The Geometry Behind the Numerical Reconstruction of Two Photos

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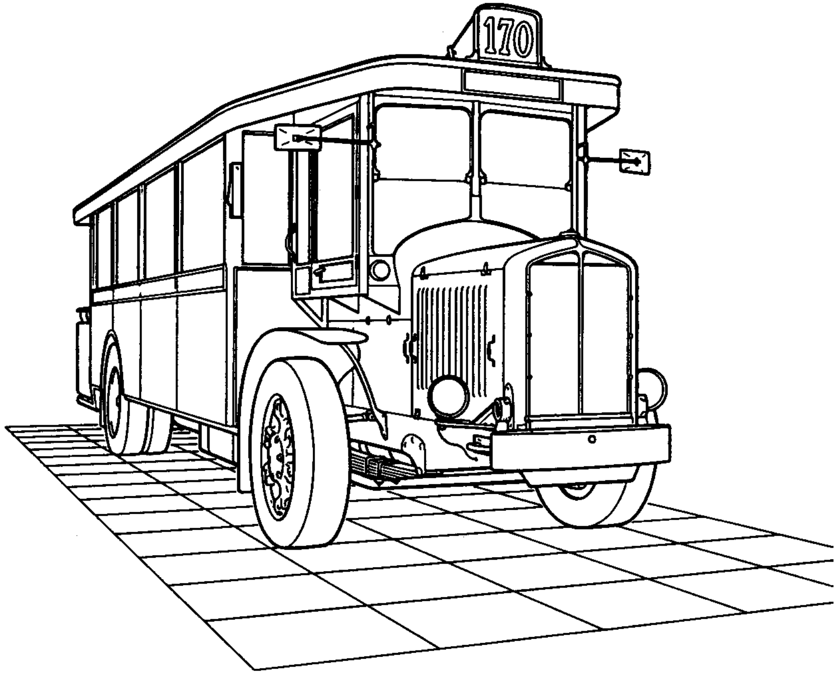
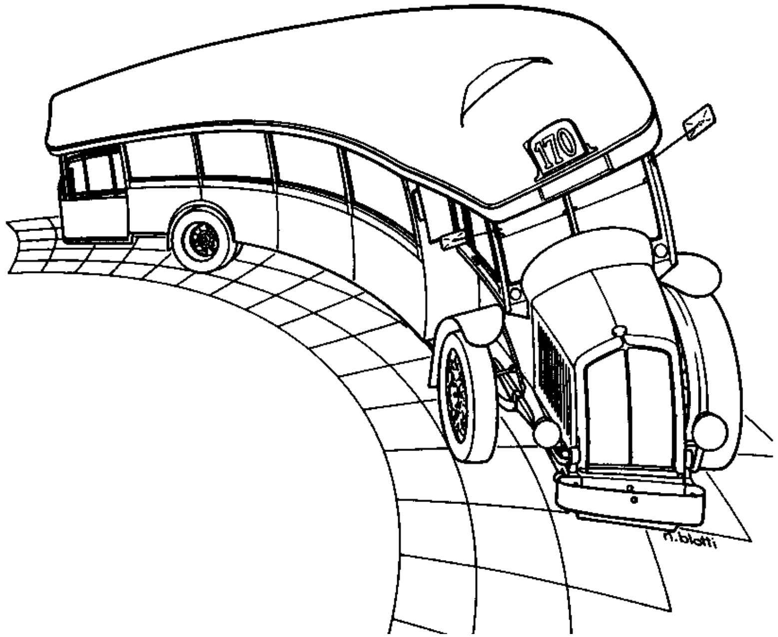


linear image



nonlinear (curved) image

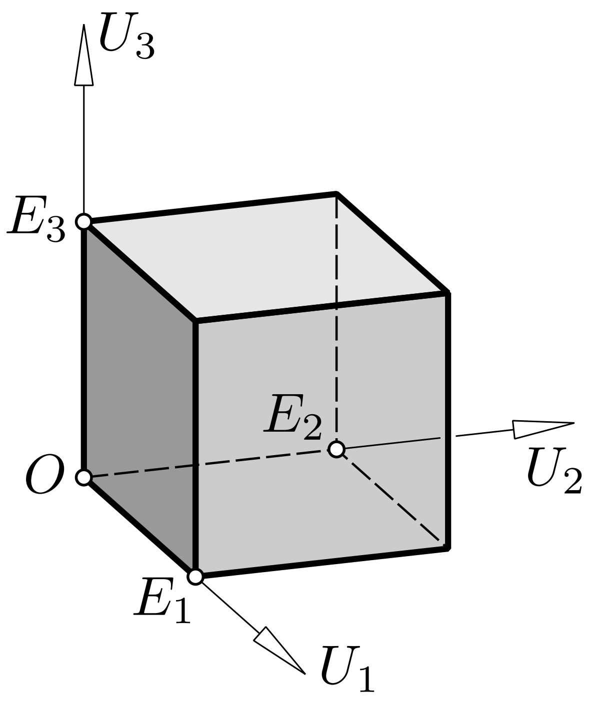
Central projection

The central projection (according to A. Dürer)

can be generalized by a central axonometry.

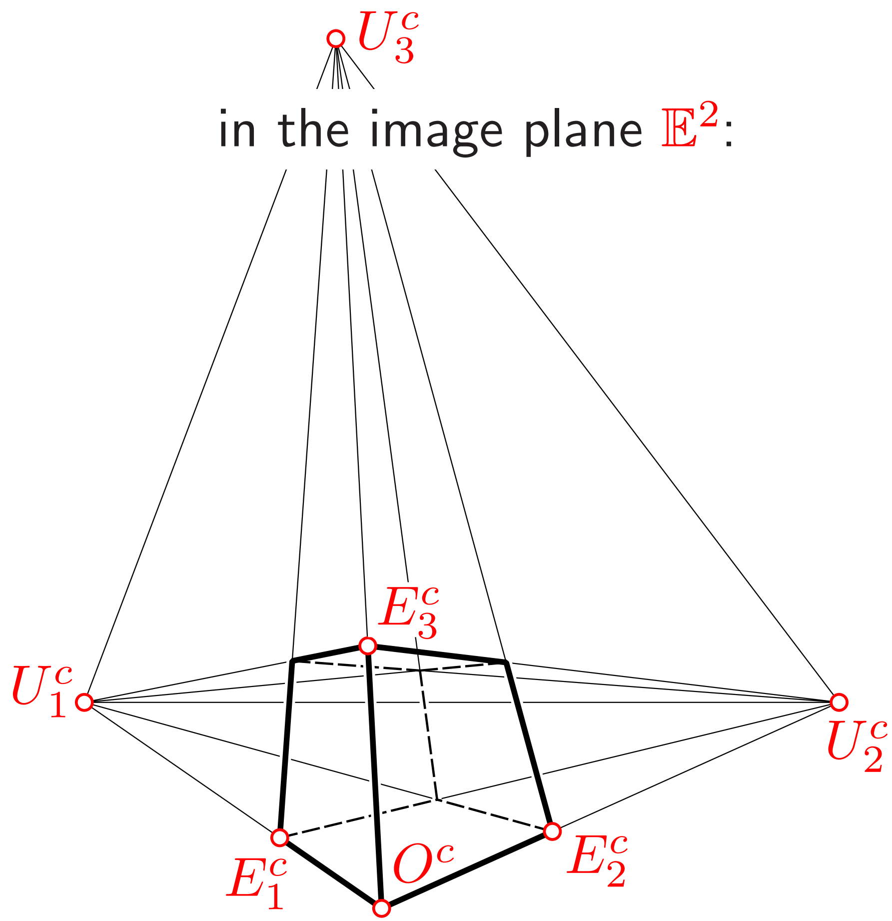
Central axonometric principle

in space $\mathbb{E}^3$:



cartesian basis $O; E_1, E_2, E_3$
and points at infinity U_1, U_2, U_3

in the image plane $\mathbb{E}^2$:



central axonometric reference system
$O^c; E_1^c, E_2^c, E_3^c; U_1^c, U_2^c, U_3^c$

Definition of linear images

There is a unique collinear transformation

$$\kappa\colon\ \mathbb{E}^3 \to \mathbb{E}^2 \ \text{ mit } \ O \mapsto O^c,\ E_i \mapsto E_i^c,\ U_i \mapsto U_i^c, \quad i = 1, 2, 3.$$

Any two-dimensional image of $\mathbb{E}^3$ under a collinear transformation is called *linear*.

$$\Longrightarrow \begin{cases} \text{collinear points have collinear or coincident images} \\ \text{cross-ratios of any four collinear points are preserved.} \end{cases}$$

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$\Longrightarrow$ {
- collinear points have collinear or coincident images
- cross-ratios of any four collinear points are preserved.

