Designs:

$t-(v,k,\lambda)$ - design ... (P,B)

blocks

blocks per t points

points

points per block

number of points

Examples:

Projective planes : $2-(n^2+n+1,\ n+1,\ 1)$

Affine planes : $2-(n^2,\ n,\ 1)$

Witt's $5-(12,6,1)$ design W_{12}

12 points, 132 blocks

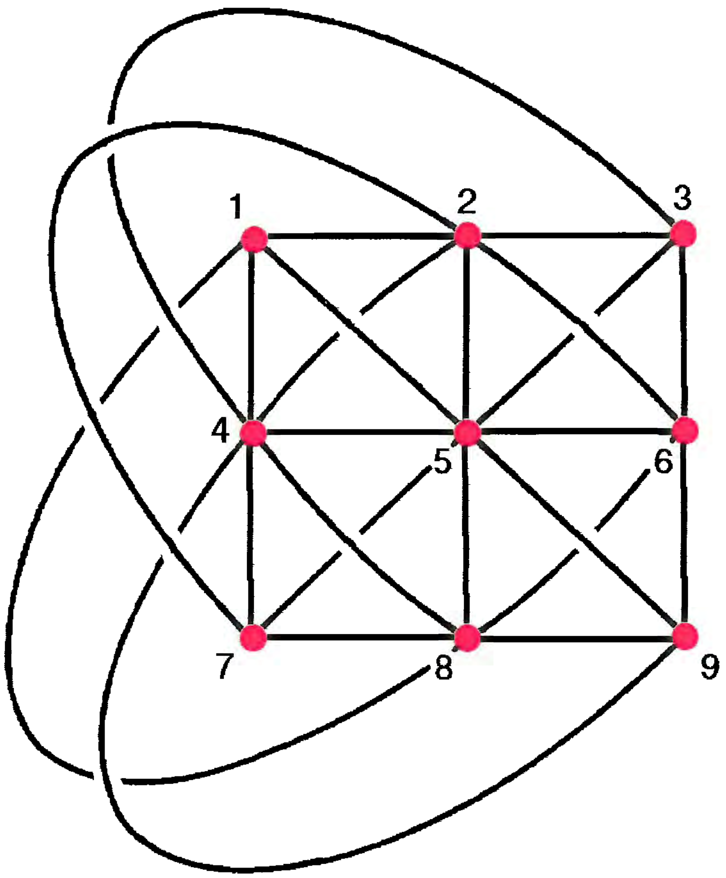
Choose 3 points $\longrightarrow$ 9 points remaining

