Comments on Helical Developables

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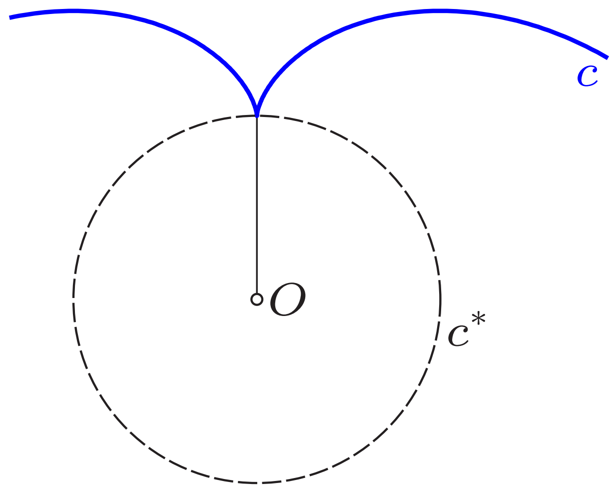
1. Introduction

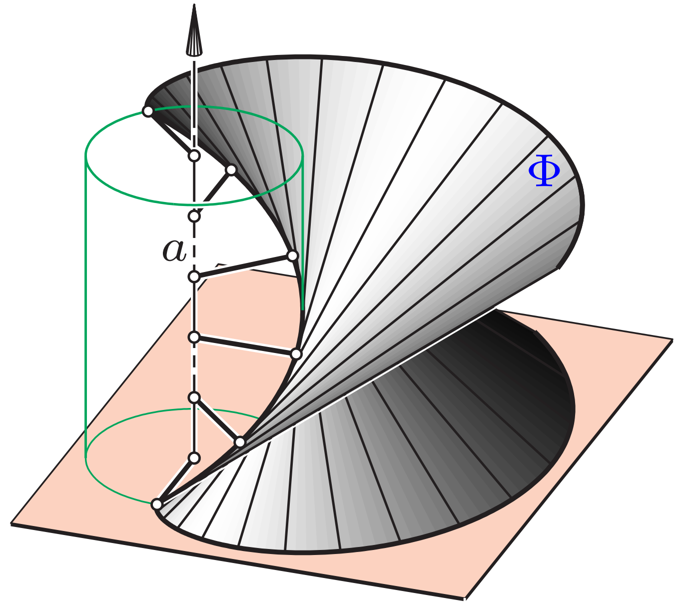
The main aim is to demonstrate

– that helical developables are the *3D-analogue* of circular involutes,

– that Jack PHILLIPS' spatial involute gearing is exactly the *3D-analogue* of Leonhard EULER's planar involute gearing.



circular involute c



helical developable Φ

1. Introduction

Ordinarily, the function of a gear set is to transmit the rotary motion from one wheel Σ_1 about axis a_1 with angular velocity ω_1 uniformly to the second wheel Σ_2, which rotates about a_2 with angular velocity ω_2, i.e., with the

transmission ratio $i := \omega_2/\omega_1 =$ *const.*

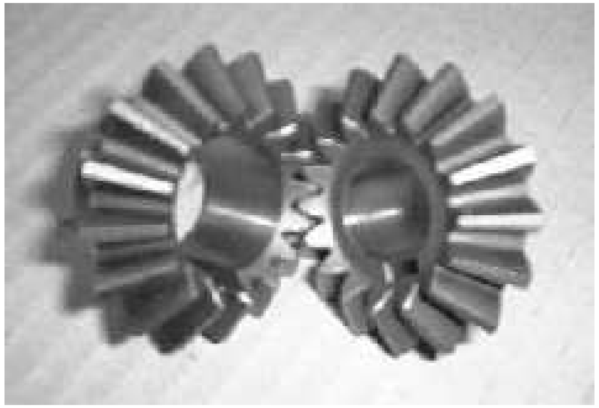
Gearing	*shafts* a_1, a_2	*gear wheels*
planar gearing	parallel	spur gears
spherical gearing	crossing	bevel gears
spatial gearing	skew	skew gears

Types of gears

spur gears

a_1, a_2 parallel

bevel gears



a_1, a_2 crossing

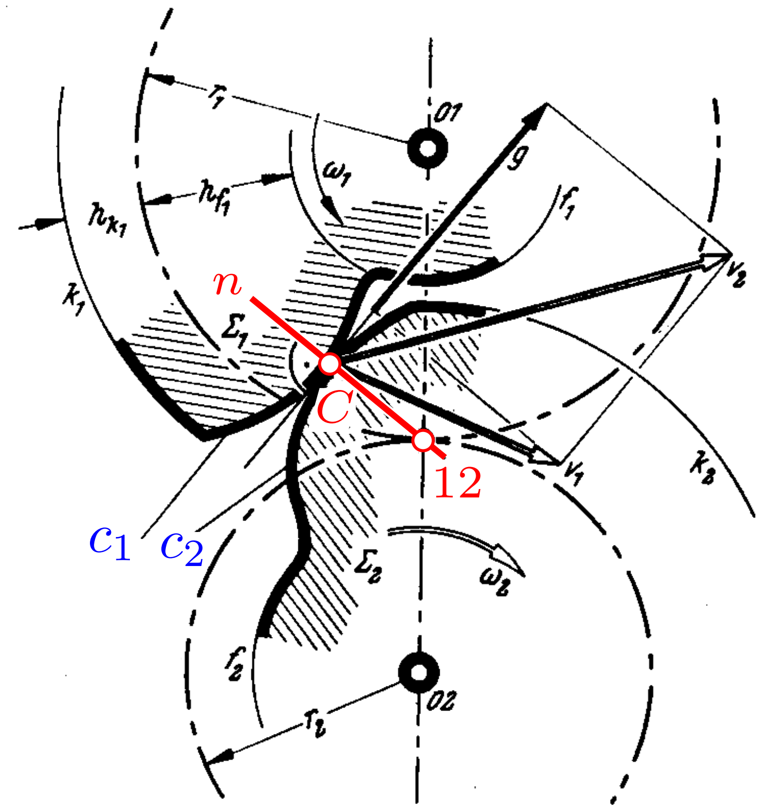
skew gears
(e.g., worm gears)

a_1, a_2 skew

2. Basic kinematics of planar gearing

Fundamental law of planar gearing:

The profiles $c_1 \in \Sigma_1$ and $c_2 \in \Sigma_2$ are conjugate if and only if the common normal n (= meshing normal) at the point C of contact (= meshing point) passes through the fixed pitch point 12.



