Reconstructing Vermeer's Perspective in 'The Art of Painting'

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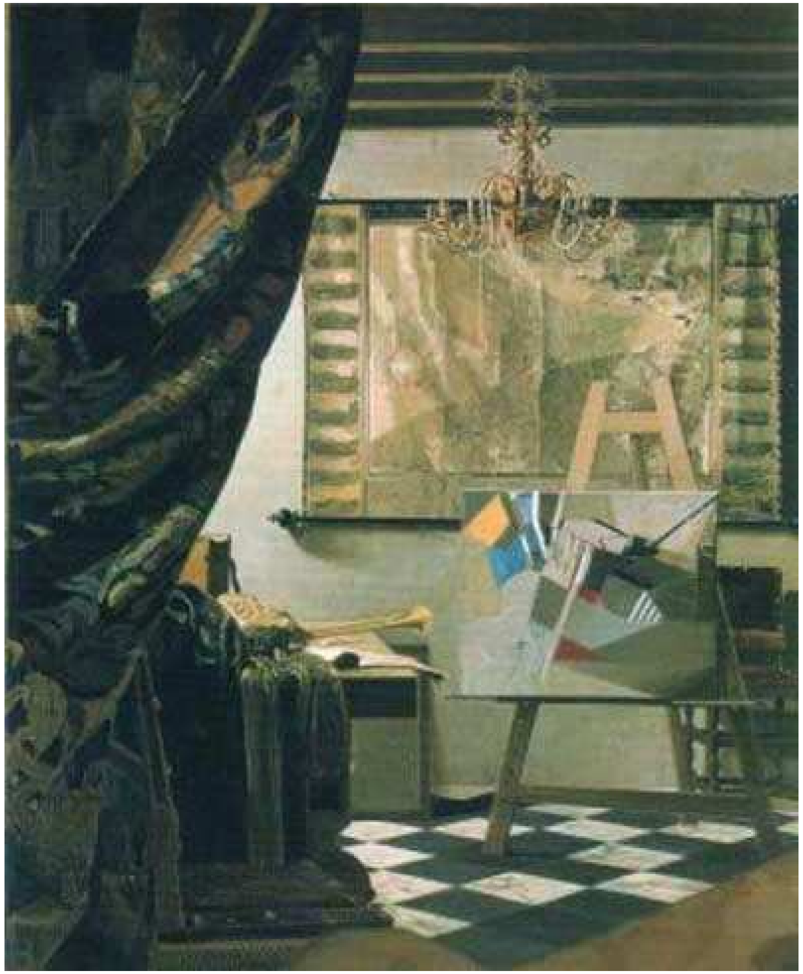
1. Introduction

Johannes Vermeer van Delft
De Schilderkunst
[The Art of Painting]
(1666/1668)

Kunsthistorisches Museum (khm)
1.00×1.20 m

Some months ago there was a particular exposition in khm only devoted to this masterpiece

1. Introduction



Gerhard Gutruf
Hommage á Vermeer
(1973/1976)

Austrian Gallery Belvedere

The idea behind:

Vermeer's imagination of art in contrast to our world of machinery

Gerhard Gutruf is my coauthor. He inspired me to this research.

1. Introduction

Gerhard GUTRUF is a prominent Austrian artist. His pictures have been exposed at numerous national and international exhibitions (e.g., Rome, Mexico City, Pretoria, Beijing, Kiew and recently in Delft).

Since almost forty years he is studying Vermeer's work and discovered many details and new elements of the construction in these paintings.

2. The reason for the reconstruction

This is GUTRUF's sketch to a new version

Gerhard GUTRUF opposes against the general opinion that Vermeer used a *camera obscura* for producing the perspective.

Philip STEADMAN: *Vermeers Camera*, New York 2001.

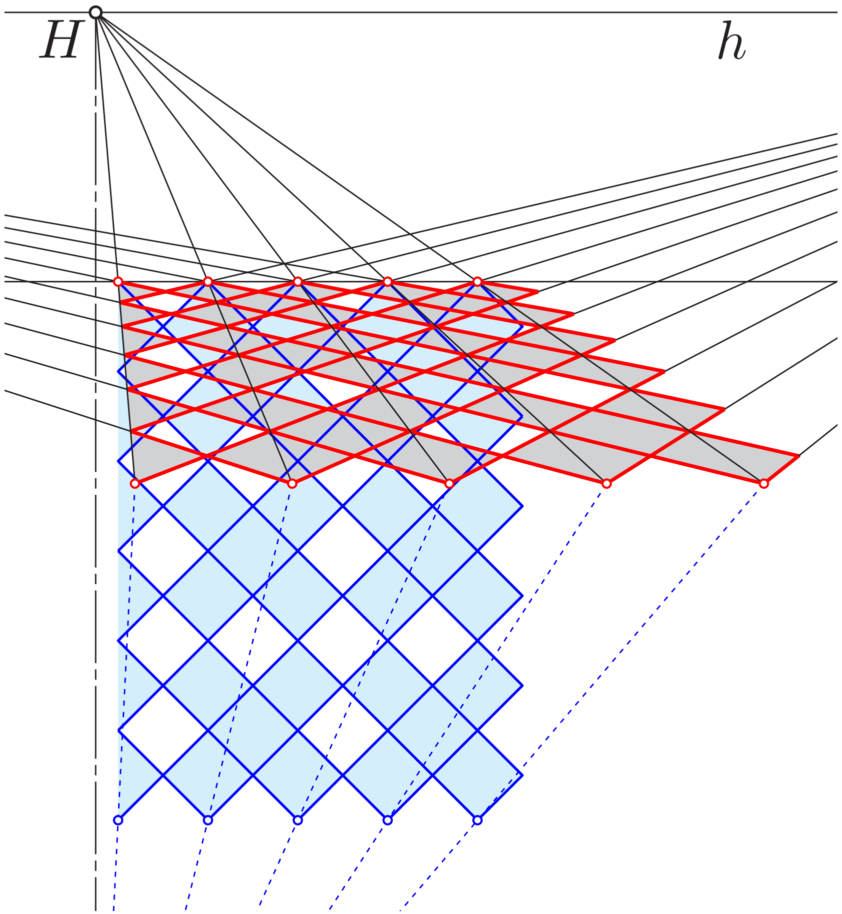
The reconstruction – analytic vs. graphical

It was GUTRUF's intention to find arguments against Steadman's camera-obscura theory.