Central projection in coordinates

Notation:

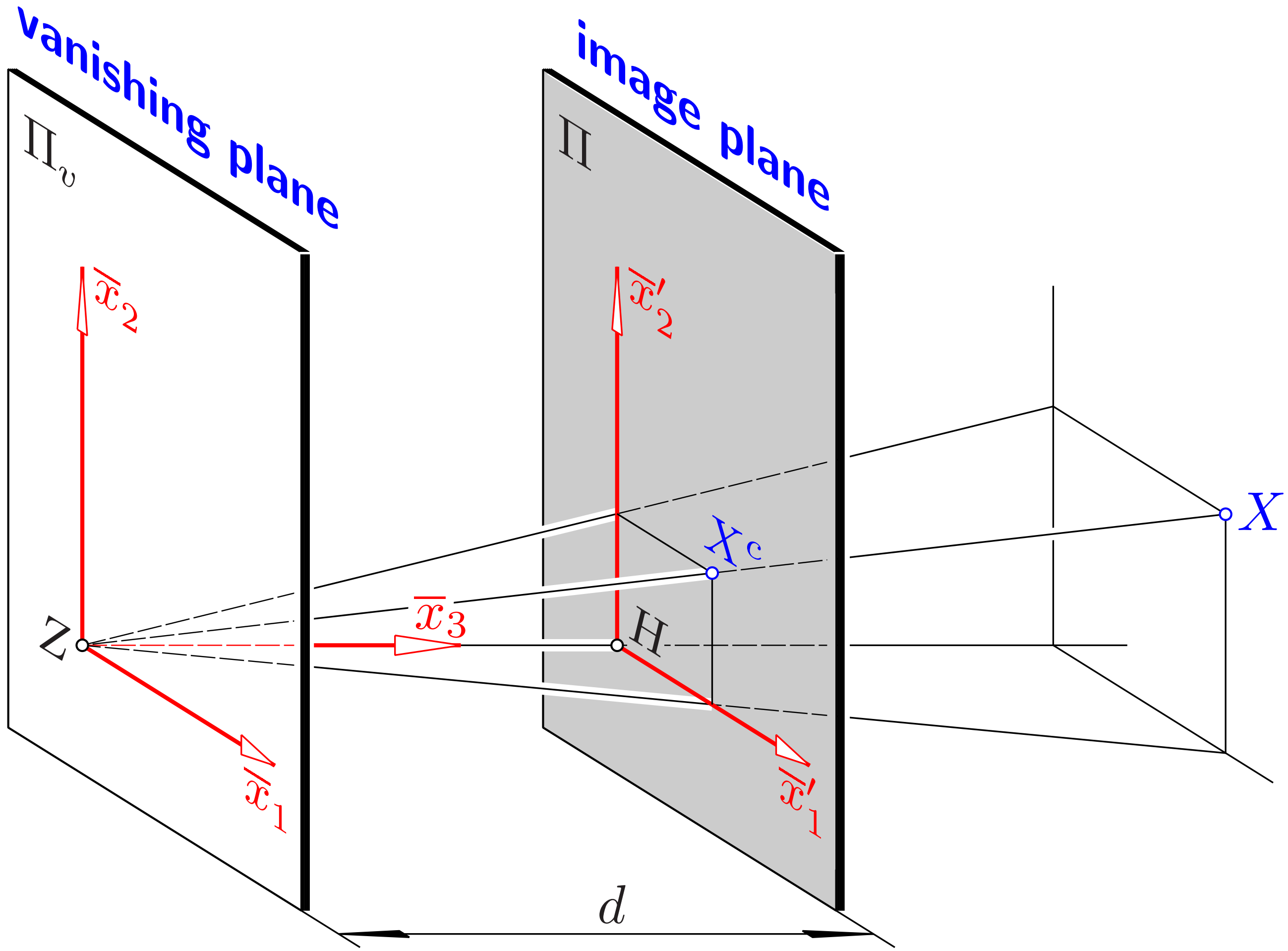
Z . . . center

H . . . principal point

d . . . focal length

$\overline{x}_1, \overline{x}_2, \overline{x}_3$ . . . camera frame

$\overline{x}'_1, \overline{x}'_2$ . . . image coordinate frame



Central projection in coordinates

$$\begin{pmatrix} \overline{x}'_1 \\ \overline{x}'_2 \end{pmatrix} = \frac{d}{\overline{x}_3} \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \end{pmatrix}, \text{ or homogeneous } \begin{pmatrix} \overline{\xi}'_0 \\ \overline{\xi}'_1 \\ \overline{\xi}'_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \end{pmatrix} \begin{pmatrix} \overline{\xi}_0 \\ \vdots \\ \overline{\xi}_3 \end{pmatrix}.$$

Transformation from the camera frame $(\overline{x}_1, \overline{x}_2, \overline{x}_3)$ into arbitrary world coordinates $(x_1,\, x_2,\, x_3)$ and translation from the particular image frame $(\overline{x}'_1, \overline{x}'_2)$ into arbitrary (x'_1, x'_2) gives in homogeneous form

$$\begin{pmatrix} \xi'_0 \\ \xi'_1 \\ \xi'_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ h'_1 & d\,f_1 & 0 \\ h'_2 & 0 & d\,f_2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \left(\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ \hline o_1 & & & \\ \vdots & & R & \\ o_3 & & & \end{array}\right)}_{\text{matrix } A} \begin{pmatrix} \xi_0 \\ \vdots \\ \xi_3 \end{pmatrix}.$$

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Central projection in coordinates

Left hand matrix: (h'_1, h'_2) are image coordinates of the principal point H, (f_1, f_2) are possible scaling factors, and d is the focal length.

These parameters are called the intrinsic calibration parameters.

Right hand matrix: R is an orthogonal matrix.

The position of the camera frame with respect to the world coordinates defines the extrinsic calibration parameters.

Photos with known interior orientation are called calibrated images, others (like central axonometries) are uncalibrated.

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Positive and negative central pespective

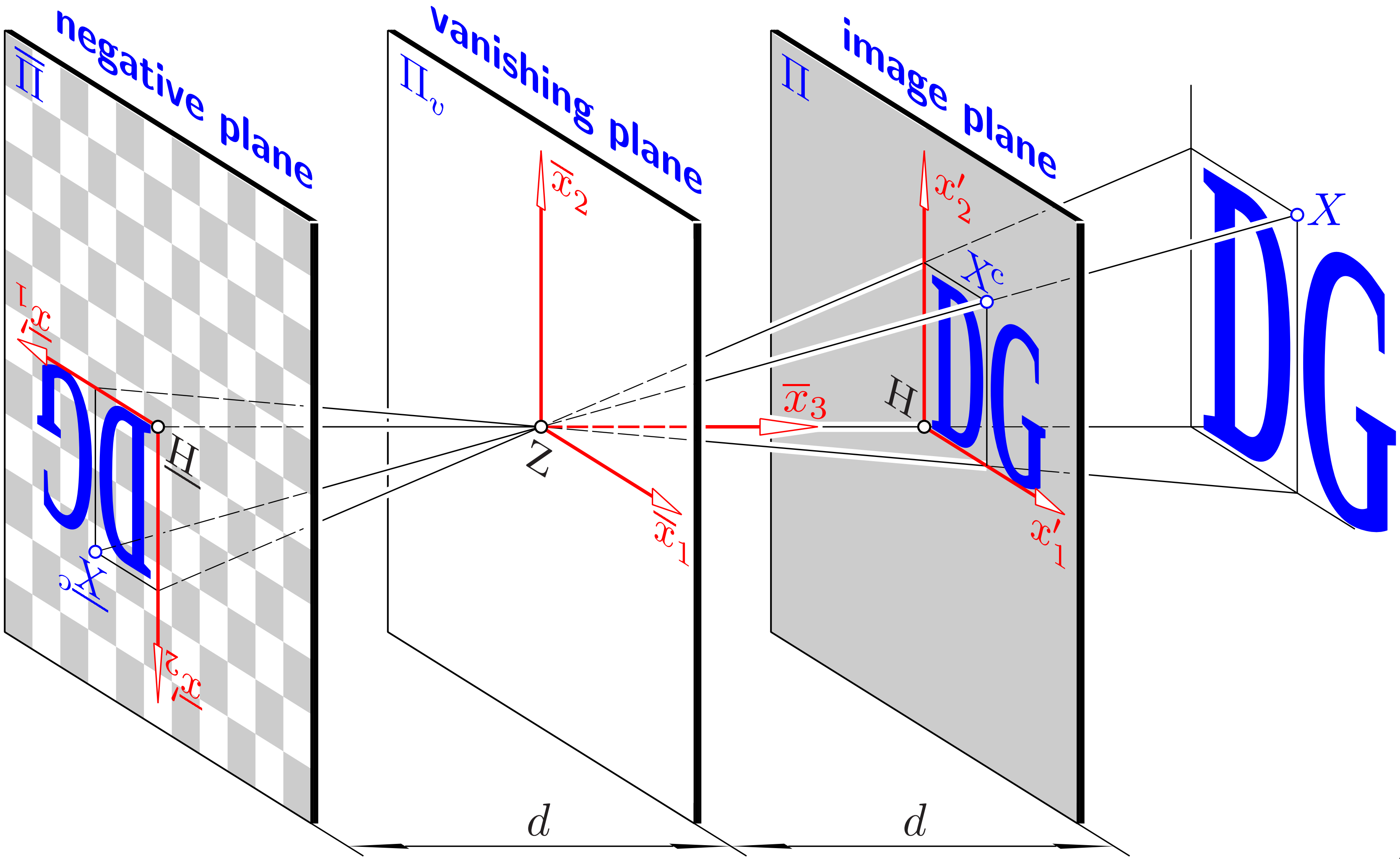
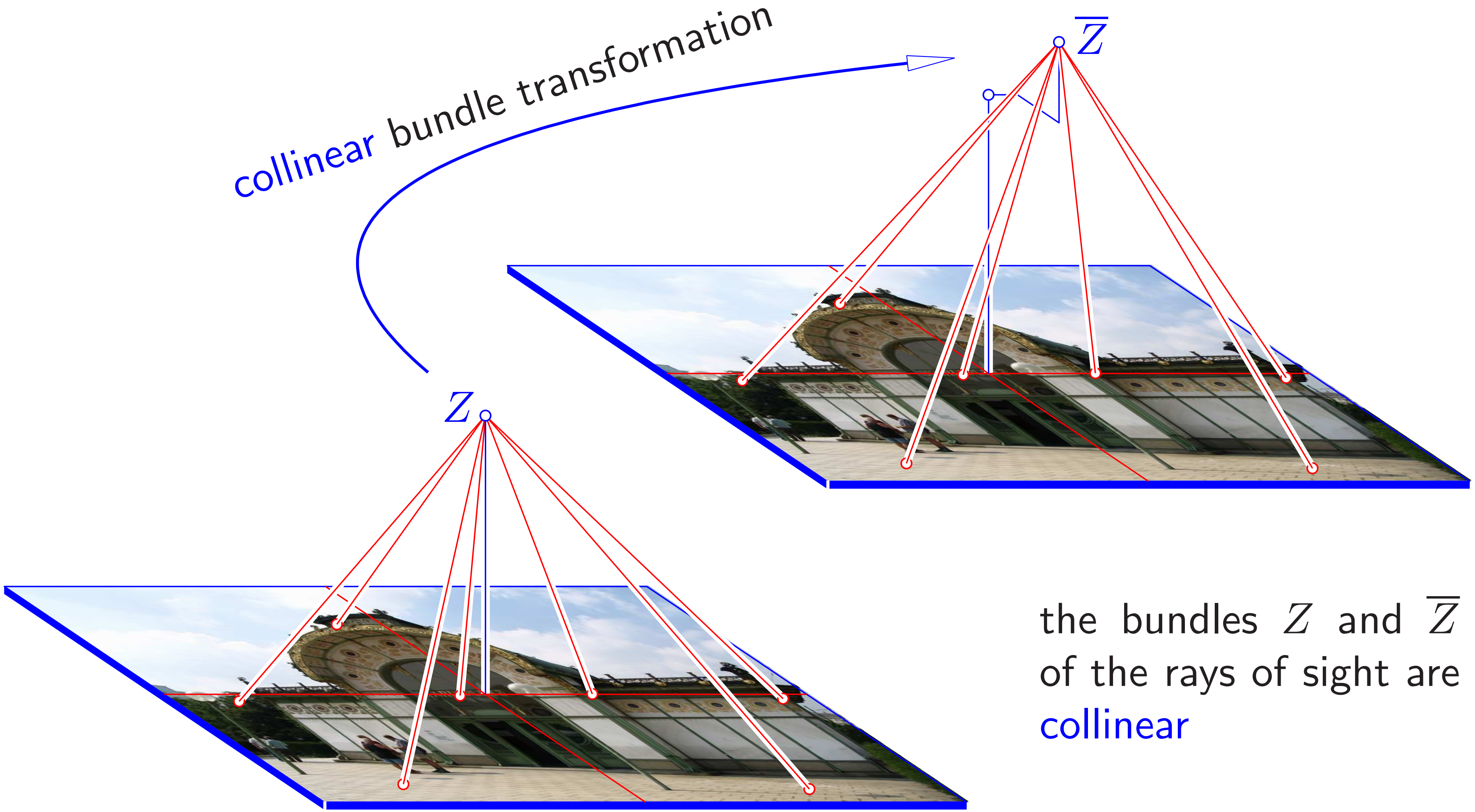


