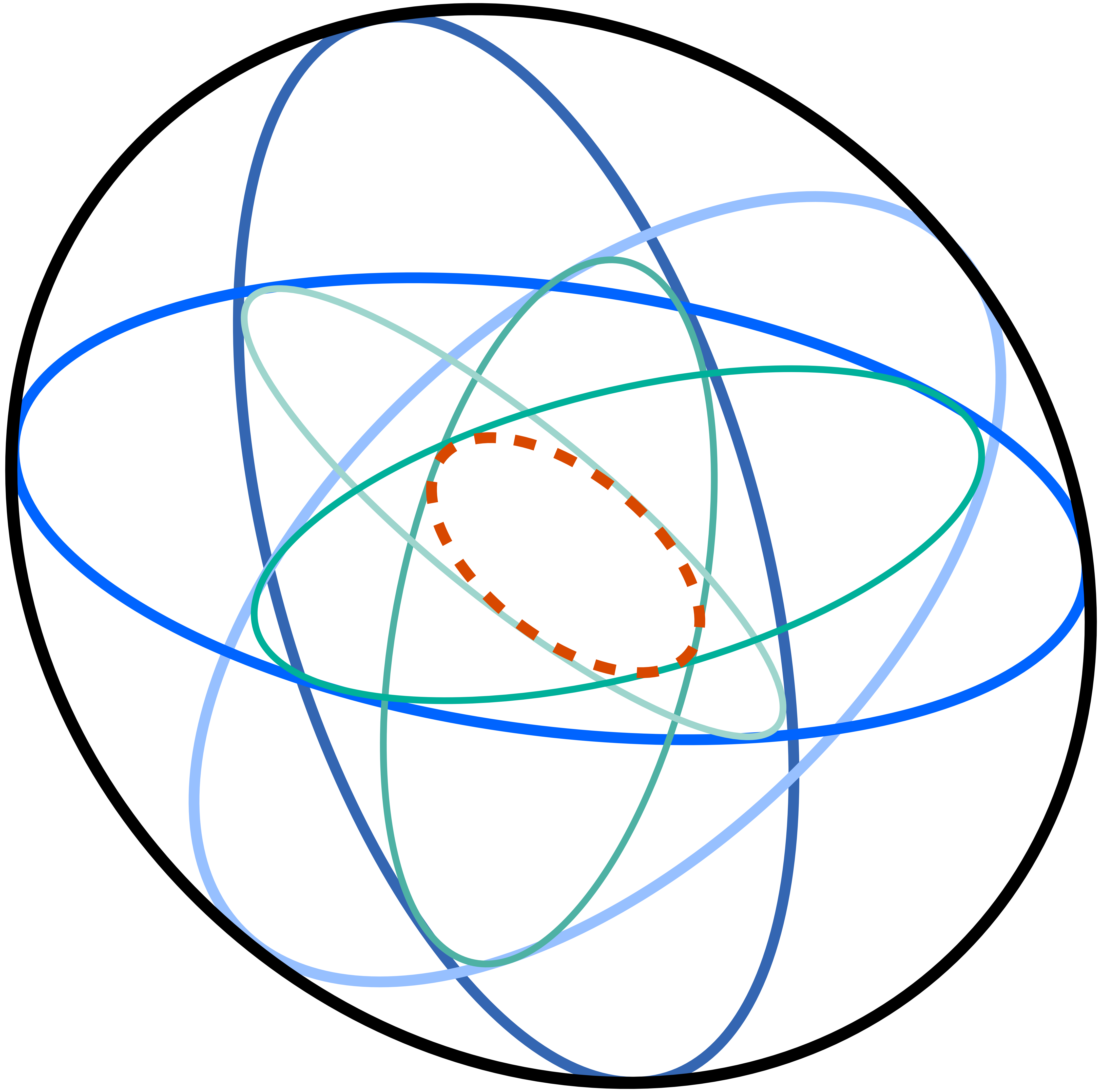
Discrete Geometric Structure
September 2, 2024

