

Smooth Freeform Facades from Spherical, Conical, and Flat Panels

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Abstract

Freeform architectural facades demand discretization methods that balance aspects of aesthetics, e.g., visual smoothness, with fabrication constraints. Doubly curved panels impose substantial manufacturing costs, causing ambitious designs to become economically unviable. Existing discretization approaches either sacrifice surface smoothness or rely on costly freeform panel geometries. We present sphere meshes, panelizations which involve spherical panels and possess triangular, quad, or hexagonal combinatorics. Using spherical panels only, even by allowing just a few sphere radii, one can nicely improve the visual smoothness compared to the use of flat panels. Our focus is on another type of facades, which are smooth, i.e., C^1 continuous, and can model any positively curved freeform surface. They are composed of spherical, conical, and flat panels and rooted in so-called Laguerre sphere geometry. We present an elementary approach to the underlying type of Laguerre meshes and focus on methods for computational design and constrained optimization and discuss further practical aspects such as repetitive elements and support structures.

Keywords: architectural geometry, freeform façade, sphere mesh, smoothness, cost reduction, geometric support structure

1. Introduction

Paneling of architectural freeform surfaces is a core topic of architectural geometry and has attracted a lot of attention. Eigensatz et al. [5] presented a solution that has been proven to be effective in various real projects. Their optimization algorithm is based on a cost model for the production of panels of different types, considering also the re-use of molds. Their method allows one to find the best tradeoff between fabrication cost, controlled deviation from the target surface and surface quality measured by visual smoothness.

If one is willing to slightly limit the class of surfaces and/or the panel layout, one can already achieve good visual smoothness. The best visual appearance of solutions with flat panels leads to quad meshes with nearly rectangular panels, such as circular or conical meshes [9]. Other alternatives study optimized shapes for potential mold re-use [10, 13] or the use of bendable materials for the panels [6].

Motivated by new machines for the fabrication of spherical panels, such as those used for the Nur Alem Museum of Future Energy in Astana, Kazakhstan [12], meshes with spherical faces and circular arcs as edges have been studied [7, 11]. A simple way to compute a quad mesh with spherical faces is to depart from a circular mesh and compute a sphere through the circum-circle of each face (see Fig. 2). A special case of this approach occurred naturally in a study of special deployable structures (see Fig. 1).

Meshes all whose elements (vertices, edges and faces) belong to sphere geometries are a rather new

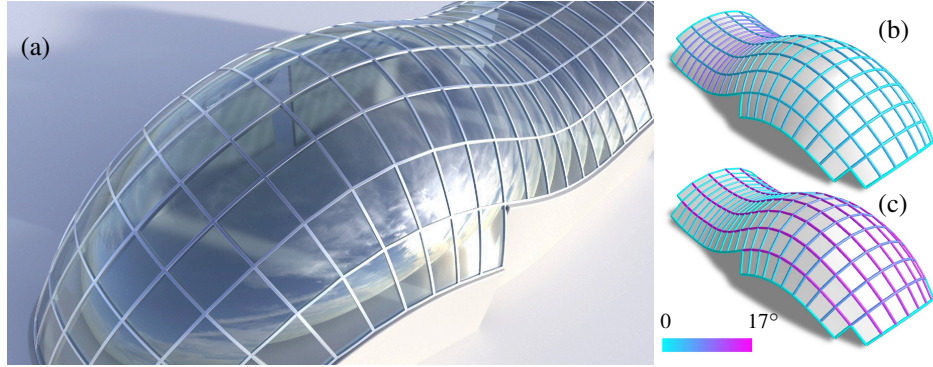


Figure 1: Paneling with spherical panels of constant radius. In a study of certain deployable structures [8], the authors derived special circular meshes, which may be materialized with planar panels (c), but also in a natural way with spherical panels of constant radius (a,b). The smoothness of the resulting surfaces is shown via angles between panels in figures b, c for spherical resp. planar panels. In negatively curved areas the angles need not be smaller, but are differently concentrated, leading to a different visual appearance. In areas of positive curvature, the spherical panels result in a much smoother surface than planar ones. Images courtesy D. Pellis; see [8].

