On the role of Descriptive Geometry in the different curricula

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0. Introduction

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In the *hierarchy of sciences* Descriptive Geometry is placed *within or next* to Mathematics, but also close to Architecture, Mechanical Engineering, and Engineering Graphics. A few examples shall highlight that Descriptive Geometry *provides a training* of the students' intellectual capability of *space perception* and *spatial reasoning.* Descriptive Geometry is therefore *of incotestable importance for all engineers, physicians and natural scientists.*

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1. What is Descriptive Geometry ?

In American textbooks Descriptive Geometry seems to be restricted to standard constructions like the determination of the true length of a line segment or the intersection of two plane polygons in 3-space.

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Descriptive geometry is the mathematical foundation of engineering graphics.

Part III provides the basics of descriptive geometry, including the important concepts of true-length lines and true size and shape surfaces, and the relationships between lines and planes.

Part III also expands on the multiview drawing concepts . . . Finally,

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. . . thus demonstrating the application of engineering graphics fundamentals to real-world requirements.

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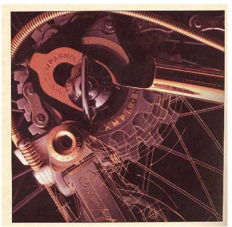

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Descriptive Geometry in Europe

Browsing through German textbooks from the last five decades:

J. Krames (Vienna, 1967):

"Descriptive Geometry" is the high art of spatial reasoning and its graphic representation.

H. Brauner (Vienna, 1986) preferred the name *'Constructional Geometry'* instead of Descriptive Geometry and defined:

"Constructional Geometry" encompasses the analysis of 3D objects by means of graphical or mathematical methods applied to 2D images.

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"Constructional Geometry" teaches how to grasp, to imagine, to design and to draw geometrical shapes.

W.-D. KLIX (Dresden, 2001) gives the following extended explanation:

"Descriptive Geometry" is unique in the way how it promotes spatial reasoning, which is so fundamental for each creative activity of engineers, and how it trains the ability to express spatial ideas graphically so that they become understandable for anybody else.

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Descriptive Geometry is a method to study 3D geometry through 2D images.

It provides insight into structure and metrical properties of spatial objects, processes and principles.

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Descriptive Geometry is a method to study 3D geometry through 2D images.

It provides insight into structure and metrical properties of spatial objects, processes and principles.

Typical for Descriptive Geometry is the interplay

- between the 3D situation and its 2D representation,
- between intuitive grasping and rigorous logical reasoning.

Descriptive Geometry in Europe, cntd.

Beside *projection theory* Descriptive Geometry courses in Europe cover

- *modeling techniques* for curves, surfaces, and solids,
- insight into a broad variety of *geometric shapes*,
- an intuitive approach to basic *differential geometry* (like curvature, osculating plane) of curves and surfaces,
- some *3D analytic geometry.*

An additional aim is to develop and refine the students' *problem-solving skills.*

'Learning by doing' is an important methodological principle.

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G. Monge's definition

La Géométrie descriptive a deux objets:

- **le premier**, de donner les méthodes pour représenter sur une feuille de dessin qui n'a que deux dimensions, savoir, longueur et largeur, tous les corps de la nature qui en ont trois, longueur, largeur et profondeur, pourvu néanmoins que ces corps puissent être définis rigoureusement.

- **Le second objet** est de donner la manière de reconnaître, d'aprés une description exacte, les formes des corps, et d'en déduire toutes les vérités qui résultent et de leur forme et de leurs positions respectives.

G. Monge (1746–1818)
Place de Monge, Beaune
Dep. Côte-d'Or, France

two main objectives of Descriptive Geometry

The *two main objectives* of Descriptive Geometry

- imaging
- analysing

3D objects

date back to its founder G. Monge.

The word *'drawing'* does not appear in Monge's definition.

In Descriptive Geometry 'drawings' are the guide to geometry but not the main aim; we teach geometry instead of construction techniques.

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Further comments on Descriptive Geometry

Only people with a profound knowledge in Descriptive Geometry are able to extended use of CAD programs.

For similar reasons the importance of mathematics is still increasing though computers take over the computational labour.

Descriptive Geometry is more than ‘descriptive’ geometry —

as well as *Geometry* is more than its literal sense, i.e., ‘measuring the earth’.

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Discussing the name 'Descriptive Geometry'

There were several attempts in the past to rename this subject:

- *'Technical Geometry'* or *'Applied Geometry'* stresses its applicability.
- *'Constructive Geometry'* — 'constructive' in its figurative sense — should indicate that manual drawings are combined with mathematical computations.

The original MONGE definition of 'Descriptive Geometry' covers all these aspects.

It might sound too old-fashioned. Hence, for strategic reasons one could replace it by *'Geometric Modeling and Visualization'* or by *'Modeling and Imaging'*.

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Nevertheless, for a field for research ‘Descriptive Geometry’ is too narrow — from my point of view.

My personal field of research is Geometry.

The name ‘Graphic Science’ used in Japan for a field of research is excellent as it covers three important aspects

theoretical — technical — cognitive

And all three are important for a research-guided education of engineers.

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2. What should remain in a student's brain

Education in Descriptive Geometry brings about the ability

- to comprehend spatial objects from given principal views (top view, front view, and side view),
- to specify and grasp particular views (auxiliary views),
- to get an idea of geometric idealization (abstraction), of the variety of geometric shapes, and of geometric reasoning.

The first two items look elementary. However, these intellectual abilities are so fundamental that many people forget how hard they were to achieve.