Penrose's 8-Conic Theorem



Albert Chern
University of California San Diego

Discrete Geometric Structure
September 2, 2024

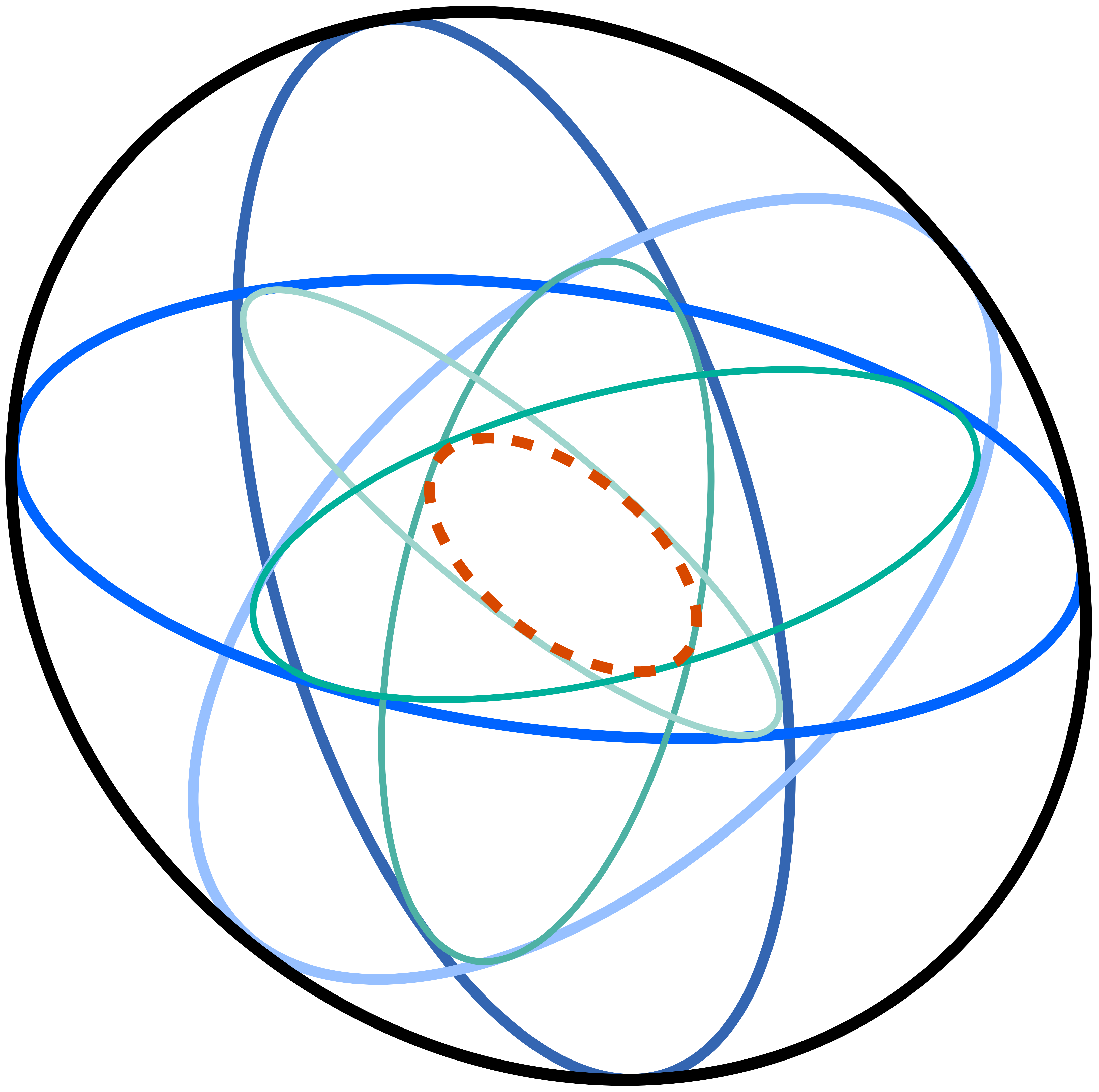