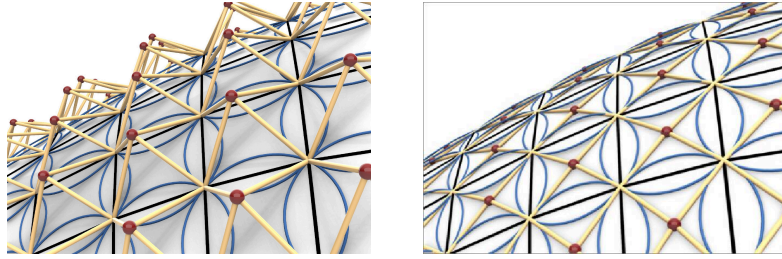


Figure 2: A simple way to turn a circular mesh C (black edges and blue circum-circles of faces) to a sphere mesh S proceeds as follows: In each face of C , choose an additional point (red) and connect with the vertices of C to obtain a triangle mesh (left). Apply a mesh smoothing technique by fixing the vertices of C (right). The new position of the additional point in each face defines the face sphere. Images courtesy C. Müller and M. Kilian.

topic. However, the connection between sphere geometries and certain meshes, such as circular and conical meshes, has been studied in discrete differential geometry [2, 3].

The sphere meshes mentioned above [7, 11] belong to so-called Möbius sphere geometry. We put together some basic insights and results on these sphere meshes in Section 2. However, the present paper focuses on meshes in another type of sphere geometry, called Laguerre geometry. We will not dive into Laguerre geometry, but present the meshes in an elementary way. The remarkable ability of these meshes is that they provide smooth panelizations of positively curved surfaces, using only spherical, conical and flat panels. We will discuss these meshes in Section 3.

2. Sphere meshes

This section provides a short discussion of a few central aspects of sphere meshes; many more results and details are found in [7, 11]. Sphere meshes generalize so-called polyhedral meshes, where all faces are planar. *In a sphere mesh, faces are spherical patches, which implies that edges are circular arcs.* As one is working in Möbius geometry, the limit cases of planar faces and straight edges are included as special cases. A sphere mesh can come with any combinatorics. For applications, one will be mainly interested Striangle, Squad, and Shex meshes with regular combinatorics apart from isolated combinatorial singularities.

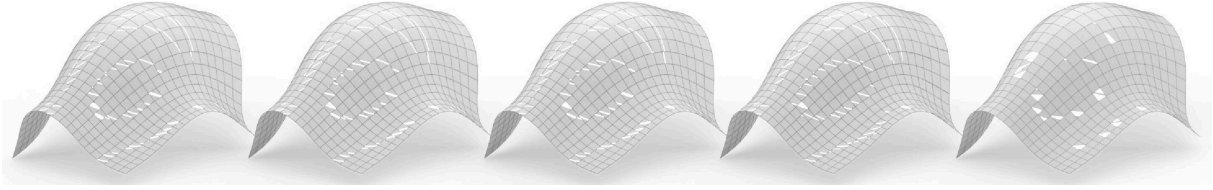


Figure 3: A circular mesh is extended to sphere meshes with a different number of used sphere radii. From left to right: one radius per face, 24 radii, 12 radii, 6 radii, flat faces. We see that even with a small number of radii the visual smoothness as apparent from the reflection pattern is higher than for the mesh with planar faces (right). Image courtesy C. Müller and M. Kilian.

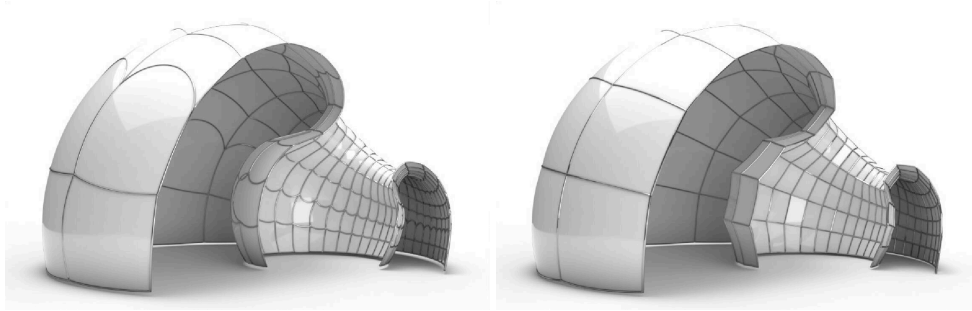


Figure 4: Optimal sphere radii in view of smoothness may lead to undesirable positions of circular edges (left). Hence, one includes good positions for edges, e.g. in bisecting planes of planar faces, into sphere mesh optimization. Image courtesy C. Müller and M. Kilian, taken from [7].

Geometry of sphere meshes In a sphere mesh, each face defines a sphere, which we refer to as a *face sphere*. The edges in form of circular arcs are part of the intersection of two neighboring face spheres. All face spheres of a sphere mesh define a discrete version of a 2-parameter family of spheres, also called a *discrete sphere congruence*. If the mesh S is a good representation of a smooth surface f , the face spheres are discrete versions of spheres which are tangent to f , i.e., f appears as envelope of the set of face spheres. Hence, we may view the sphere mesh S as a discrete envelope of the set of face spheres.