Photo versus linear image

Photo (= central perspective) or photo of a photo (= linear image) ?

unknown interior calibration parameters



the bundles Z and $\overline{Z}$ of the rays of sight are collinear

2. Geometry of two images

GIVEN: Two linear images or two photographs.

WANTED: Dimensions of the depicted 3D-object.

Historical 'Stadtbahn' station Karlsplatz in Vienna (Otto Wagner, 1897)

2. Geometry of two images

The geometry of two images is a classical subject of Descriptive Geometry. Its results have become standard (FINSTERWALDER, KRUPPA, KRAMES, WUNDERLICH, HOHENBERG, TSCHUPIK, BRAUNER, HAVLICEK, H.S., . . .).

Why now ? Advantages of digital images:

- less distorsion, because no paper prints are needed,
- exact boundary is available, and
- precise coordinate measurements are possible using standard software.

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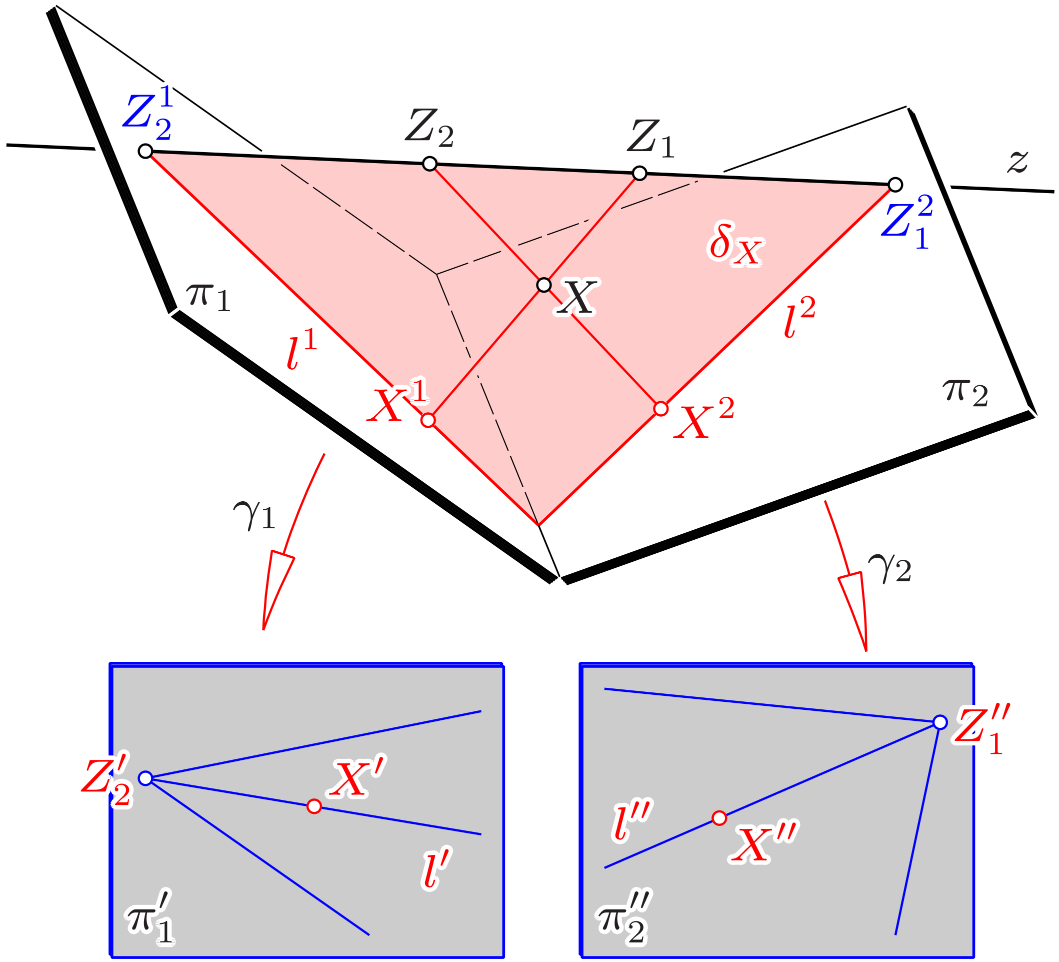
Geometry of two images (epipolar geometry)

viewing situation

collinear transformations

two images



Geometry of two images (epipolar geometry)

Notations:

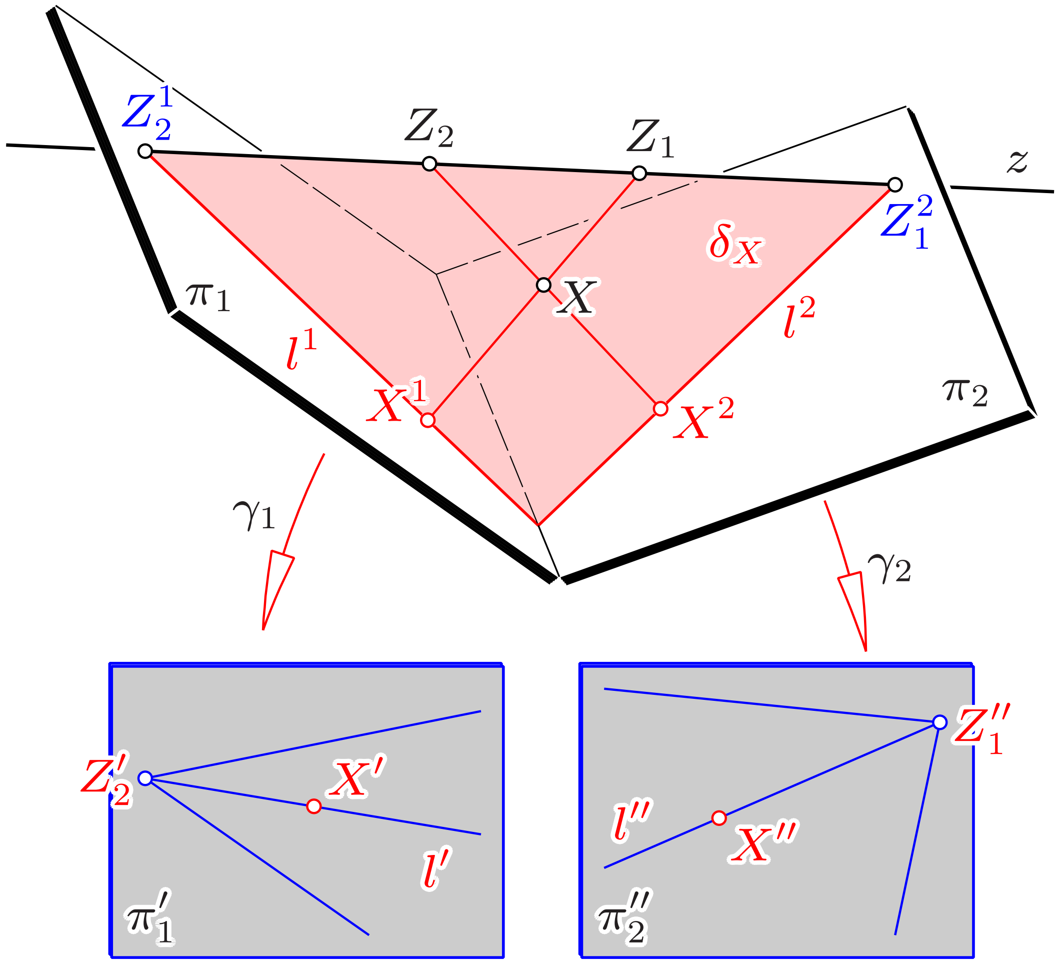
line $z = Z_1 Z_2$ . . . baseline,

Z_2', Z_1'' . . . epipoles
(German: Kernpunkte),

δ_X . . . epipolar plane (it is twice projecting),

l', l'' . . . pair of epipolar lines
(German: Kernstrahlen),

(X', X'') . . . corresponding views.



