THE STATUS OF TODAYS DESCRIPTIVE GEOMETRY RELATED EDUCATION (CAD/CG/DG) IN EUROPE

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ABSTRACT: The goal of my presentation is to explain what Descriptive Geometry is good for and in which way the Descriptive Geometry education is carried out in European countries.
By definition, Descriptive Geometry is a method to study 3D geometry through 2D images. It provides insight into structure and metrical properties of spatial objects, processes and principles. According to this, Descriptive Geometry courses in central Europe cover not only projection theory, but also modeling techniques for curves, surfaces, and solids thus offering insight into a broad variety of geometric shapes. 'Learning by doing' is an important methodological principle in this subject, and one traditional goal is to develop and to refine the students' problem-solving skills.
Drawings are the guide to geometry but not the main aim. As the drawing tools have drastically changed in the last 15 years, this had consequences for the Descriptive Geometry education. CAD packages replace manual drawings. This made the subject more interesting and attractive for pupils and students because they now can produce high-quality rendered graphics as output. Of course, this development takes place at the cost of the training in geometric reasoning.

Keywords: Graphics education, Descriptive Geometry, spatial abilities, geometry applications

1. INTRODUCTION

In the hierarchy of sciences Descriptive Geometry is placed somewhere within or next to the field of Mathematics, but also near to Architecture, Mechanical Engineering, and Engineering Graphics. I start with definitions and continue with a few examples in order to highlight that Descriptive Geometry provides a training of the students' intellectual capability of space perception and is therefore of incontestable importance for all engineers, physicians and natural scientists. The paper ends with an outlook on recent developments and future needs.

2. HOW TO DEFINE 'DESCRIPTIVE GEOMETRY'

In many American textbooks on Engineering Graphics the subject Descriptive Geometry seems to be restricted to standard constructions like the determination of the true length of a line segment or the intersection of two plane polygons in 3-space. From this point of view it must look rather strange that prominent geometers devoted their whole academic life to promote this subject

2.1 Descriptive Geometry in Europe

In order to explain the meaning of 'Descriptive Geometry' in central Europe, let us look for different definitions presented in German textbooks from the last five decades:

J. Krames defined in [2]: *"Descriptive Geometry is the high art of spatial reasoning and its graphic representation"*.
My teacher F. Hohenberg formulated in [1]: *"Descriptive Geometry teaches how to grasp, to imagine, to design, and to draw geometrical shapes."*

I prefer the following

Definition: *'Descriptive Geometry' is a method* *to study 3D geometry through 2D images. It provides insight into structure and metrical properties of spatial objects, processes and principles. Typical for Descriptive Geometry is the interplay*

- *between the 3D situation and its 2D representation, and*
- *between intuitive grasping and rigorous logical reasoning.*

According to this, Descriptive Geometry courses in central Europe cover not only projection theory, but also modeling techniques for curves, surfaces, and solids thus offering insight into a broad variety of geometric shapes. Besides, an intuitive approach to elementary differential-geometric properties of curves and surfaces and some 3D analytic geometry is included

2.2 G. Monge's definition

Gaspard Monge (1746–1818) is declared the founder of the science of Descriptive Geometry. This does not mean that he himself developed all the graphical methods. In contrary, most of them can already be found in earlier books, e.g., in those of A. F. Frezier.

However, G. Monge was a most effective scientist and manager who spread his ideas on Descriptive Geometry with the publication of his 'Leçons de géométrie descriptive' (1799) from France over whole Europe. We find in [3], p. 1, the following introductory statement:

"*La Géométrie descriptive a deux objets:*
le premier, de donner les méthodes pour représenter sur une feuille de dessin qui n'a que deux dimensions, savoir, longueur et largeur, tous les corps de la nature qui en ont trois, longueur, largeur et profondeur, pourvu néanmoins que ces corps puissant être définis rigoureusement.
Le second objet est de donner la manière de reconnaître, d'aprés une description exacte, les formes des corps, et d'en déduire toutes les vérités qui résultent et de leur forme et de leurs positions respectives."

This proves that the two main objectives of Descriptive Geometry – imaging and analyzing 3D objects – date back to its founder.

2.3 Further comments on the definition

It is remarkable that the word '*drawing*' does not appear in Monge's definition. In Descriptive Geometry drawings[1] are the *guide to geometry* but not the main aim. We teach geometry instead of construction techniques. Note that the French *'descriptive'* means 'describing' or 'representing' but not necessarily 'graphically depicting'.