A sphere mesh may not look as good as the one in Fig. 5, right. To obtain such high visual smoothness, one wants to minimize the intersection angle between neighboring face spheres. That, however, may lead to edges whose planes are too tangential (see Fig. 4, left). Thus, another optimization target is the position of the planes of edge circles; they should be roughly orthogonal to a reference surface f , as in Fig. 4, right.

A simple way to obtain an Squad mesh, is to start from a circular mesh. Each face has a circum-circle, which lies in an infinite number of spheres. An easy algorithm for the selection of a good face sphere is illustrated in Fig. 2. That may have to be combined with an optimization aiming at good locations of circular edges. Results are shown in Figures 1, 5 and 3, where Fig. 3 illustrates another practical aspect, namely the limitation to just a few different sphere radii. One sees that even with a small number of sphere radii, the result may look smoother than a mesh with planar faces.

Geometric support structure A torsion-free support structure of a sphere mesh S consists of parts of the planes in which the edge circles are lying. Being torsion-free, requires the existence of a node-axis $a(v)$ through each vertex v which lies in the planes of edge circles that meet at v (Fig. 5, middle). Since two circles through v on a face sphere meet at a second point $\bar{v}$, this point $\bar{v}$ lies on the node axis and on all face spheres through v (Fig. 5, left and middle). In other words, we have a second sphere mesh $\bar{S}$ with vertices $\bar{v}$. By symmetry, the centers of face spheres form a mesh with planar faces. The planes

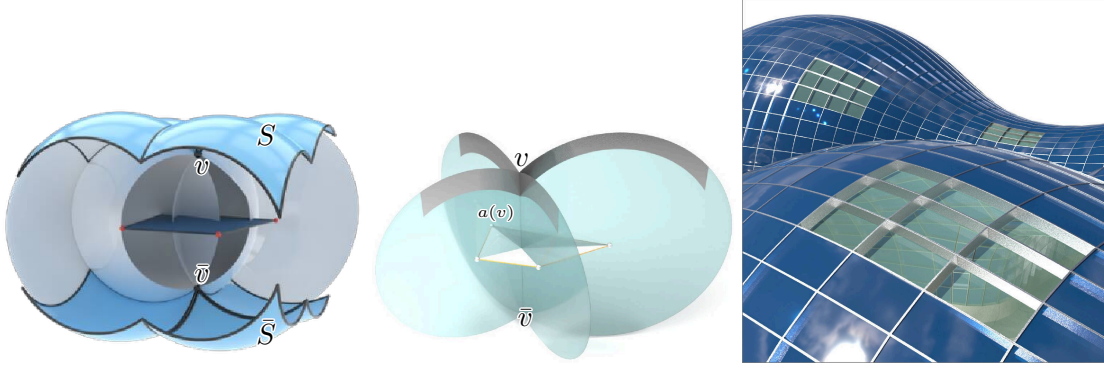


Figure 5: The face spheres of a sphere mesh S are a discrete version of a 2-parameter family of spheres. The given mesh S discretizes one envelope, but the face spheres may simultaneously represent the face spheres of the second envelope $\bar{S}$ (left). This requires that spheres through a vertex v of S also pass through a vertex $\bar{v}$ of $\bar{S}$. Then, the sphere centers lie in the bisecting plane of $v\bar{v}$, and the planes of circular edges through $v, \bar{v}$ pass through the line $v\bar{v}$ (middle). One obtains a torsion-free support structure formed by circular annuli (right; image courtesy C. Müller and M. Kilian).

are bisecting planes of corresponding points $v\bar{v}$ of meshes $S, \bar{S}$. The geometric support structure can be realized via annuli as illustrated in Fig. 5. The circular edges of the support structure which are not part of S form the edges of another sphere mesh S' , whose face spheres have the same centers as the corresponding face spheres of S . The transformation $S \mapsto S'$ is a generalization of mesh parallelism, since parallel planes may be viewed as concentric spheres with infinite radius.

Summarizing, we have shown that *a sphere mesh S has a torsion-free support structure if it simultaneously represents the second envelope of the family of face spheres and equivalently if centers of its face spheres form a polyhedral mesh* (of dual combinatorics to the one of S). The most restricted case is the one of Striangle meshes (see [7]), while sphere meshes all whose vertices have valence 3 (e.g. Shex meshes of regular combinatorics) always have a torsion-free support structure.

