Inscribable polytopes, routed trajectories, and reflection arrangements

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Steiner posed the question if any 3-dimensional polytope had a realization with vertices on a sphere. Steinitz constructed the first counter examples and Rivin gave a complete answer to Steiner's question. In dimensions 4 and up, the Universality Theorem indicates that certifying inscribability is difficult if not hopeless. In this talk, I will address the following refined question: Given a polytope P, is there a continuous deformation of P to an inscribed polytope that keeps corresponding faces parallel? In other words, is there an inscribed polytope P? that is normally equivalent (or strongly isomorphic) to P?

This question has strong ties to deformations of Delaunay subdivisions and ideal hyperbolic polyhedra and its study reveals a rich interplay of algebra, geometry, and combinatorics. In the first part of the talk, I will discuss relations to routed trajectories of particles and reflection groupoids and show that that the question is polynomial time decidable.

In the second part of the talk, we will focus on class of zonotopes, that is, polytopes representing hyperplane arrangements. It turns out that inscribable zonotopes are rare and intimately related to reflection groups and Grünbaum's quest for simplicial arrangements. This is based on joint work with Sebastian Manecke.