Nevertheless, in the public meaning Descriptive Geometry has falsely become synonymic for 'manually drawing images of 3D objects'. As in the last decades manual drawing with traditional instruments has been replaced by CAD or mathematical software with graphic output, 'people on the road' frequently conclude that therefore Descriptive Geometry has become obsolete. However, this is totally wrong, in contrary:

- Only people with a profound knowledge in Descriptive Geometry are able to make extended use of CAD programs as the communication is usually based on views only. For similar reasons the importance of mathematics is still increasing though computers take over the computational labour.
- The more powerful and sophisticated a modeling software, the higher the required geometric knowledge (cf. Figure 5).
- A poor designer will never become perfect only by using CAD instead of traditional tools.
- *Descriptive Geometry* is more than 'descriptive' geometry as well as *Geometry* is more than its literal sense, i.e., 'measuring the earth'.

Another misinterpretation of Descriptive Geometry is to consider it only as a theoretical, more or less 'academic' subject. F. Hohenberg could disprove this opinion in his textbook [1] in

[1] It is said that Felix Klein once stated: "*Among all mathematicians, geometers have the advantage to see what they are studying.*"

a convincing way. In many examples he demonstrated application of Descriptive Geometry to real-world requirements.

In order to defend the true meaning of Descriptive Geometry, there were various attempts to rename this subject. Its applicability is stressed by using the names 'Technical Geometry' or 'Applied Geometry'. Another choice is 'Constructive Geometry' – 'constructive' in its figurative sense. It should indicate that not only manual drawings but also mathematical computations are used in this subject.

Anyway, the original Monge definition of 'Descriptive Geometry' with its wide meaning covers all these aspects. So, in my opinion the original name is still appropriate. However, some find this name old-fashioned. For strategic reasons they are seeking for more attractive designations which make evident that temporary courses on Descriptive Geometry include also some methods from computer science like '*geometric modeling*' or '*visualization techniques*' and of course the usage of any CAD program. In this sense '*Geometric Modeling and Visualization*' or more briefly '*Modeling and Imaging*' could be appropriate and perhaps more in fashion.

3. WHAT SHOULD REMAIN IN A STUDENT'S BRAIN

In order to estimate the educational effect of any single subject in a curriculum, one should try to figure out what remains in the students' brains after all details are already forgotten. I would like to state that even for poor students the education in Descriptive Geometry brings about the ability

1) to comprehend spatial objects from given principal views, and

2) to specify and grasp particular views. In addition,

3) the students get an idea of geometric idealization (abstraction), of the variety of geometric shapes, and of geometric reasoning.

The first two items look rather elementary. However, these intellectual abilities are so fundamental that many people forget later how hard it was to achieve these abilities.

3.1 The importance of principal views

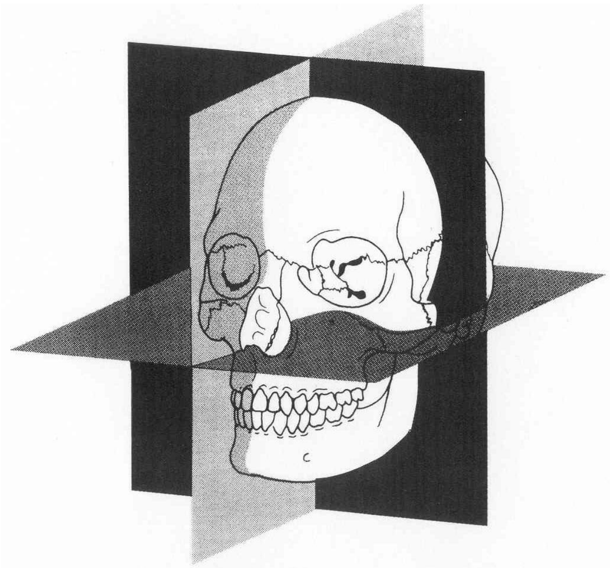

Figure 1 Explanation of the principal views taken from a textbook for dentists

Familiarity with the principal views – *top view, front view, and side view* (see Figure 1) – are substantial for several reasons, e.g.,

- they are more or less abstract as they do not correspond to our personal visual impression. But abstraction simplifies.
- In the majority of cases they better make evident the essential properties of spatial structures, and
- inspecting these planar views is much easier than to concentrate on the original spatial structure.

However, it needs training to become familiar with this kind of representation and to grasp the shape of any 3D object just by looking at its principal views. Nobody questions the necessity of a permanent training for sportsmen. But in case of Descriptive Geometry, people often neglect this necessity and they speak of a purely academic subject, when, e.g., in introductory exercises two triangles in space are to intersect.

3.2 The art of specifying particular views

Axonometric views are important and well understandable for everybody. And they are appropriate to remember on a known object or to compare with a real object nearby. However, no angle, no length, no planar shape appears in true size. Orthogonality can be figured out only because of some additional assumptions based on experience or estimation. So, these views are never sufficient for a '*description exacte*' as required in Monge's definition.