Epipolar constraint

Theorem (synthetic version): For any two linear images of a scene, there is a projectivity between two line pencils

$$Z_2'(\delta_X') \barwedge Z_1''(\delta_X'')$$

such that the points X', X'' are corresponding $\iff$ they are located on (corresponding $=$) epipolar lines.

Theorem (analytic version): Using homogeneous coordinates for both images, there is a bilinear form β of rank 2 such that two points $X' = \mathbf{x}'\mathbb{R} = (\xi_0' : \xi_1' : \xi_2')$ and $X'' = \mathbf{x}''\mathbb{R} = (\xi_0'' : \xi_1'' : \xi_2'')$ are corresponding

$$\iff \beta(\mathbf{x}', \mathbf{x}'') = \sum_{i,j=0}^{2} b_{ij}\, \xi_i'\, \xi_j'' = (\xi_0'\ \xi_1'\ \xi_2') \cdot (b_{ij}) \begin{pmatrix} \xi_0'' \\ \xi_1'' \\ \xi_2'' \end{pmatrix} = \mathbf{x}'^T \cdot B \cdot \mathbf{x}'' = 0\,.$$

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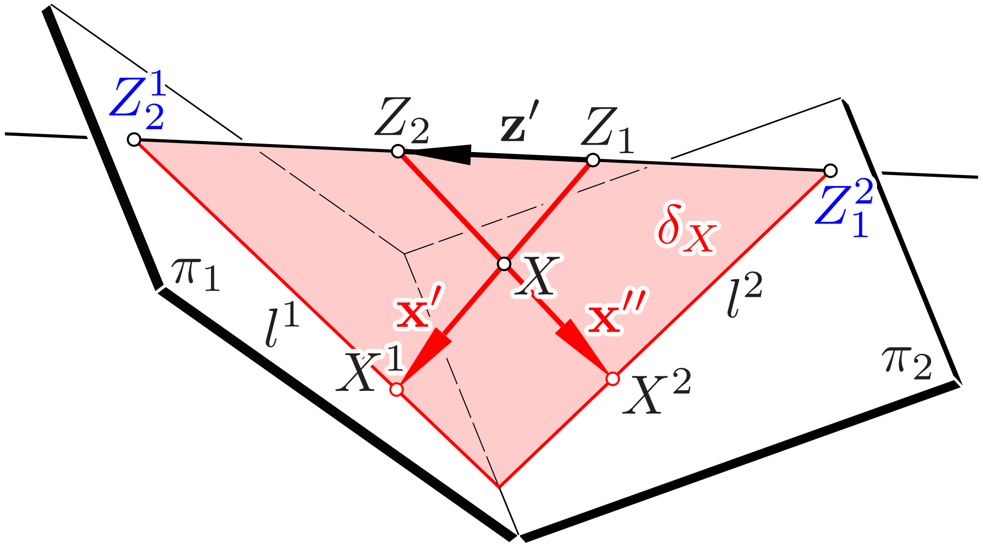
Epipolar constraint in the calibrated case

Theorem: *In the calibrated case the essential matrix* $B = (b_{ij})$ *is the product of a skew symmetric matrix and an orthogonal one, i.e.,*

$$B = S \cdot R\,.$$



Proof: We use both camera frames and the homogeneous coordinates

$$\mathbf{x}' = \overrightarrow{Z_1 X'}, \quad \mathbf{x}'' = \overrightarrow{Z_2 X''}.$$

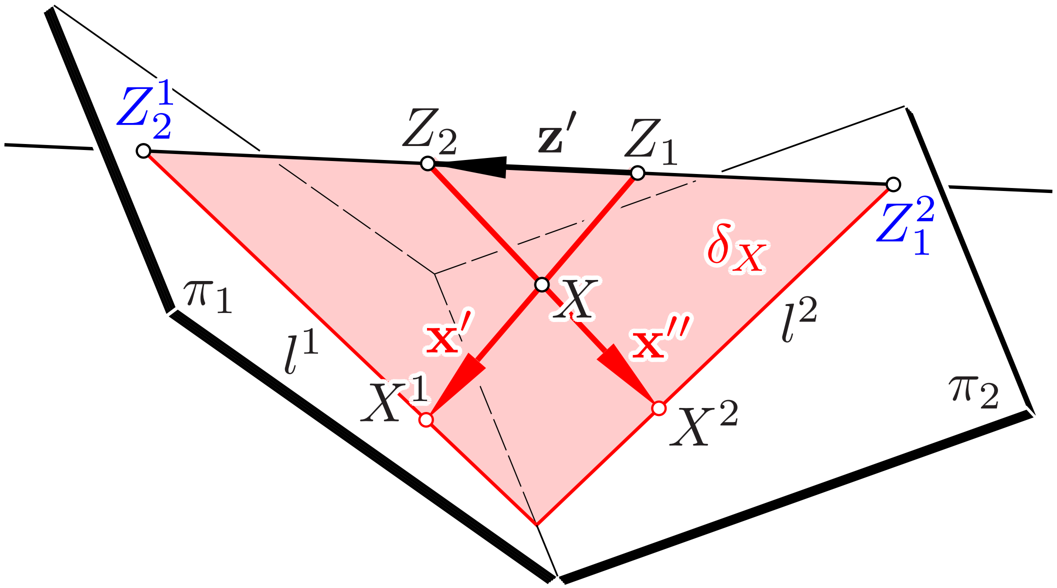
Epipolar constraint in the calibrated case

For transforming the coordinates from the second camera frame into the first one, there is an orthogonal matrix R such that

$$\mathbf{x}''_1 = \mathbf{z}' + R\cdot\mathbf{x}'' \text{ with } R^T = R^{-1} \text{ and } \mathbf{z}' = (z'_1,\, z'_2,\, z'_3)^T = \overrightarrow{Z_1 Z_2}.$$

The points X^1, X^2, Z_1, Z_2 are coplanar $\iff$ the triple product of the vectors $\mathbf{x}'$, $\mathbf{z}'$ and $\mathbf{x}''_1 = \overline{Z_1 X^2}$ vanishes, i.e.,

$$\det(\mathbf{x}', \mathbf{z}', \mathbf{x}''_1) = \mathbf{x}' \cdot (\mathbf{z}' \times \mathbf{x}''_1) = 0.$$



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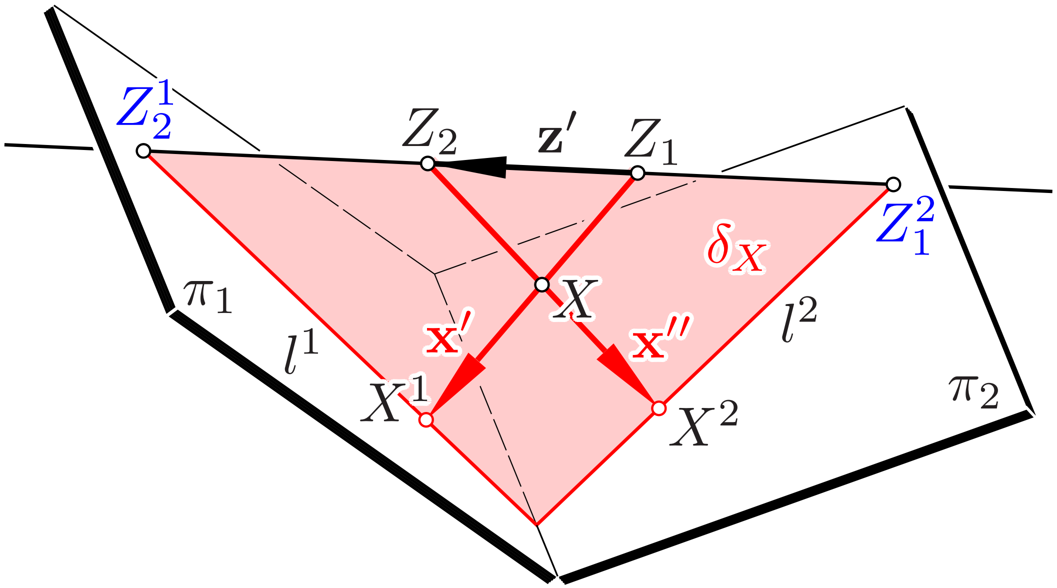

Epipolar constraint in the calibrated case

We replace the vector product $(\mathbf{z}' \times \mathbf{x}_1'')$ by

$$\mathbf{z}' \times (\mathbf{z}' + R \cdot \mathbf{x}'') = \mathbf{z}' \times R \cdot \mathbf{x}'' = S \cdot R \cdot \mathbf{x}'' \quad \text{mit} \quad S = \begin{pmatrix} 0 & -z_3' & z_2' \\ z_3' & 0 & -z_1' \\ -z_2' & z_1' & 0 \end{pmatrix}.$$

Matrix S is skew symmetric and R is orthogonal.

Hence, the coplanarity of $\mathbf{x}'$, $\mathbf{x}''$ and $\mathbf{z}'$ is equivalent to

$$0 = \mathbf{x}' \cdot (\mathbf{z}' \times \mathbf{x}_1'') = \mathbf{x}'^T \cdot \underbrace{S \cdot R}_{B} \cdot \mathbf{x}'', \quad \text{also} \quad B = S \cdot R\,. \qquad \square$$

The decomposition of the fundamental matrix B into these two factors defines the relative position of the second camera frame against the first one !