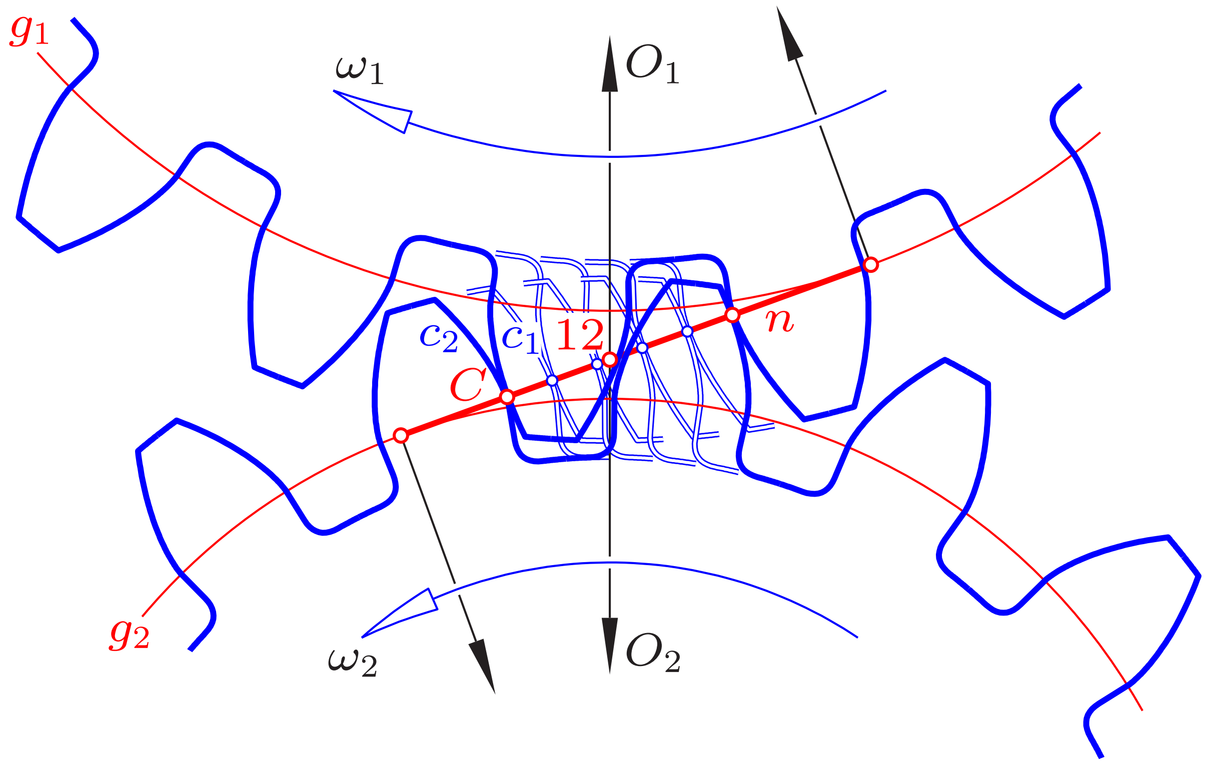
Planar involute gearing

L. Euler (1765):

Characterization of involute gearing:

All meshing normals n are coincident.

$\Longrightarrow$ the tooth profiles c_1, c_2 are involutes of the base circles g_1, g_2, respectively.



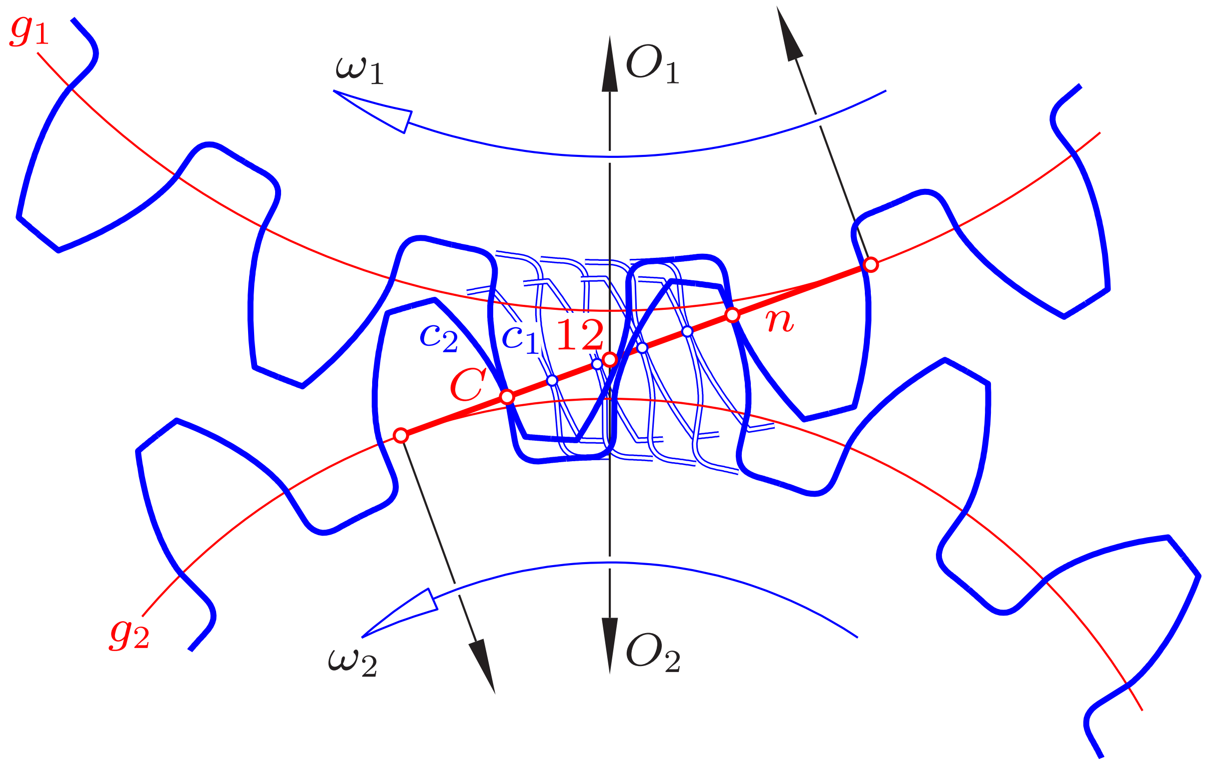
planar involute gearing

Advantages of planar involute gearing

(i) The transmitting force has a fixed line of action.

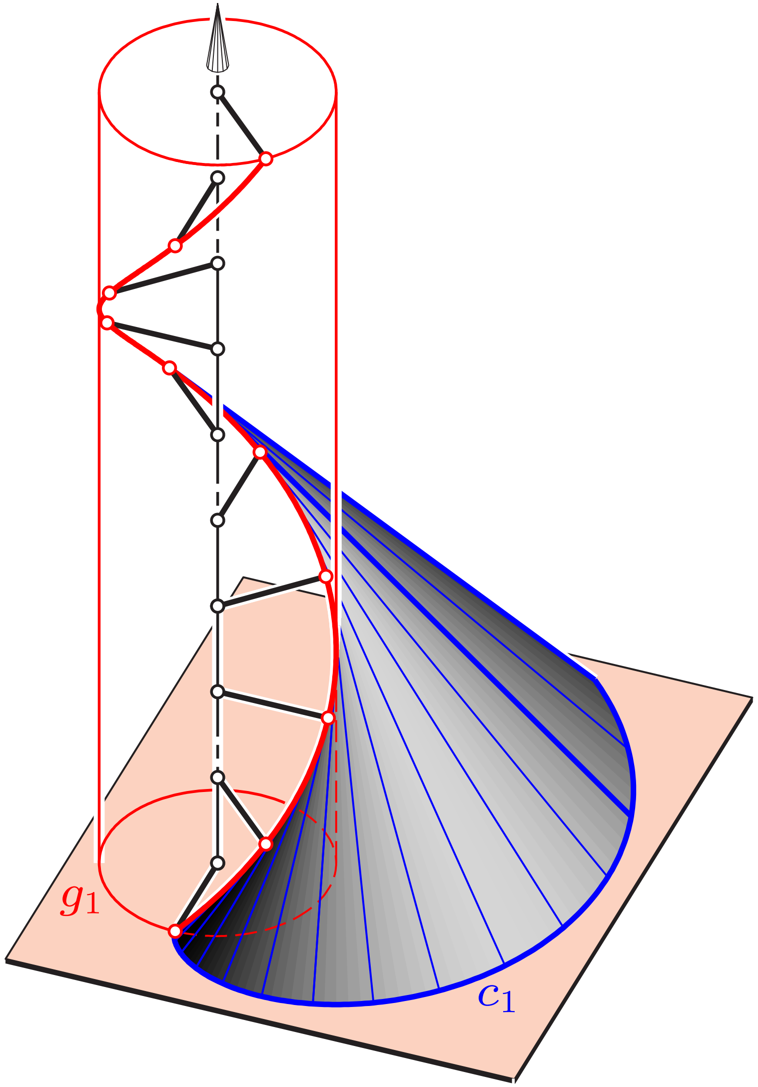
(ii) The transmission ratio ω_2/ω_1 depends only on the dimensions of c_1 and c_2; it is therefore independent of errors upon assembly.

(iii) The tooth flanks of helical gears are helical developables.



planar involute gearing

What is a helical developable ?