Computing sphere meshes The computation of sphere meshes is largely based on numerical optimization. One has a system of constraints which is solved by a Gauss-Newton algorithm including proper regularization. Spheres may be represented by center c and radius r . If they are allowed to become planes, one uses the implicit representation $f(x, y, z) = 0$ with $f = A(x^2 + y^2 + z^2) + B_1x + B_2y + B_3z + C$. It should be normalized by $B_1^2 + B_2^2 + B_3^2 - 4AC = 1$. Then f behaves like a distance function near the sphere and numerical errors are close to geometric errors. Kilian et al. [7] show how all essential constraints are formulated, ideally by at most quadratic equations, and propose regularizers and initial guesses for various tasks. A rather difficult problem is the approximation of a given surface by an Squad mesh with support structure, which is treated by Ramos-Cisneros et al. [11].

3. Smooth facades from spherical, conical and flat panels

We now turn to the main topic of the present paper, which introduces another type of sphere meshes. We work in so-called Laguerre sphere geometry and call the meshes *Laguerre sphere meshes*, or shortly *L-meshes*.

In Laguerre geometry one has two classes of basic objects: oriented spheres (shortly or-spheres) and oriented planes (or-planes). An or-sphere is given by its center $c \in \mathbb{R}^3$ and signed radius $r \in \mathbb{R}$. Here, $r > 0$ corresponds to an orientation of the sphere with inward pointing normals, $r < 0$ belongs to outwards pointing normals, and $r = 0$ describes points (no orientation). Thus, points are spheres with radius zero.

An oriented plane p is a plane with the same unit normal vector n attached to each of its points. The equation in Hesse normal form reads

$$\langle n, x \rangle + h = 0, \quad (1)$$

with $\langle \cdot, \cdot \rangle$ denoting the scalar product in $\mathbb{R}^3$. Here, the left side is the *signed distance* from a point x to p .

The essential relation between or-spheres and or-planes is *oriented contact*. It means tangency in the usual sense and agreement of unit normals at a contact point. An or-sphere s with center c and signed radius r is in oriented contact with the or-plane p of equation (1) if and only if c has signed distance r from p , i.e.

$$\langle n, c \rangle + h = r. \quad (2)$$

Laguerre geometry is based on *Laguerre transformations*, which act in a bijective way on the set of or-spheres and the set of or-planes, and preserve oriented contact. A simple example is the *d-offsetting operation*. It adds the same constant d to the signed radius r of each or-sphere and parallel translates each or-plane by the signed distance d . Like in this example, Laguerre transformations need not preserve points. Points may be mapped to spheres with radius $r \neq 0$ and thus are not considered basic objects in Laguerre geometry.

Geometry of L-meshes This leads us to an illustrative special case of L-meshes, namely *offset surfaces of polyhedral surfaces* P . We view the vertices of P as or-spheres and orient its face planes in a consistent way. Then, we apply offsetting by a distance d . Naturally filling the gaps arising from edges by cylindrical patches, we obtain a surface composed of spherical, cylindrical and planar patches. If P is locally convex, normals point to the convex side and $d > 0$, the offset surface is even smooth, i.e. a C^1 surface.

L-meshes are more general than these special L-meshes and do not just arise from polyhedral surfaces by Laguerre transformations. Our interest is in L-meshes which are good approximations of smooth surfaces Φ . However, we need the restriction to local convexity, i.e., approximations of surfaces Φ with positive Gaussian curvature. The restriction to positive curvature is essential since otherwise we will obtain singularities (see Fig. 7, right).

L-meshes are smooth compositions of spherical patches (faces), planar patches (vertices), and conical patches (edges) as shown in Fig. 6, left. We speak of face spheres, vertex planes and edge cones; all of them are oriented, and they are in or-contact as follows. Face spheres around a vertex plane are in or-contact with that plane. Adjacent face spheres are in or-contact with a common edge cone. The conical edge patches are bounded by two straight line segments along which they tangentially meet the vertex plane and by two circular arcs along which they tangentially meet the face spheres. The cone may be a cylinder as in the offset example from above.

For practical purposes, we exclude L-meshes with points as face spheres, since then we lose smoothness. We prefer to have large spherical patches, as in the example in Fig. 6, right.

Large spherical patches should go along with comparatively thin conical patches. The width of a conical patch between two face spheres s_1, s_2 equals their tangential distance. With $s_i = (c_i, r_i), i = 1, 2$, this distance is computed as

$$d(s_1, s_2)^2 = \|c_1 - c_2\|^2 - (r_1 - r_2)^2. \quad (3)$$

The tangential distance of neighboring face spheres (cone strip width measured along rulings) is a Laguerre-geometric counterpart to the intersection angle between neighboring face spheres in the sphere meshes of Section 2.

