,e2^3 - 20172\,e2^2\,e3 - 8540\,e1^3 + 31804\,e1\,e3^2 - 20172\,e1^2\,e3) \,/\,($$
$$11767\,(e2^2 + e3^2 + e1^2))\}$$

```
> assign(fs);
> F:=8525*e3^2*e1+5070*e3^2*e2+1708*e2*e1^2+1708*e2^3-1561*e1*e2^2
  -2173*e2^2*e3-1561*e1^3-2173*e1^2*e3+2870*e3^3;
```

$$F := 8525\,e1\,e3^2 + 5070\,e2\,e3^2 - 1561\,e1\,e2^2 - 2173\,e2^2\,e3 - 2173\,e1^2\,e3 - 1561\,e1^3$$
$$+ 1708\,e2^3 + 2870\,e3^3 + 1708\,e2\,e1^2$$

```
>
> # Test:
> gcd(F,factor(numer(Pi5)))-F;
```

$$0$$

```
> gcd(F,factor(numer(f0)))-F;
```

$$0$$

```
>
> # Now we make the following ansatz:
> Gamma:=numer(simplify(Lambda)):nops(%);
```

$$161$$

```
> ansatz:=simplify(F*(eta1*e1+eta2*e2+eta3*e3)-Gamma):
>
> # The set of equations implied by the coefficients with respect
  to e1,e2,e3 read as follows:
> c400:=coeff(coeff(coeff(ansatz,e1,4),e2,0),e3,0);
```

$$c400 := -808500 - 164738\,Y\,y + 164738\,X\,x + 239120\,Y + 401800\,Z - 82369\,x^2 + 239120\,y + 218540\,x + 401800\,z - 218540\,X - 82369\,Y^2 - 82369\,z^2 - 164738\,Z\,z - 1561\,\eta1 - 82369\,y^2 - 82369\,Z^2 + 82369\,R^2 - 82369\,X^2$$

```
> c040:=coeff(coeff(coeff(ansatz,e1,0),e2,4),e3,0);
```

$$c040 := -808500 + 164738\,Y\,y - 164738\,X\,x - 239120\,y + 401800\,z - 82369\,Y^2 - 218540\,x + 401800\,Z - 218540\,X + 239120\,Y - 164738\,Z\,z + 1708\,\eta2 + 82369\,R^2 - 82369\,x^2 - 82369\,y^2 - 82369\,z^2 - 82369\,Z^2 - 82369\,X^2$$

```
> c004:=coeff(coeff(coeff(ansatz,e1,0),e2,0),e3,4);
```

$$c004 := 164738\,Z\,z - 2972660 - 164738\,X\,x - 164738\,Y\,y + 82369\,R^2 - 82369\,x^2 - 82369\,X^2 - 890512\,y - 890512\,Y + 158004\,x - 82369\,y^2 + 401800\,Z - 82369\,Z^2 - 401800\,z - 82369\,z^2 + 158004\,X - 82369\,Y^2 + 2870\,\eta3$$

```
> c013:=coeff(coeff(coeff(ansatz,e1,0),e2,1),e3,3);
```

$$c013 := 5070\,\eta3 + 2870\,\eta2 - 706020\,Y + 180712\,z - 564816\,X - 564816\,x + 1600312\,Z + 329476\,Z\,y + 329476\,Y\,z - 7177952 - 1509620\,y$$

```
> c103:=coeff(coeff(coeff(ansatz,e1,1),e2,0),e3,3);
```

$$c103 := -1509620\,x - 706020\,X + 564816\,Y - 1351504\,z + 8525\,\eta3 + 1035496\,Z + 2870\,\eta1 + 564816\,y + 1204744 + 329476\,X\,z + 329476\,Z\,x$$

```
> c310:=coeff(coeff(coeff(ansatz,e1,3),e2,1),e3,0);
```

$$c310 := -1561\,\eta2 + 1708\,\eta1 + 437080\,y + 329476\,X\,y + 329476\,Y\,x - 478240\,x$$

```
> c301:=coeff(coeff(coeff(ansatz,e1,3),e2,0),e3,1);
```

$$c301 := -2173\,\eta1 - 706020\,X + 564816\,Y + 1035496\,Z - 1561\,\eta3 - 97580\,x + 1472576\,z + 564816\,y + 329476\,X\,z + 329476\,Z\,x - 4282040$$

```
> c130:=coeff(coeff(coeff(ansatz,e1,1),e2,3),e3,0);
```

$$c130 := -1561\,\eta2 + 1708\,\eta1 + 437080\,y + 329476\,X\,y + 329476\,Y\,x - 478240\,x$$

```
> c031:=coeff(coeff(coeff(ansatz,e1,0),e2,3),e3,1);
```

$$c031 := -2173\,\eta2 + 1122072\,z - 97580\,y + 1708\,\eta3 - 564816\,x + 1600312\,Z - 564816\,X - 706020\,Y + 329476\,Z\,y + 329476\,Y\,z - 3627680$$

```
> c022:=coeff(coeff(coeff(ansatz,e1,0),e2,2),e3,2);
```

$$c022 := -2173\,\eta3 + 5070\,\eta2 - 164738\,X^2 - 9731900 - 164738\,Y^2 - 164738\,x^2 - 164738\,y^2 - 164738\,z^2 - 164738\,Z^2 + 164738\,R^2 - 2070992\,y + 1412040\,z - 60536\,x + 803600\,Z - 60536\,X - 651392\,Y - 329476\,X\,x$$

```
> c202:=coeff(coeff(coeff(ansatz,e1,2),e2,0),e3,2);
```

