3-dim. <u>PAPPIAN</u> projective space   5-dim. proj. space

lines ——————γ—————▷ PLÜCKER quadric

KLEIN-map

Properties of γ:

(I) injective

(II)

Such a γ does not exist in the non-PAPPIAN case.

H.H. (1981)

1

3-dim. PAPPIAN projective space   5-dim. proj. space

lines $\xrightarrow{\quad \gamma \quad}$ PLÜCKER quadric

| | | not in the tangent hyperplane in A | • $A = a^\gamma$ within the tangent hyperplane in A |
| Skew to a | not skew to a | | |

$\psi \Big\downarrow$ stereographic projection through A

λ

| affine 4-space $\quad\rightsquigarrow\quad$ ruled quadric |

4-dim. proj. space

$\beta := \lambda^{-1}$
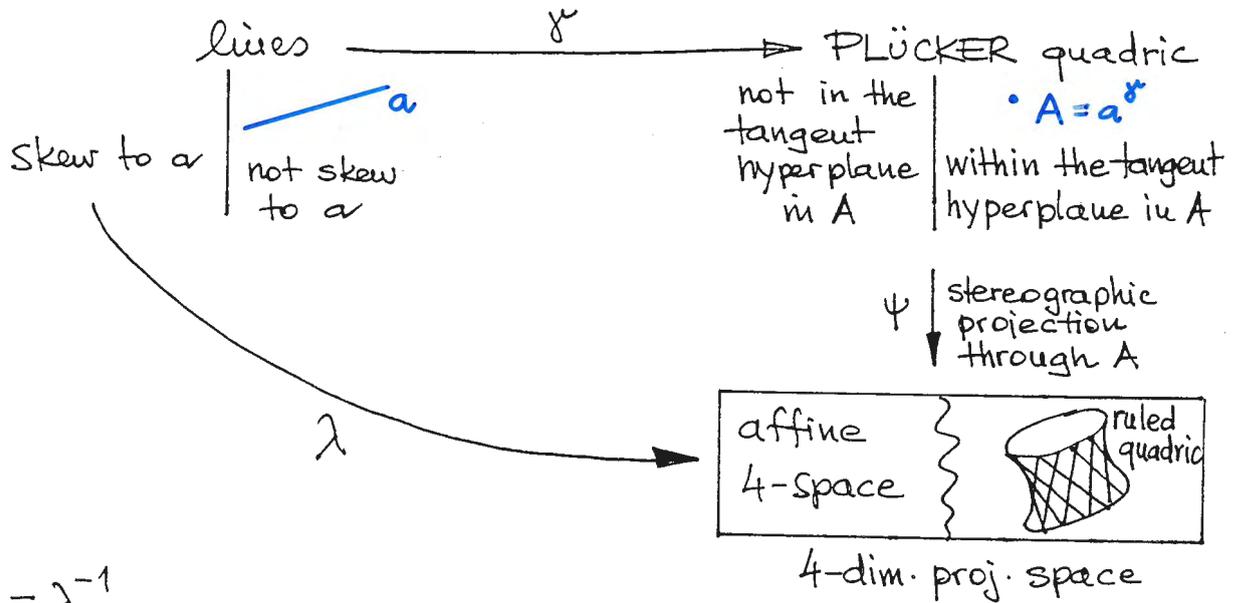
R. METZ, A. HERZER.
(1981)   (1984)

3-dim. projective space $\mathcal{P}(W_K)$ 4 dim. projective space $\mathcal{P}(V_K)$

(K... not necessarily commutative field)

$p_0, p_\infty \in \text{Hom}_K(W, W)$ $p_j := p_0 + j \cdot p_\infty$ $j \in Z$ (centre of K)