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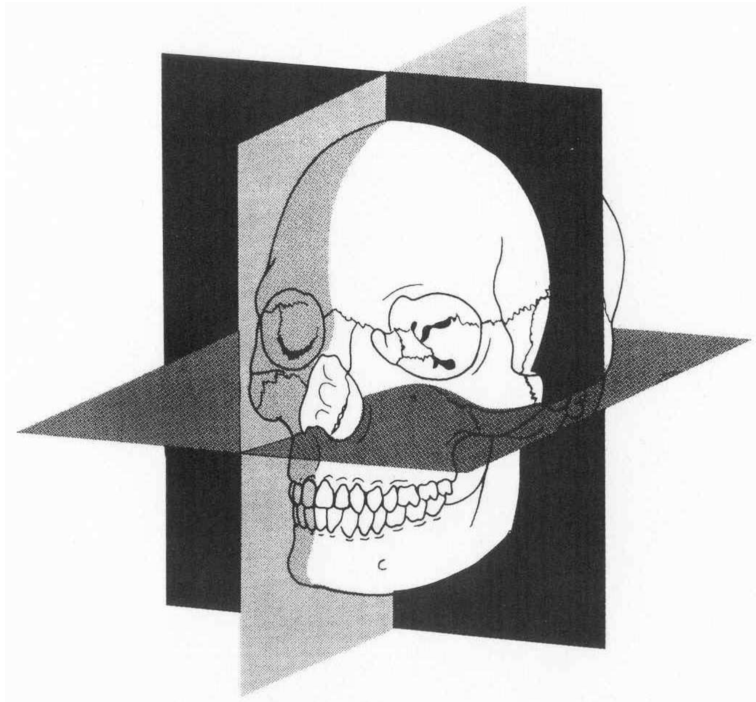
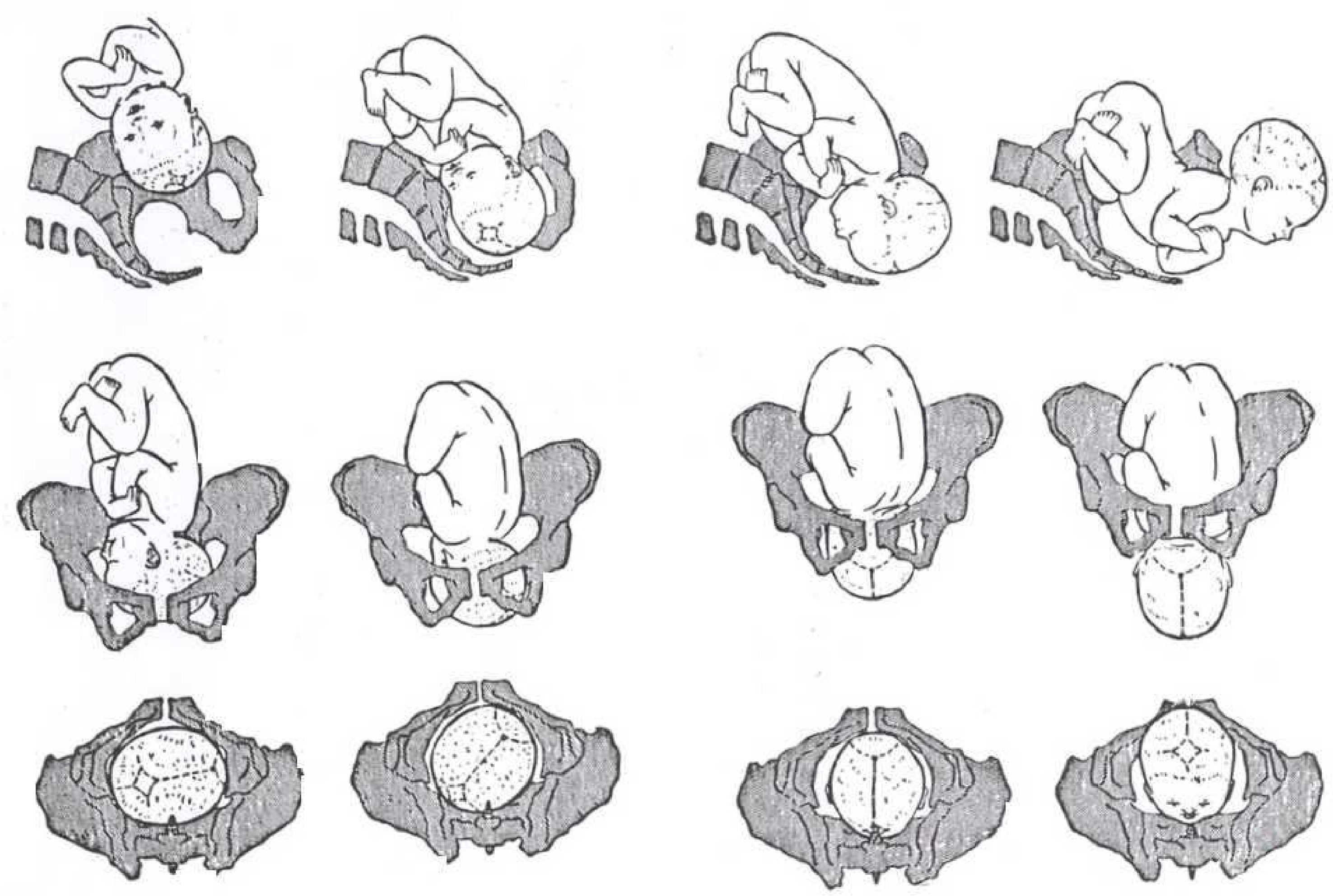
2a) The importance of principal views

- Principal views are *more or less abstract*. But abstraction simplifies.
- Inspecting these plane views is much easier than to concentrate on the true 3D structure or process.
- It needs *training* to become familiar with this kind of representation.
- *Medical doctors* often hold in esteem their Descriptive Geometry education.



Explainig the principal views to dentists

Principal views for gynaecologists



2b) The art of specifying appropriate views

For a detailed 3D analysis particular views (auxiliary views) often reveal the spatial situation.

Views showing planes in edge view or lines in point view can be the key for the solution of a 3D problem.

Appropriate views make the sublime art of Descriptive Geometry.

Only in such courses students are trained to specify and to grasp such views.

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An example

Where does the sun rise earlier on June 21, in Oslo or in Vienna?

city	*Eastern longitude*	*Northern latitude*
Oslo	10.6°	59.9°
Vienna	16.4°	48.2°

We specify a front view with sun rays parallel to the image plane.

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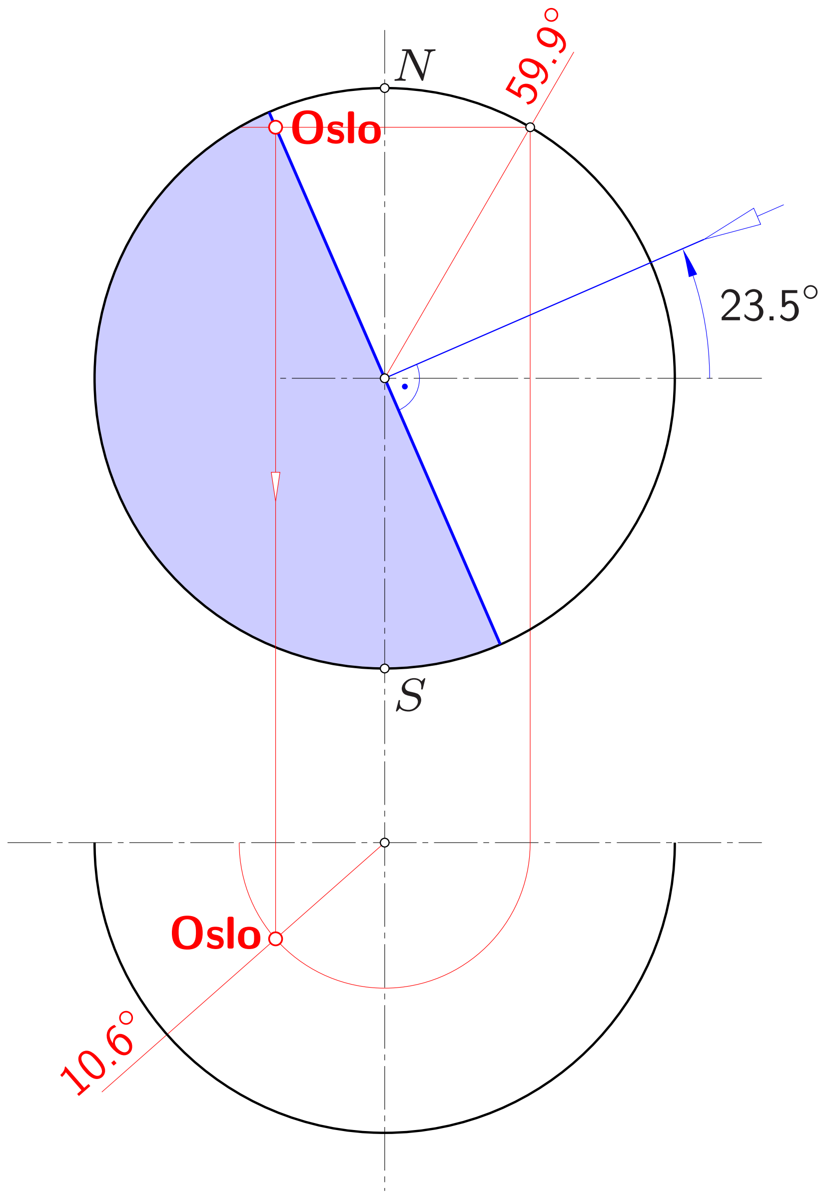