This can be done by detecting some flaws in the perspective or by proving that the depicted objects are in different scales.

Note that about 20% of the area are hidden by the left curtain.

The reconstruction – analytic vs. graphical

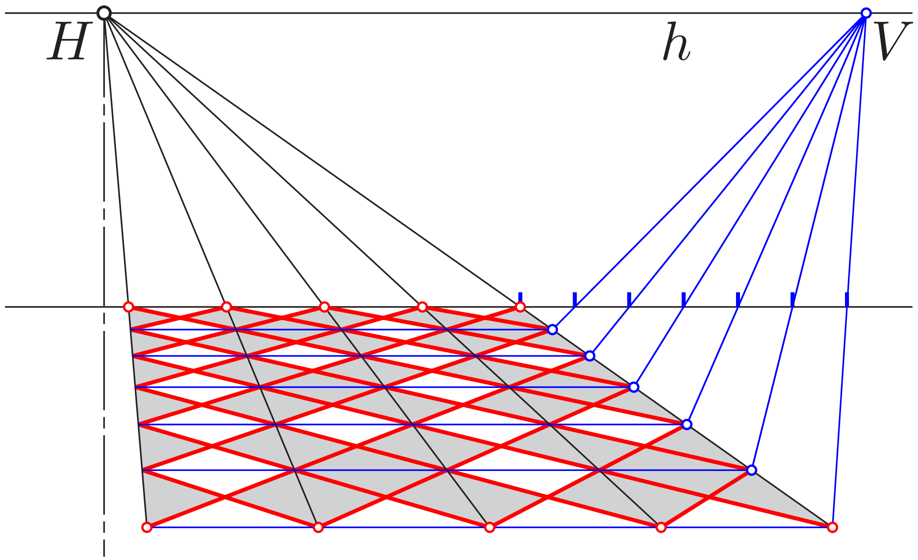


One strong argument against the camera-obscura theory is that the construction of the perspective is not hard.

Obviously, it is a frontal perspective with a vertical image plane. The grid formed by the tiles can easily be determined by a central collineation.

However, important vanishing points are far outside the canvas (about 1 m).

The reconstruction – analytic vs. graphical

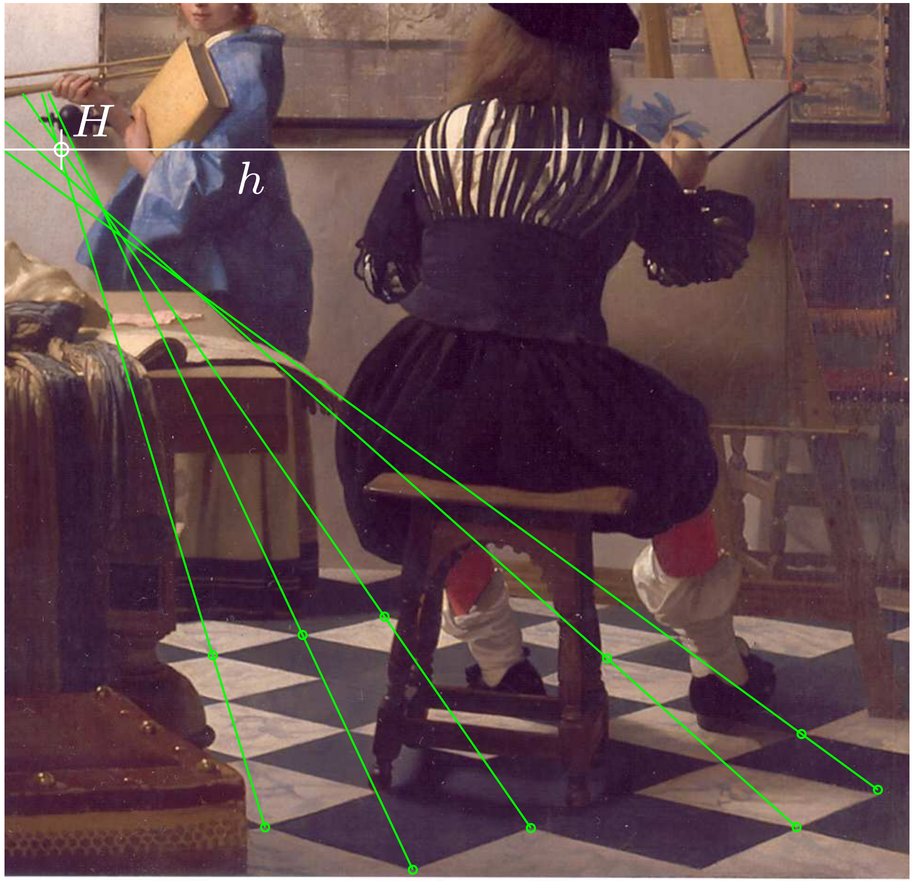


The method displayed on the left-hand side shows how to avoid the non-reachable vanishing points.

This method was known already in the Italian renaissance.

Just recently on the original painting some defect in the canvas was detected in the area of V. Up to now a reasonable explanation for this was missing.