... blocks through them $\longrightarrow$ 12 blocks $\longrightarrow$ 12 lines

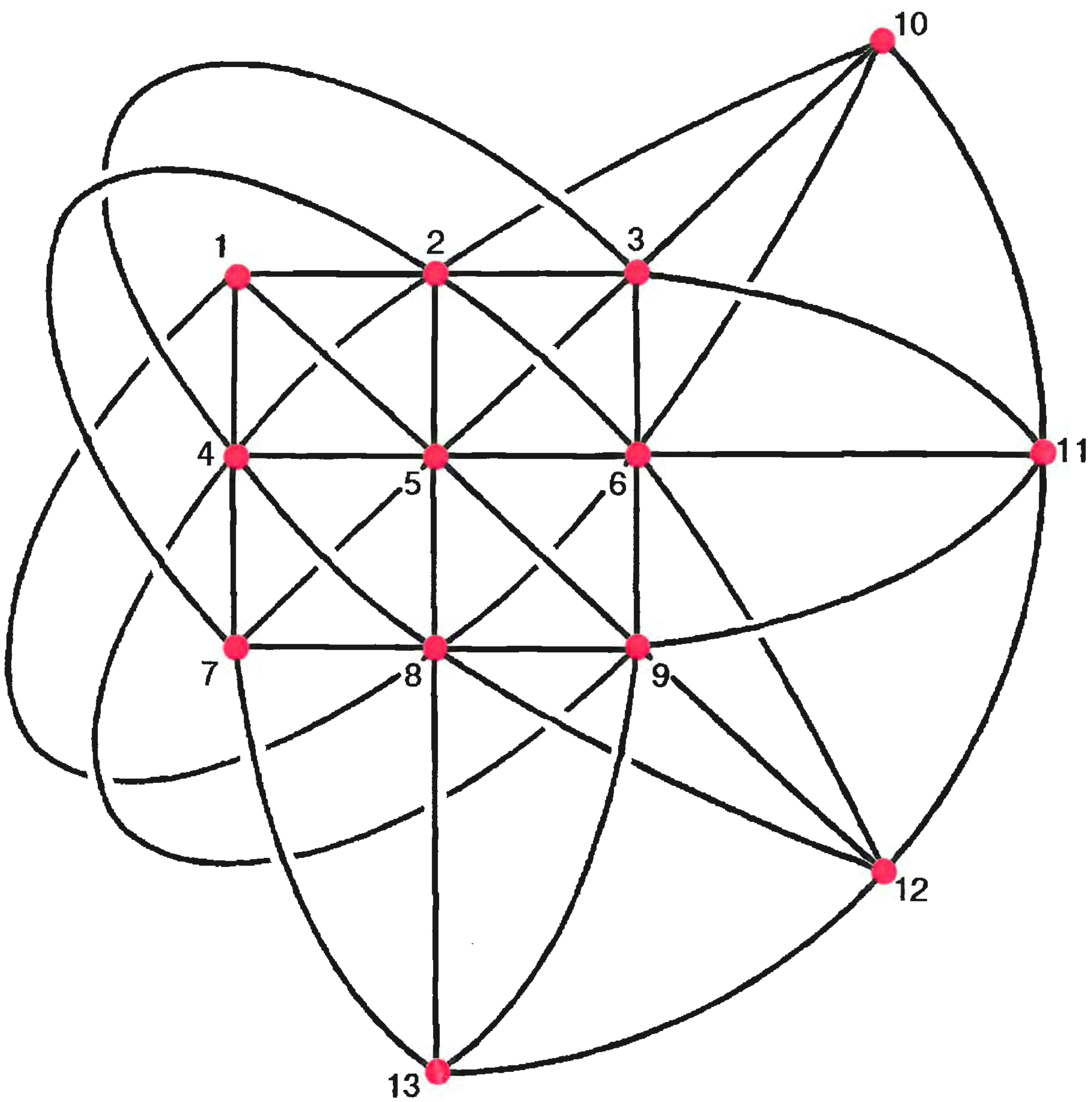
affine plane $AG(2,3)$

$\Big\downarrow$ 3 fold extension

W_{12}



The affine plane AG(2,3)



The projective plane PG(2,3)

Point model of W_{12} in $PG(5,3)$

$\mathcal{K}$ ... 12 points

5 points in $\mathcal{K} \Rightarrow \exists^*$ hyperplane $\mathcal{H}$

$$\#(\mathcal{H} \cap \mathcal{K}) = 6$$

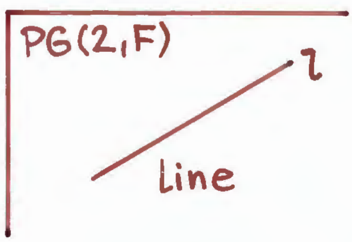
H.S.M. Coxeter, G. Pellegrino, J.A. Todd

Veronese surface:

Veronese mapping:

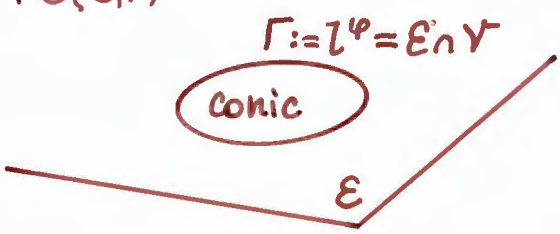
$$\underbrace{F(x_0, x_1, x_2)}_{PG(2,F)} \xmapsto{\varphi} \underbrace{F(x_0^2, x_0x_1, x_0x_2, x_1^2, x_1x_2, x_2^2)}_{PG(5,F)}$$

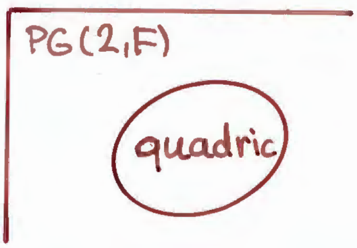
$\operatorname{im}\varphi =: \mathcal{V}$ ... Veronese surface



$\longleftrightarrow$
(#F > 2)

PG(5,F)





$$\sum_{i \le j} a_{ij} x_i x_j = 0$$

$\longleftrightarrow$

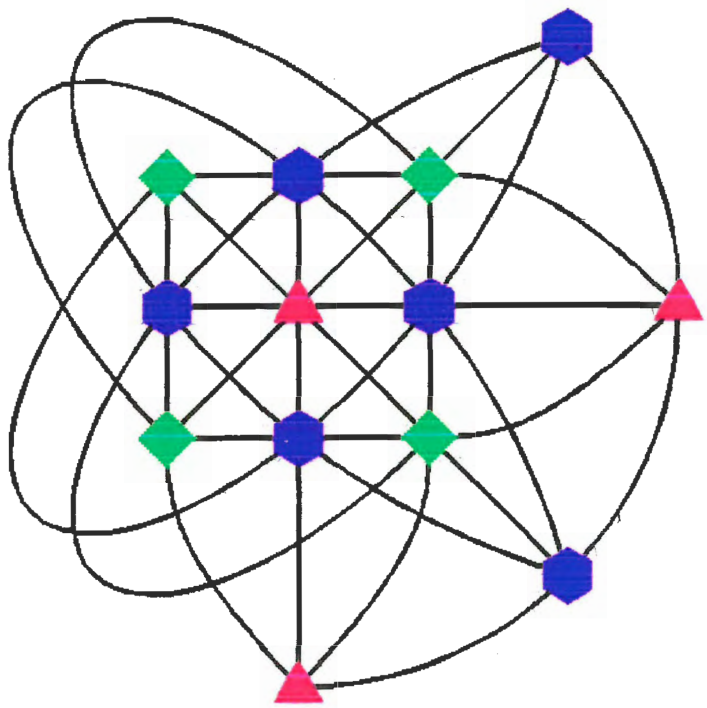
PG(5,F)