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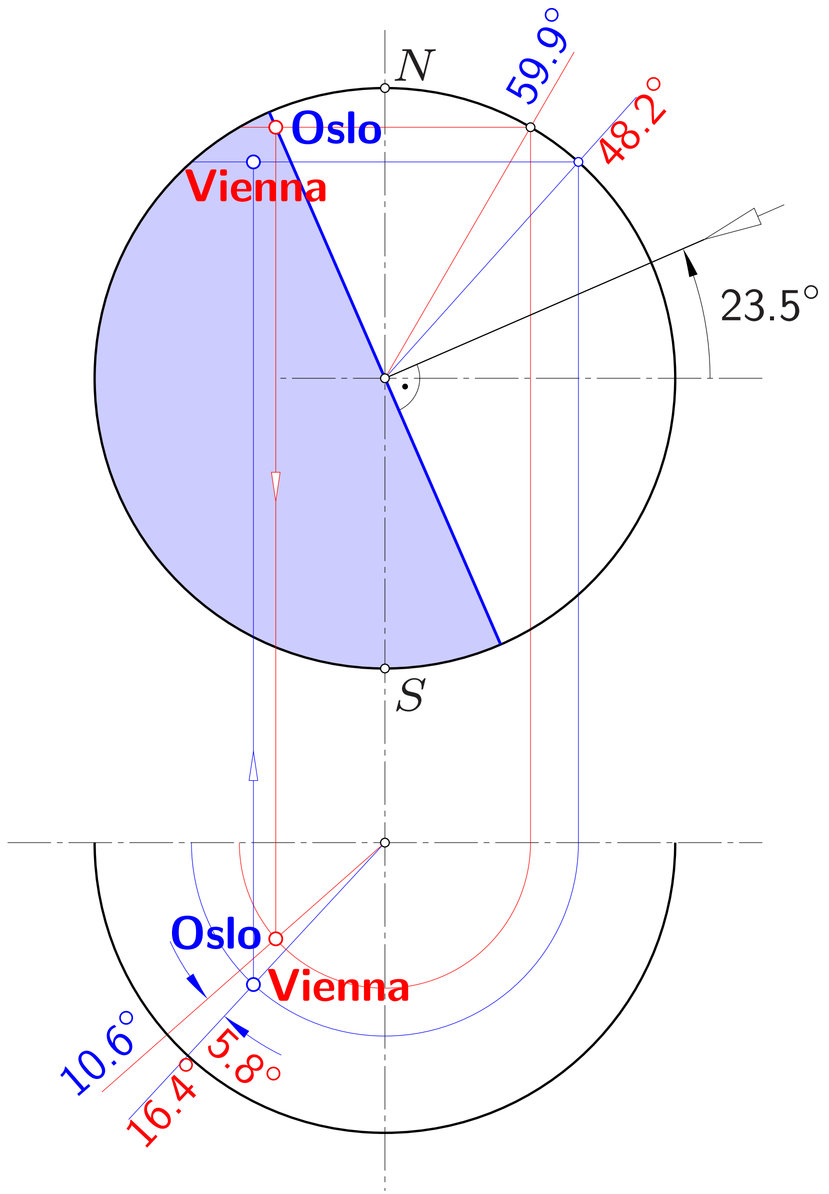
We seek Vienna relatively to Oslo.

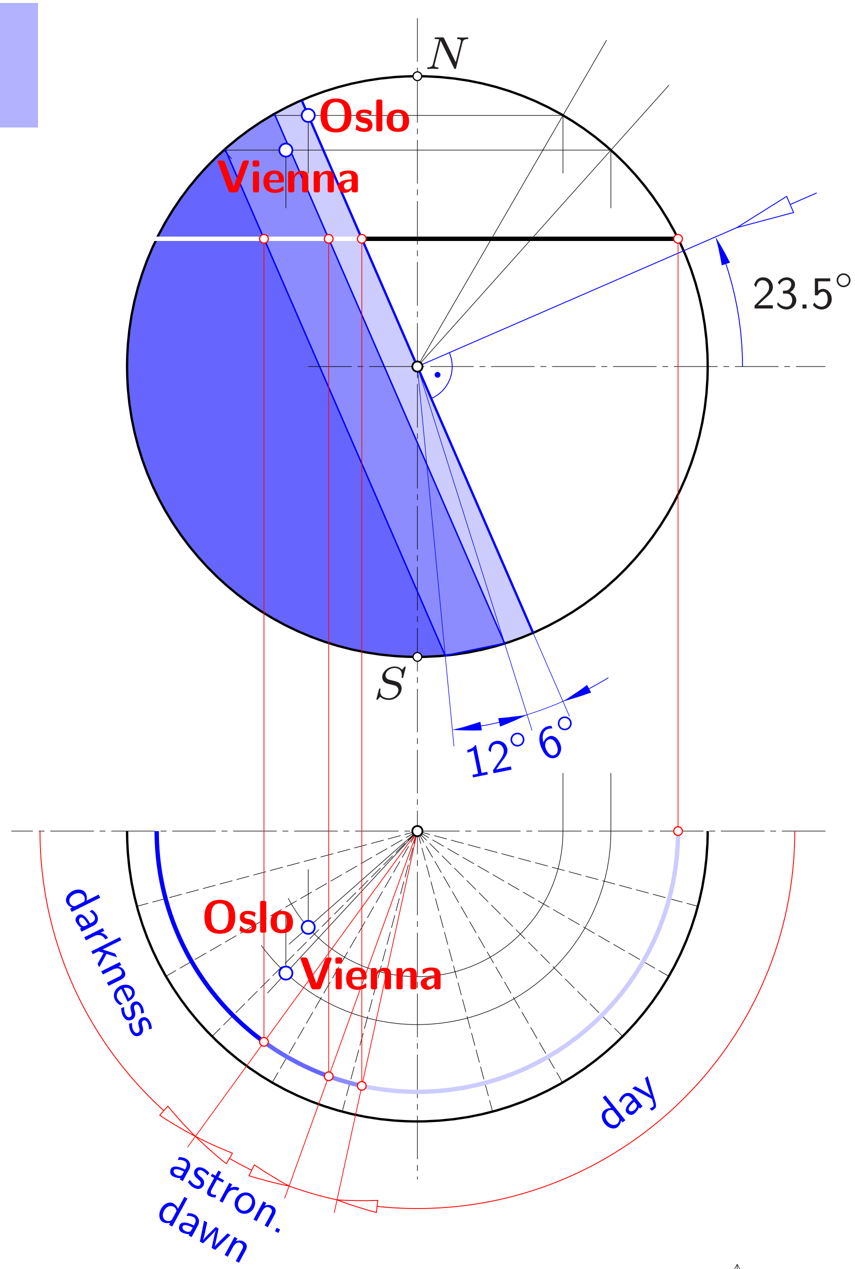


additional problems

a) We can increase the precision by paying attention to the refraction in the atmosphere: For an observer the sun seems to rise while it is still approx. $0,6^\circ$ under the local horizon.

b) In the zone of astronomic dawn the sun is between 6° and 18° under the local horizon.

c) By inspection we observe that the period of dawn is shorter in the neighborhood of the equator.

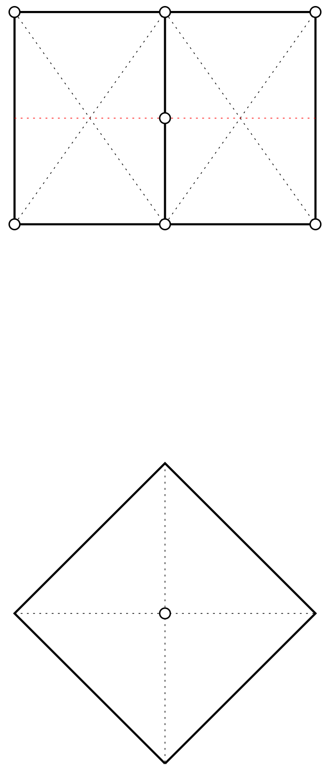


Mentally manipulating 3D objects?

The *rhombic dodecahedron* can be built by erecting quadratic pyramides with 45° inclined planes over each face of a cube.

Any two coplanar triangles can be glued together forming a rhomb.

Question:
How does this polyhedron look like from above when it is resting with one face on a table?



