

discrete geometric structures 2022

TU Wien
Aug 29 - Sep 2, 2022

invited speakers

Ulrike Bücking (FU Berlin)
Albert Chern (UCSD)
Andrew Sageman-Furnas (NCSU)



FWF

organizers

Niklas Affolter (TU Berlin)
Christian Müller (TU Wien)
Jan Techter (TU Berlin)

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Contact us

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Jan Techter, techter@tu-berlin.de

Locations

Building:

TU Wien
Freihaus (green area)
Wiedner Hauptstraße 8-10
1040 Vienna

Registration:

 in front of *FH Hörsaal 6* (2nd floor)

Talks:

 *FH Hörsaal 6* (2nd floor)

Coffee breaks:

 short coffee break: in front of *FH Hörsaal 6* (2nd floor)

 long coffee break: *Zeichensaal 3* (7th floor)

Rooms for discussions:

 *Dissertantenzimmer* (8th floor)

 *Zeichensaal 1* (8th floor)



8th floor

7th floor

2nd floor

Participants

Niklas Affolter (TU Berlin/ENS Paris)	Georg Nawratil (TU Wien)
Peter Albers (Universität Heidelberg)	Mason Pember (FSU London)
Fabian Bittl (TU Berlin)	Martin Peternell (TU Wien)
Alexander Bobenko (TU Berlin)	Ulrich Pinkall (TU Berlin)
Cédric Boutillier (Sorbonne Université)	Dominik Pint (TU Wien)
Ulrike Bücking (FU Berlin)	Denis Polly (Kobe University)
Albert Chern (UCSD)	Konrad Polthier (FU Berlin)
Joseph Cho (TU Wien)	Helmut Pottmann (KAUST)
Hana Dal Poz Kouřimská (ISTA)	Matteo Raffaelli (TU Wien)
Felix Dellinger (TU Graz)	Sanjay Ramassamy (IPHT of CEA Saclay)
Adam Doliwa (UWM)	Arvin Rasoulzadeh (TU Wien)
Kostiantyn Drach (ISTA)	Wayne Rossman (Kobe University)
Alexander Fairley (TU Berlin)	Andrew Sageman-Furnas (NCSU)
Oliver Gross (TU Berlin)	Simon Schwarz (Uni Göttingen)
Ran Gutin (ICL)	Kiumars Sharifmoghaddam (TU Wien)
Christian Hafner (ISTA)	Nina Smeenk (TU Berlin)
Ivan Izmistiev (TU Wien)	Yousuf M. Soliman (CALTECH)
Aditya Kapilavai (TU Wien)	Gudrun Szewieczek (TU Wien)
Felix Knöppel (TU Berlin)	Sergei Tabachnikov (PSU)
Carl Lutz (TU Berlin)	Jan Techter (TU Berlin)
Paul Melotti (Université Paris-Saclay)	Jonas Tervooren (TU Wien)
Ruzica Mijic (TU Wien)	Johannes Wallner (TU Graz)
Christian Müller (TU Wien)	Masashi Yasumoto (Tokushima University)

Abstracts

Monday

Peter Albers (Universität Heidelberg)

Monday 11:00

Symplectic billiards

After giving a short introduction to the general theory of symplectic billiards I will focus on the 2D polygonal case. The focus will be on explicit examples. I will present some theorems and some open questions. This is joint work with G. Banhatti, F. Sadlo, R. Schwartz, and S. Tabachnikov.

Nina Smeenk (TU Berlin)

Monday 11:30

Geometry of the discrete time Euler top and related 3-dimensional birational maps

In my talk we will consider a specific discretization of the Euler top given by Hirota and Kimura. Explicit solutions can be described either by a bi-rational map or by using Jacobi Elliptic functions.

The conserved quantities of this system geometrically describe quadrics in real 3-space. They span a pencil of quadrics with a base curve consisting of two real connected components. Ruled quadrics in this pencil can be used to describe maps interchanging points on the two components. The composition of two such (suitable) maps coincides with iterations of the discrete time Euler top.

During the talk we will investigate how to describe these maps either by complex involutions, using Jacobi elliptic functions, or by real involutions. We will determine the (degenerate) cases where the real involutions become bi-rational maps. Finally this will give a description of the solutions of the discrete time Euler top as the composition of two (complex / real / rational) involutions.