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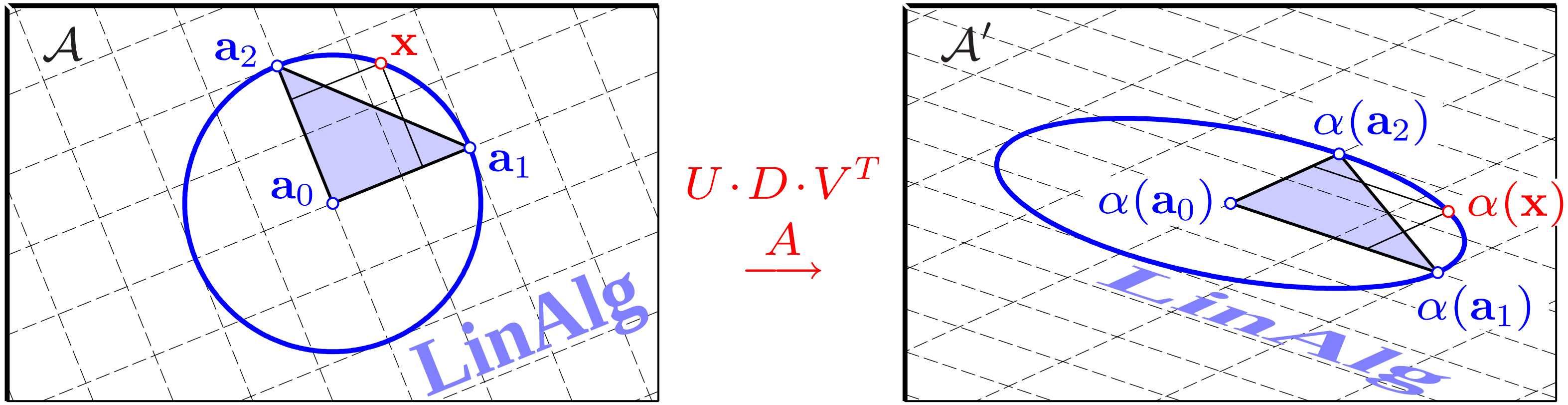
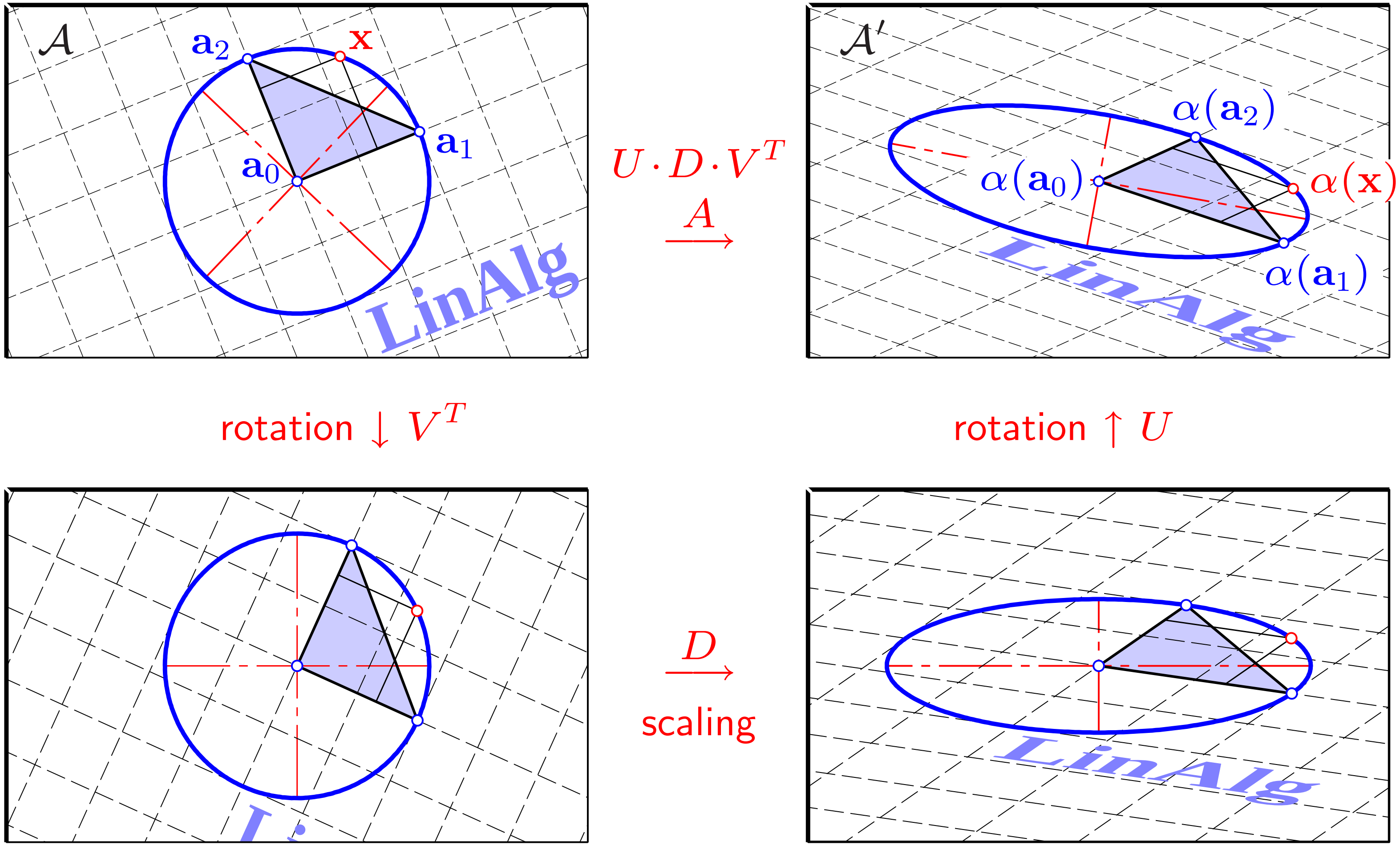
Singular value decomposition (SVD)



Singular value decomposition (SVD)



Singular value decomposition (SVD)

Theorem: [Singular value decomposition]
Any matrix $A \in M(m, n; \mathbb{R})$ *can be decomposed into a product*

$$A = U \cdot D \cdot V^T \text{ with orthogonal } U, V \text{ and } D = \operatorname{diag}(\sigma_1, \ldots, \sigma_p)$$

with $D \in M(m, n; \mathbb{R})$, $\sigma_i \geq 0$, *and* $p = \min\{m, n\}$.
The positive entries in the main diagonal of D are called singular values of A.

The singular values of A can be seen as principal distortion factors of the affine transformation represented by A, i.e., the semiaxes of the affine image of the unit sphere.

E.g., the singular values of an orthogonal projection are $(0, 1, 1)$ as the unit sphere is mapped onto a unit disk.

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Singular values of the essential matrix

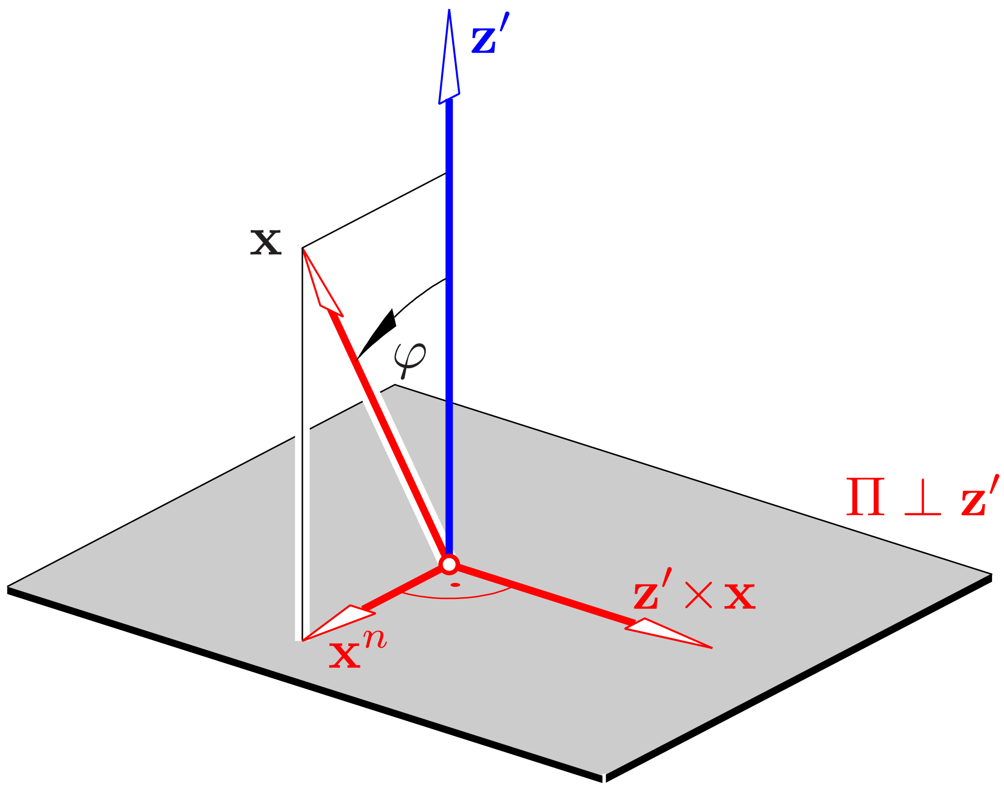
Theorem:
The essential matrix B has two equal singular values $\sigma := \sigma_1 = \sigma_2$.

