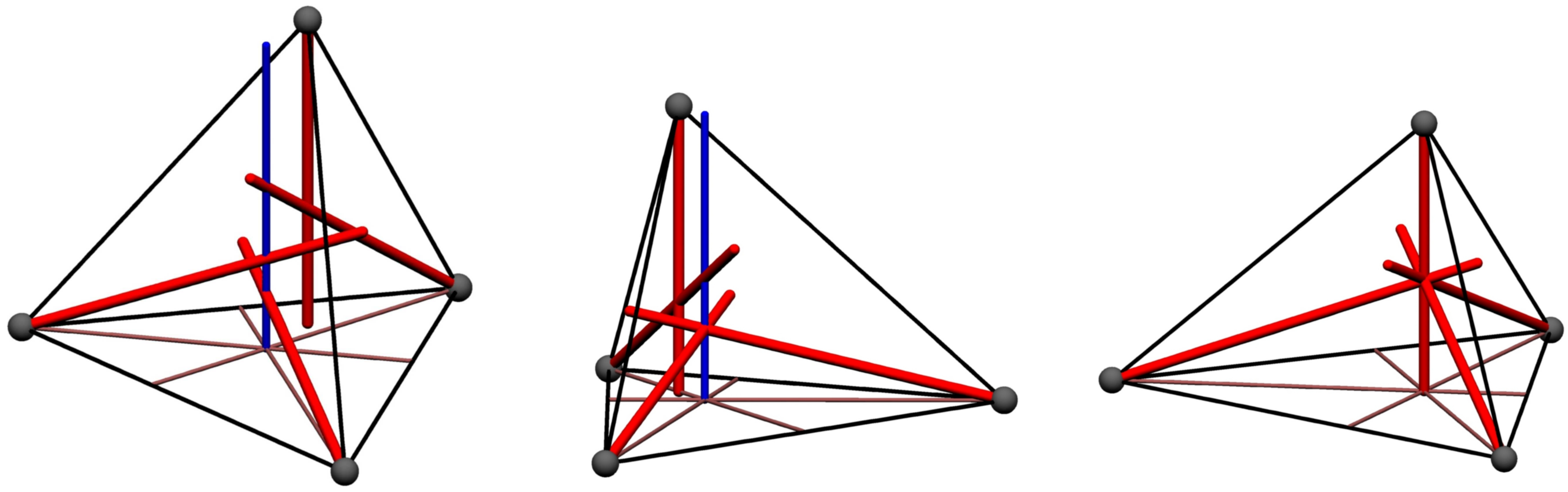
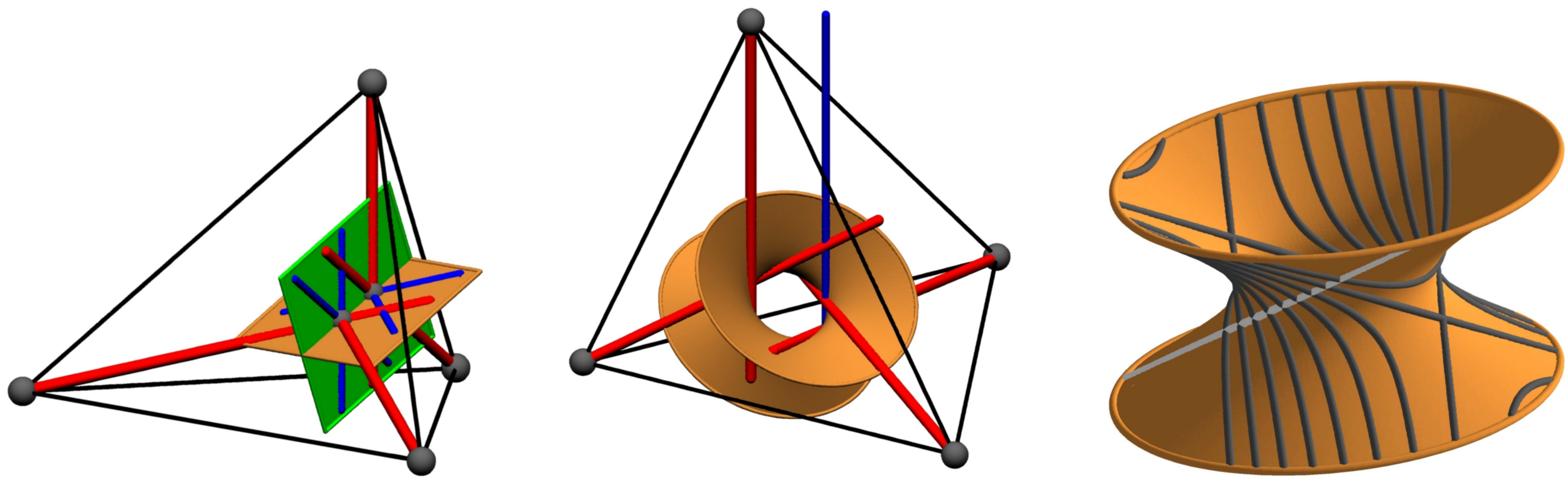
Forschungsbereich

ELEMENTARGEOMETRIE

In Zusammenarbeit mit dem Institut für Geometrie der TU Dresden.



Ein allgemeines, ein semi-orthozentrisches und ein orthozentrisches Tetraeder.



Die Höhen eines nicht orthozentrischen Tetraeders liegen auf einer Quadrik.

Literaturhinweis:

Hans Havlicek und Gunter Weiß: Altitudes of a tetrahedron and traceless quadratic forms, *Amer. Math. Monthly* **110** (2003), no. 8, 679–693.

Aus den Rezensionen:

This is a very well written survey of some topics in the geometry of the tetrahedron. Although the subject belongs to that 19th century monument that has been systematically neglected called old-fashioned geometry, the method of solution employed throughout the paper is that of linear algebra, and the paper can be read as making a brilliant point for the teaching of linear algebra with its geometric counterpart in mind. The subjects treated are: the Monge point of a tetrahedron, the Euler line in space, Steiner's hyperboloid (generated by the altitudes) of a tetrahedron. A wealth of figures make for enjoyable reading.

Zentralblatt für Mathematik

This self-contained article provides a clean, vector-based approach for proving several classic results concerning the altitudes of a tetrahedron. The authors motivate their discussion by observing that, unlike many other special points associated with a triangle, the orthocenter does not have a readily apparent counterpart for tetrahedra, because in general the altitudes are mutually skew lines in space. This state of affairs is redeemed by the Monge point M, the point of concurrency of four lines which are parallel to the altitudes. (In two dimensions, the Monge point is equivalent to the orthocenter, so it provides a reasonable generalization to higher dimensions.) In the process of constructing the Monge point several other well-known results are revisited, such as the fact that if a pair of altitudes in a tetrahedron intersect, then so must the other pair; or that the four altitudes are concurrent if and only if all opposite pairs of edges are orthogonal. One is then led to consider the Euler line in space, which passes through the circumcenter, centroid, and Monge point of an arbitrary tetrahedron. Finally, there is a nice treatment of the quadric surface that contains and is generated by the four altitudes. This surface, called an equilateral hyperboloid, is the level set of a certain traceless (trace equals zero) quadratic form. Crisp illustrations generated by Maple appear throughout the paper, providing an effective means for visualizing these results.

Mathematical Reviews

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