L quaternion skew field

Z centre of L

K maximal commutative subfield of L

$$[L:Z]=4 \quad , \quad [L:K]=[K:Z]=2$$

$\Sigma(K,L)$ .... points : projective line over L
$\mathcal{P}_L(L^2)$

blocks : K-sublines , chains

cf. BENZ

Problems :

1.) point model for $\Sigma(K,L)$ in some projective space over K

2.) affine point of view

L^2 over L $\rightsquigarrow$ L^2 over K (4-dim.)

$\mathcal{P}_K(L^2)$ … 3-dim. proj. space

$\mathcal{P}_L(L^2)$ $\rightsquigarrow$ spread of $\mathcal{P}_K(L^2)$ … $\mathcal{S}_{L|K}$

chain $\rightsquigarrow$  … chain

$t \notin \mathcal{S}_{L|K}$

transversal line

cf. KIST-REINMIEDL, H.H. (STUDY)

\# of transversal lines of a chain = $\#\mathrm{Gal}(K/Z)$

KLEIN mapping $\gamma : \mathcal{L} \to \widehat{\mathcal{P}}_K \quad (\cong PG(5,K))$

lines of $\mathcal{P}_K \; (L^2)$

$\mathcal{L}^\gamma =: Q$ ... KLEIN quadric

γ-image of the spread $\mathcal{S}_{L|K}$:

$\exists^*$ Baer-subspace Π_z of $\widehat{\mathcal{P}}_K$ such that

- $\mathcal{S}_{L|K}{}^\gamma = \Pi_z \cap Q$ ... oval quadric of Π_z

γ-image of a chain $\mathcal{C} \subset \mathcal{S}_{L|K}$:

$\exists^*$ 3-dim. subspace $\mathfrak{X} \subset \widehat{\mathcal{P}}_K$ such that

- $\mathfrak{X} \cap \Pi_z$ ... 3-dim. subspace of Π_z
- $\mathfrak{X} \cap \Pi_z \cap Q = \mathcal{C}^\gamma$ ... elliptic quadric of $\Pi_z \cap \mathfrak{X}$
- $\mathfrak{X} \cap Q$ ... contains a line of $\widehat{\mathcal{P}}_K$

(These conditions are necessary and sufficient.)

cf. HOTJE, (STUDY), H.H.

$\downarrow$

$\Sigma(Z,L)$

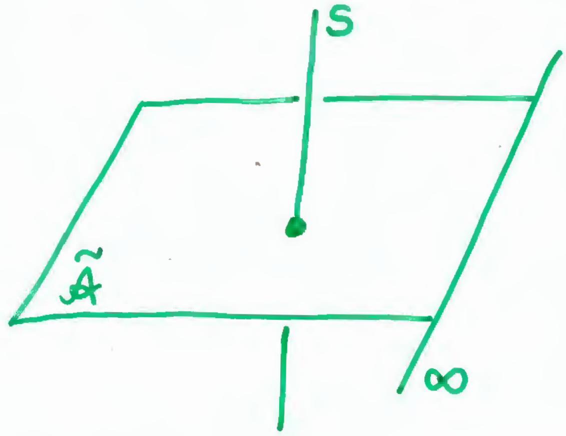
$\infty \in \mathcal{S}_{L|K}$

$\tilde{\mathcal{A}}$ ... plane through ∞

$\tilde{\mathcal{A}} \setminus \infty =: \mathcal{A}$ ... affine plane

$$\varrho : \mathcal{S}_{L|K} \setminus \{\infty\} \to \mathcal{A}$$
$$s \mapsto s \cap \tilde{\mathcal{A}}$$

is a bijection



Let ∞ ... $(0,1)L$, $(1,0)K \in \tilde{\mathcal{A}}$ $\Rightarrow$

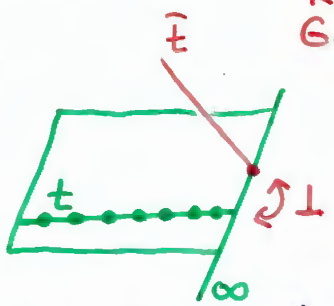
$$(l_0, l_1)L \overset{\varrho}{\longmapsto} \underbrace{(1, l_1 l_0^{-1})K}_{\in \mathcal{A}} \equiv l_1 l_0^{-1} \in L$$

ϱ-image of a chain $\mathcal{C}$ with transversal line t: ($\bar{t}$)

K|Z Galois

1) $\bar{t} \subset \tilde{\mathcal{A}}$: $(\mathcal{C} \setminus \{\infty\})^{\varrho}$ ... affine line

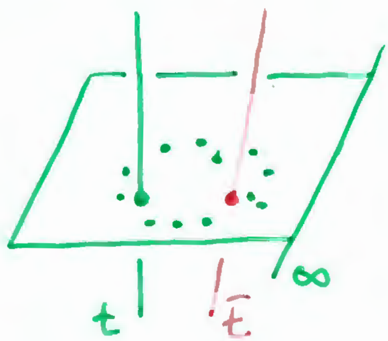
orthogonality in $\mathcal{A}$
ruled by $t \cap \infty$, $\bar{t} \cap \infty$ ($\Leftrightarrow$ norm)



2) $\bar{t} \not\subset \tilde{\mathcal{A}}$, $\infty \in \mathcal{C}$: $(\mathcal{C} \setminus \{\infty\})^{\varrho}$ ... degenerate circle

degenerate circles are affine Baer subplanes (over Z)

3) $\infty \notin \mathcal{C}$: $\mathcal{C}^{\varrho}$ ... non-degenerate circle



All non-degenerate circles are in one orbit of $AGL(1,L)$

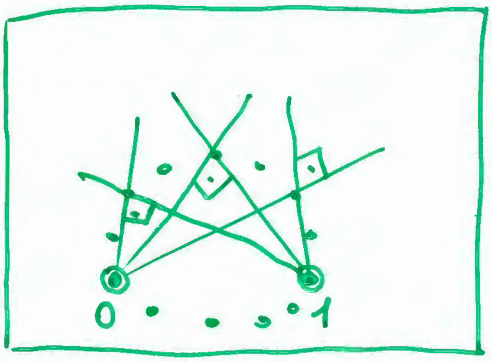
Γ_0 : $N(u+iv) = u$ $\qquad u,v \in K$, $i \in L \setminus K$ with...

KIZ Galois:

$\mathcal{H}_e$: $N(u+iv) = eu + \bar{e}\bar{u}$ $\qquad e,u,v \in K \qquad e+\bar{e}=1$

affine Hermitian variety containing Γ_0

$$\Gamma_0 = \bigcap_{\substack{e \in K \\ e+\bar{e}=1}} \mathcal{H}_e$$



0 and 1 are, however, the only points of Γ_0 with this property

alternative approach: „stereographic (?) projection" of $\Pi_Z \cap Q = \mathcal{J}_{L|K}^{\gamma}$

cf. H.H.