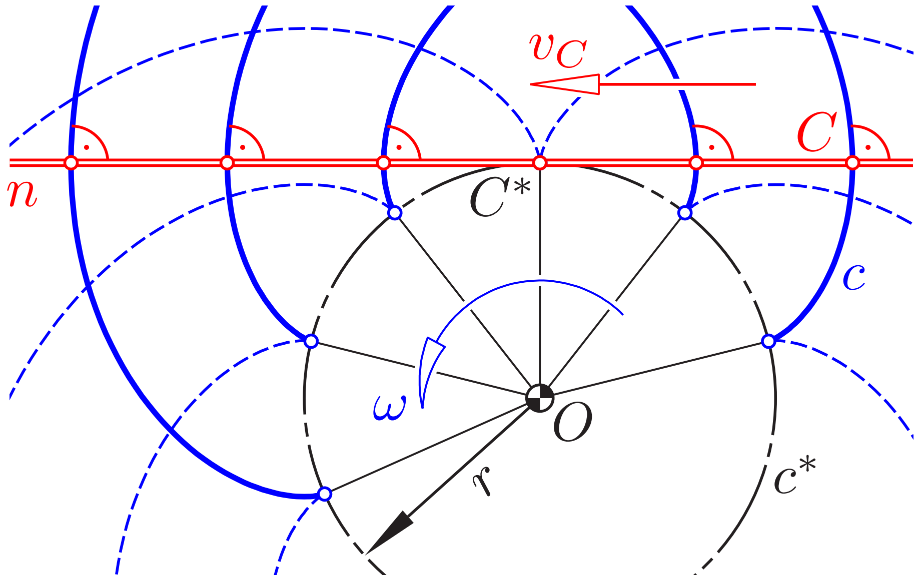


To recall: The *helical developable* is the tangent surface of a helix and helical tooth flank based on involute spur gears

2. Circular involutes

Lemma 1: *Suppose, n is the normal line of the circular involute c at any point $C \in c$.*

When c rotates about its center O with constant angular velocity ω and n is kept fixed, then the point C of intersection with n runs with constant velocity $v_C = r\,\omega$ along n while c remains orthogonal to n.



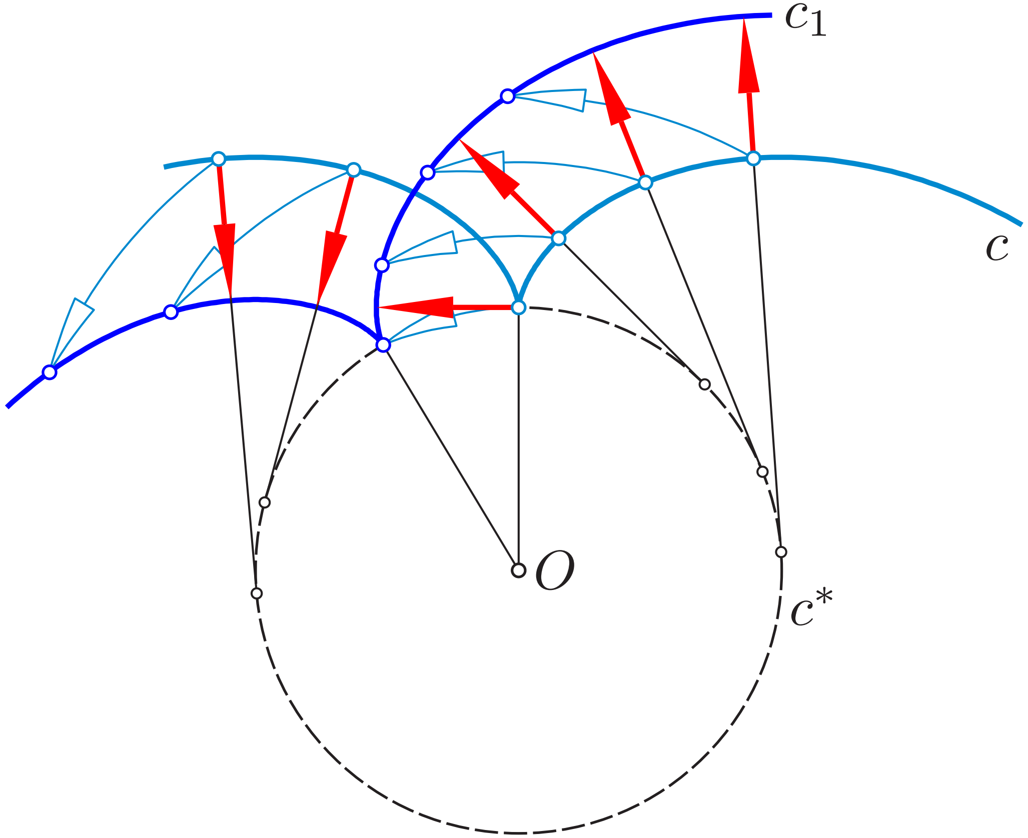
The point C^* of contact between n and c^* is the common curvature center of C at each posture of c.

2. Circular involutes

The deeper reason for Lemma 1:

A circular involute c can be transformed into an offset c_1 by rotation about its center O.

Remark: Circular involutes c are the only curves with this property as their evolute c^* must be moved onto itself by rotation.



Rotation of the circular involute c

Why circular involutes are tooth profiles

Let two planar involutes c_i rotate simultaneously about their centers O_i by ω_i.

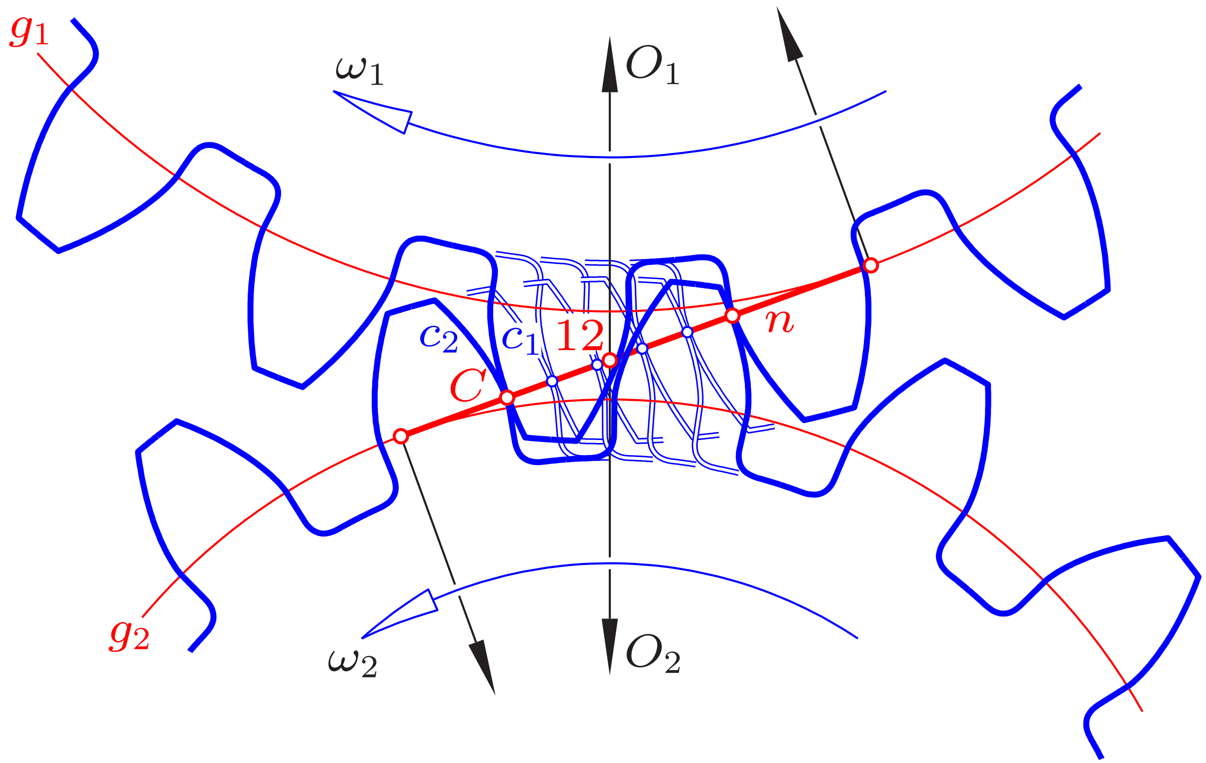
If they are initially in contact at C and