Cube

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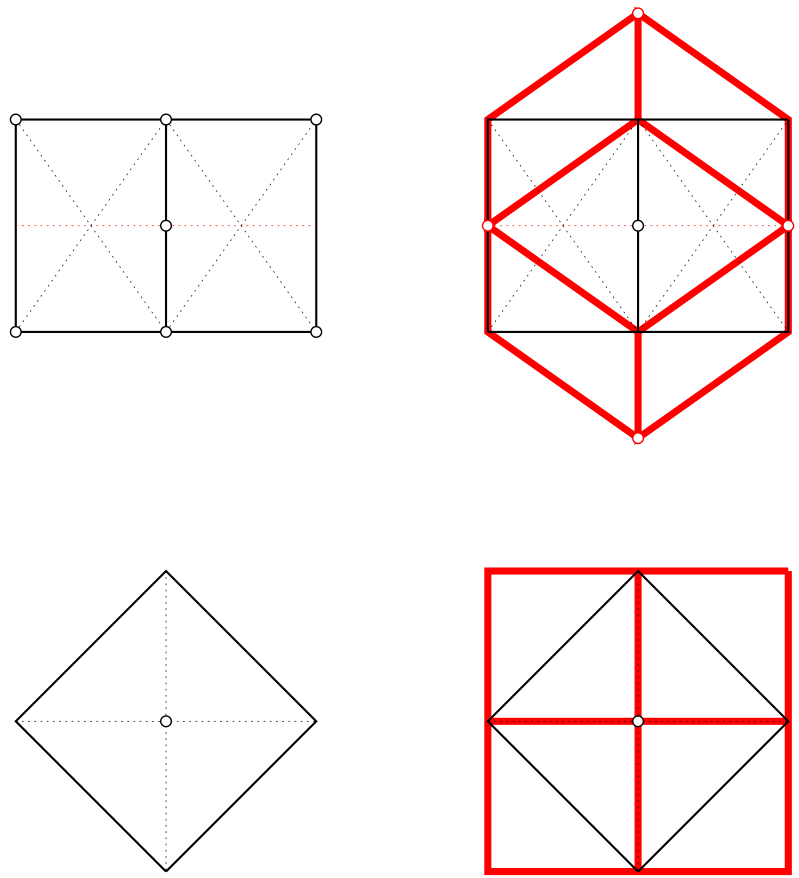
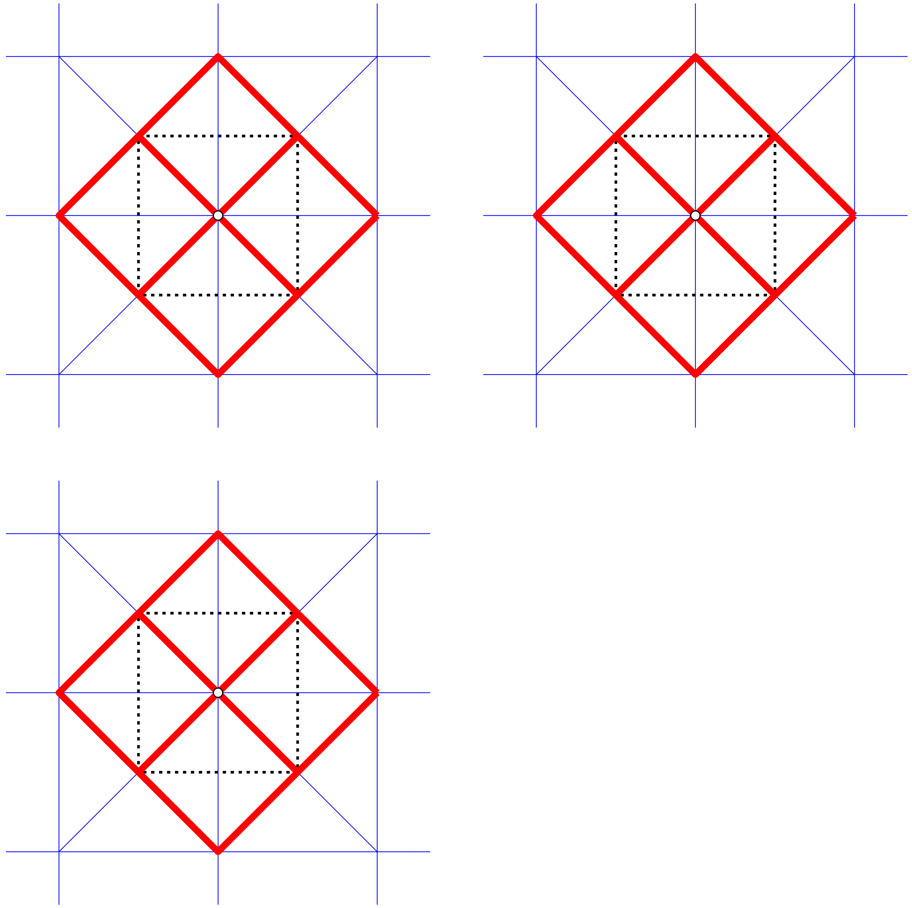


Cube and rhombic dodecahedron

Special views reveal 3D properties

The rhombic dodecahedron is the *intersection of three quadratic prisms* with pairwise orthogonal axes.

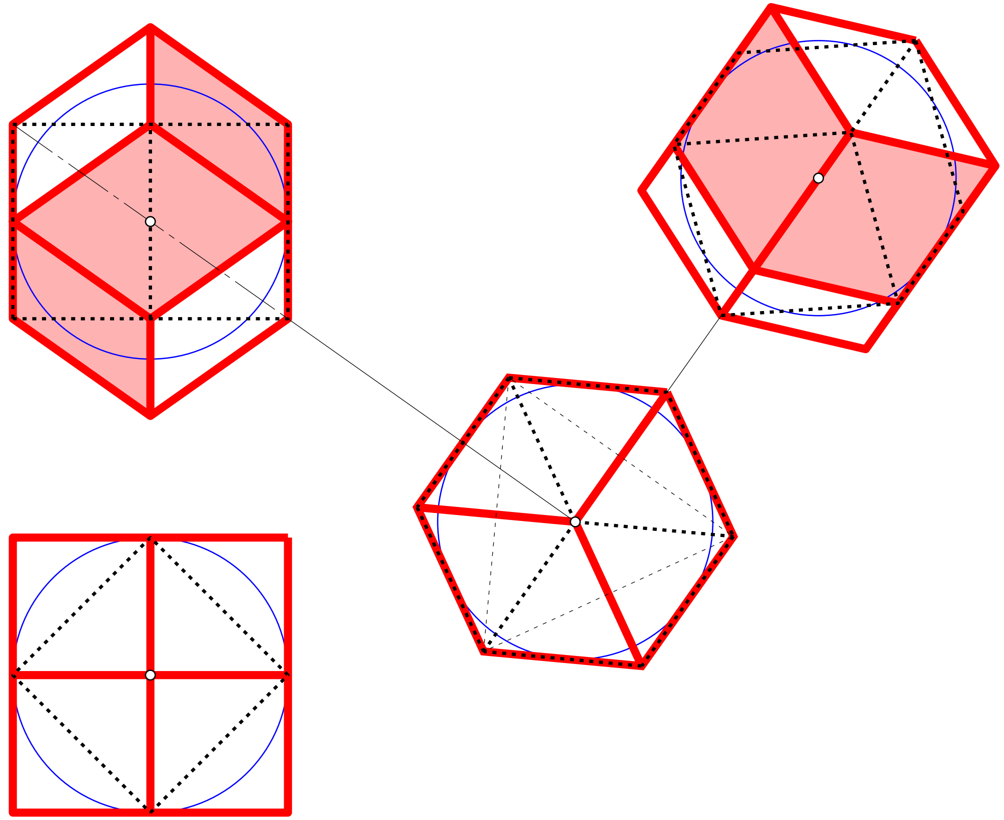


Special views reveal 3D properties

The rhombic dodecahedron is the *intersection of three hexagonal prisms* with axes in direction of the cube-diagonals.

The side and back walls of a *honey comb* belong to a rhombic dodecahedron.

Each *dihedral angle* makes $120°$, and there is an in-sphere (contacting all faces of the initial cube).



A simple 3D operation carried out mentally

The rhombic dodecahedron is a space-filling polyhedron.