The reconstruction – analytic vs. graphical

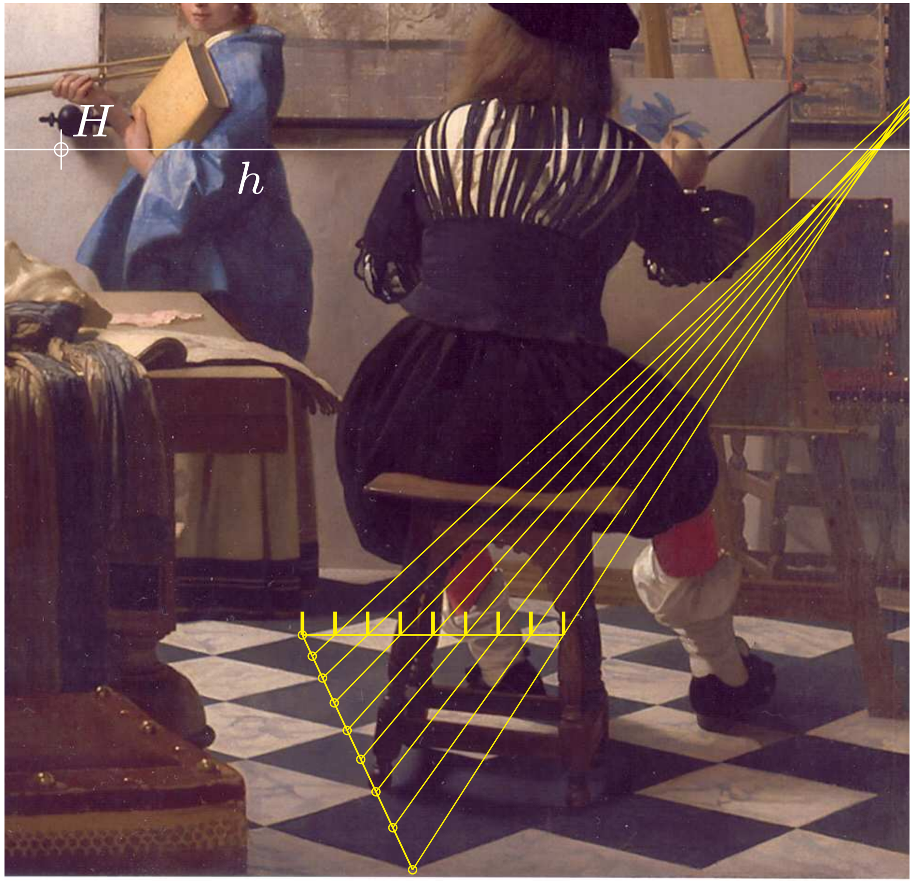


The most striking objects in the perspective are the tiles on the floor.

The graphical method is based on H and h, but because of scattered lines H and h are hard to determine.

What is the best choice of the central vanishing point H and the horizon h ?

The reconstruction – analytic vs. graphical



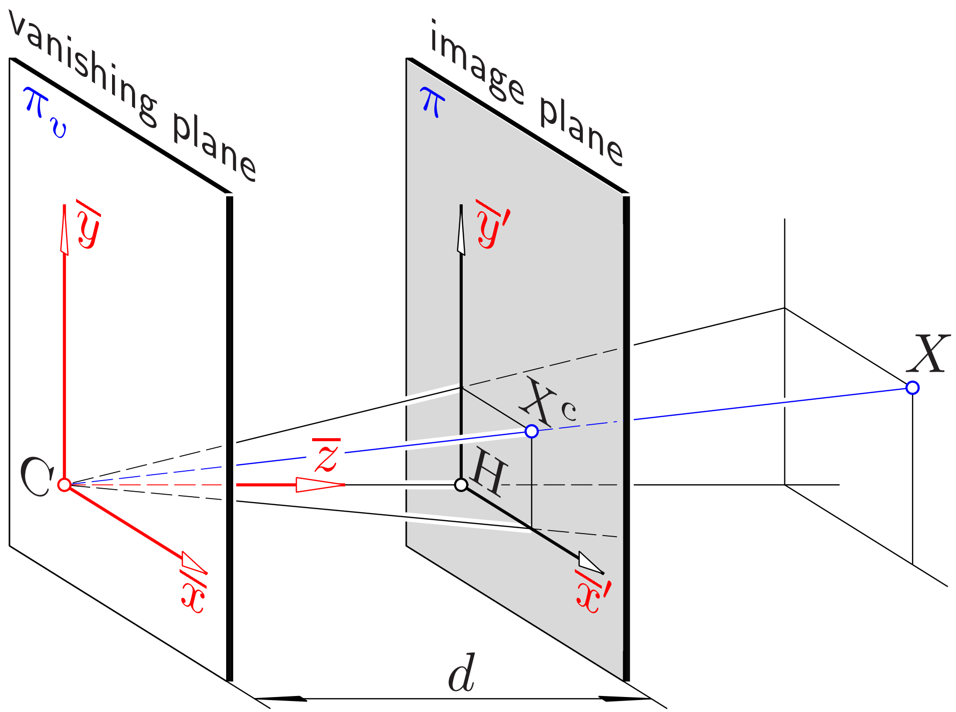
On lines through the central vanishing point the vertices of the tiles form an equidistant scale (parabolic projectivity in the image).

The yellow lines should meet at a vanishing point on the horizon.

However, also this method fails because of the scattering.

3. The analytic reconstruction

Using the camera frame, the mapping equations $X = (\overline{x}, \overline{y}, \overline{z}) \mapsto (\overline{x}', \overline{y}') = X^c$ are very simple.

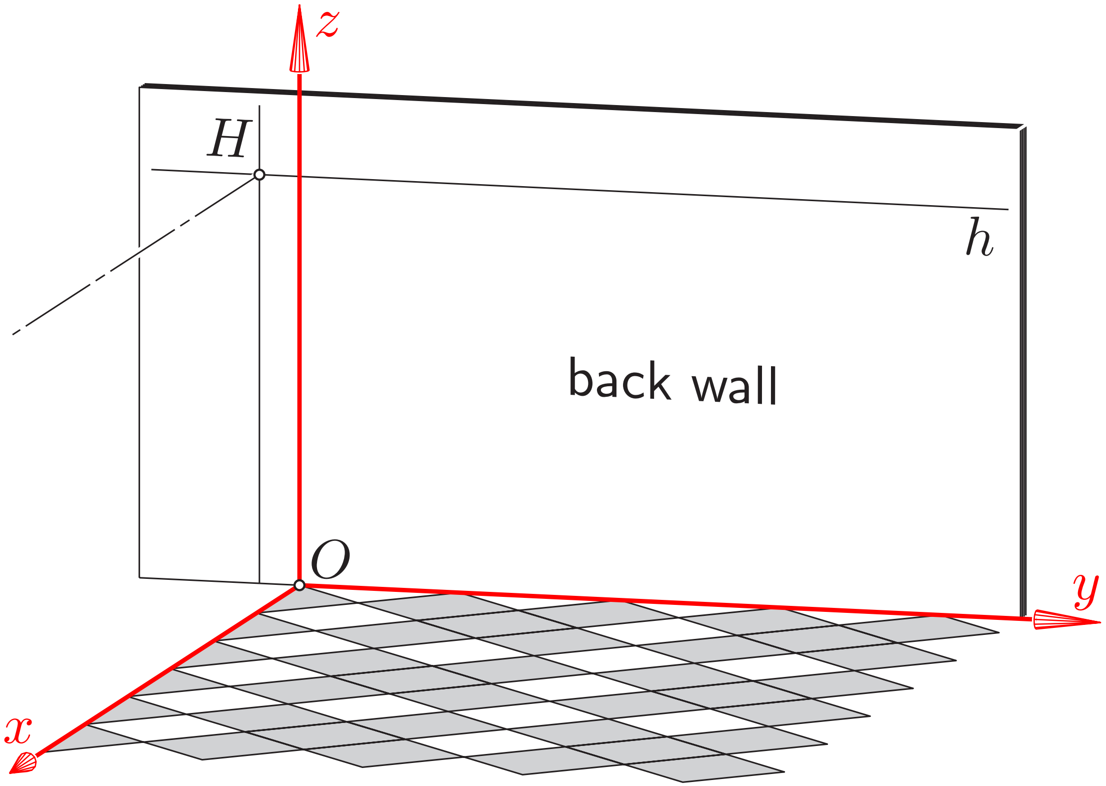


$$\begin{pmatrix} \overline{x}' \\ \overline{y}' \end{pmatrix} = \frac{d}{\overline{z}} \begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix}$$