Andy Sageman-Furnas (NCSU)

Monday 14:00

Constructing isometric tori with the same curvatures

Which data determine an immersed surface in Euclidean three-space up to rigid motion? A generic surface is locally determined by only a metric and mean curvature function. However, there are exceptions. These may arise in a family like the isometric family of vanishing mean curvature surfaces transforming a catenoid into a helicoid, or as a so-called Bonnet pair of surfaces.

For compact surfaces, Lawson and Tribuzy proved in 1981 that a metric and non-constant mean curvature function determine at most one immersion with genus zero, but at most two compact immersions (compact Bonnet pairs) for higher genus.

In this talk, we discuss our recent construction of the first examples of compact Bonnet pairs. It uses a local classification by Kamberov, Pedit, and Pinkall in terms of isothermic surfaces. Moreover, we describe how a structure-preserving discrete theory for isothermic surfaces and Bonnet pairs led to this discovery.

The smooth theory is joint work with Alexander Bobenko and Tim Hoffmann and the discrete theory is joint work with Tim Hoffmann and Max Wardetzky.

Jonas Tervooren (TU Wien)

Monday 16:00

Smooth and discrete Cone-nets

Cone-nets are conjugate nets on a surface such that along each individual curve of one family of parameter curves there is a cone in tangential contact with the surface. The corresponding conjugate curve network is projectively invariant and is characterized by the existence of particular transformations. We study properties of that transformation theory and illustrate how several known surface classes appear within our framework. We present cone-nets in the classical smooth setting of differential geometry as well as in the context of a consistent discretization with counterparts to all relevant statements and notions of the smooth setting. Special emphasis deserve smooth and discrete tractrix surfaces as those cone-nets which are characterized as principal nets with constant geodesic curvature along one family of parameter curves.

Johannes Wallner (TU Graz)

Monday 16:30

Developable Quad Meshes

Developable surfaces constitute a prominent class of surfaces, besides being important for applications. Computing with developables is notoriously difficult, and there has been a great number of individual contributions to this topic. Nearly all of the well-known geometric properties of developables have been pressed into service for characterizing developability for different kinds of surface representations. This presentation reports on progress made in recent years, in particular on so-called checkerboard patterns associated with quad meshes. We give definitions of both isometric mappings and of developability in terms of checkerboard patterns, and show how they be effectively used for computational tasks such as modeling.

Adam Doliwa (UWM)

Tuesday 10:00

Incidence geometric structures behind the Padé and Hermite-Padé approximation

Approximation by rational functions (the Padé approximation) proves to be very useful in numerical calculations and works especially well for functions with singularities. It turns out that the nominators and denominators of the approximants satisfy integrable equations (known in the numerical algorithms community as the Frobenius identities) of the Toda lattice family, with discrete variables being degrees of the polynomials. The rational approximants themselves satisfy in turn the so called Wynn recurrence (the missing identity of Frobenius), whose geometric interpretation we are going to present. The meaning is valid in the projective line over arbitrary division ring - in the described context it is the field of rational functions (embedded into the field of bounded Laurent series).

In the case of the division ring of non-commutative series we obtain the corresponding generalization of the Padé approximants described some time ago by Gel'fand and Retakh in terms of quasi-determinants. For the field of complex numbers with the standard embedding of the real line (the Moebius chain geometry) we can use the additional structure to reduce the geometric construction. In particular, with appropriate change of the initial data we obtain that the same equation (the Wynn recurrence) which governs the Padé approximation theory applies to the cyclic patters of discrete analytic function theory.

Hermite used multi-series generalization of the Padé theory to prove the transcendency of the Euler number. Recently we found that the corresponding integrable difference equations can be described as admissible reductions of Hirota's discrete Kadomtsev-Petviashvili system. Their geometric meanig is provided by the so called Desargues maps. It is worth of mentioning that the same equations (with simple linear change of independent variables) appeared recently in the theory of multi-orthogonal polynomials.

