Navigating a Family of Particular Linear Weingarten Surfaces

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In this talk, I will discuss results I have recently obtained about surfaces of revolution that have an affine-linear relationship between their principal curvature radii, $\rho_1 + m\rho_2 = c$, with an outlook towards more-thoroughly investigating questions about this family of surfaces within Lie sphere Geometry.

These surfaces were originally found by Hopf [1] with parametrizations given by an integral equation. However, the computation of this integral is, in general, difficult. The novelty of my results is that an integral is obtained that is easier to compute, quickly getting explicit parametrizations for a countably-infinite many of these surfaces, while alongside finding explicitly their asymptotic/characteristic parametrizations and being able to determine their algebracity.

Those computations gave way to questions as to the theory behind family — in particular, about how members of this family are interrelated. Coming from those computations, some relationships between members became apparent; however, there is still a lack of understanding of the parameter m. To the end of understanding this parameter better, and thusly, the family on the whole, it may be helpful to consider those questions within Lie sphere Geometry. The starting off point for this is two papers: In this direction, for another kind of linear Weingarten surfaces, there is a paper by Udo et al. [2]. And for the particular case of surfaces of revolution viewed as channel surfaces, there is a paper by Mason and Gudrun [3], which discusses channel surfaces in Lie sphere geometry.

[1] Heinz Hopf, Über Flächen mit einer Relation zwischen den Hauptkrümmungen, Math. Nachr. 4 (1951), 232-249. MR40042

[2] Francis E. Burstall, Udo Hertrich-Jeromin, and Wayne Rossman, Lie geometry of linear Weingarten surfaces, C. R. Math. Acad. Sci. Paris 350 (2012), no. 7-8, 413-416. MR2922095

[3] Mason Pember and Gudrun Szewieczek, Channel surfaces in Lie sphere geometry, Beitr. Algebra Geom. 59 (2018), no. 4, 779-796. MR3871108