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Isomorphisms of affine Plücker spaces

Let $A = (\mathcal{P},\mathcal{L},\parallel)$ be an affine space. Two lines $a,b \in \mathcal{L}$ are called related ($\sim$) if $a \cap b \neq \emptyset$. Then the *Plücker space* on A is defined as $(\mathcal{L},\sim)$. An (affine) *Plücker-transformation* is a bijection $\varphi : \mathcal{L} \to \mathcal{L}$ preserving $\sim$ in both directions.

By a theorem of W. Benz [1], any Plücker transformation of $(\mathcal{L},\sim)$ arises from an affinity if $A = \mathbb{R}^n$, $3 \leq n \in \mathbb{N}$. The proof given there essentially makes use of the following geometric counterpart of $\operatorname{Char} \mathbb{R} \neq 2$: The diagonal lines of any parallelogram have non-empty intersection. By an alternative approach we give a simple proof for the following theorem:

Any Plücker transformation of an affine space A with $\dim A \geq 3$ *arises from a collineation of A.*

As is well-known, any collineation of an affine space of order > 2 is an affinity (i.e. preserving parallelism), whereas a collineation of an affine space of order 2 is merely a bijection on its set of points.

Possible generalizations of the theorem will be discussed.

Reference:

[1] Benz, W.: Geometrische Transformationen, BI Wissenschaftsverlag, Mannheim Leipzig Wien Zürich, 1992.