Preface

The rigidity of structures is of importance for various areas of science such as chemistry, physics, biology, engineering, and material science. In this issue, the emphasis is on basic science rather than applications. It is devoted to various flexible or rigid structures such as polyhedra, frameworks or smooth surfaces in different spaces. We recall that a polyhedron is said to be flexible if its spatial form can be changed analytically with respect to a parameter due to changes of its dihedral angles only, i.e., in such a way that every face remains congruent to itself during the flex. Conversely, it is said to be rigid if it admits only trivial deformations, e.g., translations and rotations.

The first important result in this field claims that every convex polytope is rigid; this is due to A.L. Cauchy, in 1813. Since then, questions around this topic attracted many prominent mathematicians such as R. Bricard, H. Lebesgue, M. Dehn, W. Blaschke, N.V. Efimov, W. Wunderlich, A.D. Alexandrov, and A.V. Pogorelov. Nevertheless, a few outstanding results were proved rather recently: In 1977, R. Connelly constructed a piecewise linear flexible embedding of the 2-sphere into the Euclidean 3-space $\mathbb{E}^3$. In 1985, R. Alexander proved that every flexible polyhedron in $\mathbb{E}^3$ preserves its total mean curvature during the flex. In 1996, I.Kh. Sabitov confirmed the famous Bellows Conjecture, stating that for every flexible polyhedron in $\mathbb{E}^3$ the volume keeps constant during the flex. On the other hand, the theorem of G. Laman (1970) characterizing the generic rigidity of graphs in the plane was the starting point of the development of powerful computer algorithms for finding rigid clusters in large systems of proteins.

The solution of rigidity problems requires different techniques ranging from classical differential geometry to geometry “in the large”, convex and discrete geometry, algebraic geometry, geometric analysis, kinematics, combinatorics, scientific computations, and algorithmics.

This special issue provides a tutorial snapshot of the state of the study of rigidity in mathematics in 2006, when in Vienna, Austria, the workshop “Rigidity and Flexibility” was held at The Erwin Schrödinger International Institute for Mathematical Physics. The participants – leading researchers studying the underlying theory – were by invitation only.

The focus of this workshop was on basic questions of the geometry of rigidity and flexibility: Which metric or combinatorial properties must change or remain constant under certain classes of deformations such as bending, folding, stretching or flexing? Under which geometrical conditions is any structure flexible? There is still a wide field of open problems left, as indicated in the last contribution of this issue.

All articles included here were selected by the organizers of the cited workshop and peer-reviewed. A number of papers treat combinatorial aspects of rigidity of frameworks including tensegrity frameworks and the question of how to check their rigidity algorithmically. Others deal with infinitesimal bendings of smooth surfaces or polyhedra. Also the question is addressed whether a polyhedron can be reversed by folding only. The attempt to weaken the demand of convexity in the classical Cauchy rigidity theorem led to the theory of hedgehogs and the study of weakly convex polyhedra. A few papers provide insight into these generalizations. One paper is also included that shows that some results on the rigidity of polyhedra may even be attributed to Euclid.
The editors of this special issue would like to thank The Erwin Schrödinger International Institute in Vienna for hosting and financing the 2006 workshop on rigidity, thus offering an inspiring atmosphere for researchers from different fields with the common interest in attacking problems in rigidity.

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Available online 12 September 2009

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