or in homogeneous coordinates

$$(1 : \overline{x} : \overline{y} : \overline{z}) = (\overline{\xi}_0 : \overline{\xi}_1 : \overline{\xi}_2 : \overline{\xi}_3),$$
$$(1 : \overline{x}' : \overline{y}') = (\overline{\xi}_0' : \overline{\xi}_1' : \overline{\xi}_2')$$

$$\begin{pmatrix} \overline{\xi}_0' \\ \overline{\xi}_1' \\ \overline{\xi}_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \end{pmatrix} \begin{pmatrix} \overline{\xi}_0 \\ \overline{\xi}_1 \\ \overline{\xi}_2 \\ \overline{\xi}_3 \end{pmatrix}$$



We choose the back wall as image plane and change the coordinates for the image from $(\overline{x}', \overline{y}')$ to (x', y') by

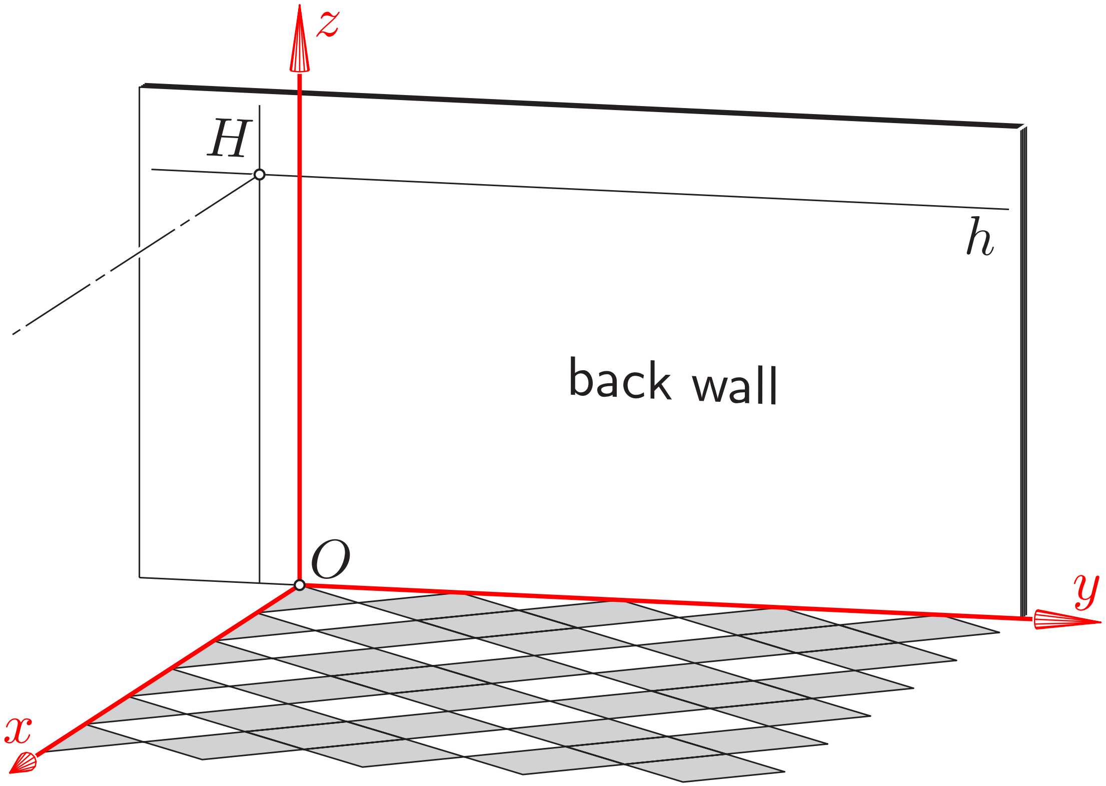
$$x' = x'_H + \sigma_x \overline{x}'$$
$$y' = y'_H + \sigma_y \overline{y}'$$

using scaling factors σ_x, σ_y .

We also replace the camera frame by the world-coordinate frame (x, y, z):

$$\begin{pmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{pmatrix} = \begin{pmatrix} -y_H \\ -z_H \\ d \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

The analytic reconstruction



We come up with the mapping equations

$$x' = x'_H + d\sigma_x \frac{-y_H + y}{d - x}$$
$$y' = y'_H + d\sigma_y \frac{-z_H + z}{d - x}$$

There are 7 unknowns:

Exterior parameters: d, y_h, z_H and the interior parameters $\sigma_x,\ \sigma_y,\ x'_H,\ y'_H$.

The analytic reconstruction

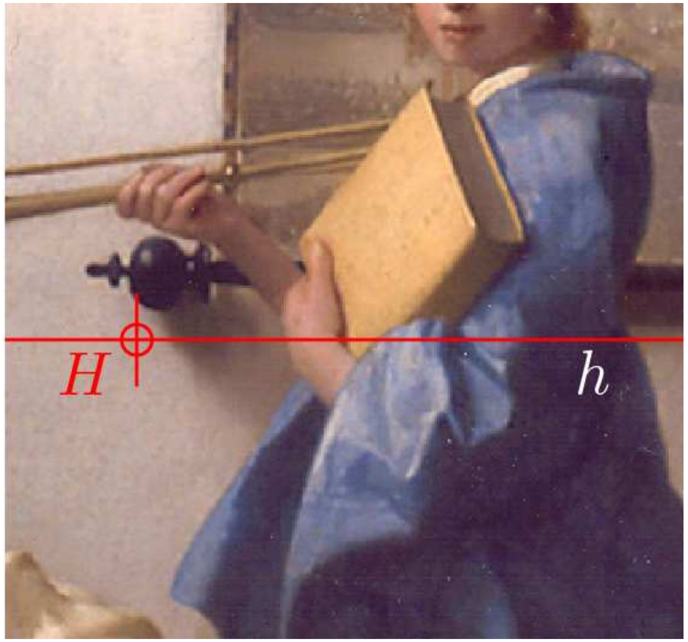
There are 18 vertices of tiles visible with known world coordinates $(x_i, y_i, 0)$ (unit $= \frac{1}{2}$ of tile diagonal).