$$v_C = r_1\,\omega_1 = r_2\,\omega_2\,,$$

then they remain in contact. And the ratio

$$\omega_2 : \omega_1 = r_1 : r_2$$

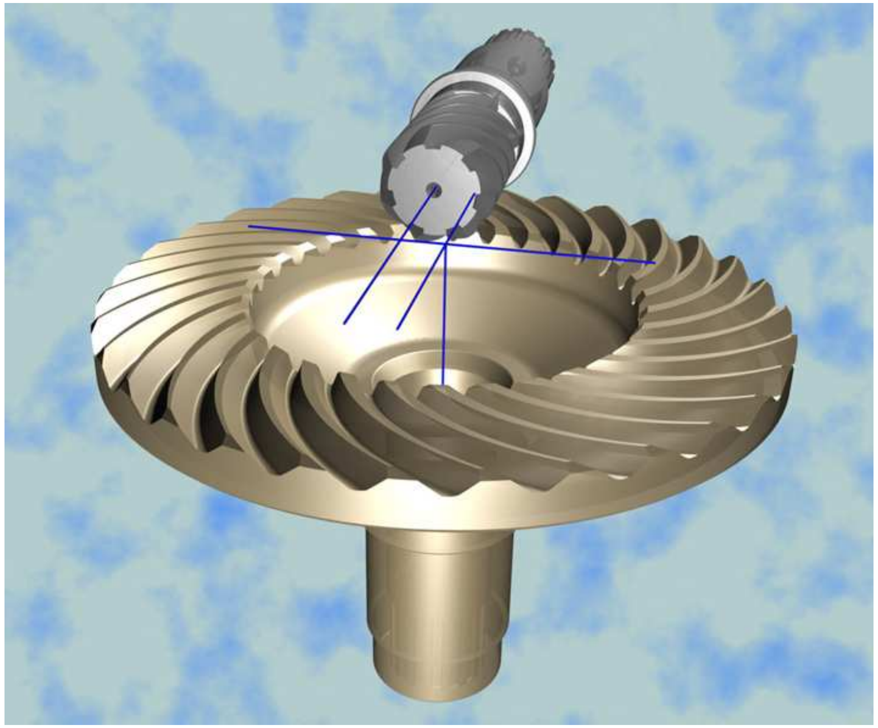
is independent of the relative position of c_1 and c_2.

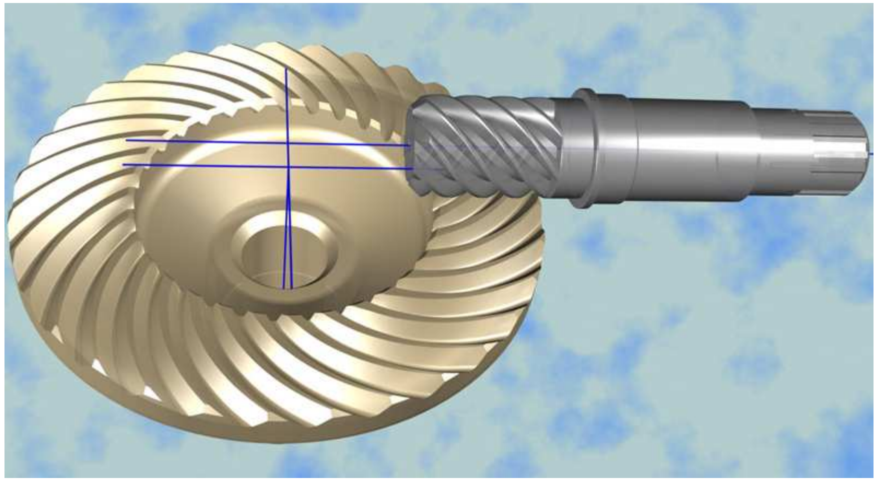


planar involute gearing

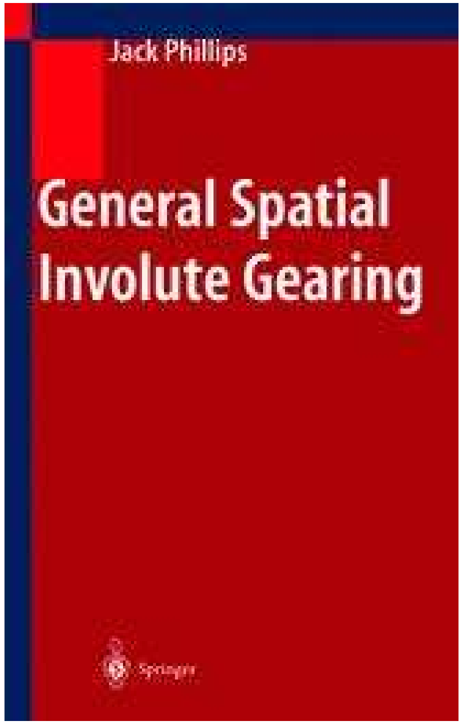
Spatial gearing

Hypoid gears (skew axes):



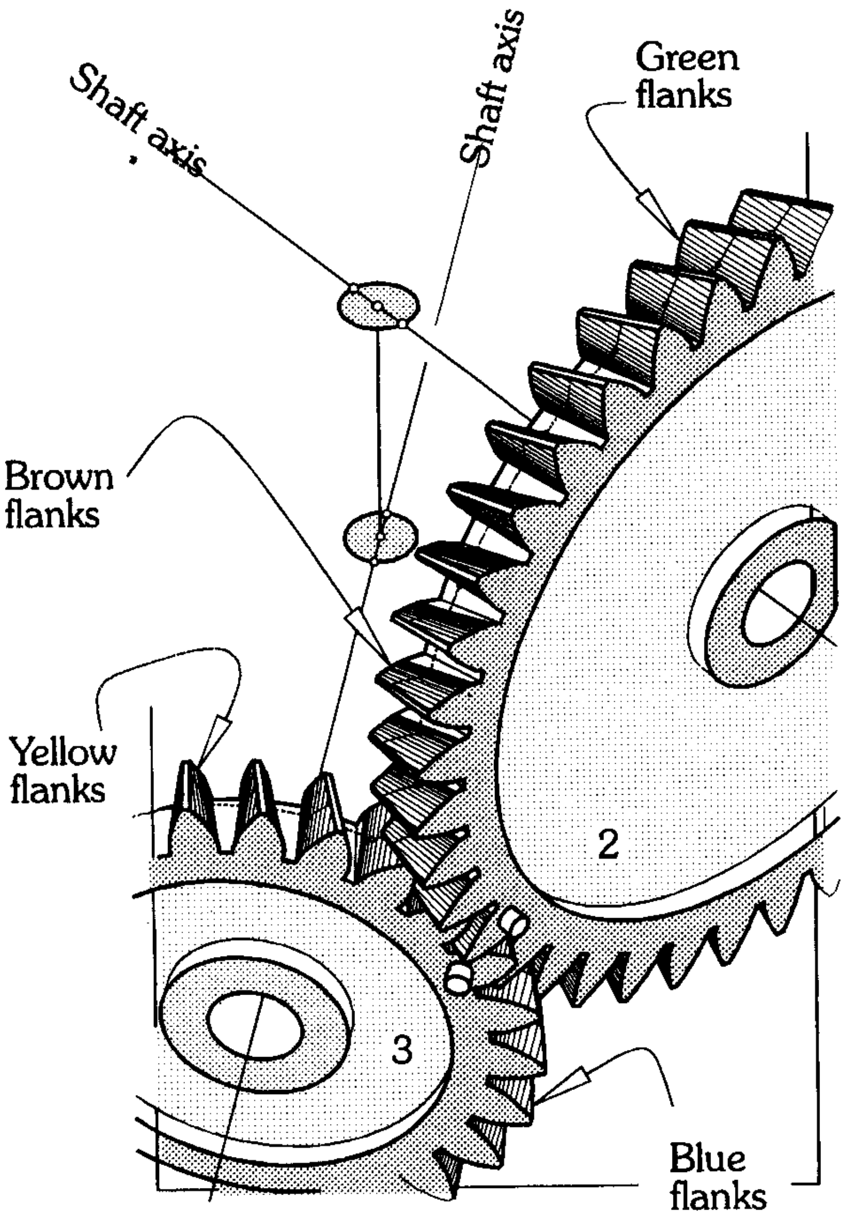


J. Phillips' new spatial involute gearing



J. Phillips:
General Spatial Involute Gearing.
Springer Verlag, New York 2003, 498 p

Jack Phillips (1923–), Sydney



J. Phillips' main results:

Helical developables serve as tooth flanks with single point contact for uniform transmission between skew axes.

The transmission ratio depends only on the dimensions of the conjugate helical developables — but not on their relative position.

