A STEINER-like approach to some BAER-subplanes

Hans Havlíček, Wien.

K ... non-commutative field, K^2 ... right vector space over K

$\mathcal{A}$ ... affine plane on K^2, $\mathcal{P} := \mathcal{A} \cup u$ ... projective closure of $\mathcal{A}$

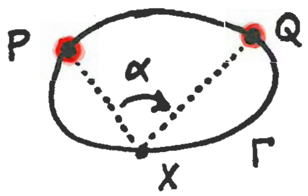
$\Gamma^\circ := \{(x,y) \in K^2 \mid y = xa\}$ $\quad a \in K \setminus Z(K)$

$Z(K)$ ... centre of K

$\Gamma := \Gamma^\circ \cup u$ .... degenerate conic

Γ° ... proper, u ... improper part of Γ

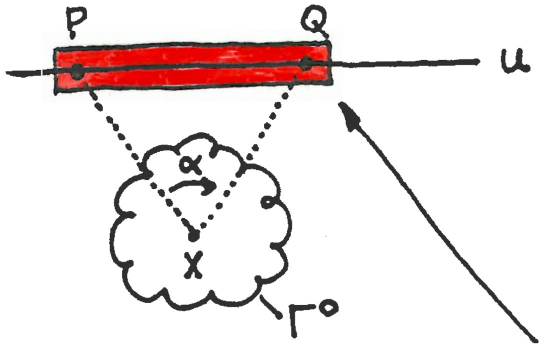
J. STEINER



$\alpha : \mathcal{L}_P \to \mathcal{L}_Q$

$XP \mapsto XQ \quad (X \in \Gamma)$

is a projectivity



$P, Q \in u$ with slopes in $Z(K) \cup \{\infty\}$

$\alpha : \mathcal{L}_P \to \mathcal{L}_Q$

$XP \mapsto XQ \quad (X \in \Gamma)$

fundamental chain of Γ (subline)

E. BERZ, B. SEGRE, R. RIESINGER, H.H. (1962 .... 1985)

R. RIESINGER: generalized polynomial identities

properties of Γ depend highly on whether

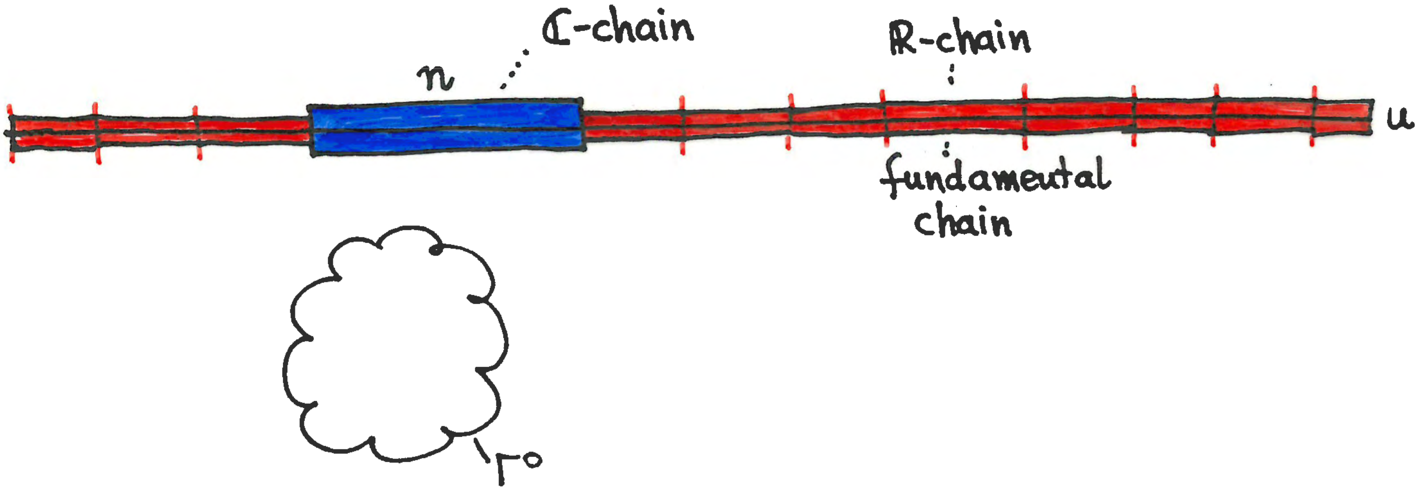
$|a : Z(K)| = 2$ or $|a : Z(K)| \neq 2$.

Example: $K = \mathbb{H}$ .. real quaternions

$Z(K) \cong \mathbb{R}$

$a \in \mathbb{H} \setminus \mathbb{R} \Rightarrow |a:\mathbb{R}| = 2$, $Z(a) \cong \mathbb{C}$.

$\Gamma^\circ \cup n$ ... complex projective plane



automorphic collineations of Γ .... collineations of $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ which can be extended to collineations of $\mathcal{P}$

fundamental chain of Γ ...... orbit of $X \in u \setminus n$ under the group Ψ of collineations of $\mathcal{P}$ fixing $\Gamma^\circ \cup n$ elementwise.

$(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ ... trace space given by Γ°

THEOREM: $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ}) \cong$ affine space on K (regarded as right vector space) over $Z(a)$

centralizer of a in K

$(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ is an affine plane, iff $|a : Z(K)| = 2$.

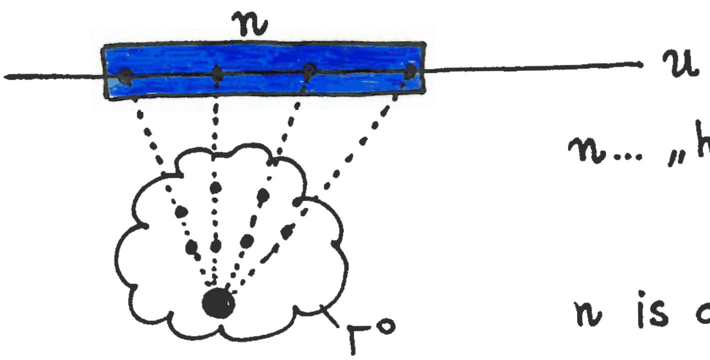
Proof. .....

$$2 = \dim(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ}) = |K : Z(a)|_r =$$

centralizer theorem

$$= |a : Z(K)| = |K : Z(a)|_\ell$$

$\Rightarrow (\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$ is an affine BAER-subplane.



n ... „hyperplane at infinity" of $(\Gamma^\circ, \mathcal{L}_{\Gamma^\circ})$

n is a $Z(a)$-chain of u iff $|a : Z(K)| = 2$.