Proof:

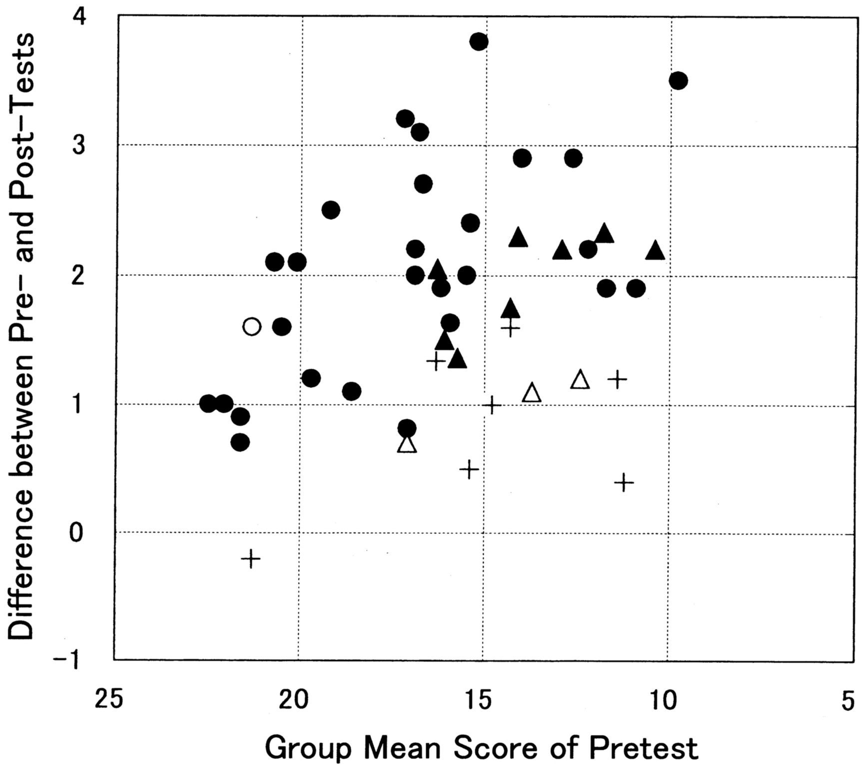
- Start with a *'3D-chessboard'* built from black and white cubes.
- Then the 'white' cubes can be *partitioned* into 6 quadratic pyramides with the vertex at the cube's center.
- Glue each pyramide to the *adjacent* 'black' cube thus enlarging it to a rhombic dodecahedron. □

3. Descriptive Geometry in presence of computers

K. Suzuki:
J. Geometry Graphics **6**, 221–229 (2002)

MCT-Test: Differences between pre- and post-test at Japanese students before and after the geometry education:

- ● Descriptive Geometry
- ○ Computergraphics,
- ▲ Engineering Graphics,
- △ 3D-CAD,
- \+ control persons



What is obsolete:

- complicated manual constructions,
- hard theoretical proofs (e.g., Pohlke's theorem)
- particular construction methods for images of particular 3D objects

What is still necessary:

- **3D-competence**, i.e.,
- capability to comprehend 3D objects and situations from given images,
- mental orientation in 3-space
- basic knowledge of 3D geometry and its applications.
- Promoting creativity and problem-solving skills,
- producing attractive illustrations.

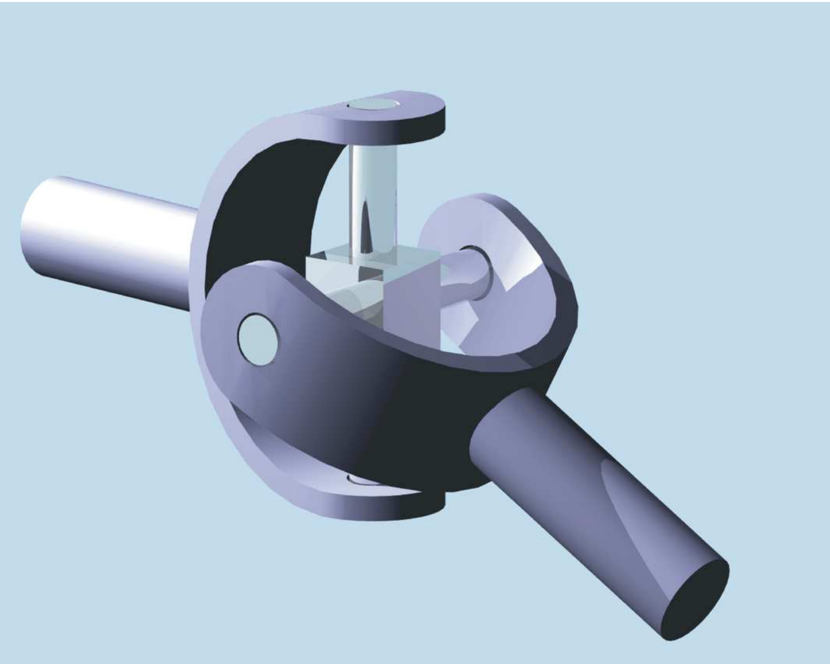
Additional demands:

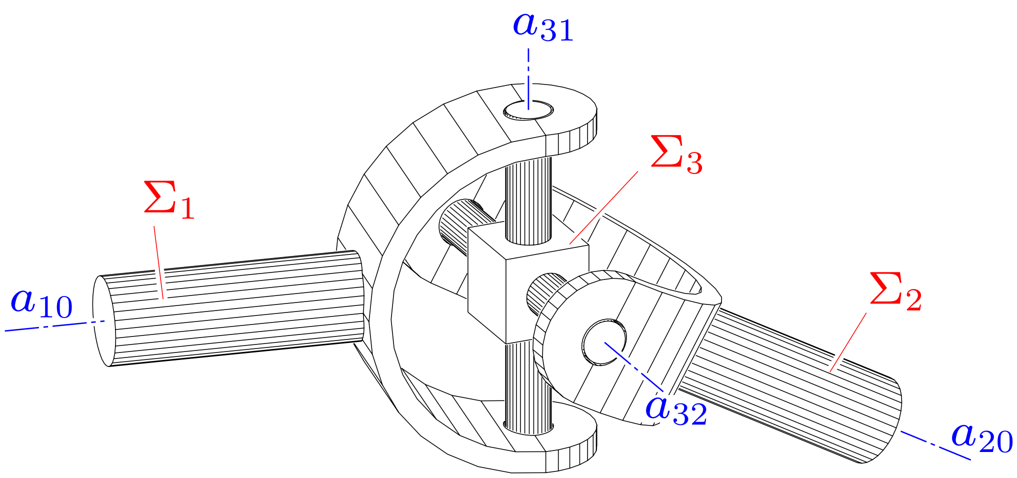
- Handling software for geometric modeling and visualization
- treating new geometric shapes (e.g., B-spline surfaces)
- new visualization techniques (e.g., curved perspectives)
- design of animations.

Needs for the future:

- mental orientation in 3-space (e.g., user coordinate system),
- reducing the flood of graphic information to the essential.

shaded or line graphics





Cardan joint

'Data compression' by line graphics ?

How to meet these demans ?

New tools must have an impact on education.

Students can solve more and more complex problems using computer software as a ‘black box’ while there is still a lack of basic understanding.

This is a problem of methodology, of the right balance between imparting knowledge on the one hand and the intelligent use of powerful computer programs on the other.