Ulrike Bücking (FU Berlin)

Tuesday 11:00

Convergence results for discrete conformal maps in the plane and discrete minimal surfaces

Discrete conformal maps as discrete analogues of smooth conformal maps may be defined in several ways. In my talk, I consider two notions, namely conformally equivalent triangulations and cross-ratio preserving mappings. Smooth conformal maps are linked to minimal surfaces via Weierstrass representation formulas, and this is also known for the two discrete notions considered in my talk. In order to make this connection more explicit, I present two convergence results. The first result concerns discrete conformal maps defined by conformally equivalent triangulations from Dirichlet boundary conditions. The second result is joint work with Daniel Matthes and explains the construction of discrete minimal surfaces based on cross-ratio preserving discrete holomorphic functions which locally approximate the solution of a given Björling problem. For both results I also explain some main ideas of the proof.

Masashi Yasumoto (Tokushima University)

Tuesday 14:00

Discrete p-holomorphic functions and discrete timelike minimal surfaces

In this talk we focus on a discrete version of p-holomrphic functions with values in the split-complex plane. They have similar properties to discrete holomorphic functions, and there are several properties that do not appear in the previous case. Using discrete p-holomorphic functions, we describe two types of Weierstrass-type representations for discrete timelike minimal surfaces. As an application, we can show that all the discrete timelike minimal surfaces can be described by a discrete version of null curves.

Felix Dellinger (TU Wien)

Tuesday 14:30

A checkerboard pattern approach to Isothermic Surfaces

A checkerboard pattern is a quadrilateral net with $\mathbb{Z}^2$ combinatorics where every second face is a parallelogram. Such a mesh can be easily obtained through midpoint subdivision from a general quadrilateral net. The structure of a checkerboard pattern is very suitable to describe discrete differential geometric properties of the net. In particular, we can use it to consistently define conjugate nets, principal curvature nets, a shape operator and Koenigs nets.

We find that the class of discrete principal curvature nets is invariant under Möbius transformations and can be studied in the projective model of Möbius geometry. Koenigs nets are exactly those nets that allow a discrete dualization. Analogously to the smooth case, they can be characterized by the existence of certain osculating conics or by the equality of their Laplace invariants.

Isothermic nets can then be characterized as Koenigs nets that are also principal curvature nets. Again we can transform them using the Möbius transformation or dualization. The combination of both allows us to easily create examples of discrete minimal surfaces and their Goursat transformations.

Hana Dal Poz Kouřimská (ISTA)

Tuesday 16:00

The geometry of thickened bisectors

In my talk I will show you what happens to perpendicular bisectors in spaces of constant curvature upon small distortions of the metric, and how we can use this to triangulate manifolds.

Simon Schwarz (Uni Göttingen)

Tuesday 16:30

Towards convergence of the heat method via large deviations

Using probabilistic techniques, we prove that the short-time behavior of the discrete heat kernel on scaling limits of polyhedral surfaces is subject to a phase transition. This provides a central step towards the convergence of the heat method for distance computation.

Wednesday

Oliver Gross (TU Berlin)

Wednesday 10:00

Filament Based Plasma

The arcs visible in our sun's atmosphere are some of the most awe-inspiring natural spectacles and their visual depiction is of great interest in scientific visualization, special effects, and games. These prominent features can be modeled as magnetic filaments whose shape is governed by the magnetohydrostatic (MHS) equation. This talk will explain how a variational problem corresponding to a new Lagrangian model for the solar atmosphere can be solved by geometry optimization.

Albert Chern (UCSD)

Wednesday 11:00

Geometric Measure Theory and Kelvin Geometry for Convexifying and Compactifying PDEs

This talk consists of two geometric techniques for computational problems, both motivated by Geometric Measure Theory. They are applied to geometric optimizations and solving partial differential equations (PDEs) on unbounded domains, which are fundamental in shape synthesis and physical simulations. Shape optimization problems often involve non-convex geometric functionals such as surface area. Directly minimizing these energies can lead to stuck at local