$\downarrow \quad \downarrow$ $\downarrow$

$\pi_0 \quad \pi_\infty$ π_j

$\mathcal{P}(W_K)$

ε_j

X^{π_∞}

X^{π_j}

X^{π_0}

ε_∞

a

$\mathcal{P}(\text{im } p_0) =: \varepsilon_0$

X^β

β

$\mathcal{P}(V_K)$

affine
4-space

$\cdot X$

$\mathcal{P}(\ker p_0) =: a_0$

a_j

$a_\infty \qquad \mathcal{J}$

$r: \text{"absolute regulus"}$
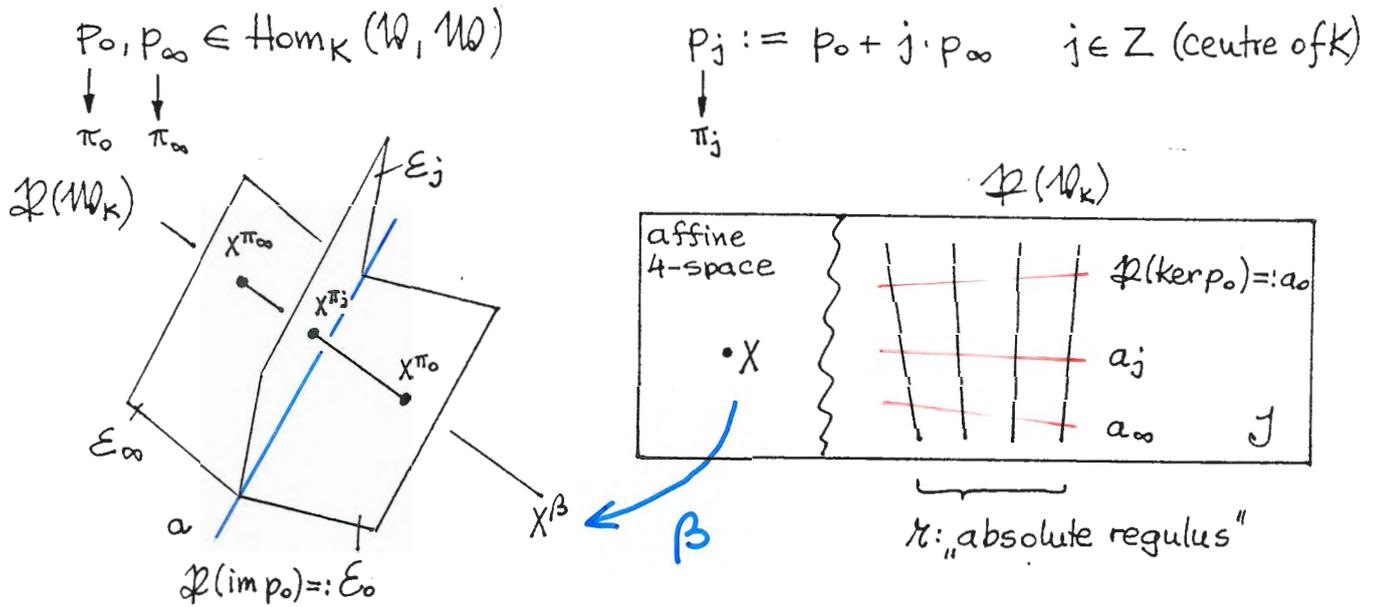
$\{\varepsilon_j \mid j \in Z \cup \{\infty\}\} =: a_Z^*$
(sub)-pencil

$\{a_j \mid j \in Z \cup \{\infty\}\}$
directrices of r

Images of lines $L = l \cap \mathcal{J}$

(I)

$l^{\pi_j} = X^{\pi_j}$

X^β

l^{π_k}

X^{π_k}

a
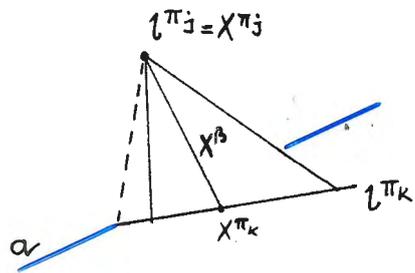
$l^\beta \subset$ pencil of lines

(II)

l^{π_j}

X^{π_j}

X^β

a X^{π_k} l^{π_k}
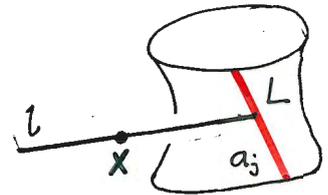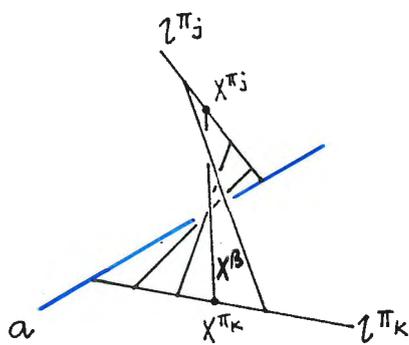
$X^{\pi_j} \mapsto X^{\pi_k}$ $(X \in l)$ .. projectivity

$l^\beta \subset$ regulus

(III)