Albert Chern

Russell Arnold

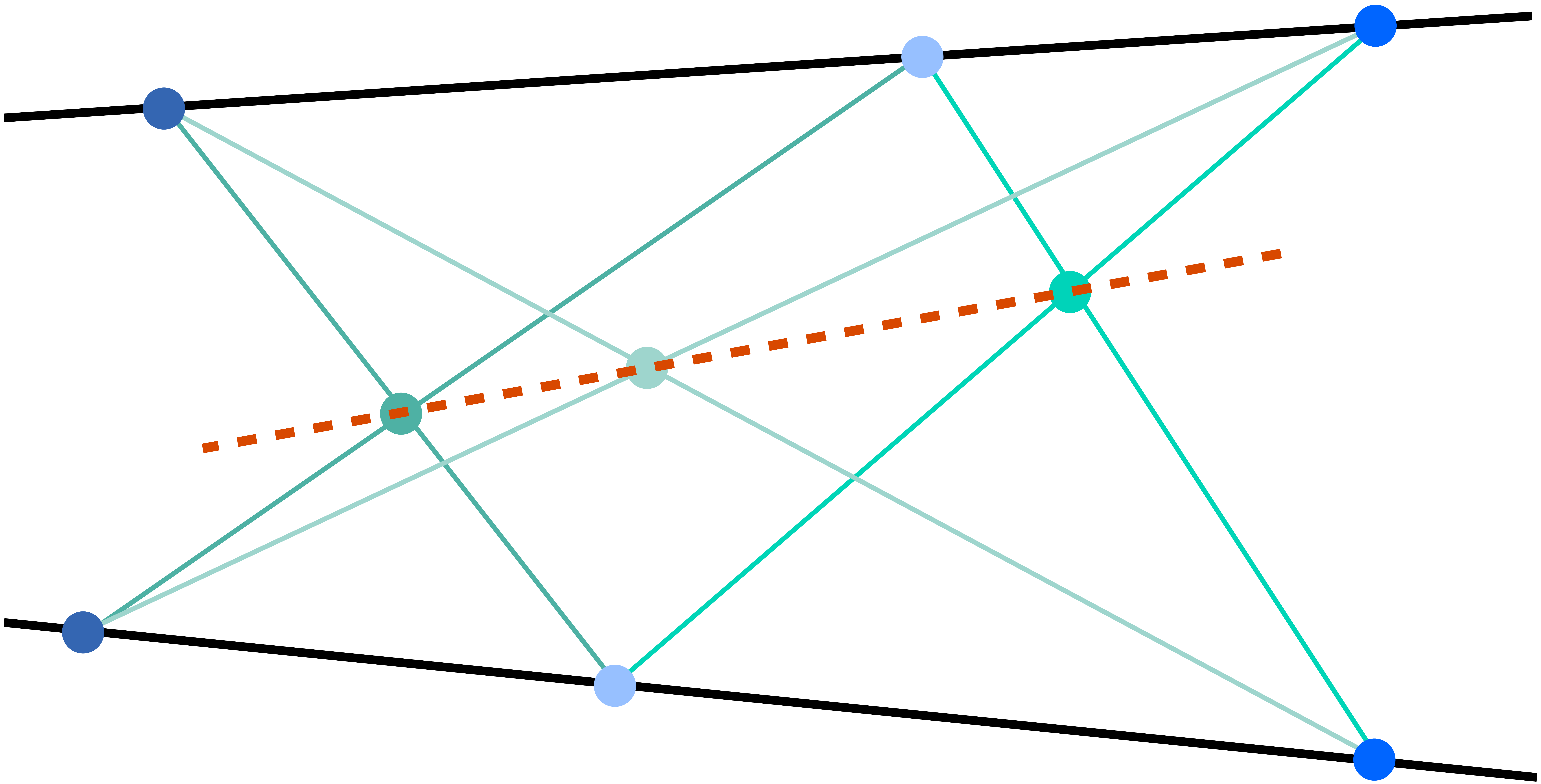
Charles Gunn