minima, and initializing a correct surface topology is challenging. Geometric measure theory provides new ways of approaching geometric optimization problems. Classical curves and surfaces are generalized to differential forms representing superpositions of infinitely many curves and surfaces. Under such a representation, the minimal surface problem becomes convex, and standard convex optimization techniques apply. The second part of the talk focuses on simulations aided by Kelvin transformation. Many physical simulation problems take place in an unbounded space, requiring solving PDEs on a non-compact domain. Standard numerical approaches rely on coordinate mapping or domain truncation, yielding coordinate singularity or artifacts on the truncation boundary. We describe a general Kelvin transformation technique, which maps the infinite domain to a bounded one without creating singularities. The method is made possible by factoring out an asymptotic of the singularity induced by the coordinate stretching. The resulting transformation of functions can be understood as the natural transformation for fractional densities in geometric measure theory. In the viewpoint of Klein's Erlangen Program, the analysis reveals a "Kelvin Geometry," where objects are functions subject to Kelvin transforms, leaving the PDE of interest invariant. The key to solving the infinite domain problem is to recognize that the boundedness quality of the domain is not a geometrically invariant notion under Kelvin Geometry. Therefore, we can transform the infinite domain problem into a compact one without sacrificing numerical accuracy.

Thursday

Sergei Tabachnikov (PSU)

Thursday 10:00

Projective evolutes of pentagons

The evolute of a curve is the envelope of its normals. A discrete analog, the evolute of a polygon, is formed by the perpendicular bisectors of the sides of the polygon. Iterations of this Euclidean construction were studied recently.

In this talk, a projectively natural analog of perpendicular bisectors of the sides of a polygon will be defined, and the map that sends a polygon to the polygon formed by the projective perpendicular bisectors of its sides will be studied in the simplest case of pentagons. The second iteration of the map, acting on the moduli space of projective pentagons, has a rational integral whose level curves are cubic curves, and the transformation on these level curves is conjugated to the map $f(x) = ? - 4x \pmod{1}$. I shall also present some experimental results in the case of projective hexagons.

Sanjay Ramassamy (IPHT of CEA Saclay)

Thursday 11:00

Integrable dynamics on polygons and the dimer integrable system

On the one hand, several discrete-time dynamical systems on spaces of polygons have been shown in the last twenty years to be integrable. On the other hand, Goncharov and Kenyon introduced ten years ago an integrable system associated with the dimer model on bipartite graphs on the torus. Building upon the notion of triple crossing diagram maps (introduced in recent works of Affolter, Glick, Pylyavskyy and myself), I will describe a framework which encompasses both the geometric dynamics on polygons and the dimer integrable system. This framework makes it possible in particular to identify the conserved quantities of both systems. I will illustrate this paradigm on the example of the pentagram map.

This talk is based on joint works with Niklas Affolter (TU Berlin and École normale supérieure), Terrence George (University of Michigan), Max Glick (Google) and Pavlo Pylyavskyy (University of Minnesota).

Paul Melotti (Université Paris-Saclay)

Thursday 11:30

Combinatorics of dSKP and geometric systems

We consider the discrete Schwarzian KP equation on a lattice. We show how its solution can be expressed via ratios of generating function of some combinatorial objects, namely oriented dimer configurations. We then represent many geometric systems as such dSKP evolutions, and using the previous solution we can prove various properties of these systems, such as the reappearance of singularities, or Devron property. Among those systems, I will mention discrete holomorphic functions and the pentagram map. Joint work with Niklas Affolter and Béatrice de Tilière.

Alexander Fairley (TU Berlin)

Thursday 14:00

Circular nets with spherical parameter lines and terminating Laplace sequences

In the context of discrete differential geometry, surfaces with spherical curvature lines are classical surfaces that motivate the study of circular nets with spherical parameter lines. An important feature of surfaces with spherical curvature lines is that they have terminating Laplace sequences. There is a similar phenomenon for circular nets with spherical parameter lines.

Gudrun Szewieczek (TU Wien)

Thursday 14:30

Discrete cyclic systems and nets in hyperbolic space

A 2-dimensional congruence of circles in 3-space is called cyclic if it admits a 1-parameter family of smooth orthogonal surfaces. Classical geometers deeply investigated the relation between the geometry of those circle congruences and their orthogonal surfaces. In particular, it was pointed out how special cyclic congruences provide orthogonal surfaces of various integrable surface classes.

In this talk we shall discuss an integrable discretization of this classical notion and point out how it can be used to construct discrete surfaces in hyperbolic space, as for example discrete flat fronts.