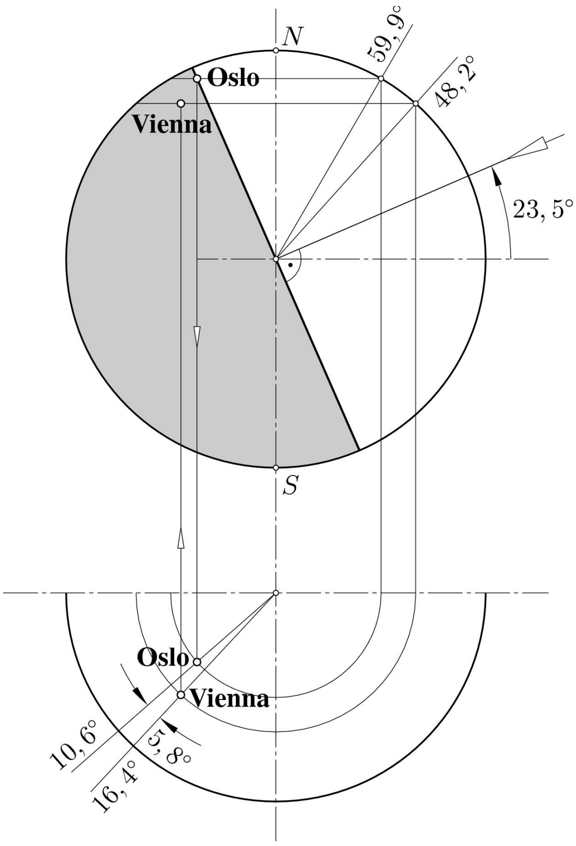

Figure 2 Where does the sun rise earlier on June 21, in Oslo or in Vienna

For a detailed analysis of any 3D object such *particular views* (auxiliary views) with planes in edge view or lines in point view really can reveal the true spatial situation. Such views often are the key to the solution of a 3D problem. As examples note the problem addressed in Figure 2 and also the complex process displayed in Figure 4.

In my opinion these particular views make the *sublime art* of Descriptive Geometry. Only in such courses the students learn what conditions can be simultaneously fulfilled in particular views and how such views can be specified.

3.3 Views are a guide to spatial geometry

I don't know if anybody is able to manipulate 3D objects mentally, i.e., in his imagination only, and to figure out how these objects look like in different postures without using paper and pencil. Maybe, sculptors or pilots have this mental ability. Actually, I myself don't – even under highest mental exertion. And the *rhombic dodecahedron* (Figure 3) serves for me as a convincing example:

This convex polyhedron can be built by erecting a quadratic pyramide with 45° inclined planes over each face of a cube. As any two coplanar triangles can be glued together forming a rhomb, this polyhedron has 12 congruent faces and seems to be well understood. Nevertheless, I'm not able to imagine (with closed eyes) how this polyhedron looks like from above when it is resting with one face on a table. Fortunately, a simple freehand sketch helps to figure out this view (upper left view in Figure 3) as well as other remarkable properties.

