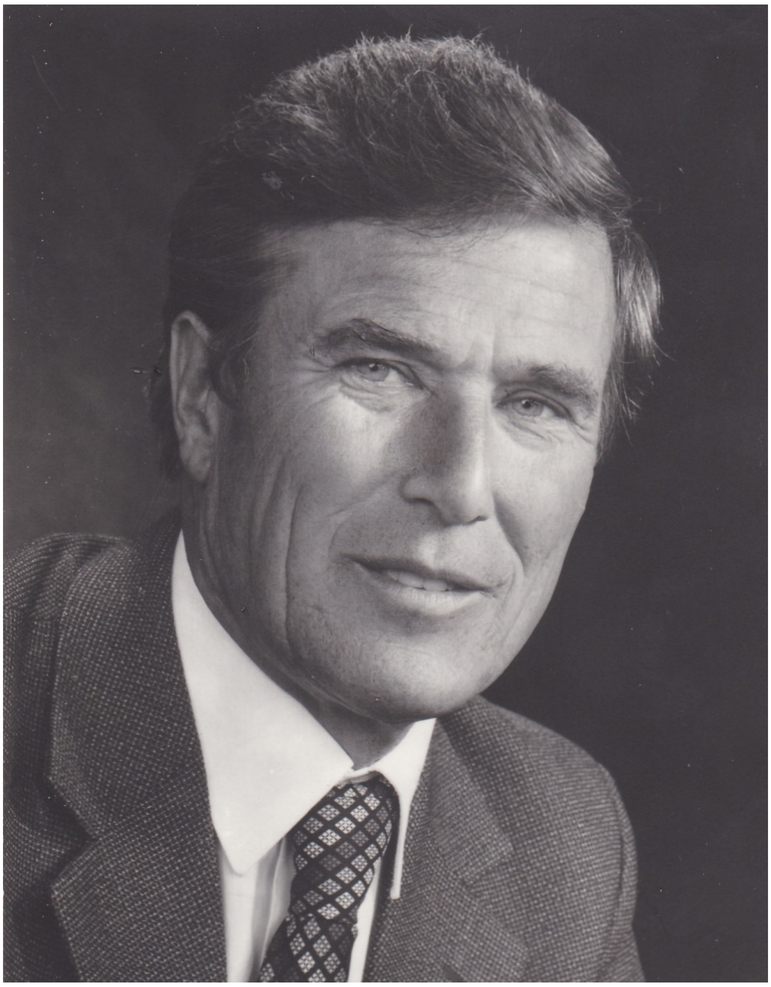

Helmut Karzel (1928–2021)

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We are saddened to report that Professor Dr. Dr. h.c. Helmut Karzel passed away on June 22, 2021, in Burgwedel, Germany, at the age of 93. He will always be in our best memories as teacher, friend and valued member of the worldwide community of mathematicians. Our deepest sympathy goes out to his three children Dörte, Herbert, Barnim and to Gisela, his partner over the last years.

Helmut Karzel was born on January 15, 1928, in Schöneck in Westpreußen, Poland. He spent his youth in Posen, where he attended school from 1934 to 1945. During the last year of World War II he had to serve as flak helper. At the end of the war he was advised to go west. So, in 1945 he reached the German town of Magdeburg, where an uncle lived. There, in 1946, he took the "Abitur" at the Otto von Guericke Schule. His mother and two of his three siblings had to flee from Posen and all of them met again in Magdeburg.

By crossing several occupation zone boundaries (at night), Karzel reached Freiburg, Germany, in 1947. He received the admission to study mathematics and physics at the University of Freiburg. Prior to his studies, he was obliged to help for eight weeks in clearing up the rubble of the bomb attacks. Among Karzel's teachers in Freiburg was Emanuel Sperner, who became Professor at the University of Bonn in 1950. Karzel moved to Bonn and obtained his doctorate under Sperner's supervision in 1951. That year he also received the Hausdorff Memorial Prize, which is awarded annually by the University of Bonn for the best dissertation in mathematics of the past academic year.

After his studies, Karzel worked as "Assistent" at the University of Bonn and, starting from 1954, at the University of Hamburg. Karzel accomplished his "Habilitation" in the year 1956. He stayed as Visiting Associate Professor at the University of Pittsburgh, Pennsylvania, USA, in 1961/62. Furthermore, he was on leave from Hamburg for a guest professorship at the Technische Universität Karlsruhe from 1967 to 1968. During the summer term 1968 he taught in Hamburg as well as in Karlsruhe. In 1968, Karzel was appointed Chair of Geometry at the Technische Hochschule Hannover, Germany. A few years later, in 1972, he accepted an offer from Technische Universität München as Chair of Geometry, a position he retained until his retirement in 1996.