Previously we turned circular meshes into sphere meshes, which offers degrees of freedom, since there are infinitely many spheres through a circle. The Laguerre geometric counterpart is to turn a conical

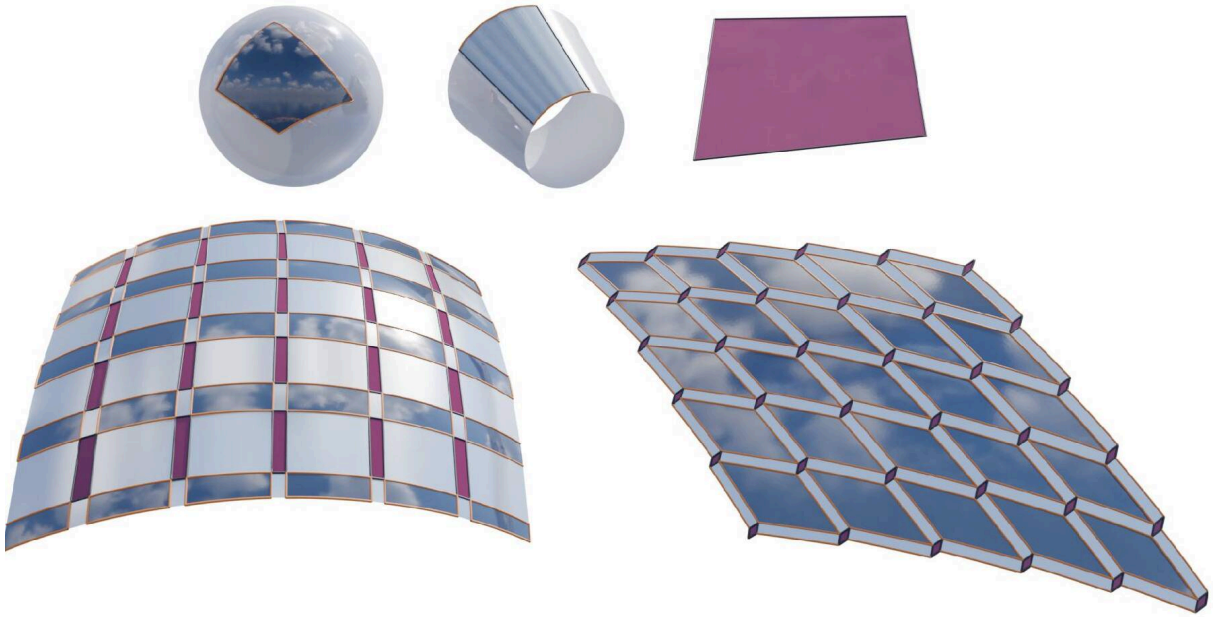


Figure 6: An L-mesh represents a smooth surface in positively curved areas, and consists of planar polygons (red), spherical faces (blue) bounded by circular arcs (orange) and pieces of rotational cones (cyan), which we view as its edges. (Left) L-mesh computed from a conical mesh. (Right) L-mesh with thin cone strips.

mesh C into an L-mesh L . We start with a locally convex conical mesh C and orient its face planes with normals pointing to the convex side. Being a conical mesh means that the four face planes meeting at an inner vertex are tangent to an or-cone, i.e., to infinitely many or-spheres that can be tangentially inscribed to the cone (Fig. 7, left). We can now select an appropriate sphere at each vertex, make sure that the tangential distance of neighboring spheres is small (but no singularities arise) and fill in the conical patches which join neighboring or-spheres and neighboring vertex planes. Note that in our definition of L-meshes, the faces of C become vertex planes of the L-mesh L , the vertices of C are replaced by face spheres of L and the edges of C are replaced by edge cones. Examples are shown in Fig. 7 and Fig. 6, left.

Quad L-meshes computed from conical meshes are special principal curvature parameterizations with a rectangular arrangement of patches. However, this is not the typical case for the appearance of a quad L-mesh. Since the conical patches are always rectangular, but the spherical and planar patches are generally not rectangular, patterns as in Fig. 6, right, or even better visible in Fig. 8, right, appear.

Clearly, L-meshes are not restricted to quad combinatorics. Fig. 8 shows an example of a hexagonal and a triangular L-mesh. This is seen in hexagonal and triangular spherical patches, respectively.

In the following, it is good to define the union of all patch boundaries in an L-mesh L as *extended L-mesh* L^* . Its edges are circular arcs or straight line segments and faces can be planar, conical or spherical. These faces are the panels in an architectural setting. For that, it is necessary to come up with an appropriate support structure, discussed next.

Support structure The simplest type of a torsion-free support structure is formed by rectangular planar strips attached orthogonally to the base surface L along each straight edge of L^* and by conical strips that are attached orthogonally along the circular edges of L^* (see Fig. 10). The strips can be of constant width d and then connect the smooth surface L with an offset at distance d , which is also an L-mesh.