$$c202 := -5213372 - 164738\,X^2 - 164738\,Y^2 - 164738\,x^2 - 164738\,y^2 - 164738\,z^2 - 164738\,Z^2 + 164738\,R^2 - 329476\,Y\,y - 651392\,Y + 803600\,Z - 2447536\,x + 1412040\,z - 651392\,y - 2173\,\eta3 - 60536\,X + 8525\,\eta1$$

```
> c220:=coeff(coeff(coeff(ansatz,e1,2),e2,2),e3,0);
```

$$c220 := -1617000 - 164738\,X^2 - 164738\,Y^2 - 164738\,x^2 - 164738\,y^2 - 164738\,z^2 - 164738\,Z^2 + 164738\,R^2 - 329476\,Z\,z + 803600\,z + 803600\,Z - 437080\,X + 478240\,Y + 1708\,\eta2 - 1561\,\eta1$$

```
> c211:=coeff(coeff(coeff(ansatz,e1,2),e2,1),e3,1);
```

$$c211 := -2173\,\eta2 + 1122072\,z - 97580\,y + 1708\,\eta3 - 564816\,x + 1600312\,Z - 564816\,X - 706020\,Y + 329476\,Z\,y + 329476\,Y\,z - 3627680$$

```
> c121:=coeff(coeff(coeff(ansatz,e1,1),e2,2),e3,1);
```

$$c121 := -2173\,\eta1 - 706020\,X + 564816\,Y + 1035496\,Z - 1561\,\eta3 - 97580\,x + 1472576\,z + 564816\,y + 329476\,X\,z + 329476\,Z\,x - 4282040$$

```
> c112:=coeff(coeff(coeff(ansatz,e1,1),e2,1),e3,2);
```

$$c112 := 329476\,Y\,x - 2387000\,y + 329476\,X\,y - 10059104 + 5070\,\eta1 + 8525\,\eta2 - 1419600\,x$$

```
>
> # Gröbner base elimination of X,Y,Z,eta1,eta2,eta3,R
> GBE:=fgb_gbasis_elim([c400,c040,c004,c310,c301,c130,c031,c013,c1
  03,c112,c121,c211,c022,c202,c220],0,[X,Y,Z,eta1,eta2,eta3,R],[X,
  Y,Z,x,y,z,eta1,eta2,eta3,R]):
> Hilbert:=fgb_hilbert(GBE,0,[],[op(indets(GBE))],T);
```

$$Hilbert := [\,4\,T^3 + 3\,T^2 + 2\,T + 1,\ 1\,]$$

```
> # Dimension of GBE:
> Hilbert[2];
```

$$1$$

```
> # Degree of GBE:
> subs(T=1,Hilbert[1]);
```

$$10$$

```
>
> # Isotropic planes through p are given by:
> epsilon1:=91*x-84*y+147*I*y-98*I*z-126*z-122+714*I;
```

$$\varepsilon1 := 91\,x - 84\,y + 147\,I\,y - 98\,I\,z - 126\,z - 122 + 714\,I$$

```
> epsilon2:=91*x-84*y-147*I*y+98*I*z-126*z-122-714*I;
```

$$\varepsilon2 := 91\,x - 84\,y - 147\,I\,y + 98\,I\,z - 126\,z - 122 - 714\,I$$

```
>
> x:=solve(epsilon1,x);
```

$$x := \frac{12\,y}{13} - \frac{21}{13}\,I\,y + \frac{14}{13}\,I\,z + \frac{18\,z}{13} + \frac{122}{91} - \frac{102}{13}\,I$$

```
> g1:=0:
> for i from 1 to nops(GBE) do
> g:=numer(factor(simplify(GBE[i]))):
> g1:=gcd(g1,g):
> end do:
> x:='x':
>
>
> x:=solve(epsilon2,x);
```

$$x := \frac{12\,y}{13} + \frac{21}{13}\,I\,y - \frac{14}{13}\,I\,z + \frac{18\,z}{13} + \frac{122}{91} + \frac{102}{13}\,I$$

```
> g2:=0:
```

```
> for i from 1 to nops(GBE) do
> g:=numer(factor(simplify(GBE[i]))):
> g2:=gcd(g2,g):
> end do:
> x:='x':
>
> factor(numer(g1));
```

$$(-14\,z+102+21\,y)\,(274400\,y^3+9573816\,I\,y\,z+3927840\,y^2+817369\,I\,z^2-30870\,y^2\,z$$
$$-1840195\,I\,y^2+15910300\,y+3374238\,y\,z-15809850\,I\,y-408170\,I\,z^3-1165514\,y\,z^2$$
$$+13169366\,z+17761620-29479660\,I+20061237\,I\,z+984410\,I\,y^2\,z-5472908\,z^2$$
$$+113190\,z^3-115248\,I\,y\,z^2)$$

```
> factor(numer(g2));
```

$$(-14\,z+102+21\,y)\,(274400\,y^3-9573816\,I\,y\,z+3927840\,y^2-817369\,I\,z^2-30870\,y^2\,z$$
$$+1840195\,I\,y^2+15910300\,y+3374238\,y\,z+15809850\,I\,y+408170\,I\,z^3-1165514\,y\,z^2$$
$$+13169366\,z+17761620+29479660\,I-20061237\,I\,z-984410\,I\,y^2\,z-5472908\,z^2$$
$$+113190\,z^3+115248\,I\,y\,z^2)$$

```
> # End.
```