A major aim of Helmut Karzel's work was to maintain contact with colleagues worldwide. Consequently, Karzel lectured as Visiting Professor at numerous universities outside Germany: Bologna, Italy (Adriano Barlotti); Brescia, Italy (Mario Marchi); Toronto, Canada (Erich Ellers); College Station, Texas, USA (Carl J. Maxson); Rome, Italy (Giuseppe Tallini); Teesside, England (Allan Oswald); Tucson, Arizona, USA (James Clay).

Helmut Karzel was editor for several mathematical journals: *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, *Jahresberichte der Deutschen Mathematiker Vereinigung*, *Journal of Geometry* (founding editor and, from 2017, honorary editor), *Mitteilungen der Mathematischen Gesellschaft Hamburg*, *The Nepali Mathematical Sciences Report* and

Results in Mathematics. Since 1959, Karzel was member of the *Mathematische Gesellschaft in Hamburg*. He served as "Jahrverwalter" (chairman) of this society from 1966 to 1971 and was awarded an honorary membership in 1978. In appreciation of his outstanding services to mathematics, Karzel received an honorary doctorate from the University of Hamburg in the year 1993.

Karzel inspired highly gifted young people for mathematics and encouraged them to do own research work. In Hamburg, Karlsruhe, Hannover and München he guided a large number of students through their theses. As a result, 30 doctorates arose under his supervision. Several of his students later became University Professors.

A list of Helmut Karzel's scientific publications, which comprises more than 170 entries, can be found at the end of this article. His scientific work is distinguished by impressive ideas and proves his widespread interests, with algebra, geometry and foundations of geometry playing a central role. In the following, we will only briefly discuss a small selection of his contributions.

Since the beginning of the 20th century, theorems of elementary geometry and absolute geometry have increasingly been proven by the use of reflections. In 1943, Arnold Schmidt found a reflection-theoretical axiom system that allows for the foundation of all classical absolute and elliptic planes. The three reflections theorem plays an essential role in this approach. Friedrich Bachmann gave a reduced version of Schmidt's axiom system in 1951. A fine detailed justification and a large collection of models can be found in Bachmann's book *Aufbau der Geometrie aus dem Spiegelungsbegriff*, first published in 1959.

A decisive new impetus was given to reflection geometry in 1954, when Emanuel Sperner examined under which minimal conditions Desargues' theorem can be proven in a group theoretic setting. Sperner based his approach on a group G with a system, say J, of involutory generators, which are to be viewed as "lines". He assumed essentially only the validity of the following *general three reflections theorem*: If for five elements $A, B, X, Y, Z \in J$, with $A \neq B$, each of the products ABX, ABY, ABZ is an involution, then $XYZ \in J$. From 1954 onwards, Karzel developed further Sperner's axiom system in a series of papers [2, 3, 6, 7, 8, 9]. The corresponding geometries fall into two classes:

1. The (previously known) *regular absolute planes*. Here different perpendiculars to a fixed line have different pedals.

2. The so-called *Lotkern-Geometrien*. These are planes in which all perpendiculars to a fixed line G share the same pedal. Together with the

line G, all perpendiculars to G form a pencil, the *Lotkern* (perpendicular pencil) of G.

The geometries of the second class turned out to be new. Karzel was the first to recognise their quite unusual properties and investigated the subject more closely. Among the *Lotkern-Geometrien* are the so-called *Zentrums-Geometrien*, where the system J of generators contains an element from the centre of the underlying group G. From an algebraic point of view, *Lotkern-Geometrien* arise from subgroups of orthogonal groups over fields of characteristic two. A common foundation of the regular and the *Lotkern-Geometrien* with the help of so-called kinematic spaces (see below) was given by Karzel in his 1963 lecture *Gruppentheoretische Begründung metrischer Geometrien* (Group theoretical foundation of metric geometries); cf. his book [B5], co-authored with Günter Graumann.

The classical example of a kinematic space is due to Wilhelm Blaschke and, independently, Josef Grünwald. In two articles, published 1911, they established a bijection between the group of proper motions (rotations and translations) of the Euclidean plane and a particular set of the points in the three-dimensional real projective space. In 1934, Erich Podehl and Kurt Reidemeister founded the elliptic plane with the help of the associated kinematic space, which in this case comprises all points of a projective space. Reinhold Baer extended this principle by assigning a geometry to a group G in such a way that its elements represent both points and hyperplanes. Given $\alpha, \beta \in G$, a "point" (α) is incident with a "hyperplane" $\langle\beta\rangle$ if $\alpha\beta$ is involutory. The space defined in this way is a projective space if and only if G is an elliptic motion group.