Thomas Neukirchner

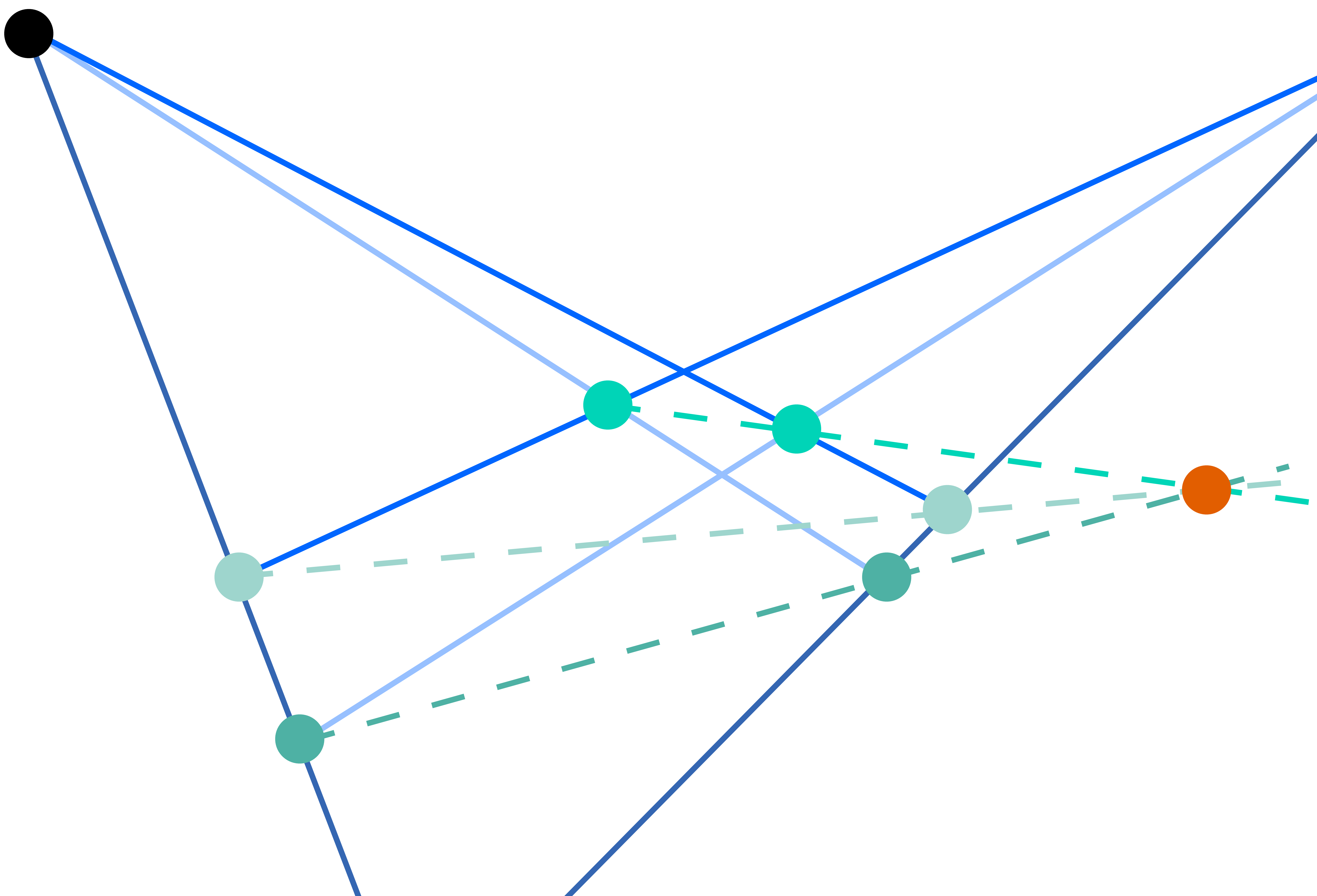
Sir Roger Penrose



