

Symplectic Plücker-Transformations

We discuss bijections of the set \mathcal{L} of lines of a symplectic space, i.e. a (not necessarily finite-dimensional) projective space with orthogonality based upon an absolute symplectic (null) quasipolarity. Following [1], two lines are called related, if they are concurrent and orthogonal, or if they are identical. A bijection of \mathcal{L} that preserves this relation in both directions is called a (symplectic) Plücker transformation. We shall show that any bijection $\mathcal{L} \rightarrow \mathcal{L}$ taking related lines to related lines is already a Plücker transformation. Moreover, a complete description of all Plücker transformations will be given: Plücker transformations of symplectic spaces with dimensions other than three are induced by orthogonality-preserving collineations. For three-dimensional symplectic spaces all Plücker transformations can be obtained - up to orthogonality-preserving collineations - by replacing some but not necessarily all non-isotropic lines by their absolute polar lines.

References

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