CONSTRUCTION OF MOVABLE HEXAPODS VIA MÖBIUS PHOTOGRAMMETRY

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This talk is devoted to the construction of a family of mechanical devices that belong to the class of so-called *hexapods* (also known as *Stewart Gough platforms*). The geometry of this kind of mechanical manipulators is defined by the coordinates of the 6 platform anchor points $p_1, \ldots, p_6 \in \mathbb{R}^3$ and of the 6 base anchor points $P_1, \ldots, P_6 \in \mathbb{R}^3$ in one of their possible configurations. All pairs of points (p_i, P_i) are connected by a rigid body, called *leg*, so that for all possible configurations the distance $d_i = \|p_i - P_i\|$ is preserved. We say that a hexapod is *movable*, or admits a *self-motion* if, once we fix the position of the base points $\{P_i\}$, then the platform points $\{p_i\}$ are allowed to move in an (at least) one-dimensional set of configurations respecting the constraints given by the legs; in this case, each p_i will move on the sphere with center P_i and radius d_i.

The classification of movable hexapods is still an open problem. Therefore it seems interesting to provide new instances of this kind of objects. This talk addresses the following question: given 6 points $\vec{P} = (P_1, \ldots, P_6)$ in $\mathbb{R}^3$, do there exist six other points $\vec{p} = (p_1, \ldots, p_6)$ and six non negative numbers $d_1, \ldots, d_6$ such that the hexapod whose base and platform points are given respectively by $\vec{P}$ and $\vec{p}$ and whose leg lengths are the numbers $\{d_i\}$ is movable? There are already examples in the literature that provide an affirmative answers (e.g. congruent hexapods, in which base and platform points differ by an isometry); we present a new family of examples that can be produced starting from a general choice of the points $\vec{P}$.

Our construction proceeds as follows. First, given a general 6-tuple $\vec{P}$, we construct, up to a scaling factor, the candidate platform $\vec{p}$ applying Möbius photogrammetry — a technique, already used by the authors to establish necessary conditions for the mobility of pentapods, that allows to associate to the 6-tuple $\vec{P}$ a curve in the moduli space of 6 points on the projective line — and then liaison theory. Secondly, we determine the right scaling factor for the platform points and the leg lengths so that the corresponding hexapod is movable; both these tasks are achieved by inspection of tangency conditions for the set of configurations of the hexapod, thought as a curve in a suitable projective variety.

Acknowledgements. The first-named and third-named author's research is supported by the Austrian Science Fund (FWF): W1214-N15/DK9 and P26607 - "Algebraic Methods in Kinematics: Motion Factorisation and Bond Theory". The second-named author's research is funded by the Austrian Science Fund (FWF): P24927-N25 - "Stewart Gough platforms with self-motions".