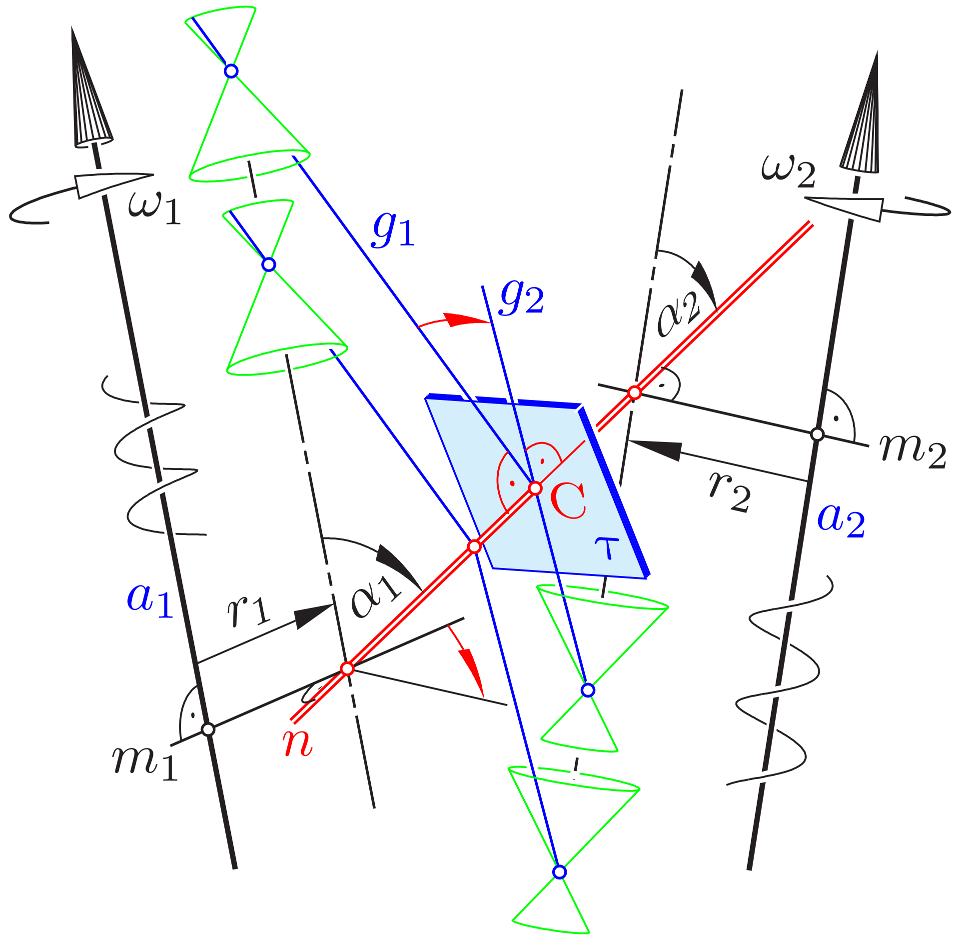
J. Phillips' new spatial involute gearing

Definition:

At spatial involute gearing all meshing normals n are coincident (skew and not perpendicular to the axes a_1 and a_2).

Consequences:

All planes τ of contact between conjugate tooth flanks Φ_1, Φ_2 are orthogonal to n. The envelopes of these planes τ are helical developables.



3. Helical developables

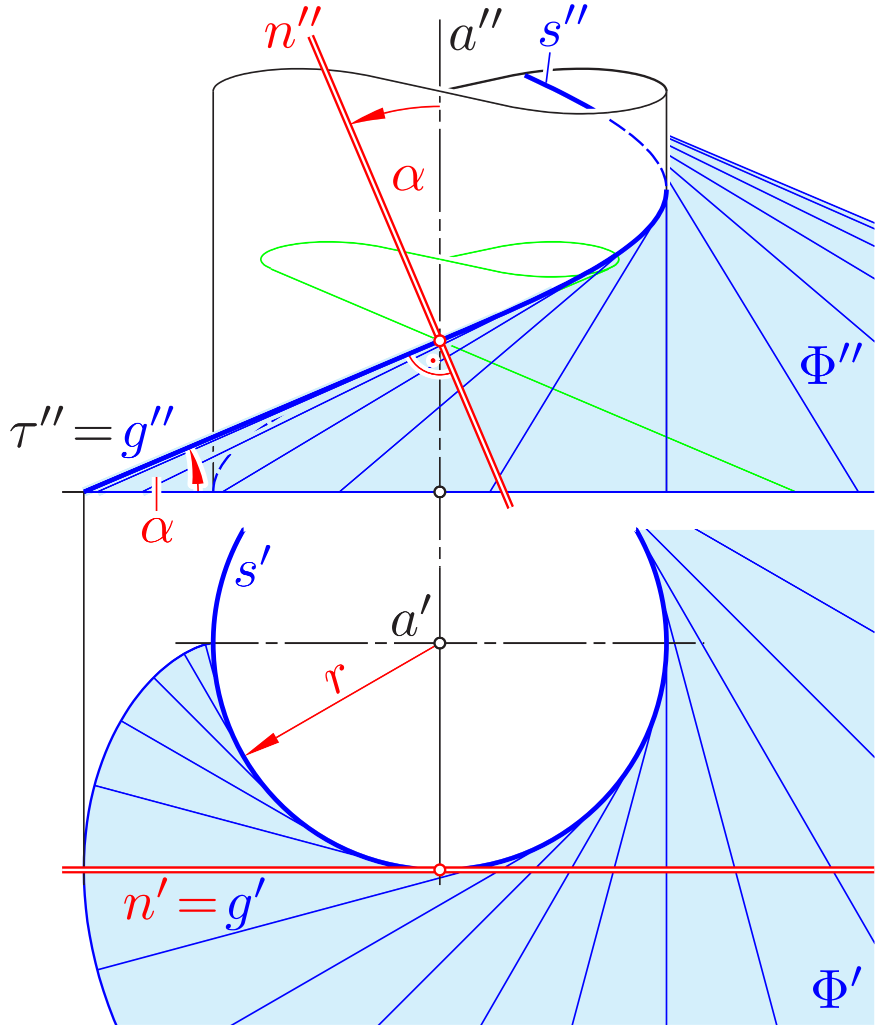
Lemma 2:

All normal lines n *of the helical developable* Φ *with radius* r *and pitch* p *have*

the same distance r *to the axis* a

and make the

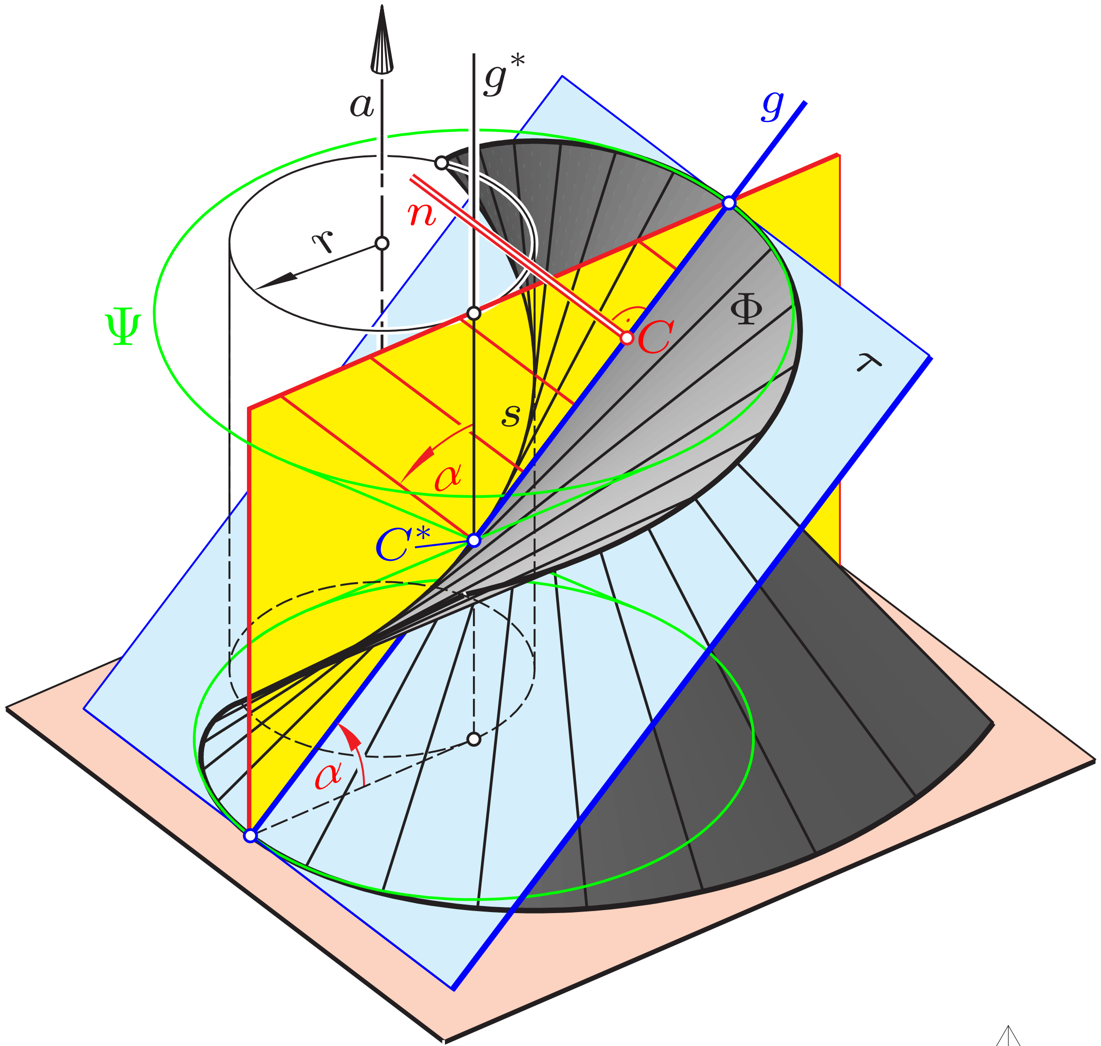
same angle $\alpha = \arctan p/r$ *with* a.



