The motion of billiards in ellipses

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A billiard in an ellipse e arises from any ray in the interior after iterated reflections in e. Due to the optical property of ellipses, all sides of the billiard contact a confocal conic, the caustic c. If any billiard closes under a finite number of reflection, then this holds wherever the inital point is chosen (=Poncelet porism). This allows a variation of periodic billiards in the ellipse e, which will be called a 'billiard motion' though it is neither an isometry nor an affine or projective motion. We focus on the cases with an ellipse as caustic.

Based on computer experiments, the Brazilian engineer Dan Reznik detected recently around 40 invariants of the motion of periodic billiards in ellipses, for example that the sum of cosines of the interior angles remains constant. Reznik's experiments inspired a couple of mathematicians (including myself) to revisit billiards and to look for proofs and new invariants.

The goal of the lecture is to present a new approach and new properties of billiards and the associated Poncelet grid, which is formed by extensions of the billiard's sides. A velocity analysis paves the way to a representation of the Lie group of transformations which induces the motion of billiards with a common caustic and of the associated Poncelet grid.