We measure their image coordinates (x'_i, y'_i) in the scanned painting. This gives two linear equations for each $i \in \{1, \ldots, 18\}$:

$$\begin{aligned} x'_i u_1 - y_i u_2 + x_i u_3 - u_4 &= x_i x'_i \\ y'_i u_1 - z_i u_5 + x_i u_6 - u_7 &= x_i y'_i \end{aligned}$$

with u_j as functions of the 7 unknowns.

The analytic reconstruction



There are standard methods to obtain the 'best' solution of this overdetermined system:

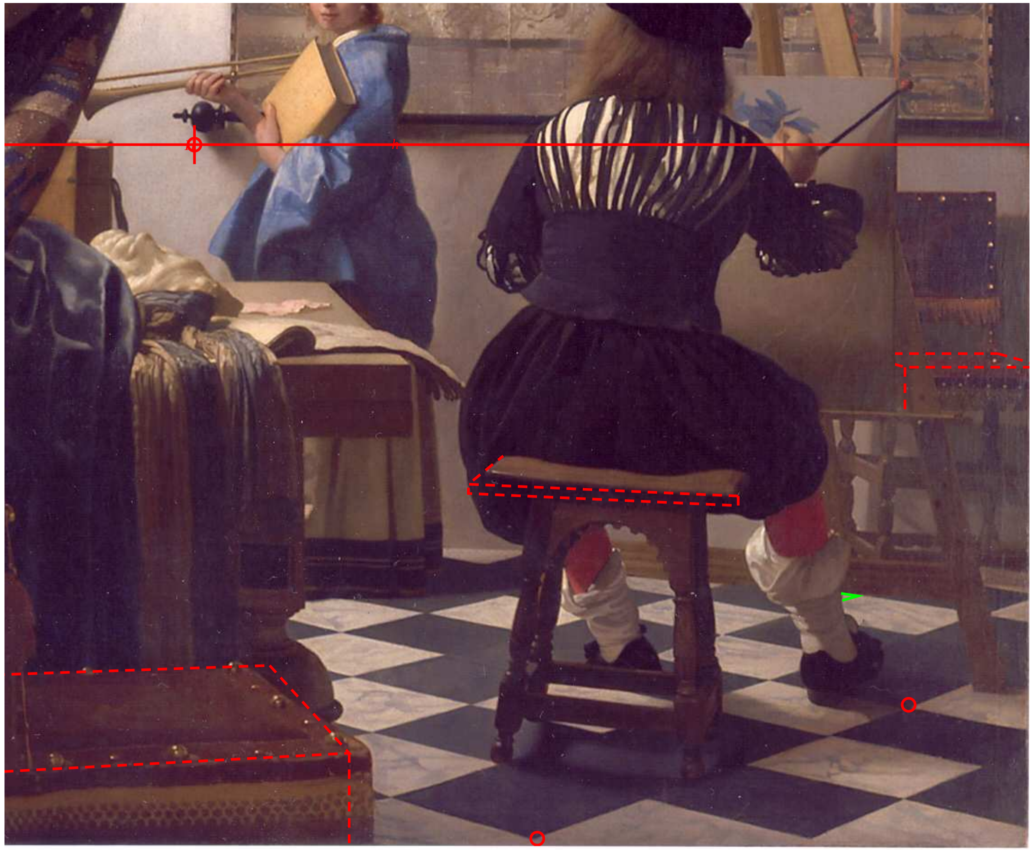
Instead of $\mathbf{A} \cdot \mathbf{u} = \mathbf{b}$ we solve the normal equations

$$(\mathbf{A}^T \cdot \mathbf{A}) \cdot \mathbf{u} = \mathbf{A}^T \cdot \mathbf{b}$$

or apply the pseudo-inverse of $\mathbf{A}$ on $\mathbf{b}$.

At the computed central vanishing point H there is a deformation in form of a hole on the canvas.

