Isothermic contrained Willmore tori

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For a given abstract closed surface M, there exist many different immersions into the euclidean space $\mathbb{R}^3$. As a natural consequence, the question arises which immersions of a given surface M, are the most 'round' or symmetric ones. An answer to this question was given in 1965 by Thomas James Willmore by defining an energy for immersions of surfaces using their mean curvature H.

The critical points of the Willmore energy are called Willmore surfaces and play an important role in differential geometry, elasticity theory and computer graphics.

If we fix a conformal structure on the surface M and consider only variations of the immersion that preserve this structure, the critical points of the Willmore functional are called constrained Willmore surfaces. The Clifford torus has a Willmore energy of $2\pi^2$ and realizes the absolute minimum among all immersions of tori into $\mathbb{R}^3$, and hence in its conformal class. To find the explicit minimizers of the Willmore energy among all tori in an arbitrary prescribed conformal class is still an open problem. Peter Li and Shing-Tung Yau proved that compact surfaces with a Willmore energy smaller than 8π are embedded. Therefore, embedded surfaces are the best candidates for minimizing the Willmore energy and it is an interesting question which constrained Willmore tori are embedded. Numerical experiments and further results of Lynn Heller et al. suggest that immersions of tori that minimize the Willmore energy in their conformal class should be isothermic. In this talk, we will present an classification of isothermic constrained Willmore tori as constant mean curvature surfaces in some spaces with constant sectional curvature. Then we will discuss which of them can be embedded, to obtain the following classification of embedded isothermic constrained Willmore tori:

After a stereographic projection, every embedded isothermic constraint Willmore torus is either an isothermic Bryant surface with smooth ends or Möbius equivalent to a surface of revolution in S^3.