Proof: We have $B = S \cdot R$ with orthogonal R. The vector

$$S \cdot \mathbf{x} = \mathbf{z}' \times \mathbf{x}$$

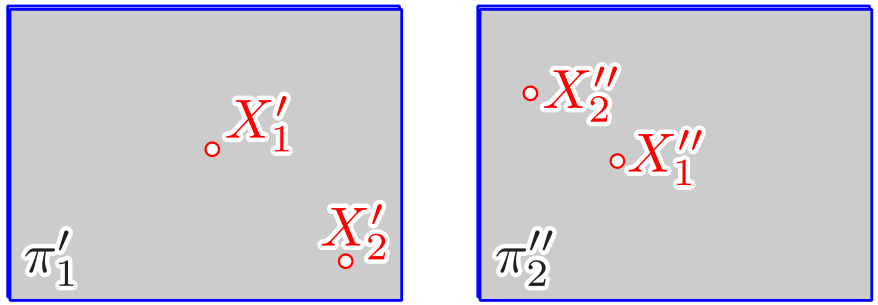
is orthogonal zu the orthogonal view $\mathbf{x}^n$, where

$$\begin{aligned} \|\mathbf{z}' \times \mathbf{x}\| &= |\sin\varphi|\, \|\mathbf{x}\|\, \|\mathbf{z}'\| = \\ &= \|\mathbf{x}^n\|\, \|\mathbf{z}'\| = \sigma\, \|\mathbf{x}^n\|. \end{aligned}$$



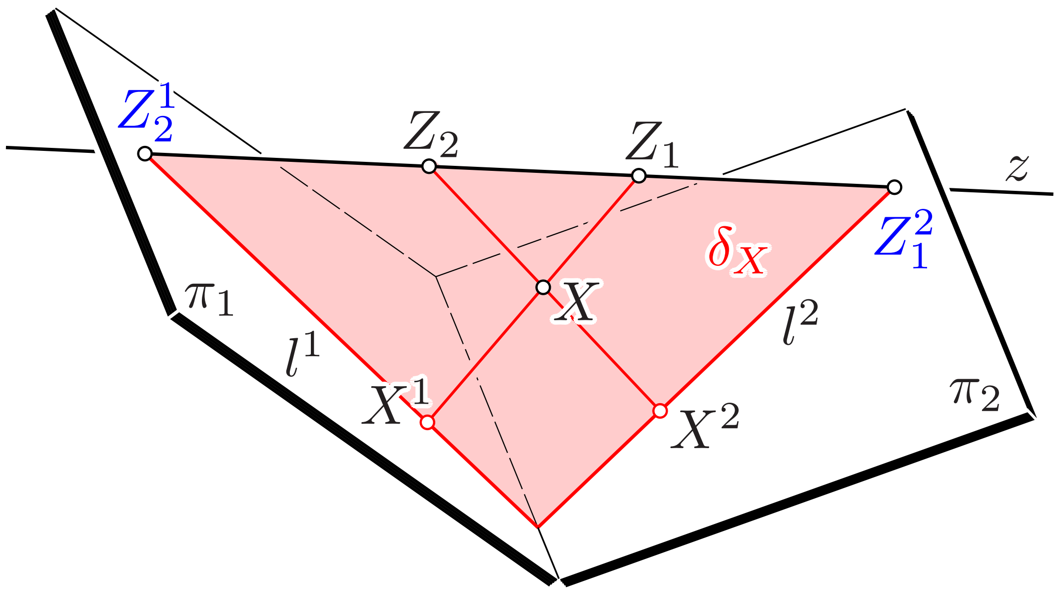
What means ‘reconstruction’

GIVEN: Two either calibrated or uncalibrated images.



WANTED: ‘viewing situation’, i.e., determine

- the relative position of the two camera frames, and
- the location of any space point X from its images (X', X'').



The fundamental theorems

Theorem 1:
From two uncalibrated images with given projectivity between epipolar lines the depicted object can be reconstructed up to a collinear transformation.

Theorem 2 (S. Finsterwalder, 1899):
From two calibrated images with given projectivity between epipolar lines the depicted object can be reconstructed up to a similarity.

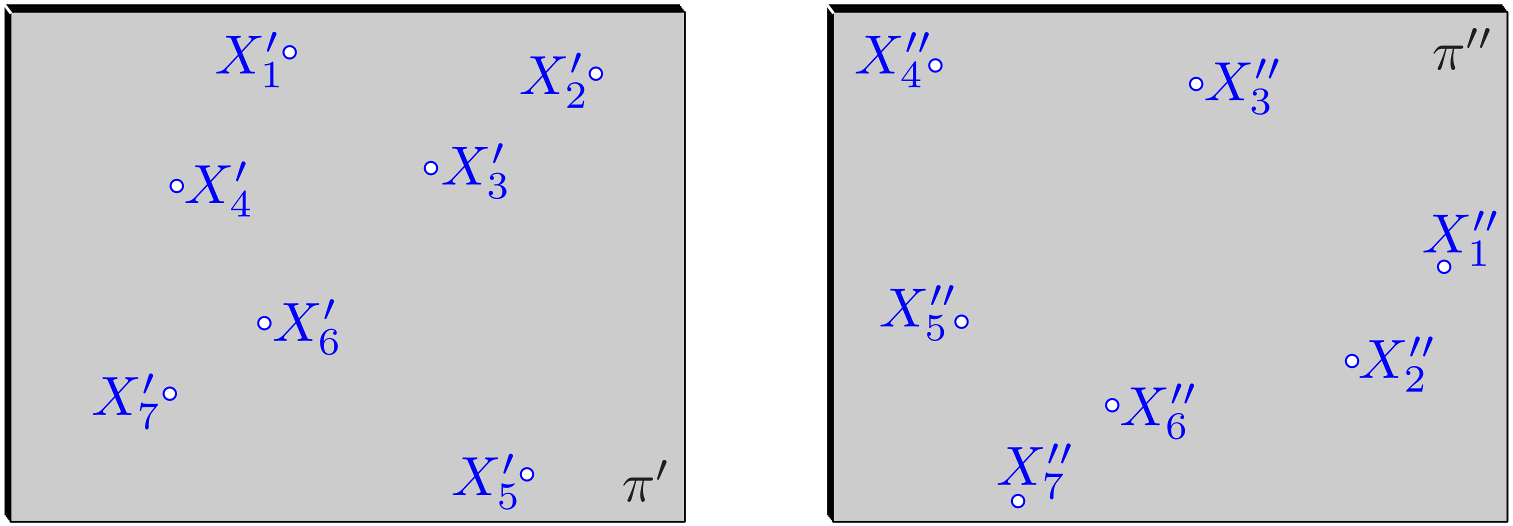
Determination of epipoles — geometric meaning

Problem of Projectivity:

GIVEN: **7** pairs of corresponding points $(X_1', X_1''), \ldots, (X_7', X_7'')$.

WANTED: A pair of points (S', S'') (= epipoles) such that there is a projectivity

$$S'([S'X_1'], \ldots, [S'X_7']) \;\barwedge\; S''([S'X_1''], \ldots, [S''X_7'']).$$



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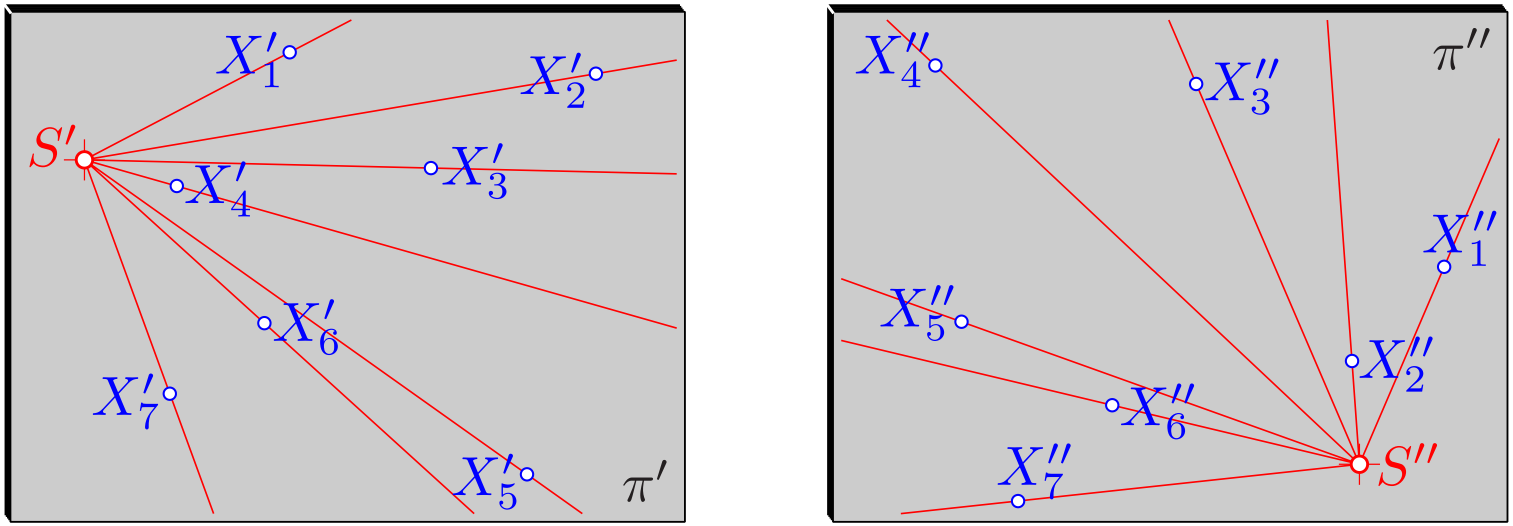

Determination of epipoles — analytic solution

Theorem: If **7** pairs of corresponding points $(X_1', X_1''), \ldots, (X_7', X_7'')$ are given, the determination of the epipoles is a cubic problem.