Karzel picked up Baer's ideas and generalised them, in collaboration with Erich Ellers, by considering a group G (with unit element 1) together with a distinguished subset D of G that is invariant under all inner automorphisms of G and such that $\xi^2 = 1$ for all $\xi \in D$. Then a geometry $D(G)$ is defined as follows: For $\alpha, \beta \in G$ the incidence of (α) and $\langle\beta\rangle$ means $\alpha\beta \in D$. Karzel and Ellers completely classified the corresponding geometries: $D(G)$ is either a projective space of dimension 3 and G is isomorphic to the motion group of an elliptic plane, or $D(G)$ is a so-called *involutory geometry* of dimension $2n - 1$, which can be represented in terms of a Clifford algebra over a field with characteristic 2 (cf. [11, 12]).

Let us take a closer look at the geometry $D(G)$ from above. For all $a \in G$, the left translations $a_\ell\colon x \to ax$ and the right translations $a_r\colon x \to xa$ are automorphisms of $D(G)$ that act regularly on the set of points. Starting from this observation, Karzel developed the concept of an *incidence group*, a group with a "compatible" geometric structure, comparable to the concept of a topological group. To be more precise, a group G is called an *incidence group* if the elements of G are the "points" of a geometry (with "lines" understood as subsets of G) and the left translations in G are automorphisms of the geometry; see the surveys [24] and [31], the latter being joint work with Irene Pieper. Motivated by previous examples, Karzel also coined the notion of an abstract *kinematic space* [40] as an incidence group satisfying two additional conditions: (i) all right translations in G are automorphisms of the geometry; (ii) lines through the neutral element of G are subgroups of G.

In numerous publications and several dissertations under his guidance, Karzel advanced the algebraic description of incidence groups in terms of representation theorems. *Normal nearfields*, i.e. nearfields which are also left vector spaces over a normal sub-nearfield, play a crucial role in some of these results.

Kinematic spaces permit two *parallelisms* by considering the left (or right) cosets the lines passing the neutral element (of the underlying group) as classes of parallel lines. It turned out that kinematic spaces can be characterised as incidence geometries with two parallelisms; see [41, 44, 45, 86, 88, 89, 93] (with Erich Ellers, Hans-Joachim Kroll, Carl J. Maxson, Kay Sörensen).

In the above mentioned nearfields, the validity of one distributive law is removed as a generalisation to fields. Leonard E. Dickson gave examples of nearfields that are not fields. Hans Zassenhaus extended Dickson's construction method and showed that in this way, up to seven exceptions, all finite nearfields can be obtained. Karzel axiomatised this construction method (Dickson's process) in [21] and characterised those nearfields that can be obtained in this way, known as *Dickson's nearfields*.

Given a nearfield F the group

$$T_2(F) := \{\tau_{a,b}\colon F \to F;\ x \to a + bx \mid a \in F, b \in F^*\}$$

of all affine translations operates sharply 2-transitively on F. Conversely, if T is a sharply 2-transitive permutation group on a finite set F, then, according to Robert D. Carmichael, two operations can be defined on F such that F is made into a nearfield and T is isomorphic to the group $T_2(F)$. In the infinite case, the situation is more intricate. Karzel introduced two operations on F, translated the double transitivity into algebraic axioms and obtained a so-called *neardomain*. In a neardomain F the additive structure is in general not a group, but only a loop with additional properties, a so-called *K-loop* (in particular, there is an additive automorphism $\delta_{a,b}$ with $a + (b + x) = (a + b) + \delta_{a,b}(x)$ for any elements $a, b \in F$). Compared to other concepts, neardomains can be characterised by the fact that isomorphic permutation groups lead to isomorphic neardomains and vice versa.

The existence of proper K-loops was an open problem for a long time. Surprisingly, the first proper example of a K-loop was found in the context of mathematical physics. Abraham A. Ungar investigated in 1988 the relativistic velocity addition $\oplus$ on $\mathbb{R}^3_c = \{x \in \mathbb{R}^3 : |x| < c\}$, which is neither commutative nor associative. However, Ungar was able to prove that $\oplus$ makes $\mathbb{R}^3_c$ into a K-loop. Investigations on K-loops and relativistic velocity addition are in focus of Karzel's papers [100, 104, 106] (with Bokhee Im, Heinrich Wefelscheid).

Last but not least, let us take a glimpse at the remaining areas of Karzel's work. These include, among others, *circle geometries*, intensively worked on by Karzel in [36, 37, 38, 39, 48, 66, 68, 91] (with Werner Heise, Hans-Joachim Kroll, Helmut Mäurer, Rotraut Stanik, Heinz Wähling), *coding theory* (in particular applications of circle geometry in coding theory) [90, 91, 99] (with Alan Oswald, Carl J. Maxson), questions of the *order relation* in algebra and geometry [1, 4, 5, 13, 14, 16, 30, 48, 94] (with Hanfried Lenz, Rotraut Stanik, Heinz Wähling) and the foundation of *metric planes* [47, 55, 56, 57, 61, 65, 77] (with Günter Kist, Rotraut Stanik, Monika König).

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