3. Helical developables

The normal line n at $C \in g$ of the helical developable Φ makes the distance r and the angle α with the axis a.

Ψ is the osculating cone of Φ along g with apex at the cuspidal point $C^* \in s$ and with axis g^*.



3. Helical developables

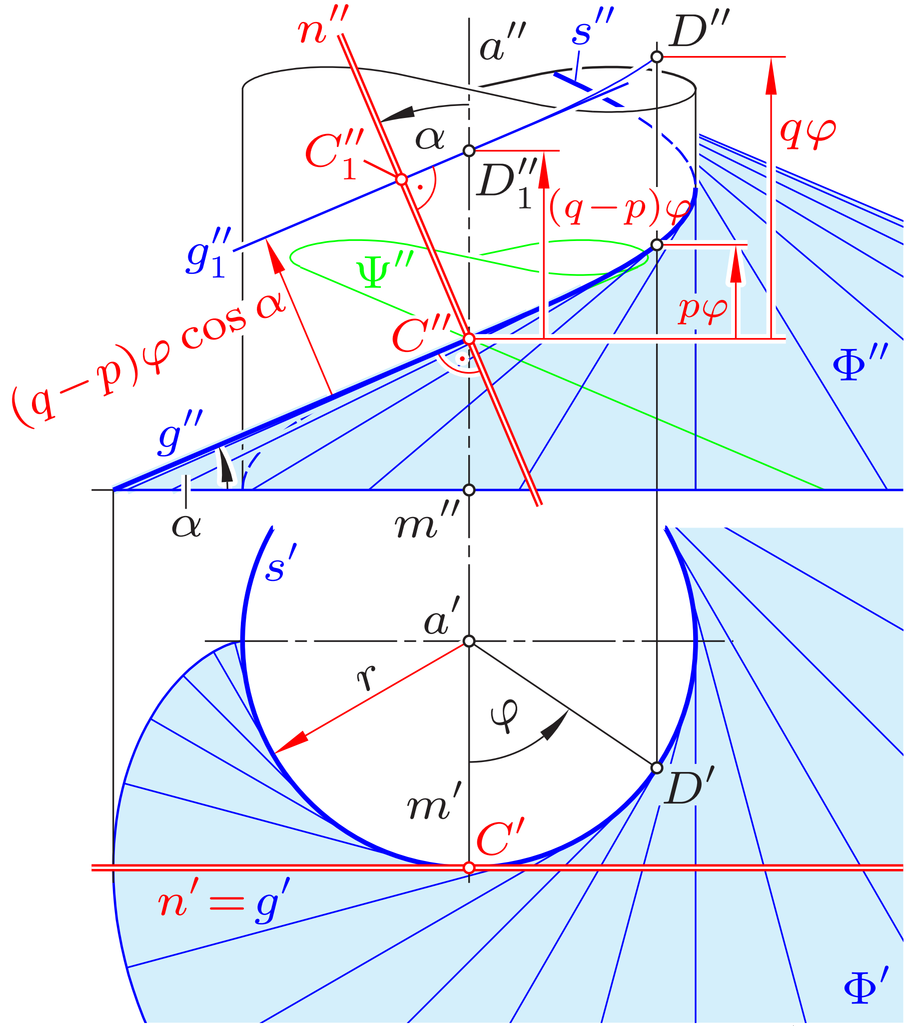
Lemma 3:
When Φ performs a helical motion with pitch q and angular velocity ω along its axis a, while the normal line n is kept fixed, then the point

C of intersection between Φ and n moves along n with constant velocity

$$v_C = (p-q)\omega\cos\alpha$$

independent from the choice of C.

Φ remains orthogonal to n at C, and the generators g passing through C are mutually parallel.

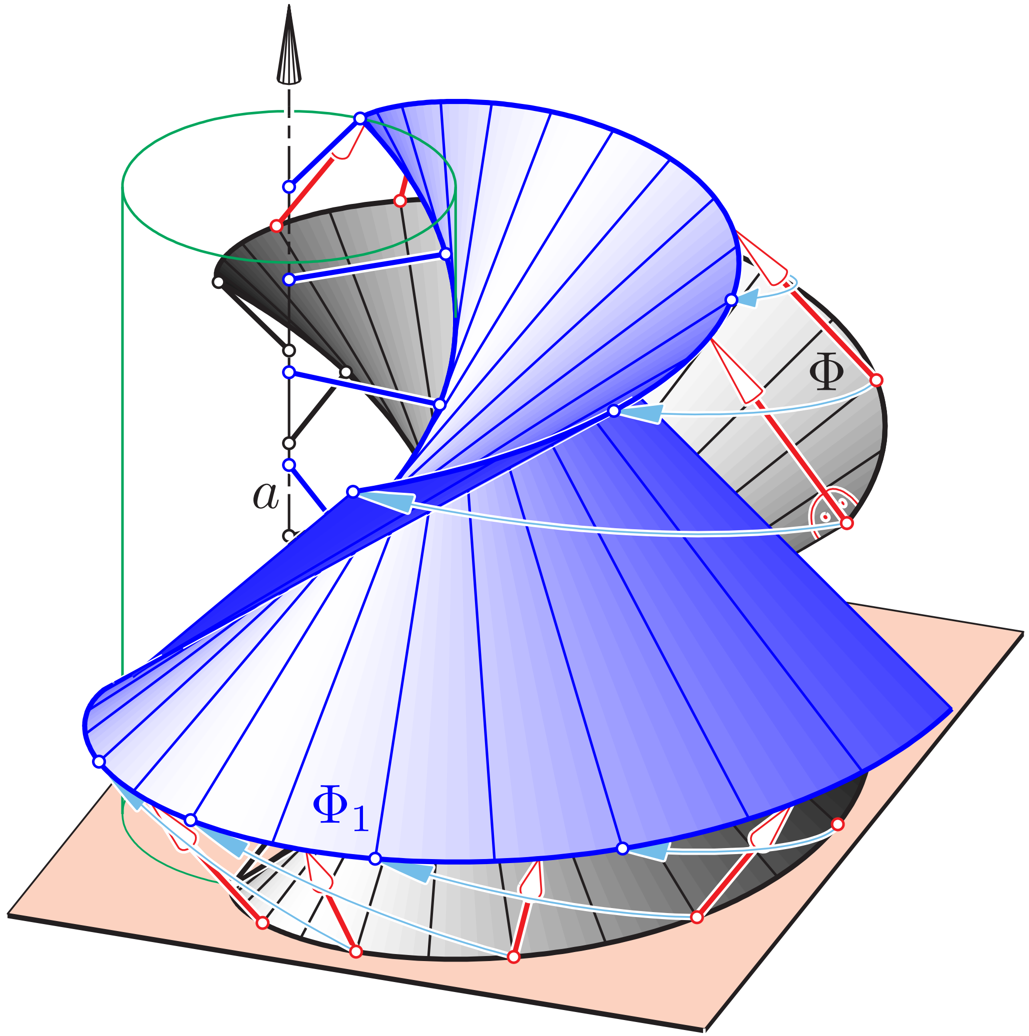


3. Helical developables

The deeper reason for Lemma 3:

A helical developable Φ can be transformed into an offset Φ_1 by rotation about and translation along its axis a.