Proof: 7 pairs of corresponding points give 7 linear homogeneous equations

$$\beta(\mathbf{x}_i', \mathbf{x}_i'') = \mathbf{x}_i'^T \cdot B \cdot \mathbf{x}_i'' = 0, \quad i = 1, \ldots, 7,$$

for the 9 entries in the (3×3)-matrix $B = (b_{ij})$ — called essential matrix.

$\det(b_{ij}) = 0$ gives an additional cubic equation which fixes all b_{ij} up to a common factor. □

For noisy image points it is recommended to use more than 7 points and methods of least square approximation for obtaining the *'best fitting matrix'* B:

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Determination of epipoles — analytic solution

1) Let A denote the coefficient matrix in the linear system for the entries of B. Then the 'least square fit' for this overdetermined system is an eigenvector for the smallest eigenvalue of the symmetric matrix $A^T \cdot A$.

---

2) As an essential matrix needs to have rank 2, we use the 'projection into the essential space'. This means, the singular value decomposition of B gives a representation

$$B = U \cdot \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3) \cdot V^T \text{ with orthogonal } U, V \text{ and } \sigma_1 \geq \sigma_2 \geq \sigma_3 \,.$$

Then in the uncalibrated case $B = U \cdot \mathrm{diag}(\sigma_1, \sigma_2, 0) \cdot V$ is optimal (with respect to the Frobenius norm) and in the calibrated case

$$B = U \cdot \mathrm{diag}(\sigma, \sigma, 0) \cdot V^T \quad \text{with} \quad \sigma_1 = (\sigma_1 + \sigma_2)/2.$$

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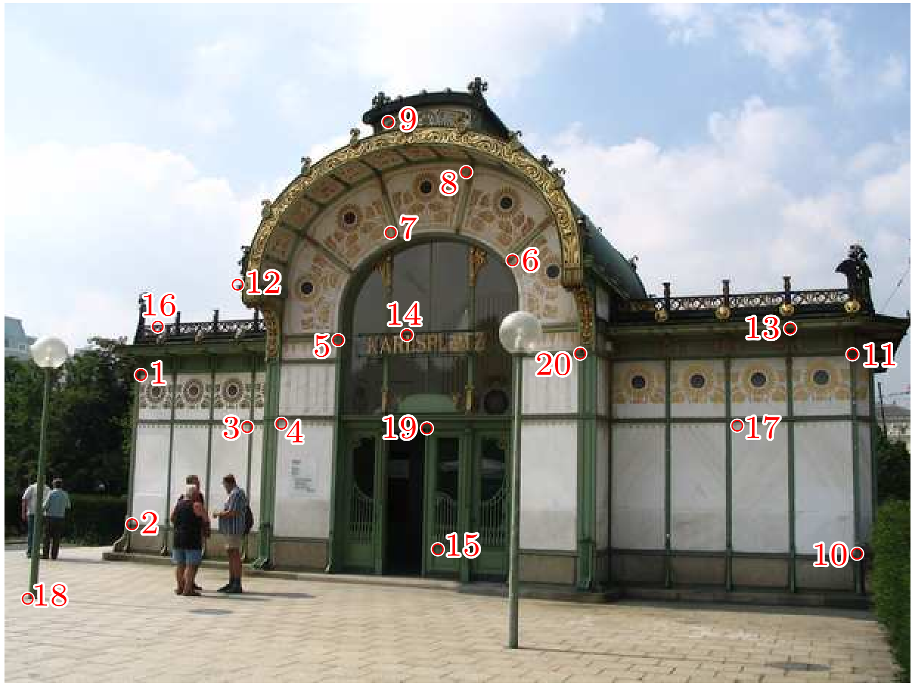
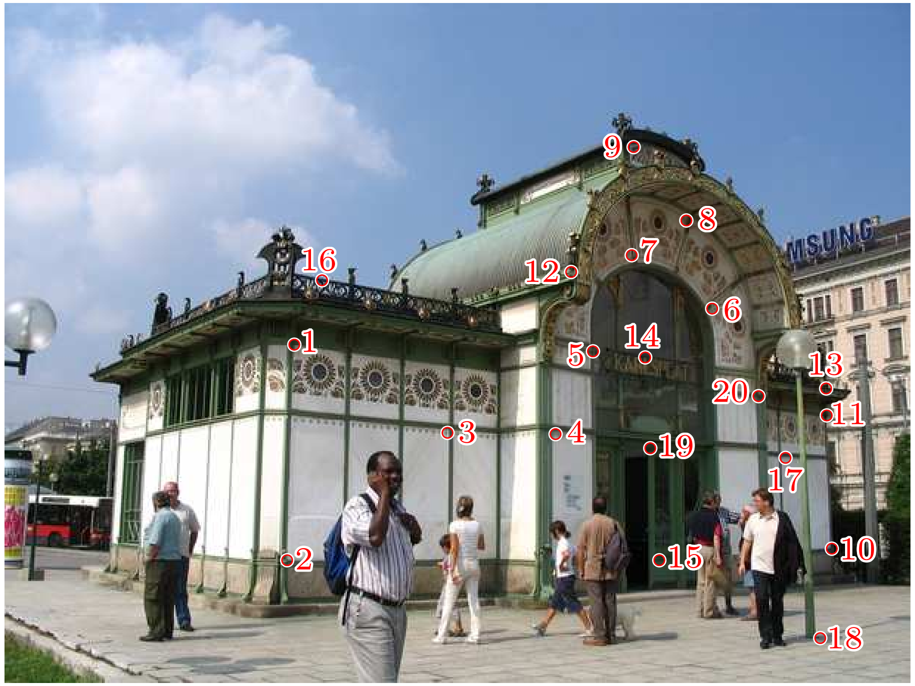
3. Numerical reconstruction of two images

Step 1: Specify at least 7 reference points





. . . manually — or automatically by methods of pattern recognition

Step 2: Compute the essential matrix

Step 2: Compute the essential matrix B — including the pairs of epipolar lines

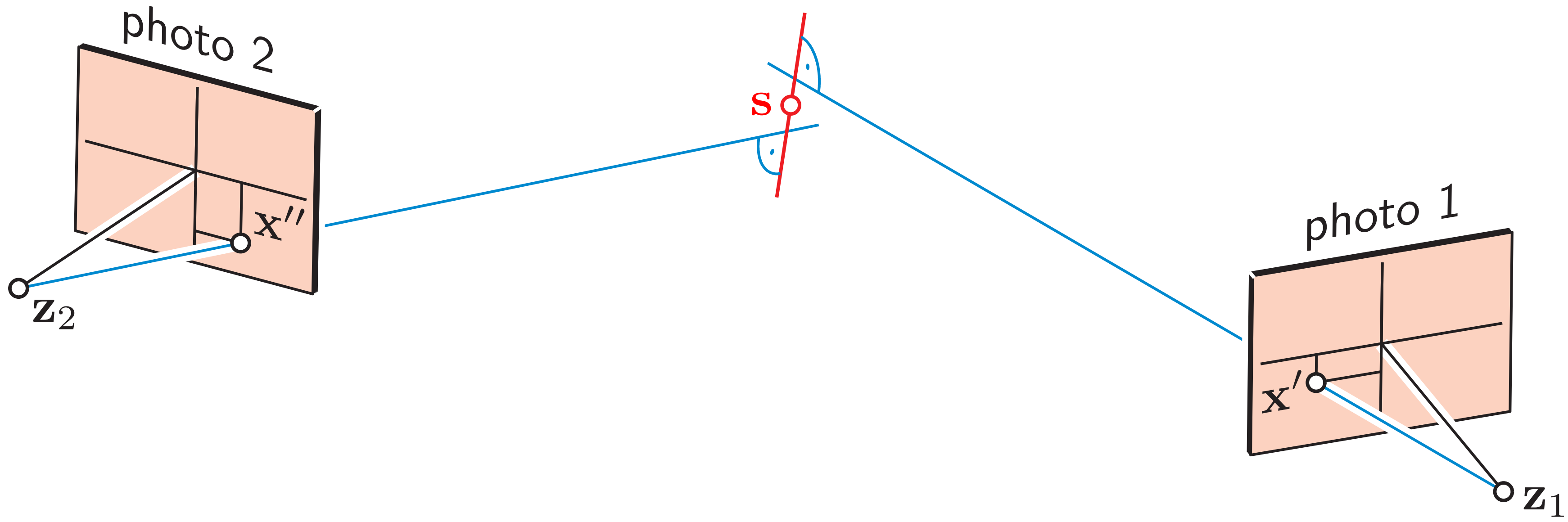
Step 3: Factorize $B = S.R$

Theorem: There are exactly two ways of decomposing $B = U \cdot D \cdot V^T$ with $D = \mathrm{diag}(\sigma, \sigma, 0)$ into a product $S \cdot R$ with skew-symmetric S and orthogonal R:

$$S = \pm U \cdot R_+ \cdot D \cdot U^T \quad \text{and} \quad R = \pm U \cdot R_+^T \cdot V^T \quad \text{with} \quad R_+ = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Step 4: Intersecting corresponding rays

In one of the frames compute the approximate point of intersection between corresponding rays.



For the center of the common perpendicular line segment the sum of squared distances is minimal.

Summary of algorithm

1) Specify $n > 7$ pairs (X_i', X_i''), $i = 1, \ldots, n$.

2) Set up linear system of equations for the essential matrix B and seek best fitting matrix (eigenvector of the smallest eigenvalue).

3) Compute the closest rank 2 matrix B with two equal singular values.

4) Factorize $B = S \cdot R$; this reveals the relative position of the two camera frames.