The analytic reconstruction

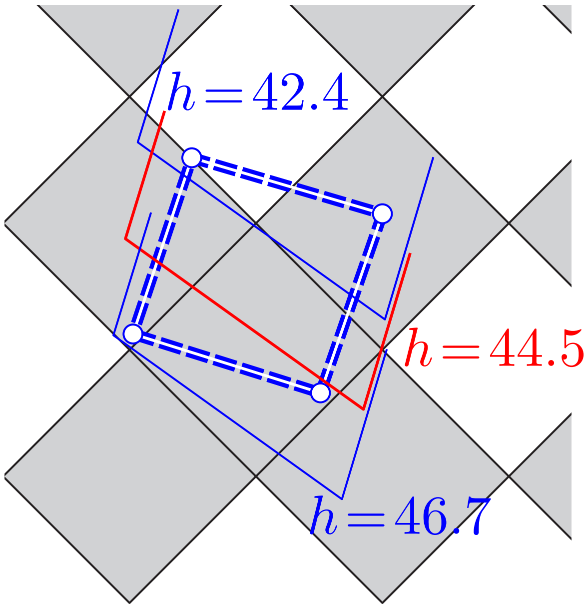


The perspective of the tiles is of remarkable precision

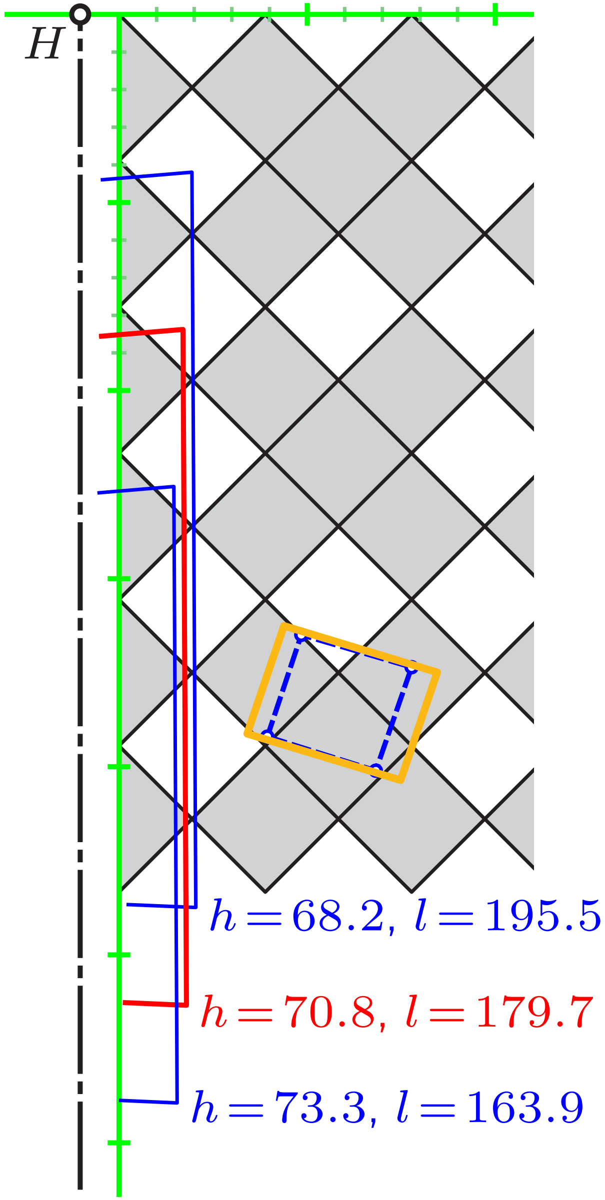
mean error	1.0 mm
maximum error	2.5 mm.

- One corner of a black tile is missing.
- The tiles are not continued in front.
- There are flaws at the chairs
- and at the stool.