Mason Pember (FSU London)

Thursday 16:00

Constrained elastic curves and spherical curvature lines.

Constrained elastic curves are curves that minimise the bending energy for conformal and area preserving variations. They appear on many integrable surfaces, for example as the curvature lines of the Wente torus. In this talk we show how we can characterise such curves in Lie sphere geometry and then use this characterisation to generate Lie applicable surfaces with one family of spherical curvature lines.

Denis Polly (TU Wien)

Thursday 16:30

Discrete channel linear Weingarten surfaces

Channel linear Weingarten surfaces have been an important field of study since the 19th century. Lie sphere geometry provides the tools to understand them in hyperbolic, spherical and Euclidean space forms simultaneously. These descriptions lend themselves to define discrete analogues of linear Weingarten and channel surfaces within the realm of (integrable) discrete differential geometry (cf [Burstall, Hertrich-Jeromin, Rossmann, Nagoya math, 2018], [Hertrich-Jeromin, Rossmann, Szewieczek, Math Z, 2020]). In this talk, we discuss discrete channel linear Weingarten surfaces in space forms and try to recover a result from the smooth theory: non-tubular channel linear Weingarten surfaces are always rotational. We will see, however, that the definition of discrete non-tubularity needs refinement in the discrete category.

Carl Lutz (TU Berlin)

Friday 10:00

Canonical tessellations of decorated hyperbolic surfaces

A decoration of a hyperbolic surface is a choice of circle, horocycle or hypercycle about each cone point, cusp or flare of the surface, respectively. Each decoration induces a canonical tessellation and dual decomposition of the underlying surface. They exhibit a rich geometric structure. Amongst others: a characterisation in terms of the hyperbolic geometric analogues of Delaunay's empty discs and Laguerre's tangent-distance and connections to convex hulls in Minkowski space. This talk will give an overview of the hyperbolic geometric tools needed to analyse decorated hyperbolic surfaces and sketch the derivation of the properties mentioned above.

Ran Gutin (ICL)

Friday 11:00

Clifford algebras and CK geometry: A middle-ground between Euclid and Descartes

Why linear fractional transformations (LFTs) over hypercomplex numbers are interesting, with applications to the pedagogy of iterative matrix algorithms. This is a presentation on:

- Why it's worth looking at the LFTs (that is, analogues of the Moebius transformations) over hypercomplex numbers. Connections with Laguerre and Cayley-Klein geometries are discussed here.
- Why this can make complicated numerical linear algebra more visual, with the example of the QR algorithm.
- Briefly, LFTs over the dual numbers.
- Briefly, LFTs over the double numbers.
- Why LFTs over the real numbers represent both the transformations and the cycles of hyperbolic geometry.
- Reducing the geometry of the double and dual number cases to the real case.
- Discussion about visualising the QR algorithm again.
- Generalisation via Clifford algebras.

Georg Nawratil (TU Wien)

Friday 11:30

Generalizing continuous flexible Kokotsakis belts of the isogonal type

Antonios Kokotsakis studied the following problem in 1932: Given is a rigid closed polygonal line (planar or non-planar), which is surrounded by a polyhedral strip, where at each polygon vertex three faces meet. Determine the geometries of these closed strips with a continuous mobility. On the one side, we generalize this problem by allowing the faces, which are adjacent to polygon line-segments, to be skew; i.e to be non-planar. But on the other side, we restrict to the case where the four angles associated with each polygon vertex fulfill the so-called isogonality condition that both pairs of opposite angles are equal or supplementary. In more detail, we study the case where the polygonal line is a skew quad, as this corresponds to a (3×3) building block of a so-called V-hedra composed of skew quads. The latter also gives a partial answer to a question posed by Robert Sauer in his book "Differenzengeometrie" of 1970 whether continuous flexible skew quad surfaces exist.

Ivan Izmestiev (TU Wien)

Friday 14:00

Discrete Laplacians and infinitesimal isometric deformations

It is a classical fact that the vertical component of an infinitesimal isometric deformation (IID) of a paraboloid of revolution is a harmonic function. In this talk we prove a discrete analog: the vertical component of an IID of a polyhedron inscribed in the paraboloid is a discrete harmonic function, with respect to the cotangent Laplacian.