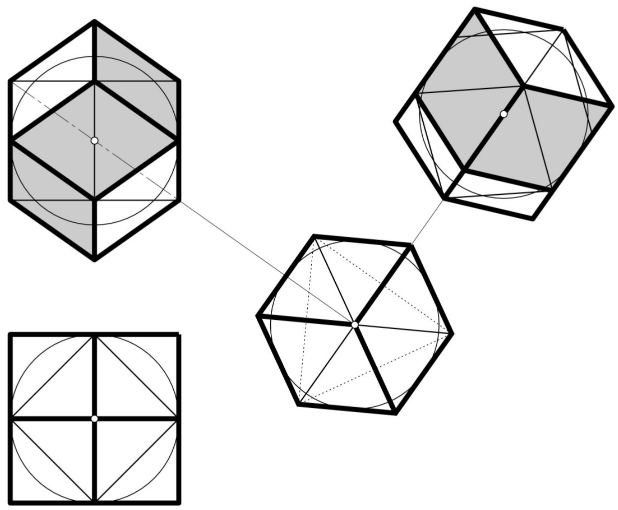

Figure 3 Different views of the rhombic dodecahedron

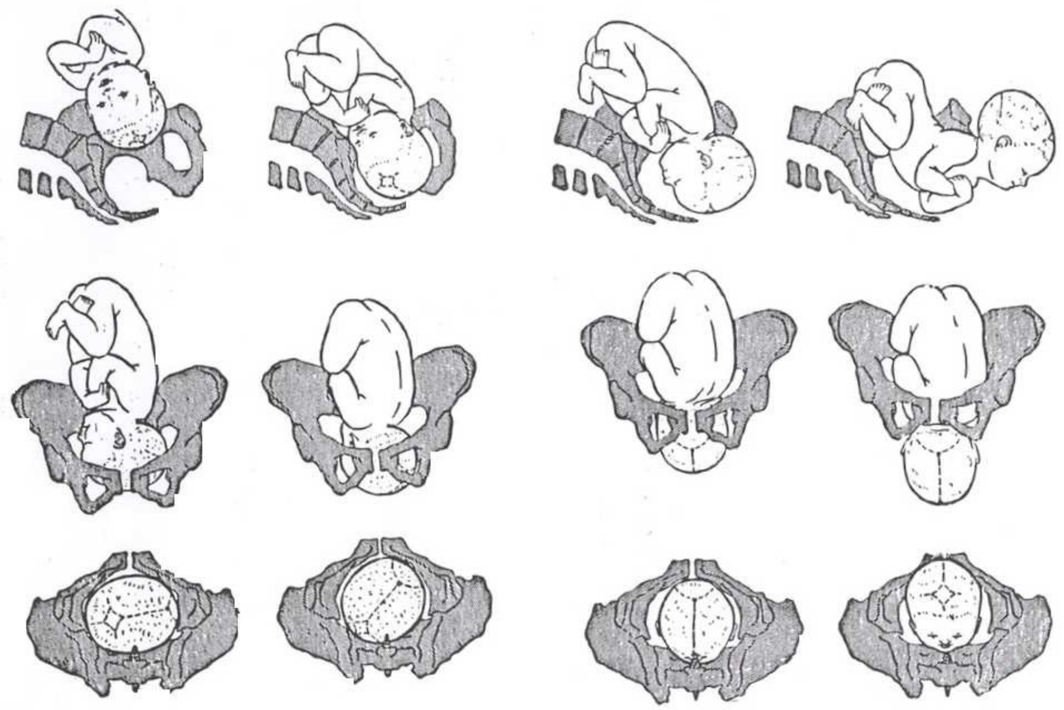

Figure 4 There is no better way to explain the baby's spatial movement when being born

4. DESCRIPTIVE GEOMETRY IN PRESENCE OF COMPUTERS

The statistics in [5], Fig. 5, reveals the high effect of traditional Descriptive Geometry education on improving spatial ability. However, in the frame of permanently changing curricula one has to present the topics in actualized form such that they are attractive for the majority of young people. So compromises are necessary.

4.1 Pros and Cons in modern Descriptive Geometry education

Let me summarize in abbreviated form:

What is obsolete:

- complicated manual constructions,
- hard theoretical proofs,
- the theory of how to obtain images of particular 3D objects.

What is still necessary:

- '3D-competence', i.e.,
- the capability to comprehend 3D objects and situations from given images,
- mental orientation in 3-space (e.g., user coordinate system),
- basic knowledge of 3D geometry,
- promoting creativity and problem-solving skills,
- applications of geometry,
- producing attractive illustrations.

Additional demands:

- Handling software for geometric modeling and visualization,
- treating new geometric shapes (e.g., B-spline surfaces),
- competence in handling graphics files (in different format),
- design of animations.

4.2 How to meet these demands

New tools must have an impact on education. This is not typical for Descriptive Geometry alone. Similar effects show up in other subjects like Mathematics or Mechanics. Students can solve more and more complex problems using computer software as a ‘black box’ while there is still a lack of basic understanding. But for me this seems to be a problem of methodology, of the right balance between imparting knowledge on the one hand and the intelligent use of powerful computer programs on the other. I’m convinced that there will be a way in the near future to combine the benefits of educational CAD programs with the training of spatial reasoning in an optimized way.

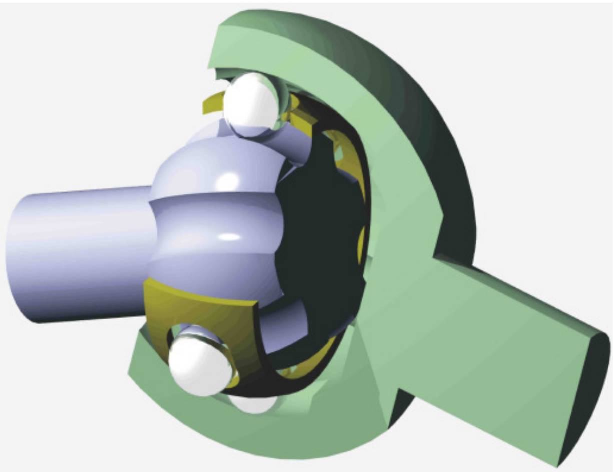

Figure 5 A constant-velocity ball joint, an interesting exercise for solid modeling

Within the last 15 years the Descriptive Geometry education in central and East European universities has changed like everywhere. It has been reduced – in particular in Mechanical Engineering, while in Civil Engineering and Architecture this subject is still in a strong position. CAD packages combined with free-hand sketches are more and more replacing manual constructions, and often the introduction into CAD programs is included. If for students of Computer Science some geometry courses are held then they preferably introduce into Analytic Geometry and B-spline-techniques. In addition, in Austria there is still the optional subject Descriptive Geometry in secondary schools.

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