The analytic reconstruction

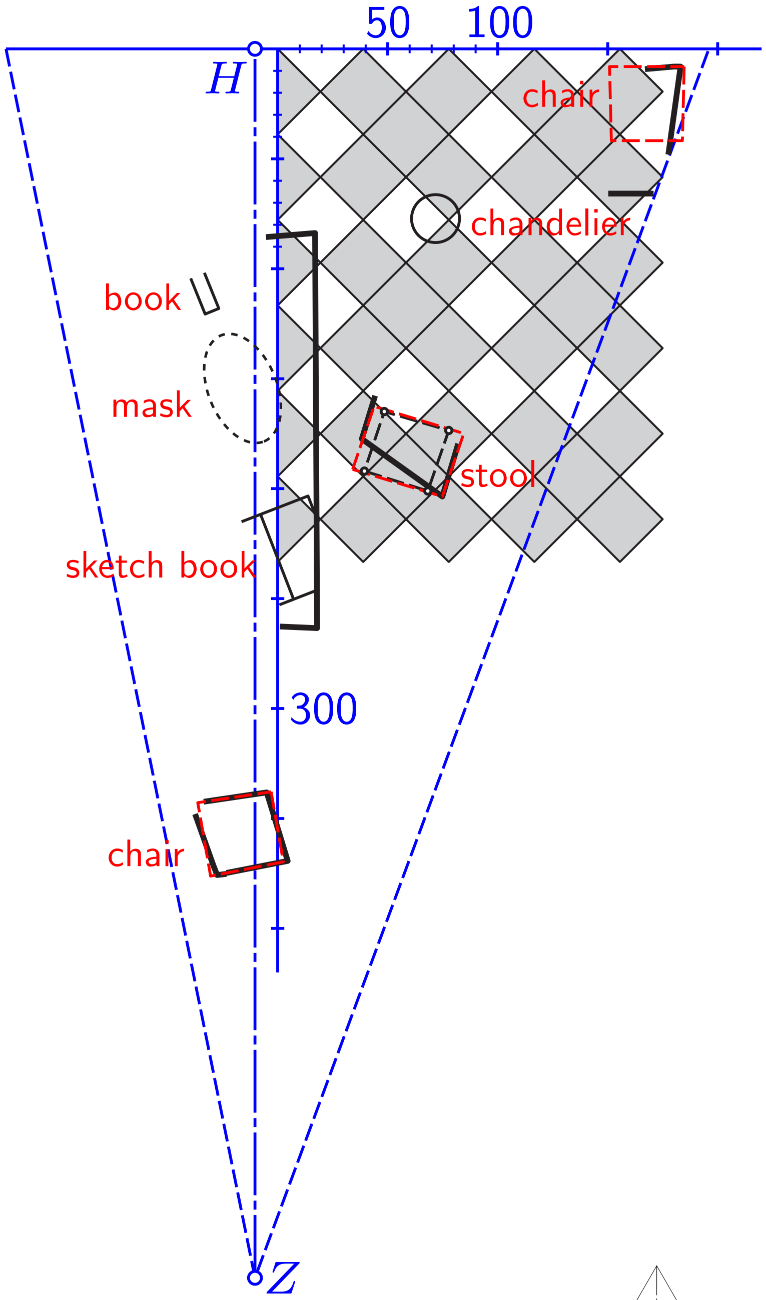


Recovering the height of the stool



Recovering the height of the table

an estimate of the top view →



4. Vermeer's hidden laws of composition

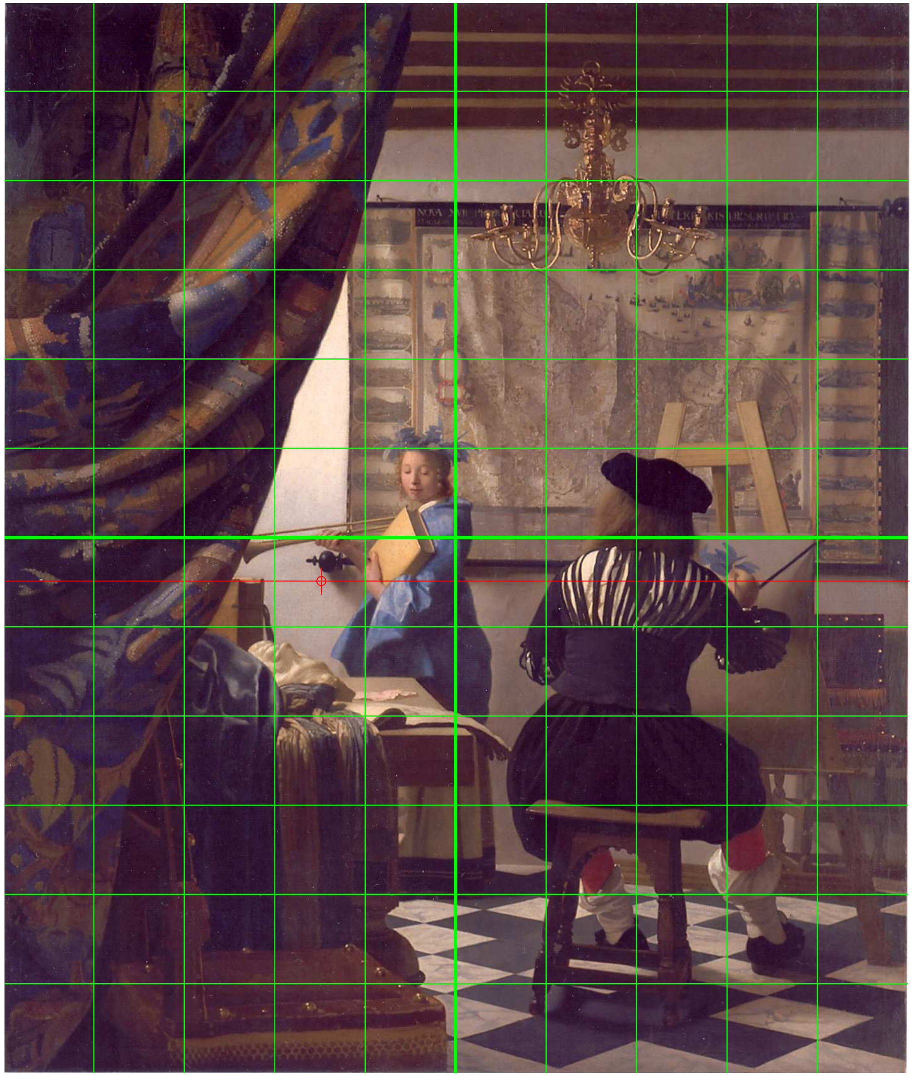
Vermeer himself called this painting 'The Art of Painting'

different to previous titles like *'Woman in blue reading a letter'*, *'Girl with a red hat'*, *'Woman with a pearl necklace'* etc.

G. Gutruf:
'It was a designed masterpiece'.

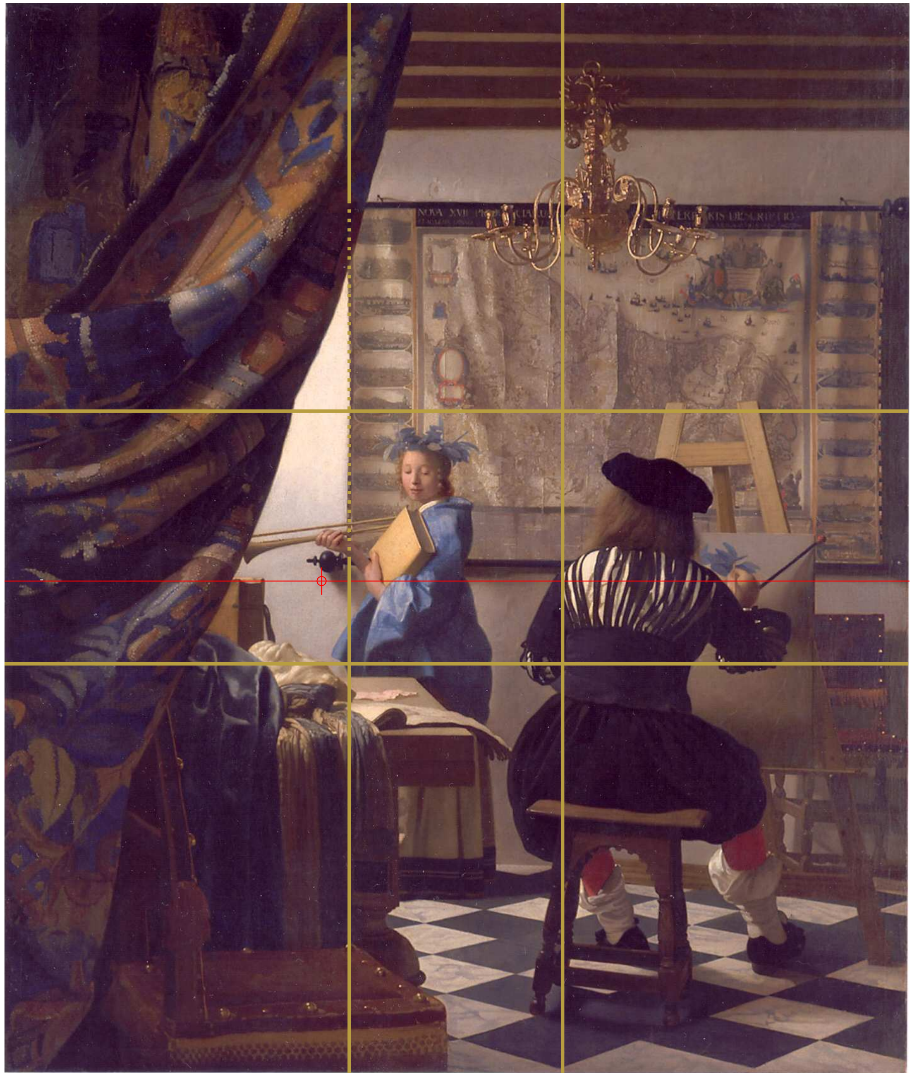
What is meant with 'Art of Painting' ?

Obviously, it is not a 'real' scene like *'The painter and his model'*.