Similar, but less known is the fact that the radial component of an IID of a spherical domain is an eigenfunction of the spherical Laplacian. We establish a discrete analog: the radial component of an IID of an inscribed polyhedron is an eigenfunction of the discrete spherical Laplacian.

The talk is based on a joint work with Roman Prosanov.

Joseph Cho (TU Wien)

Friday 14:30

Semi-discrete mKdV equations and Darboux deformations

In this talk, we show an efficient method to obtain semi-discrete modified Korteweg-de Vries equation as the compatibility condition using Darboux deformations arising from semi-discrete isothermic surfaces. This work is based on the joint work with Wayne Rossman and Tomoya Seno.

Excursion and Dinner

General Information

Requirement to wear an **FFP2 mask** in Vienna:

- In public transport and in enclosed stops, stations, and platforms
- In pharmacies
- In the health sector

Excursion

Option 1: Walk from Cobenzl to Dinner (Buschenschank Fuhrgassl-Huber)

Meeting point in front of Freihaus main entrance **at 2pm**

Public transport to starting point of excursion:

U4 Karlsplatz (direction Heiligenstadt) → Heiligenstadt

Bus 38A (direction Kahlenberg) → Cobenzl Parkplatz

Walk to Dinner (see map below)

Important Information

Distance: $\approx 4\text{km}$ Time: $\approx 60\text{ min}$

Ascend: $\approx 90\text{m}$ Descend: $\approx 190\text{m}$ (partially steep descend on paved road)

Option 2: Public transport directly to dinner

Public transport to dinner (*without excursion*):

U4 Karlsplatz (direction Heiligenstadt) → Spittelau

Bus 35A (direction Salmansdorf) → Neustift am Walde

Dinner at 5pm

Buschenschank Fuhrgassl-Huber

Neustift am Walde 68, 1190 Wien

www.fuhrgassl-huber.at

Public transport back to Karlsplatz:

Bus 35A (direction Spittelau) → Spittelau

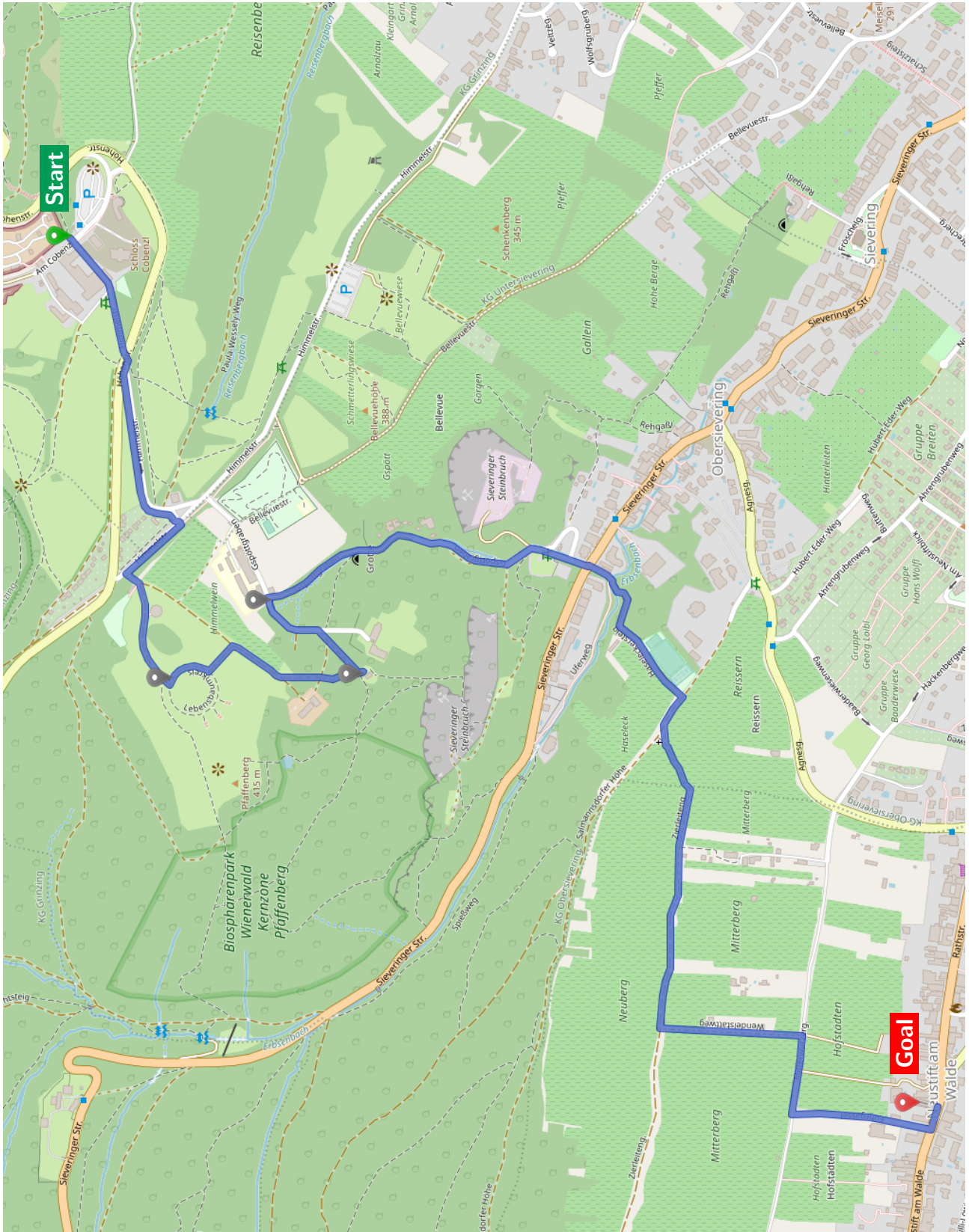
U4 Spittelau (direction Hütteldorf) → Karlsplatz