Remark: Helical developables Φ are the only surfaces with this property as their normal congruence must be moved onto itself by these rotations and translations.



Why helical developables are tooth flanks

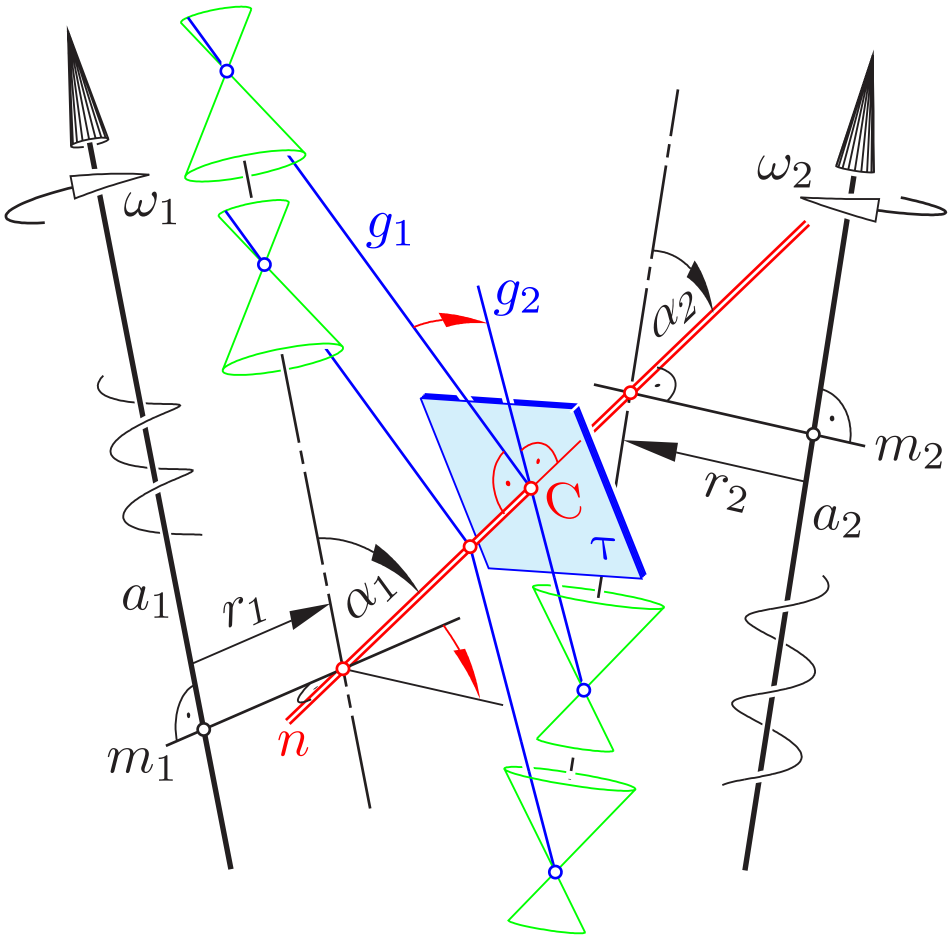
Let two helical developables Φ_i simultaneously perform helical motions (pitch q_i) about their axes a_i by ω_i.

If they are initially in contact at C and

$$v_C = (p_1 - q_1)\omega_1 \cos\alpha_1 = (p_2 - q_2)\omega_2 \cos\alpha_2$$

they remain in contact.

And $\omega_2 : \omega_1$ is independent from errors of assembly (J. Phillips, 2003).