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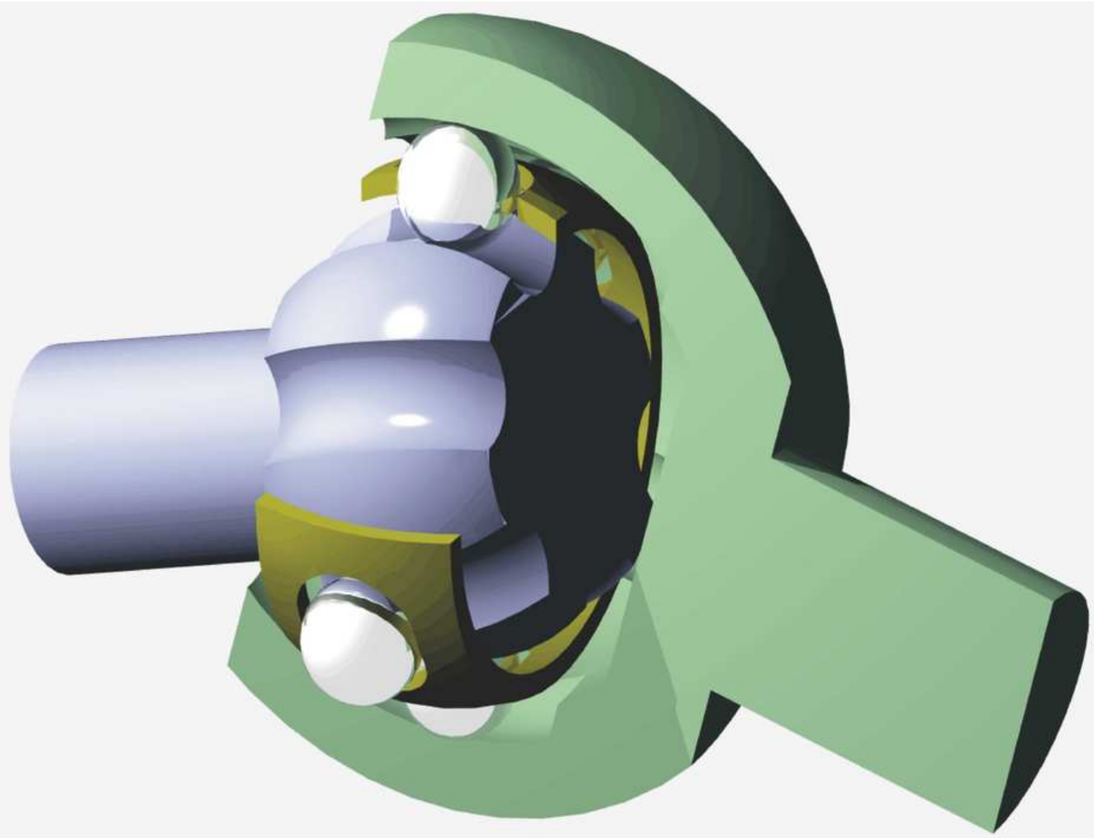
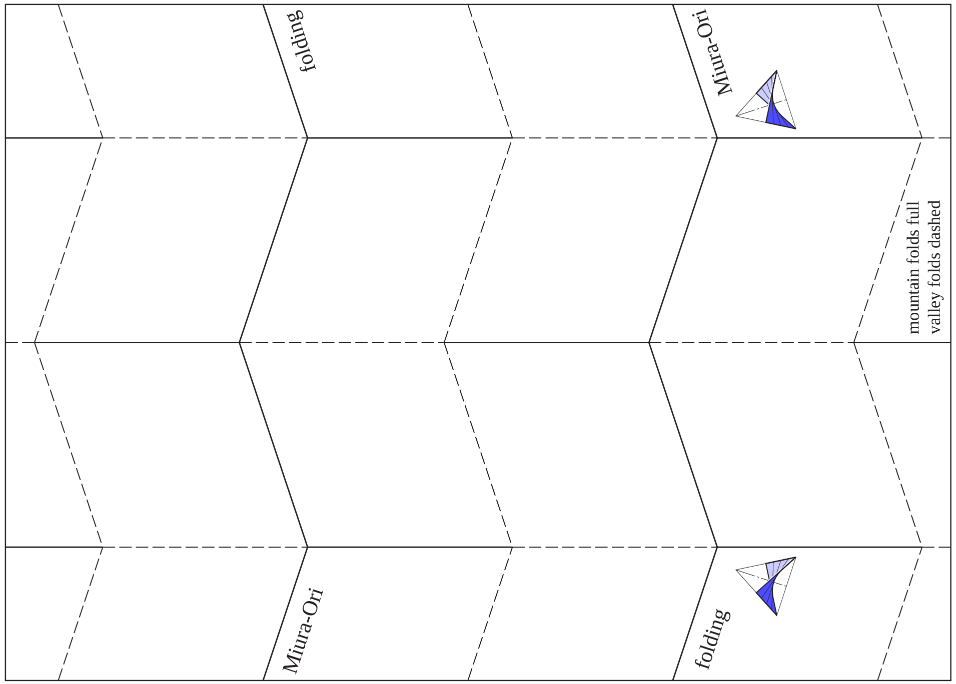


constant velocity ball joint

There will be a way in the near future to combine the benefits of educational CAD programs with the training of spatial reasoning in an optimized way.

An Example: Why is Miura-ori flexible ?



Unfolded miura-ori;
dashs are *valley folds*,
full lines are *mountain folds*

Miura-ori is a Japanese folding technique named after Prof. Koryo Miura, The University of Tokyo.

It is used for solar panels because it can be unfolded into its rectangular shape by pulling on one corner only.

On the other hand it is used as kernel to stiffen sandwich structures.

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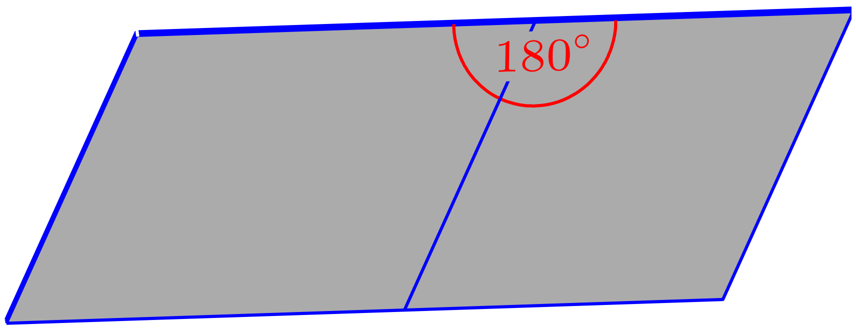

we start with two parallelograms sharing one edge . . .

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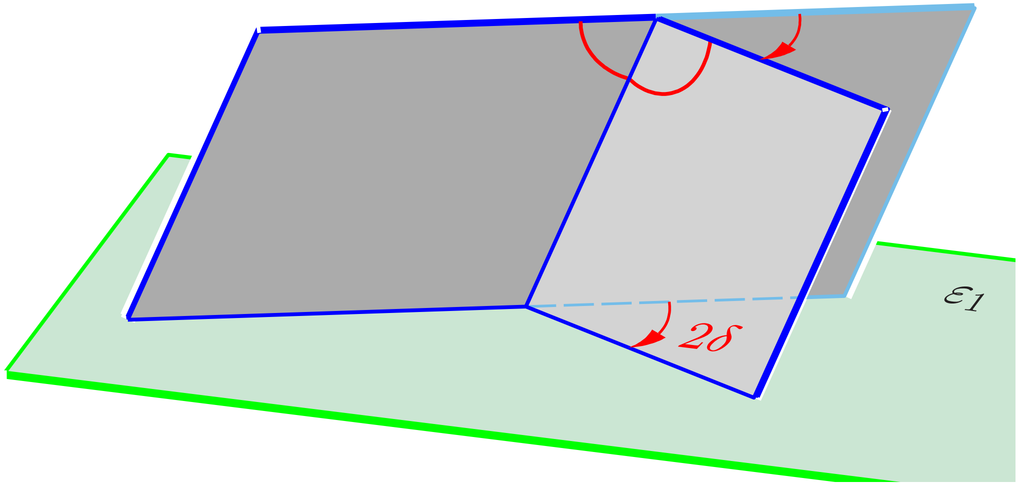

and rotate the right one against the left one through the angle 2δ.

The lower sides span a plane ε_1, the upper sides a plane ε_2 parallel ε_1 .