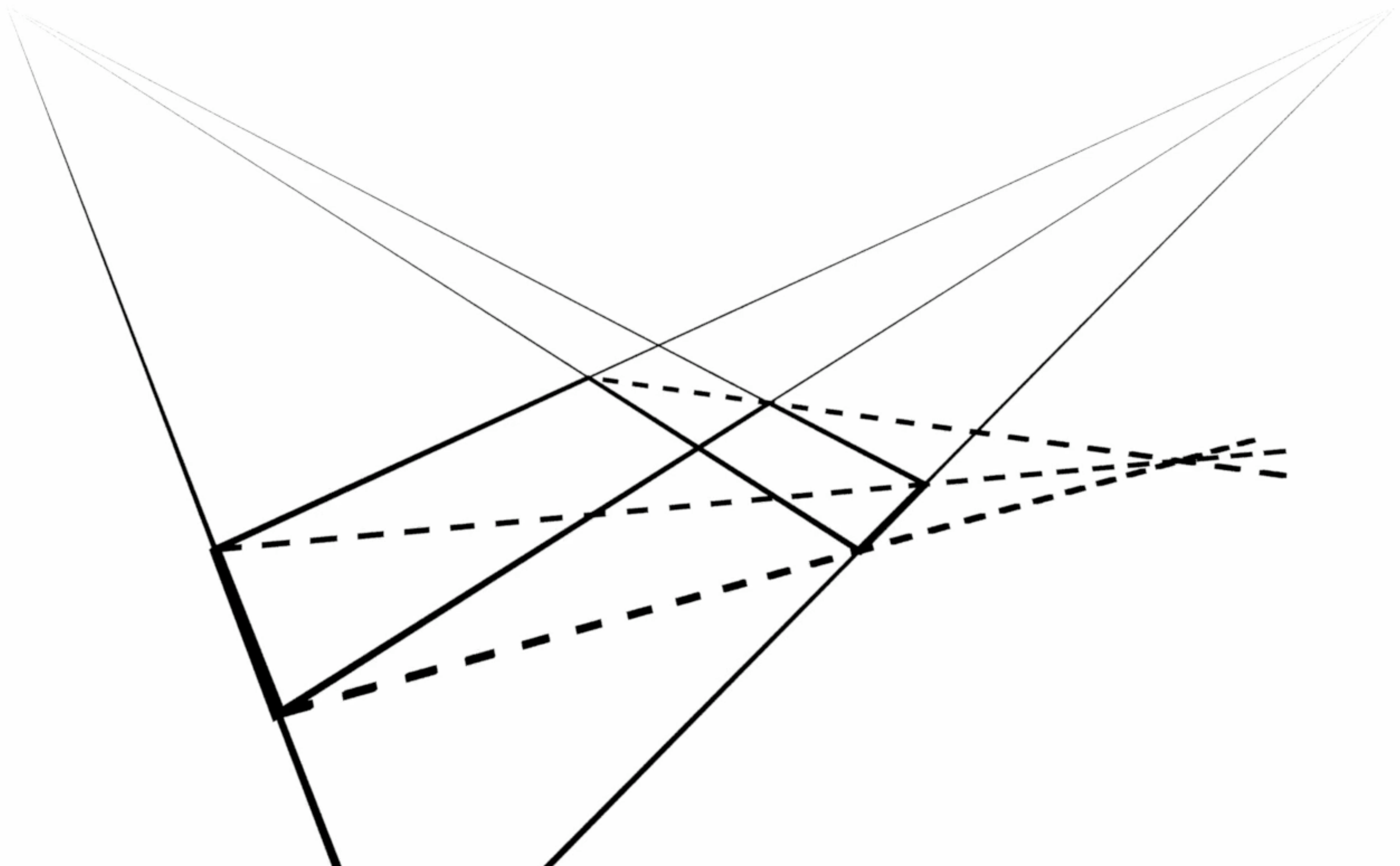
Pappus' Theorem (340AD)

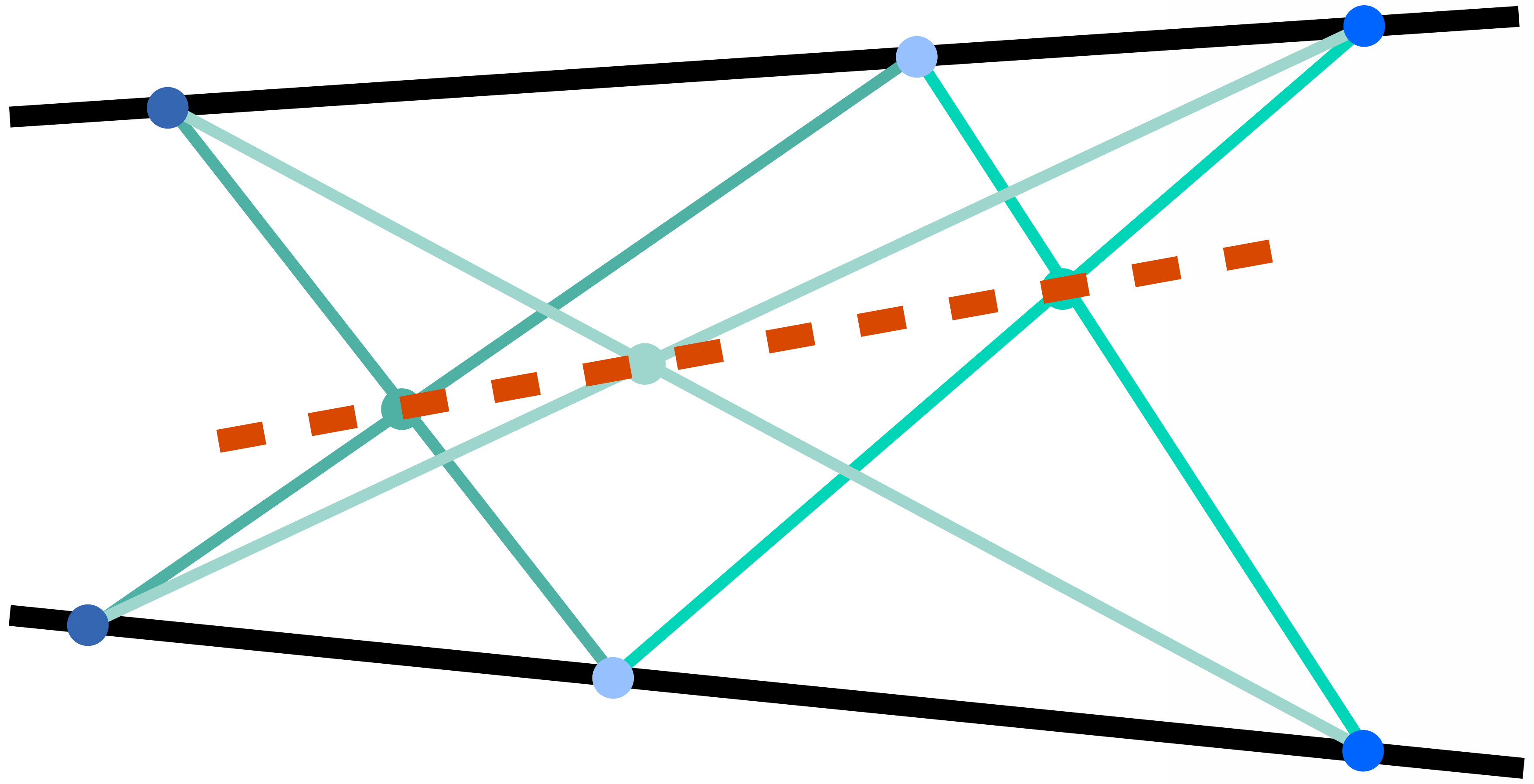