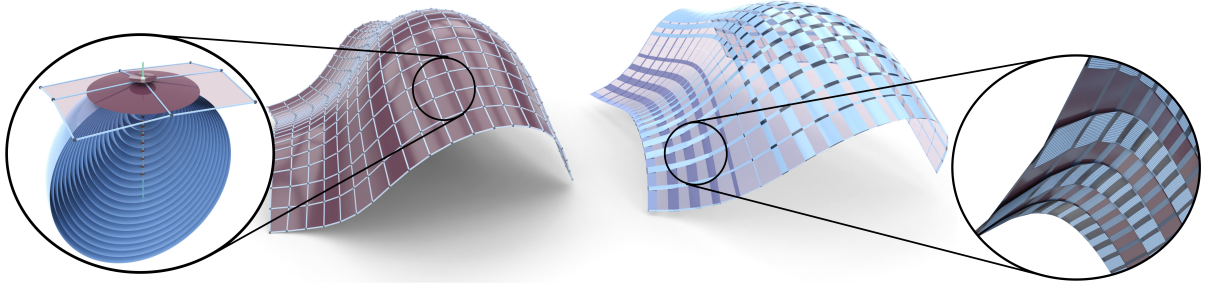


Figure 7: A conical mesh (left) and its extension to an L-mesh (right). At each inner vertex of the conical mesh, infinitely many oriented spheres exist that are in oriented contact with the oriented cone tangent to the four adjacent face planes (left inset) Singularities arise in regions of negative curvature(right inset).

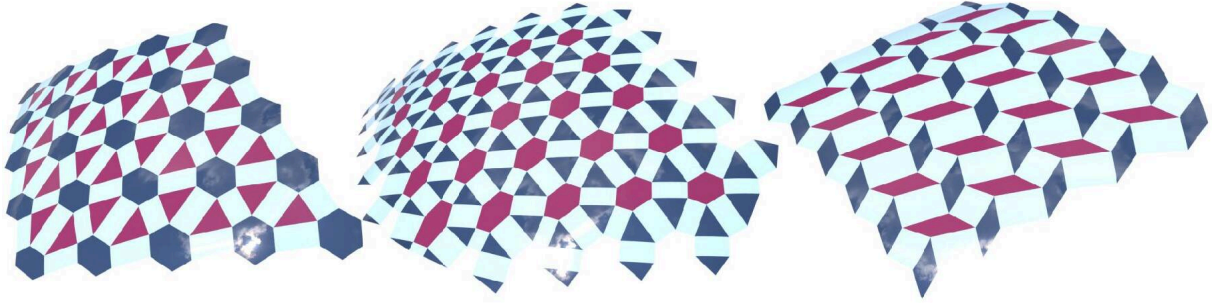


Figure 8: L-meshes with varying face combinatorics. From left to right: a hexagonal L-mesh, a triangular L-mesh, and a quad L-mesh.

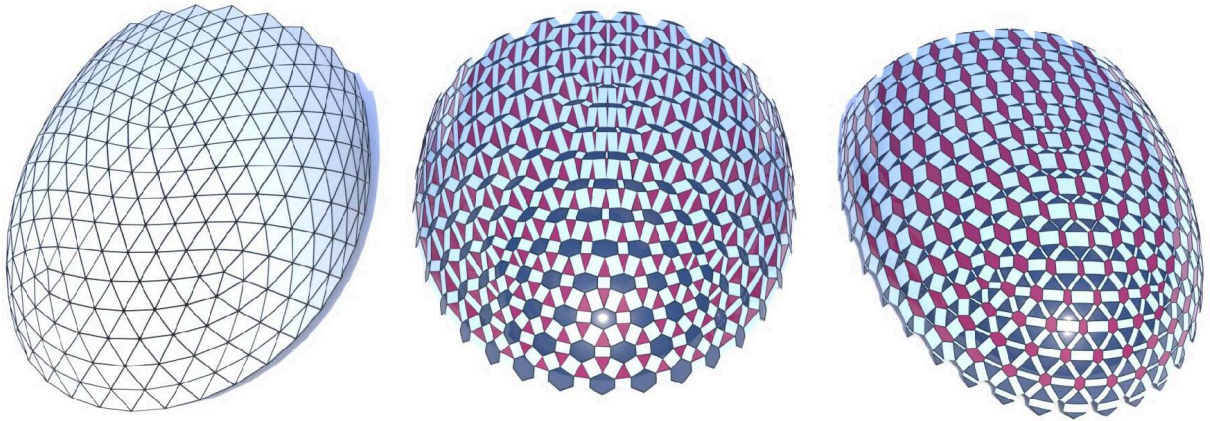


Figure 9: L-meshes with hexagonal and triangular combinatorics computed from a reference triangulation (left). In the hexagonal case, or-spheres are assigned to vertices and or-planes to faces; each or-sphere is initialized with radius $\mu \in (0, 1)$ times the smallest principal curvature radius along the vertex normal (mid). In the triangular case, or-planes are initialized at vertices using position and normal. Or-spheres are assigned to faces by interpolating both the normals and the smallest principal curvature radius at the barycenter of each triangle, scaled by a factor μ (right).