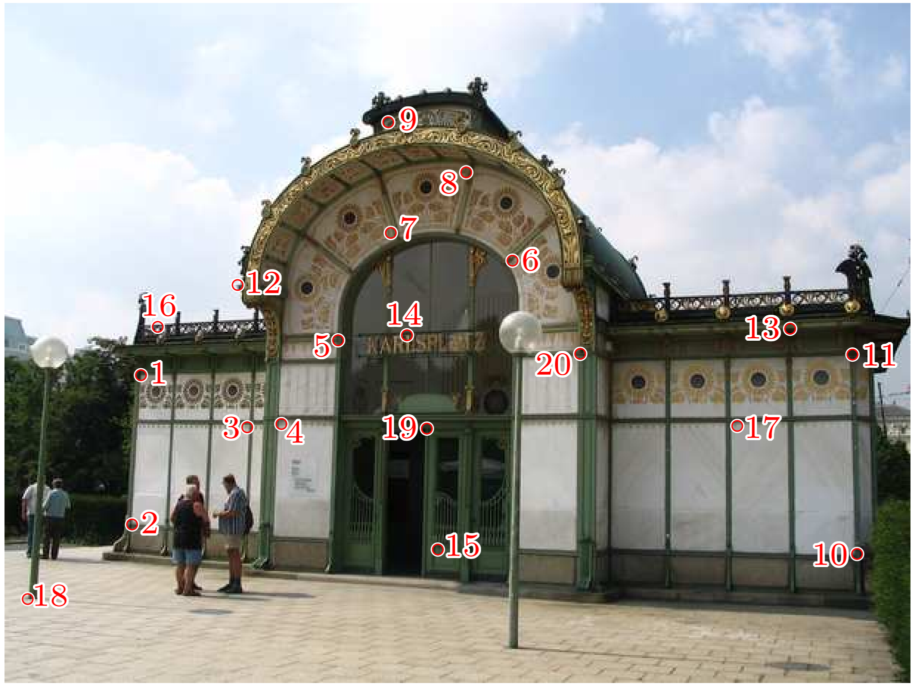
5) In one of the frames compute the approximate point of intersection between corresponding rays.

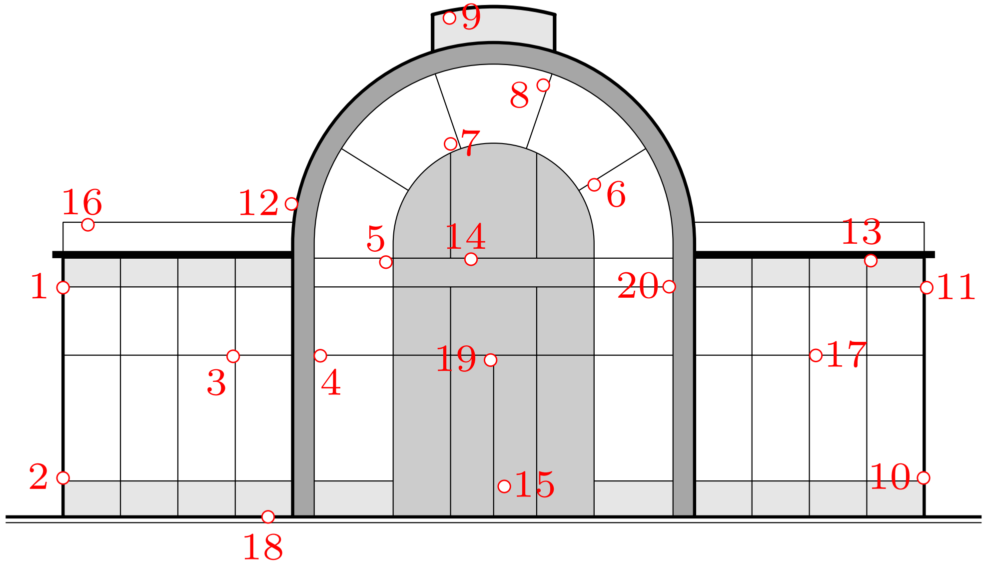
6) Transform the recovered coordinates into world coordinates.

Remaining problems

- Analysis of precision,
- automated calibration (autofocus and zooming change the focal distance d),
- critical configurations.

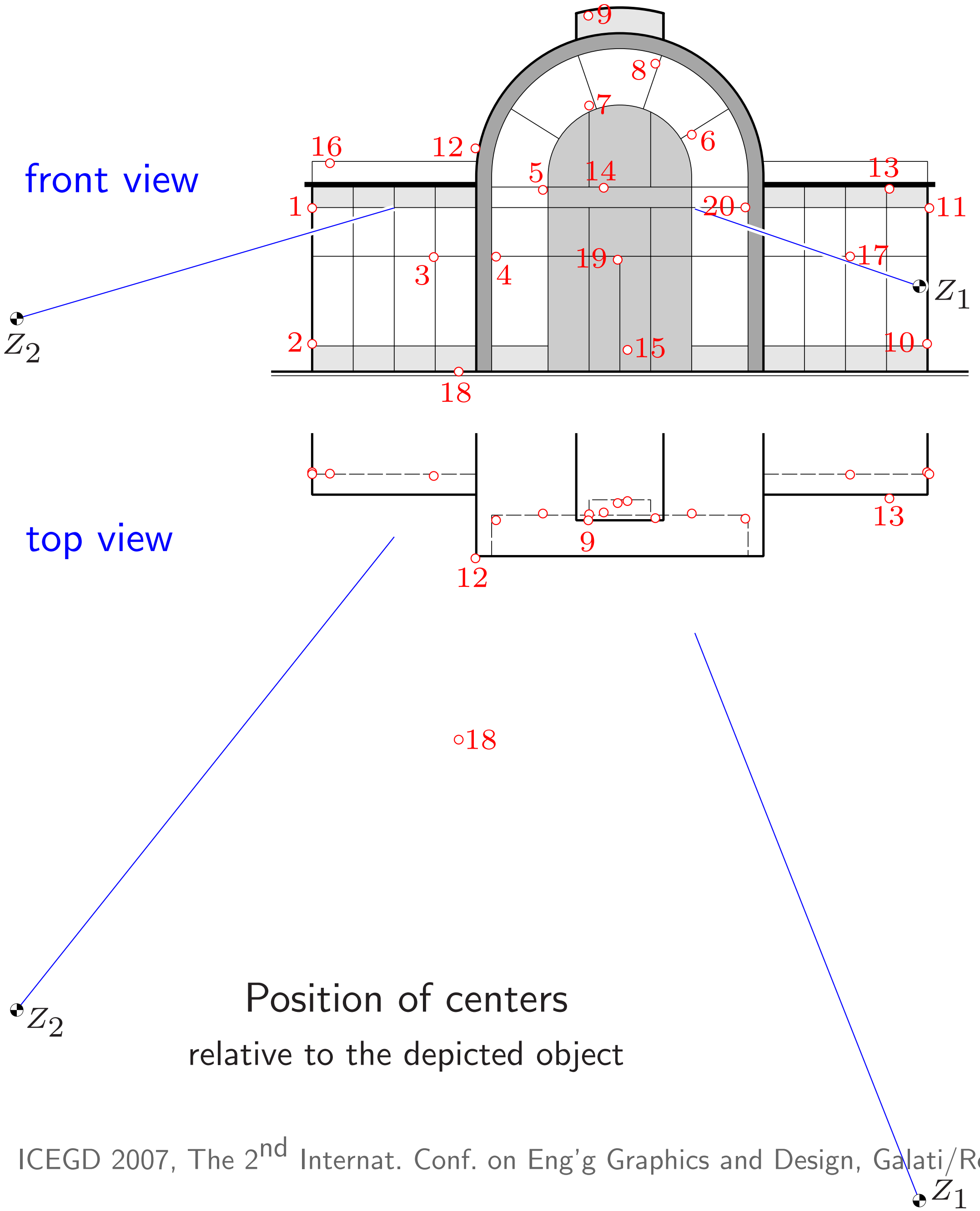
The solution





original image

the reconstruction (M $\sim 1 : 100$)



Position of centers

relative to the depicted object

Photo 1

Photo 2

Literatur

- H. Brauner: *Lineare Abbildungen aus euklidischen Räumen*. Beitr. Algebra Geom. **21**, 5–26 (1986).
- O. Faugeras: *Three-Dimensional Computer Vision. A Geometric Viewpoint*. MIT Press, Cambridge, Mass., 1906 .
- O. Faugeras, Q.-T. Luong: *The Geometry of Multiple Images*. MIT Press, Cambridge, Mass., 2001.
- R. Harley, A. Zisserman: *Multiple View Geometry in Computer Vision*. Cambridge University Press 2000.
- H. Havlicek: *On the Matrices of Central Linear Mappings*. Math. Bohem. **121**, 151–156 (1996).

- E. Kruppa: *Zur achsonometrischen Methode der darstellenden Geometrie*. Sitzungsber., Abt. II, österr. Akad. Wiss., Math.-Naturw. Kl. **119**, 487–506 (1910).

- Yi Ma, St. Soatto, J. Košecká, S. Sh. Sastry: *An Invitation to 3-D Vision*. Springer-Verlag, New York 2004.

- H. Stachel: *Zur Kennzeichnung der Zentralprojektionen nach H. Havlicek*. Sitzungsber., Abt. II, österr. Akad. Wiss., Math.-Naturw. Kl. **204**, 33–46 (1995).

- H. Stachel: *Descriptive Geometry Meets Computer Vision — The Geometry of Two Images*. J. Geometry Graphics **10**, 137–153 (2006).

- J. Szabó, H. Stachel, H. Vogel: *Ein Satz über die Zentralaxonometrie*. Sitzungsber., Abt. II, österr. Akad. Wiss., Math.-Naturw. Kl. **203**, 3–11 (1994).

- J. Tschupik, F. Hohenberg: *Die geometrische Grundlagen der Photogrammetrie*. In Jordan, Eggert, Kneissl (eds.): *Handbuch der Vermessungskunde III a/3.* 10. Aufl., Metzlersche Verlagsbuchhandlung, Stuttart 1972, 2235–2295.