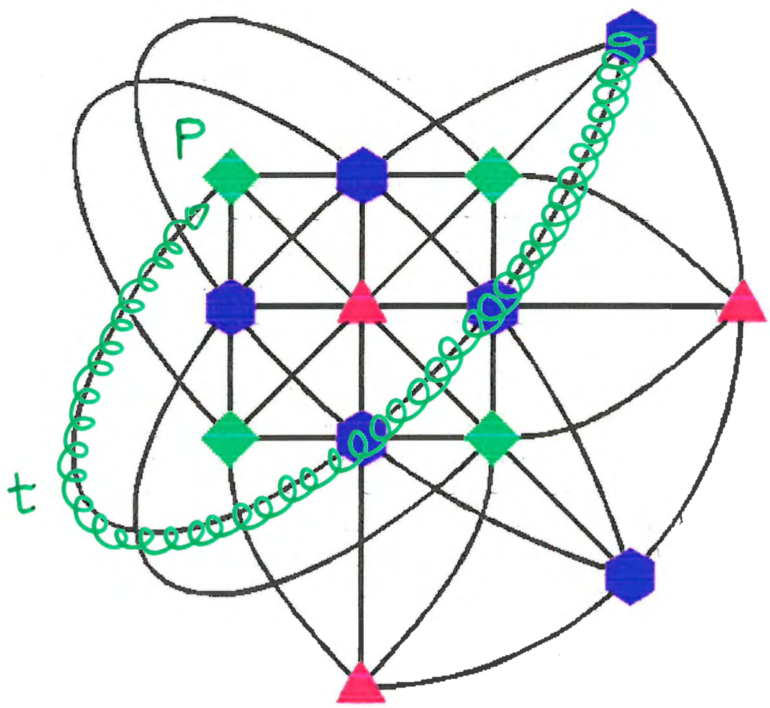
hyperplane section of $\mathcal{V}$

$$\sum_{i \le j} a_{ij} y_{ij} = 0$$

Zanella – H.

The plane of a conic (green) in PG (2,3)





$\varkappa^0 = \mathrm{id}$

$\varkappa$ ... (P,t)-elation

$\varkappa^2$ ... (P,t)-elation

General replacement in PG(5,3):

$F := GF(3) = \{0, 1, 2\}$

There are four conic planes of $\mathcal{V}$ through P

... ε_i , $i \in F \cup \{\infty\}$

$\Rightarrow$ four collineations $\varkappa_i$ (◆ ↦ ▲)

$$(p, q, r, s) \in F^4$$

$$\Downarrow$$

$$(\varkappa_0^p, \varkappa_1^q, \varkappa_2^r, \varkappa_\infty^s)$$

$$\Downarrow$$

$$\underbrace{\mathcal{V} \setminus \{P\}}_{12\text{ points}} \xrightarrow[\text{Replacement}]{} \text{12 POINTS}$$

81 distinct 12-sets in PG(5,3)

each $\subset$ algebraic hypersurface (... points on a chord of $\mathcal{V}$)

$(p,q,r,s) \in (F^4,+) \Rightarrow 12$ points

* $p+q+r+s=0$

$x_0^p, x_1^q, x_2^r, x_\infty^s$ extend to a collineation of $PG(5,3)$

$\Rightarrow$ 12-sets projectively equivalent to $\gamma \setminus \{P\}$

* $p+q+r+s=1$

$\Rightarrow$ 12-sets projectively equivalent to $\mathcal{K}$

* $p+q+r+s=2$

$\Rightarrow$ other 12-sets

(1,1,1,1) - replacement in terms of coordinates

Parametric representation of $\mathcal{K}$

$$\underbrace{F(x_0,x_1,x_2)}_{\neq F(1,0,0)} \mapsto F(x_0^2+1,\ x_0x_1,\ x_0x_2,\ x_1^2,\ x_1x_2,\ x_2^2)$$

Remark:

$\Rightarrow$ generator matrix of G_{12} (Golay code)