

On Möbius Pairs of Simplices

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Two n -simplices in the n -dimensional projective space over a field F are *mutually inscribed and circumscribed* if each point of the first simplex is in a hyperplane of the second simplex, and *vice versa* for the points of the second simplex. Two such n -simplices will be called a *Möbius pair of simplices* or shortly a *Möbius pair*. The construction of Möbius pairs in any odd dimension is a straightforward task: Given any n -simplex take the images of its hyperplanes under any null polarity as vertices of a second simplex. By this approach, it remains open, though, whether or not the simplices have common vertices.

We focus our attention to *non-degenerate* Möbius pairs. These are pairs of n -simplices such that each point of either simplex is incident with *one and only one* hyperplane of the other simplex. We present a construction of non-degenerate Möbius pairs which is based on null polarities. Furthermore, we exhibit the *nested Möbius pairs* arising from sub-simplices.

Finally, we sketch how our results can be applied to construct distinguished systems of commuting / non-commuting operators in Pauli groups.

This is joint work with Boris Odehnal and Metod Saniga.

References.

- H. Havlicek, B. Odehnal, and M. Saniga. Factor-group-generated polar spaces and (multi-)qudits. *SIGMA Symmetry Integrability Geom. Methods Appl.*, **5** (2009), paper 096, 15 pp. (electronic).
- H. Havlicek, B. Odehnal, and M. Saniga: Möbius pairs of simplices and commuting Pauli operators, *Math. Pannonica* **21** (2010), 115–128.