X^β

l^{π_j} $\mathcal{F}$

X^{π_j}

$L^{\pi_j} = L^{\pi_k}$

X^{π_k}

a l^{π_k}
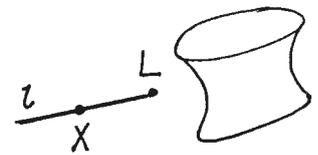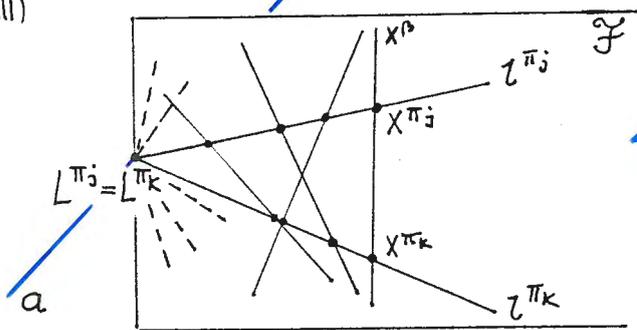
$X^{\pi_j} \mapsto X^{\pi_k}$ $(X \in l)$ .. projectivity but
                                      no perspectivity

$l^\beta \subset$ degenerate conic of lines
↑
C-configuration
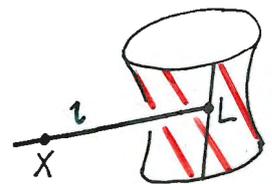
Images of hyperplanes $\quad \mathcal{H} \neq \mathcal{J}$

$\mathcal{H}^{\pi_j}$

β

$\mathcal{H} \cap \mathcal{J}$

a_j

$r \in \mathcal{R}$

2 lines

(⊂ special linear complex)

$\mathcal{H}^\beta$ less similar

↑

(⊂ general linear complex)

β

$\mathcal{H} \cap \mathcal{J}$

conic

$\mathcal{H}^\beta \subset$ union of ruled planes

(does not exist)

degenerate conic within the dual space β

$\mathcal{H} \cap \mathcal{J}$

degenerate conic
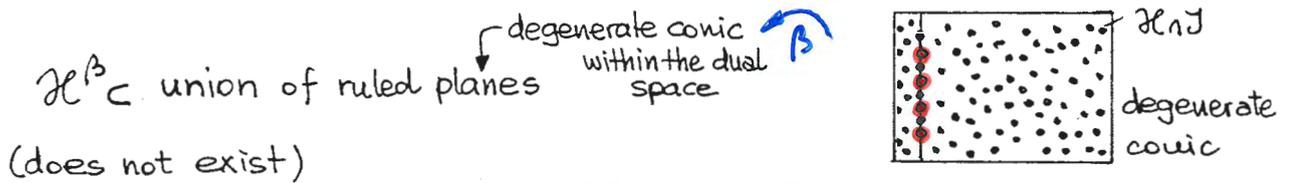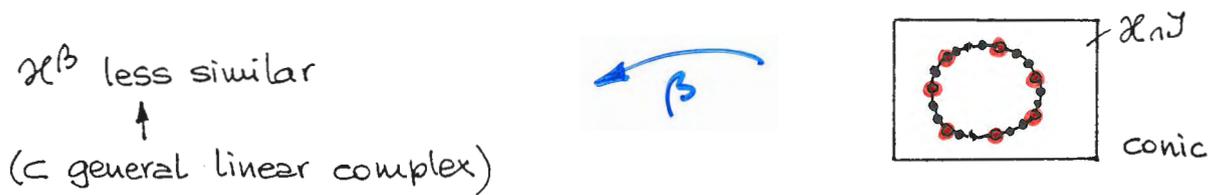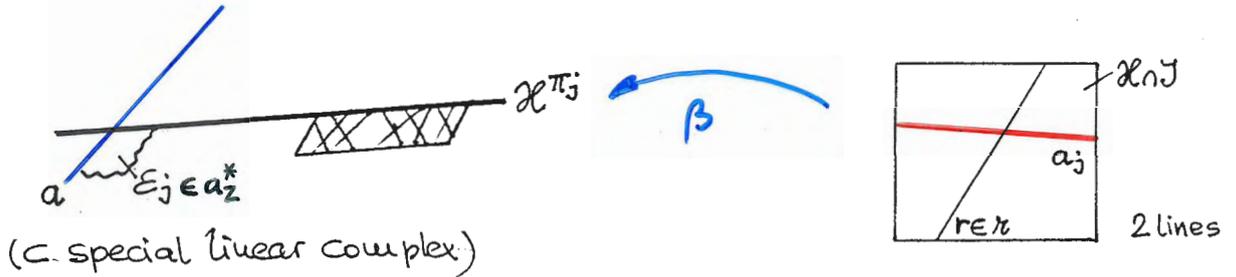
Images of planes $\quad m \neq m \cap \mathcal{J} =: \mathfrak{m}$

---

9 possibilities for $\mathfrak{m}^\beta$

Discussion of the possibile positions of $\mathfrak{m}$ with respect to the absolute regulus $\mathcal{R}$