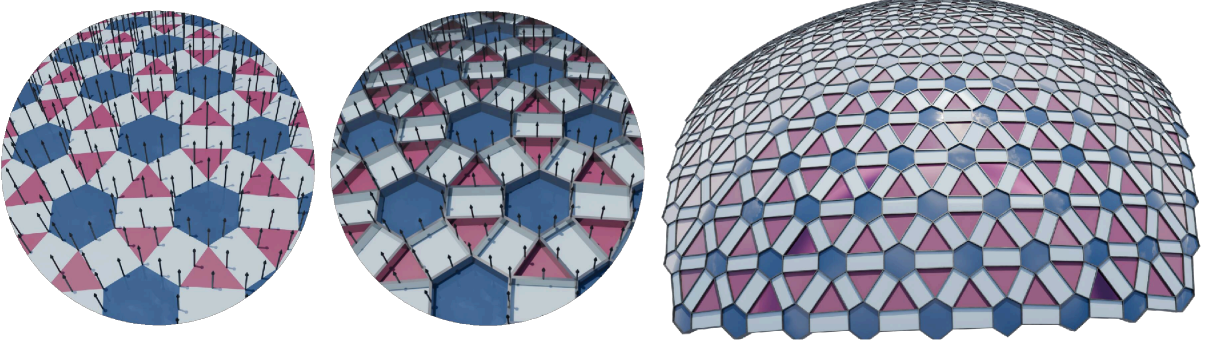


Figure 10: The simplest support structure can be obtained by extruding the edges of the extended L-mesh along their normals by a constant width.

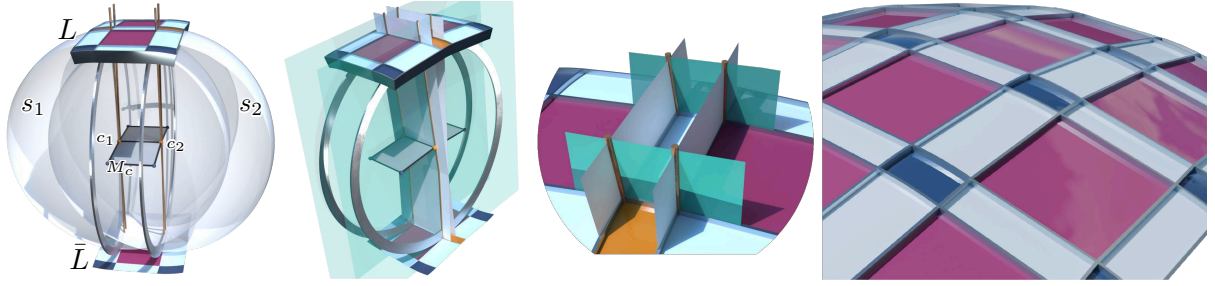


Figure 11: L-meshes simultaneously representing two envelopes posses support structures formed by planar elements: node axes (copper lines) connect corresponding vertices of both envelopes L and $\bar{L}$ and are orthogonal to the faces of the sphere-center mesh M_c (left). The circular arcs of a cone patch connecting face spheres s_1, s_2 with centers c_1, c_2 lie on planes (teal) orthogonal to the edge c_1c_2 of M_c ; node axes along the rulings of the cone patch are parallel, hence coplanar (white planes), and orthogonal to the same face of M_c (center-left). Close up to the torsion free configuration at the vertices of an L-mesh (center-right). Resulting L-mesh with planar support structure (right).

