Plücker space $(L, \sim)$:

L ... any set,
$\sim$ ... a binary relation on L such that

1. $\sim$ is reflexive and symmetric,
2. $(L, \sim)$ is connected.

$a \sim b$ ... *related* elements
$a \approx b :\Leftrightarrow a \sim b$ and $a \neq b$ ... *adjacent* elements.

Isomorphism of Plücker spaces $(L, \sim)$, $(L', \sim')$:

A bijection $\varphi : L \to L'$ such that

$$a \sim b \Leftrightarrow a^{\varphi} \sim' b^{\varphi} \text{ for all } a, b \in L.$$

$\mathsf{A} = (\mathcal{P}, \mathcal{L}, \parallel)$ ... affine space.

Affine Plücker space:

$(\mathcal{L}, \sim)$ with

$$a \sim b :\Longleftrightarrow a \cap b \neq \emptyset \;\; (a, b \in \mathcal{L})$$

Plücker space on A.

Assumptions:

$\mathsf{A} = (\mathcal{P}, \mathcal{L}, \parallel)$,

$\mathsf{A}' = (\mathcal{P}', \mathcal{L}', \parallel')$, $\dim \mathsf{A}' \geq 3$,

$\varphi : \mathcal{L} \to \mathcal{L}'$ ... a bijection satisfying

$$a \sim b \Longrightarrow a^{\varphi} \sim' b^{\varphi} \text{ for all } a, b \in \mathcal{L}.$$

Theorem 1 *The mapping*

$$\lambda : \mathcal{P} \to \mathcal{P}', \ a \cap b \mapsto a^{\varphi} \cap b^{\varphi} \ (a, b \in \mathcal{L}, \ a \approx b)$$

is a well-defined injection preserving collinearity and non-collinearity of points. Moreover,

$$\mathcal{L}(Q)^{\varphi} = \mathcal{L}'(Q^{\lambda}) \text{ for all } Q \in \mathcal{P}$$

and

$$\dim \mathsf{A} \geq 3.$$

Theorem 2 *If the order of* A *is not two, then for all* $Q \in \mathcal{P}$ *the restricted mapping*

$$\varphi|\mathcal{L}(Q) : \mathcal{L}(Q) \to \mathcal{L}'(Q^{\lambda})$$

is a semicollineation of the projective space A/Q *onto* A'/Q^{λ}*, i.e. a bijective mapping preserving collinearity of "points".*

Theorem 3 *Each of the following conditions is sufficient for λ to be a collineation:*

1. φ *is an isomorphism.*
2. A *or* A' *is finite.*
3. $\dim \mathsf{A} \leq \dim \mathsf{A}' < \infty$.
4. *The order of* A *is different from two and every monomorphism of an underlying field of* A *in an underlying field of* A' *is surjective.*
5. A *and* A' *are affine spaces of order two.*