Walk from Cobenzl to Dinner (Buschenschank Fuhrgassl-Huber)

Important Information

Distance: $\approx 4\text{km}$ Time: $\approx 60\text{ min}$

Ascend: $\approx 90\text{m}$ Descend: $\approx 190\text{m}$ (partially steep descend on paved road)



Lunch and Dinner Recommendations

Lunch recommendations close to the university:

- | | |
|---|--|
| 1 Swing Kitchen (Vegan)
Operngasse 24, 1040 Wien
www.swingkitchen.com | 9 Bep Viet (Vietnamese)
Favoritenstraße 2, 1040 Wien
www.bep-viet.at |
| 2 BÁNH MÌ VIENNA (Vietnamese)
Faulmanngasse 1, 1040 Wien
www.banh-mi4.eatbu.com | 10 Pizzeria RIVA Favorita (Pizzeria)
Favoritenstraße 4/6, 1040 Wien
www.riva.pizza |
| 3 SHU Restaurant Vienna (Sichuan)
Operngasse 28, 1040 Wien | 11 Santos Wieden (Mexican)
Favoritenstraße 4-6/2, 1040 Wien
www.santos-bar.com |
| 4 Matcha Komachi (Japanese)
Operngasse 23, 1040 Wien
www.matchakomachi.com | 12 BOK Restaurant (Pan-Asian)
Favoritenstraße 8, 1040 Wien
www.bok.co.at |
| 5 El Burro (Mexican)
Margaretenstraße 9, 1040 Wien
www.elburro.at | 13 Wieden Bräu (Austrian)
Waaggasse 5, 1040 Wien |
| 6 Kojiro 2 (Japanese)
Kühnplatz 4, 1040 Wien | 14 Chang (Asian)
Waaggasse 1, 1040 Wien
www.chang.at |
| 7 Yong Streetfood (Asian)
Rechte Wienzeile 9A, 1040 Wien
www.yongstreetfood.at | 15 Gorilla Kitchen (Burritos)
Gußhausstraße 19, 1040 Wien
www.gorillakitchen.at |
| 8 Tofu und Chili (Chinese)
Linke Wienzeile 18, 1060 Wien | |

Dinner recommendations:

Filmfestival Rathausplatz (Delicacies from around the world)

Rathausplatz, 1010 Wien

www.filmfestival-rathausplatz.at/en/gastronomy

Waldviertlerhof (Austrian)

Schönbrunner Str. 20, 1050 Wien

www.waldviertlerhof.at