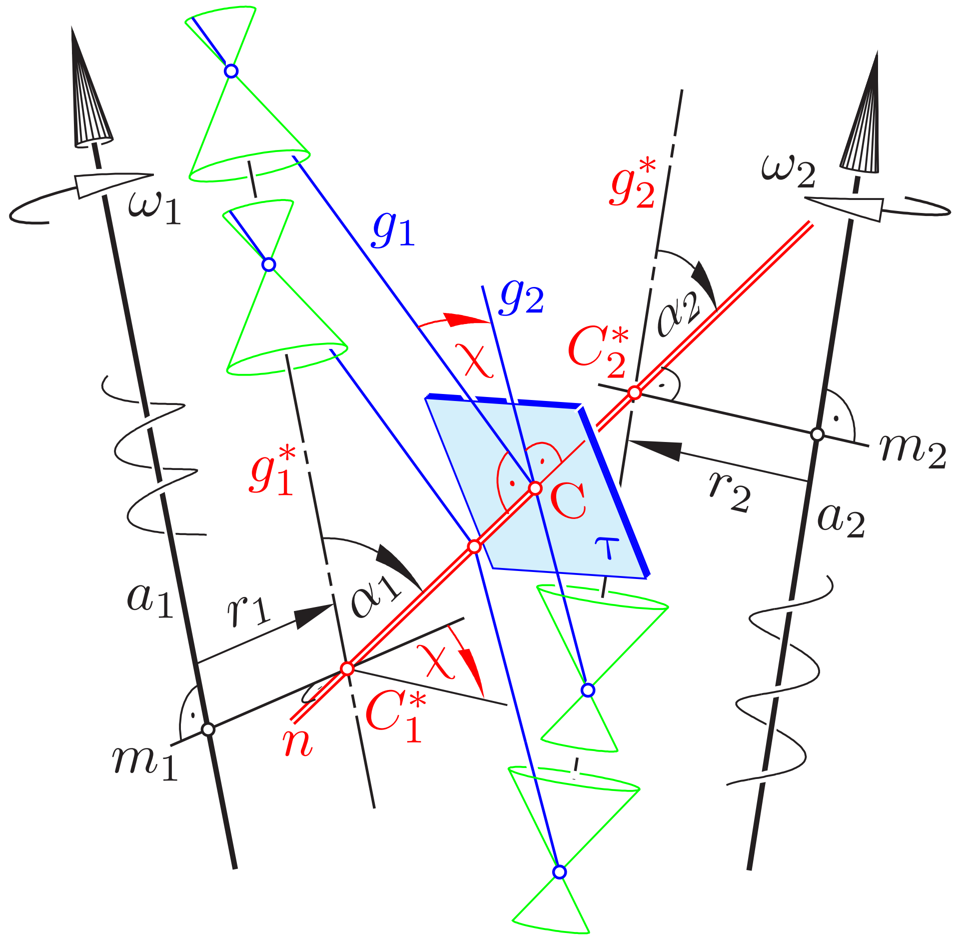


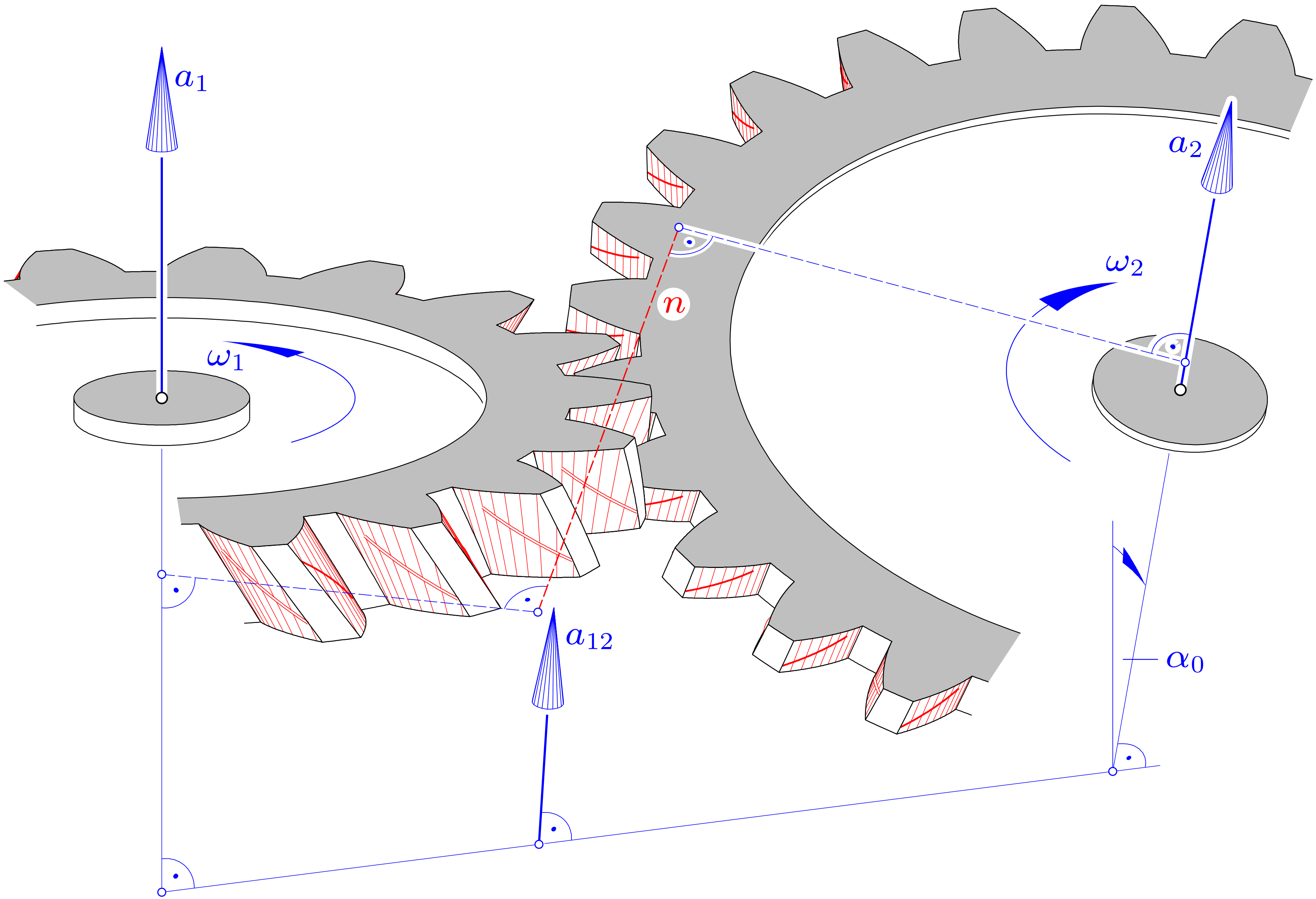
Why helical developables are tooth flanks

$$\frac{\omega_2}{\omega_1} = \frac{q_1 \cos\alpha_1 - r_1 \sin\alpha_1}{q_2 \cos\alpha_2 - r_2 \sin\alpha_2}.$$

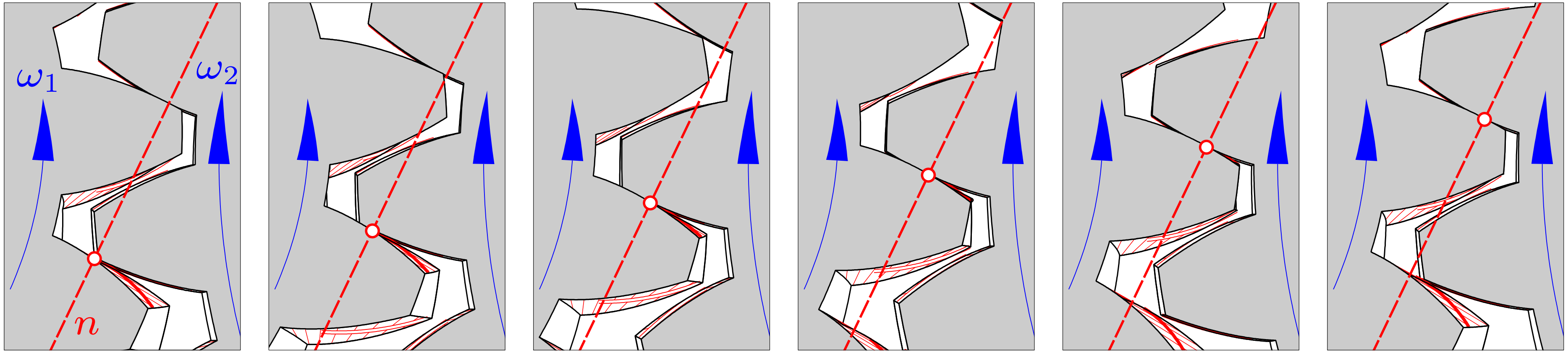
In addition: The angle χ between the two contact generators g_1, g_2 remains constant (H.S., 2004).

The specification $\chi = 0$ (a_1, a_2, n parallel to a common plane) gives skew gears with helical developables and permanent line contact.




a_1
a_2
ω_1
ω_2
n
a_{12}
α_0

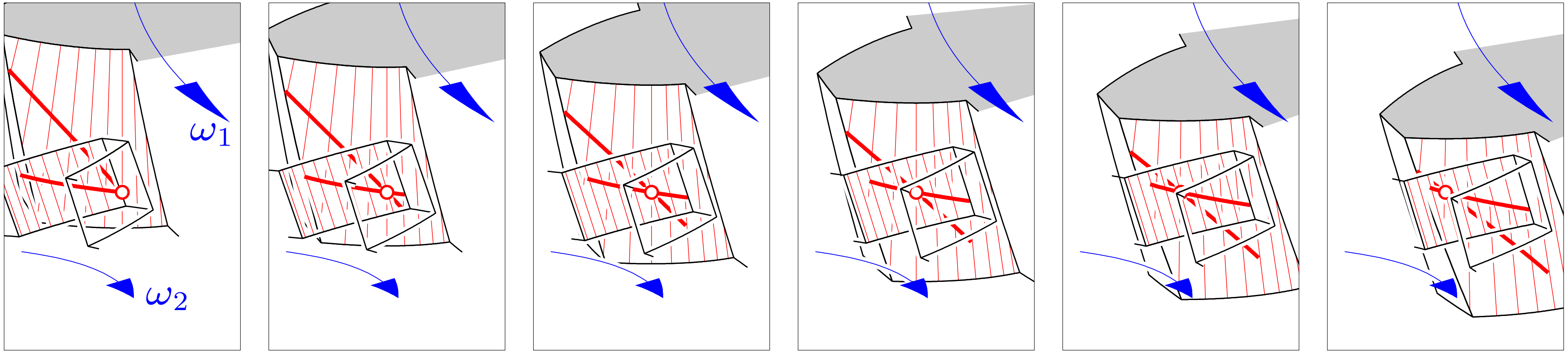
Inspecting the backlash



$\varphi_1 = 0°$ $\varphi_1 = 3.6°$ $\varphi_1 = 7.2°$ $\varphi_1 = 10.8°$ $\varphi_1 = 14.4°$ $\varphi_1 = 18.0°$

Different postures of meshing involute teeth for inspecting the backlash.
n is the line of contact.
Interval of the input angle $\Delta\varphi_1 = 3.6°$,
interval of the output angle $\Delta\varphi_2 = -2.4°$.

Inspecting the single point contact



$\varphi_1 = 0°$ $\varphi_1 = 3.6°$ $\varphi_1 = 7.2°$ $\varphi_1 = 10.8°$ $\varphi_1 = 14.4°$ $\varphi_1 = 18.0°$

Different postures of meshing involute gear flanks together with the effective slip tracks, seen in direction of the contact normal n. The second wheel is displayed as a wireframe.

5. Conclusion

The main aim was to present a *new approach* to spatial involute gearing.

It was demonstrated that the following statement is sufficient to proof all properties of planar and spatial involute gearing:

Under uniform rotations

$$\begin{cases} \text{of a circular involute } c \text{ about its center or} \\ \text{of a helical developable } \Phi \text{ about its axis} \end{cases}$$

the point C of intersection with any fixed normal line n runs with constant velocity v_C — depending only on the dimensions of c or Φ, respectively.

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