There is another type of support structure formed by planar elements only; see Fig. 11. This case is very similar to the one for sphere meshes illustrated in Fig. 5. It exists if the face spheres in the L-mesh L simultaneously are the face spheres of the second envelope $\bar{L}$ of the set of spheres. Corresponding vertex planes of L and $\bar{L}$ must be in or-contact with the same adjacent face spheres and thus the sphere centers lie in a plane, namely a bisecting plane of the two vertex planes. Hence, *the mesh M_c of sphere centers must have planar faces.*

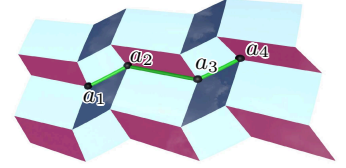
We show that we obtain a torsion-free support structure whose node axes connect corresponding vertices of the two extended L-meshes L^* and $\bar{L}^*$. Node axes are orthogonal to the planar faces of M_c and can be end points of straight line segments or of circular arcs. Consider now a cone patch, connecting two face spheres s_1, s_2 with centers c_1, c_2 . Each circular arc of the cone patch lies in a normal plane of the edge c_1c_2 of M_c . This plane contains the node axes at the end points of the arc, because each node axis is orthogonal a face of M_c and thus orthogonal to the edge c_1c_2 . Moreover, the node axes along a ruling of the cone patch are parallel, namely orthogonal to the same face of M_c . Thus, we also have planar elements of the support structure that belong to the straight line segments in the extended L-mesh. The geometric support structure can be a union of planar quads and parts of annuli that connect L with another L-mesh L' whose face spheres are concentric with those of L .

Computation A core part of computation is a numerical optimization algorithm which ensures that all hard constraints such as tangency of adjacent patches are satisfied and that the result is aesthetically pleasing. The latter is a soft constraint and guides the algorithm to a good solution. This approach has been used in many papers on architectural geometry and computational design, e.g. [5, 6, 7, 11, 13].

Basic variables are the parameters of face spheres (center c and radius r) and of vertex planes (unit normal n and origin distance h). Normalization of n via $\langle n, n \rangle - 1 = 0$ has to be added as hard constraint. Oriented contact of a vertex plane $p_i = (n_i, h_i)$ and an adjacent face sphere $s_j = (c_j, r_j)$ is expressed via (2) as $\langle n_i, c_j \rangle + h_i - r_j = 0$.

For the computation of fair results, the vertices of the extended L-mesh have to be added as additional variables. They are contact points a_{ij} of vertex planes p_i and face spheres s_j and expressed as $a_{ij} = c_j - r_j n_i$.

Fairness terms may not be just the standard ones if one has a pattern as in general quad L-meshes. Here, a good appearance of a polyline with vertices $a_1, a_2, a_3, a_4, \dots$ requires that edge vector $a_2 - a_1$ is not far away from $a_4 - a_3$, etc., giving rise to soft constraints $a_2 - a_1 - a_4 + a_3 = 0$, etc.



The final objective function is based on a least squares formulation. It squares the left hand side of all constraints of the form $C_k = 0$ and sums them up. Expressing the contact mentioned above with its contact point leads to a term

$$(\langle n_i, c_j \rangle + h_i - r_j)^2 + (\langle n_i, n_i \rangle - 1)^2.$$

The sum E of all such terms arising from hard constraints must get very close to 0. However, analogous terms from fairness, e.g. $(a_2 - a_1 - a_4 + a_3)^2$ shall just be small, but not tend to zero. Their sum E_{fair} is added to E with a small weight w as $E + wE_{\text{fair}}$. The weight w is stepwise reduced during optimization and set to zero in the final iterations. For more details and other constraints such as proximity to a reference surface, we refer to the literature, specifically to [4].

Any numerical optimization must come with an initial choice. This step may be difficult and require further knowledge. This is the case for the approximation of a given surface with targeted sphere radii by quad L-meshes, which requires a new form of conjugate parameterizations of surfaces with respect to a congruence of tangent spheres [4].

Conclusion and future research

After a short introduction to recent work on meshes with spherical faces, we presented another type of sphere meshes, called L-meshes, that have their geometric basis in Laguerre sphere geometry. While they are restricted to the representation of positively curved freeform surfaces, they possess some attractive features: They are smooth surfaces composed of planar, conical and spherical patches and have very simple torsion-free support structures. Their computation can be based on standard optimization techniques in computational design, and they are accessible via elementary geometric knowledge.

More details on the optimization and the approximation of a given positively curved surface by quad L-meshes are treated in a forthcoming article [4]. We have started to study L-meshes in a systematic way, including important special types and the use of other models of Laguerre geometry. This shall include practical aspects such as control over the width of conical strips (large spherical patches), the optimal selection of sphere radii or the limitation to just a few radii.

Sphere meshes and L-meshes are discrete surface representations within the sphere geometries of Möbius and Laguerre, respectively. One may also go into the overarching area of Lie sphere geometry and define corresponding meshes. Examples are provided by images of L-meshes under Möbius transformations.

This eliminates the restriction to positive curvature, but introduces patches on Dupin cyclides. For all types of sphere geometries and a study of smooth surfaces within these geometries we refer to Blaschke [1]. Discrete counterparts are largely missing.

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