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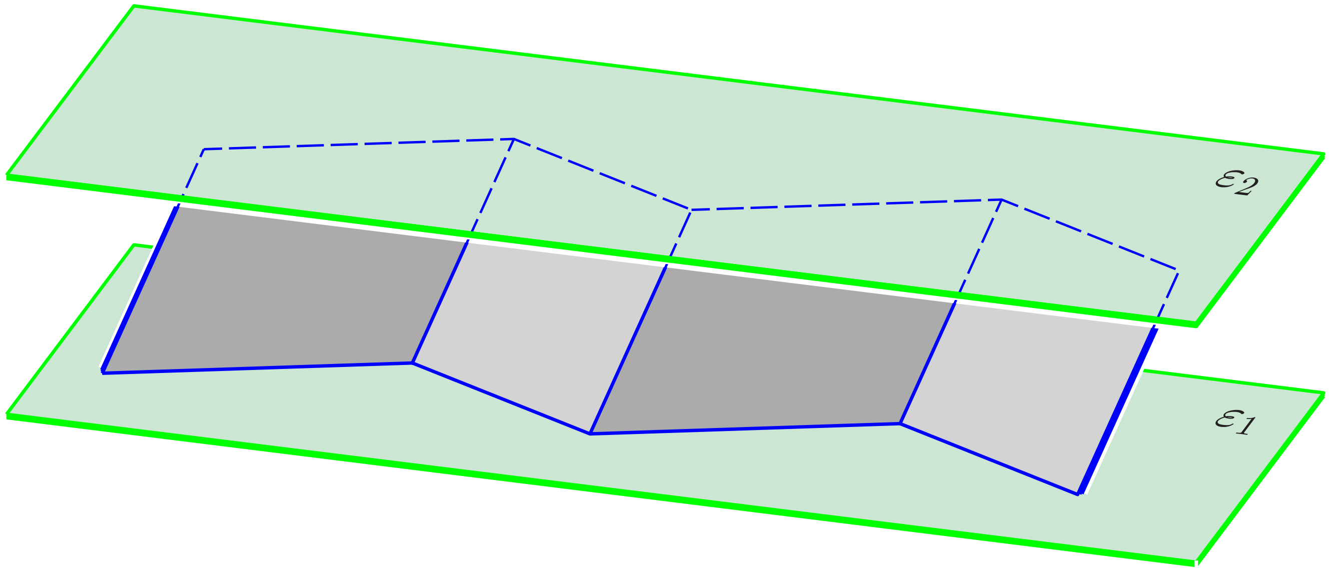

By translations we generate a zig-zag strip of parallelograms between the two parallel planes ε_1 and ε_2 .

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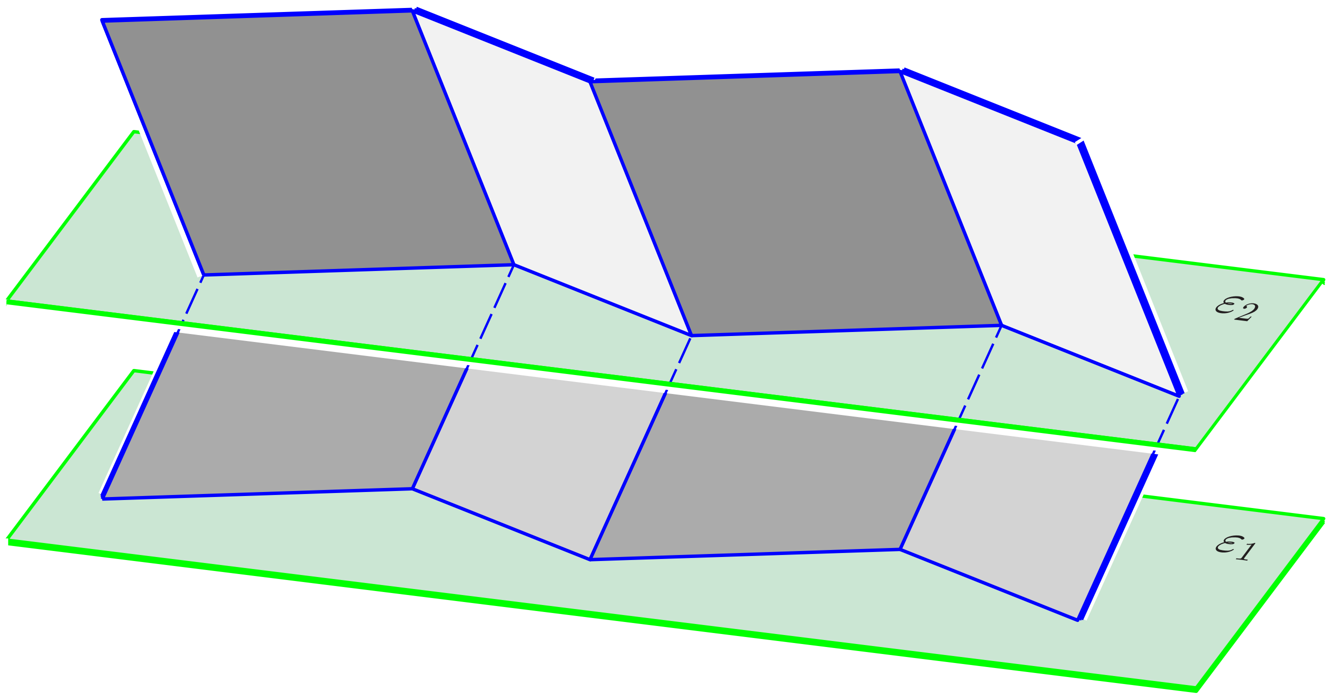

By reflection in ε_2 we generate a second zig-zag strip of parallelograms sharing the border line in ε_2 with the initial strip

— and we iterate . . .

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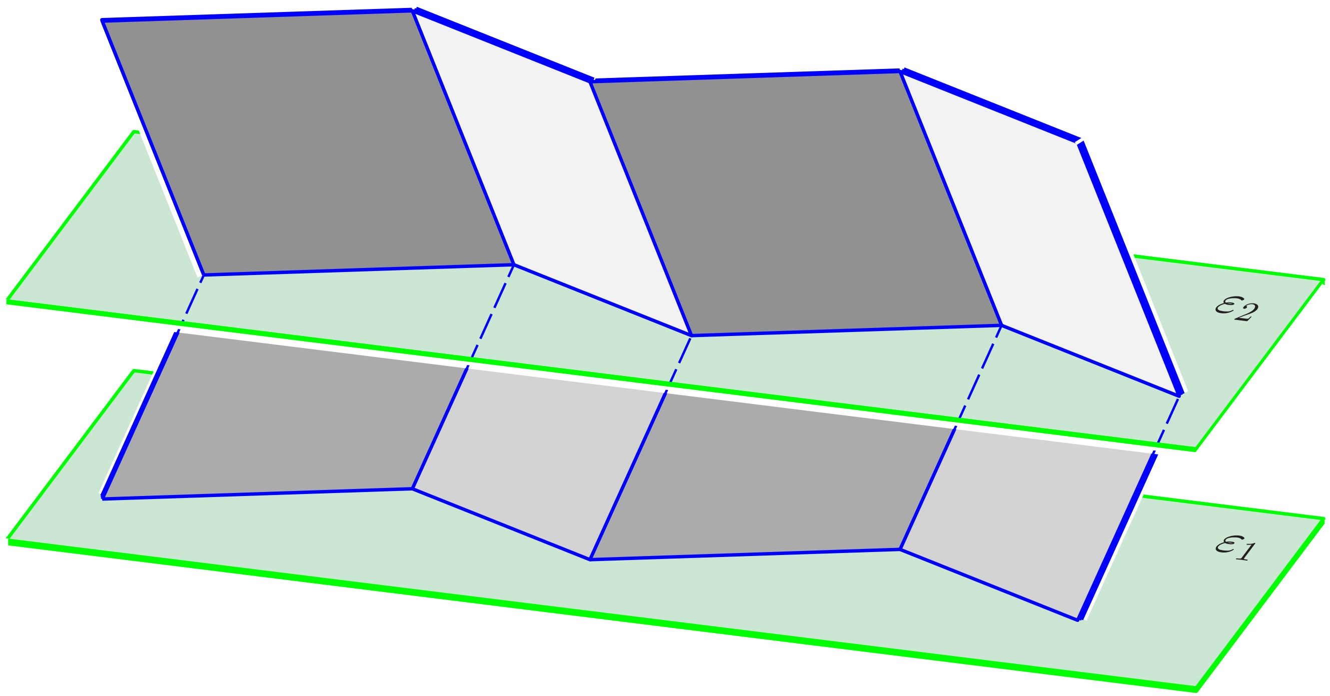