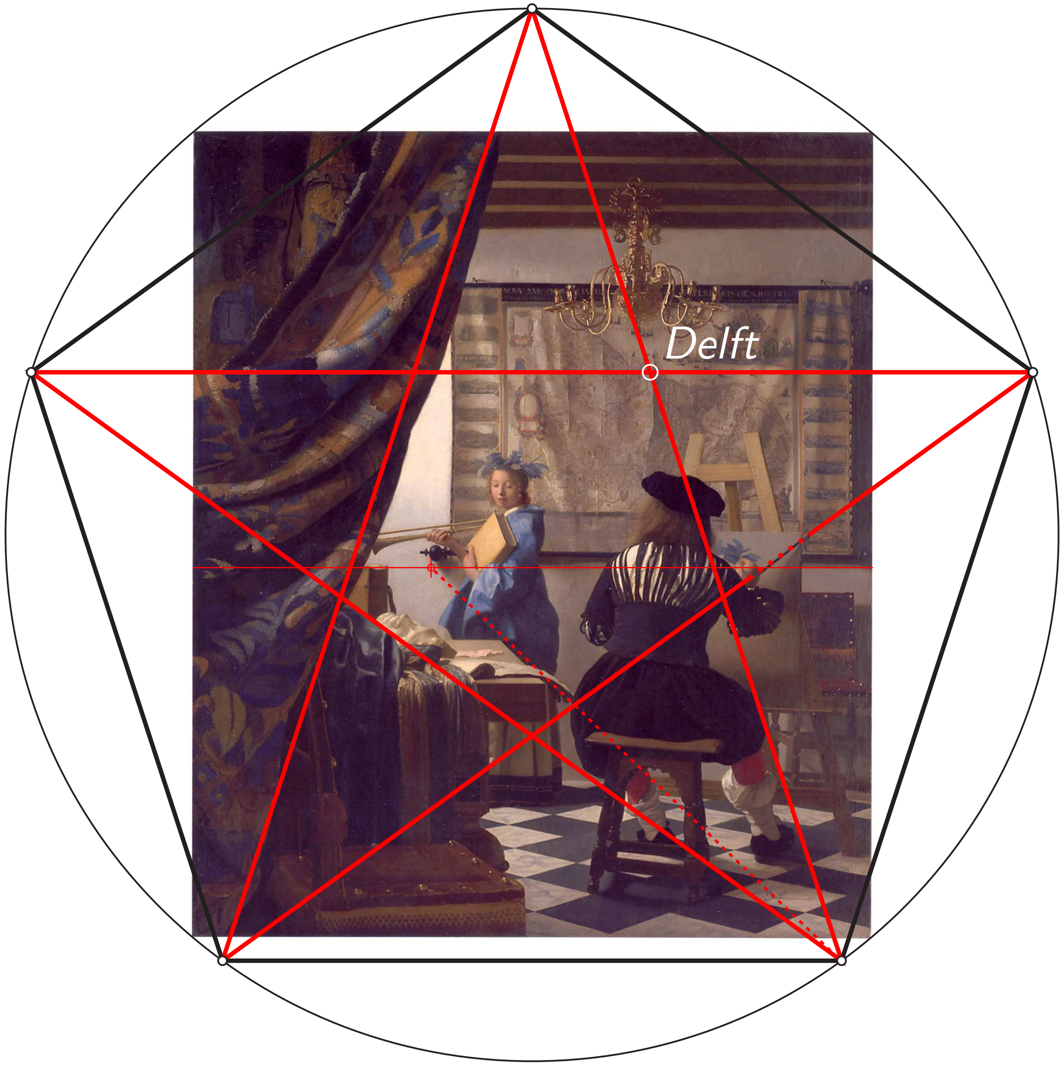
Noting the ratio 5:6:

- the central horizontal line touches the knob of the painting-stick and passes through the trumpet holding hand
- Vertical lines pass through grid points of the tiles
- The central vertical line cuts the roman number XVII of the map — thus reminding on the separation of the 17 provinces of the Netherlands



Noting the golden ratio:

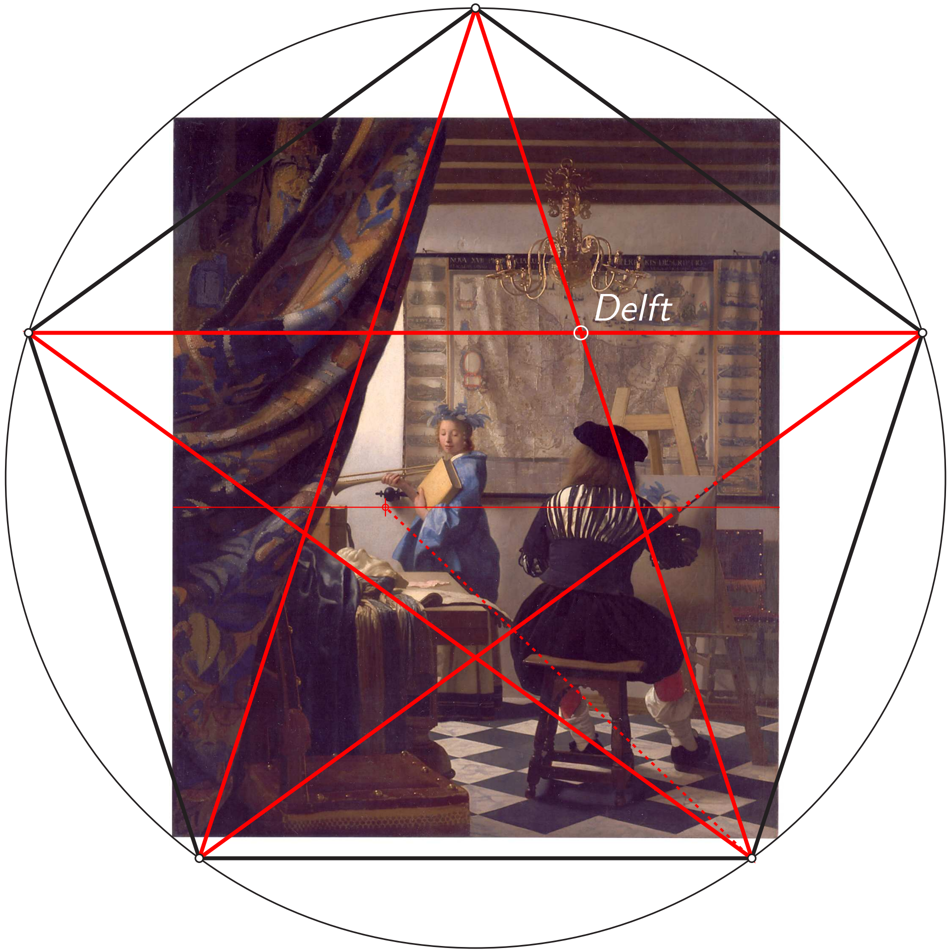
- the left line passes through the left border of the wall-map
- The painter seems to paint the central motive on his canvas



Noting the pentagon:

- the curtain follows the left-hand diagonal
- the right-hand diagonal passes exactly through the painter's stick
- the city of Delft on the map is an intersection point of diagonals

5. Conclusion



The main aim of this research was

- to disclose some of the secrets hidden in Vermeer's masterpiece, and
- to contradict Philip Steadman's camera-obscura theory