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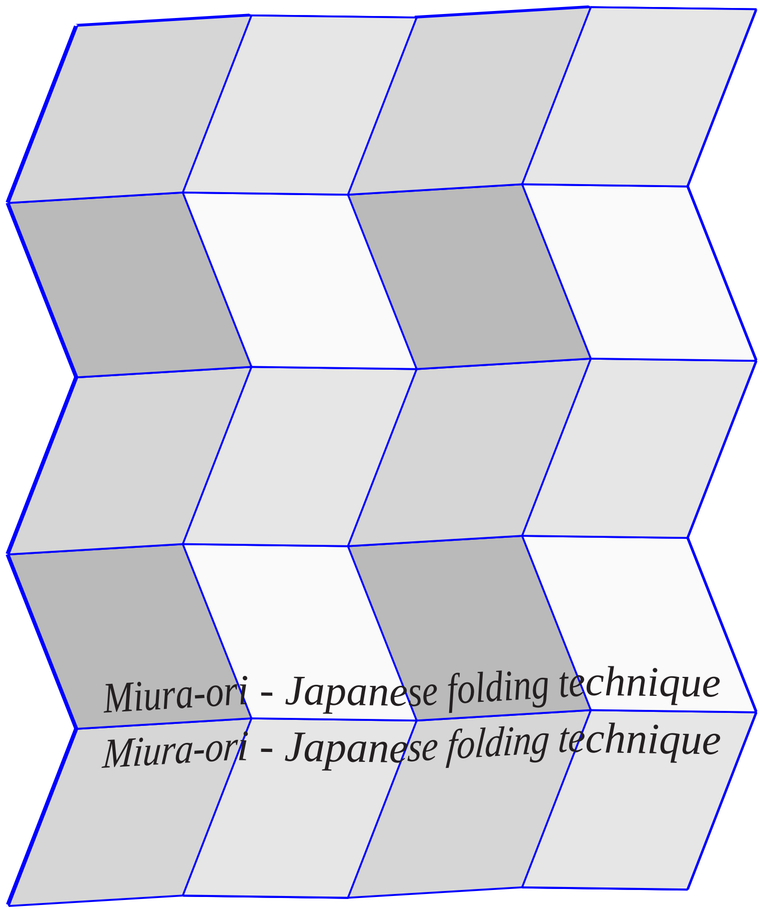

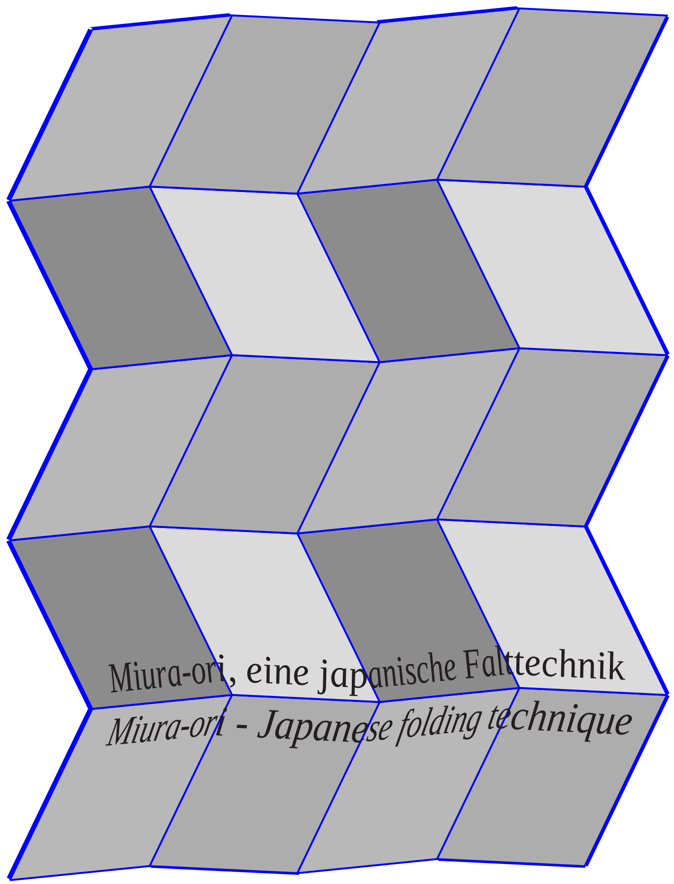

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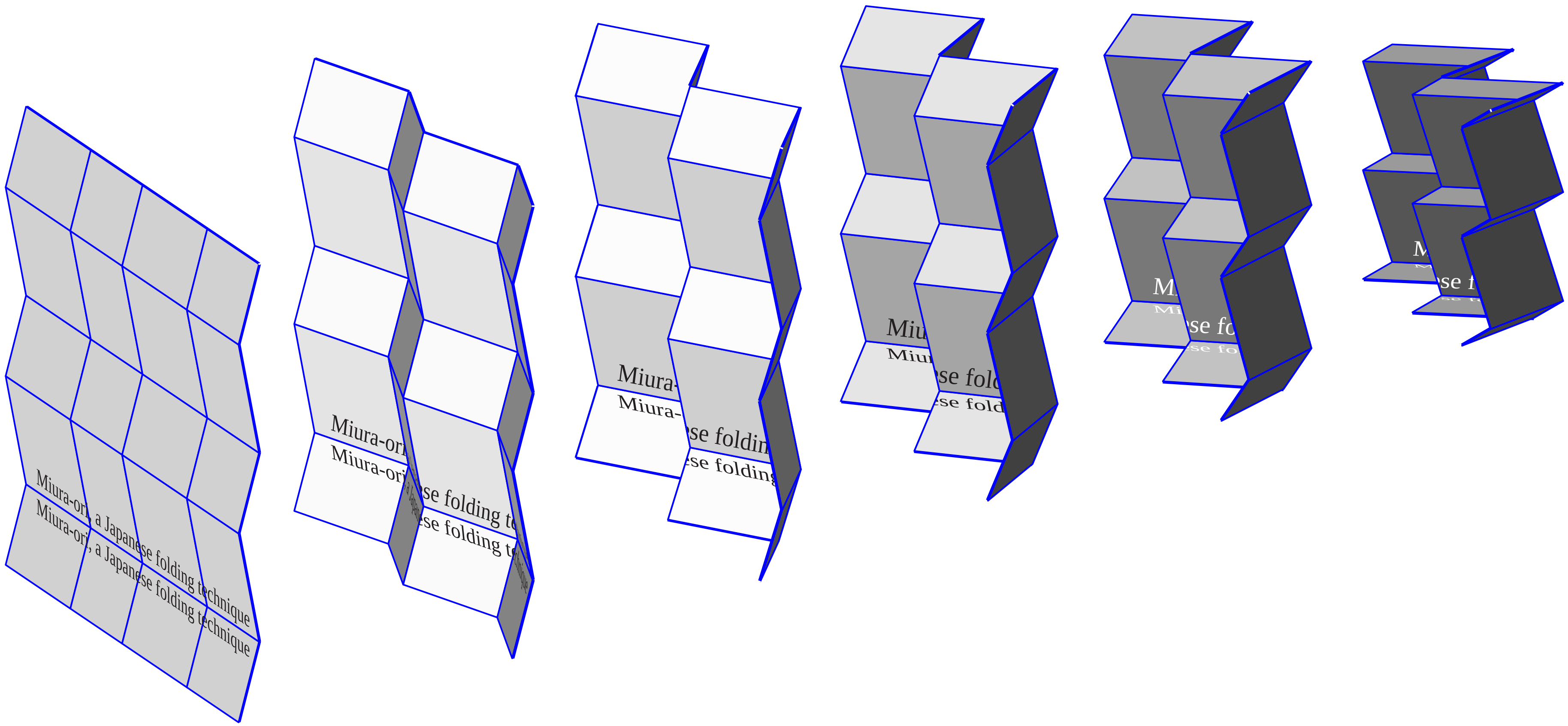

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