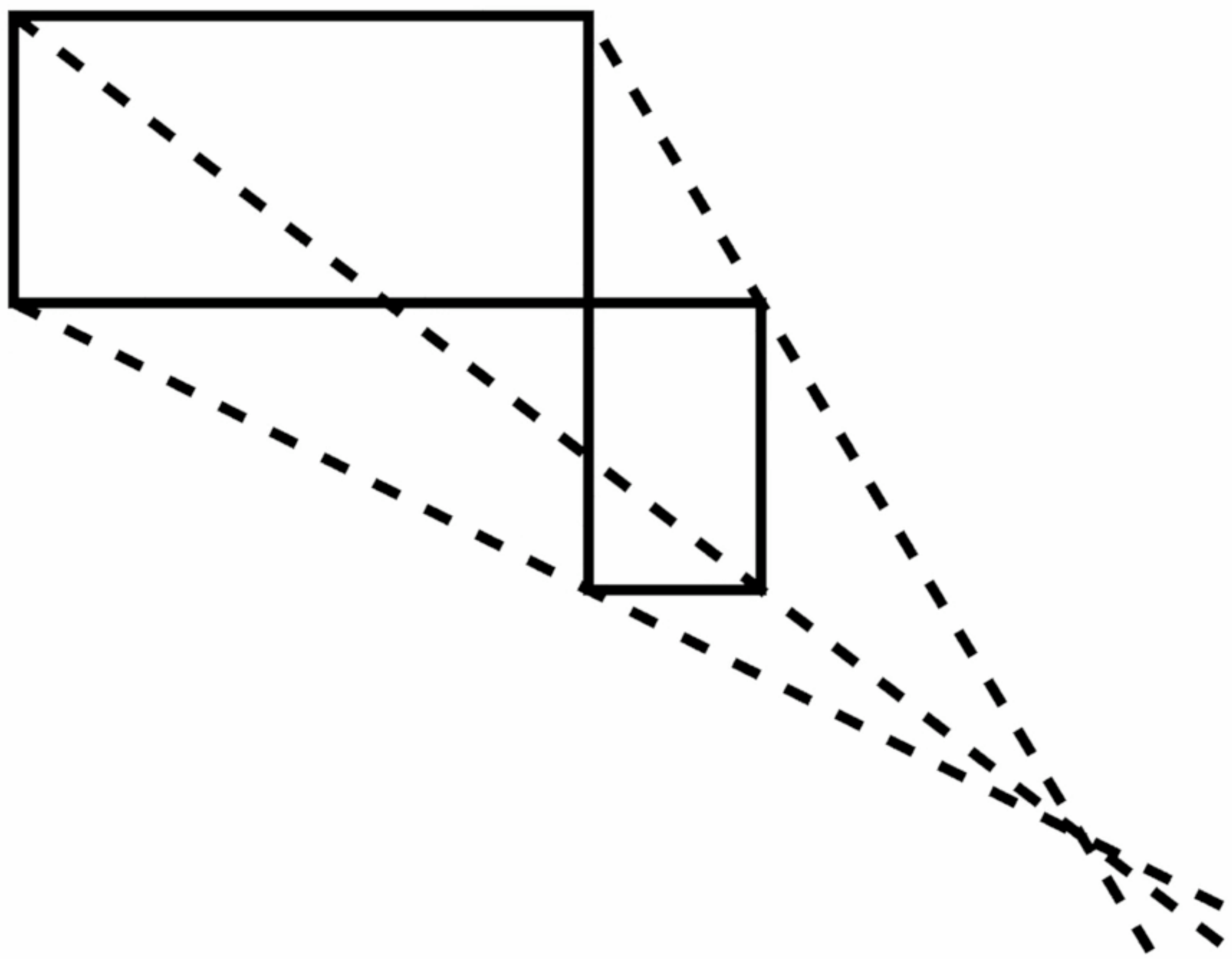
Dual Pappus' Theorem



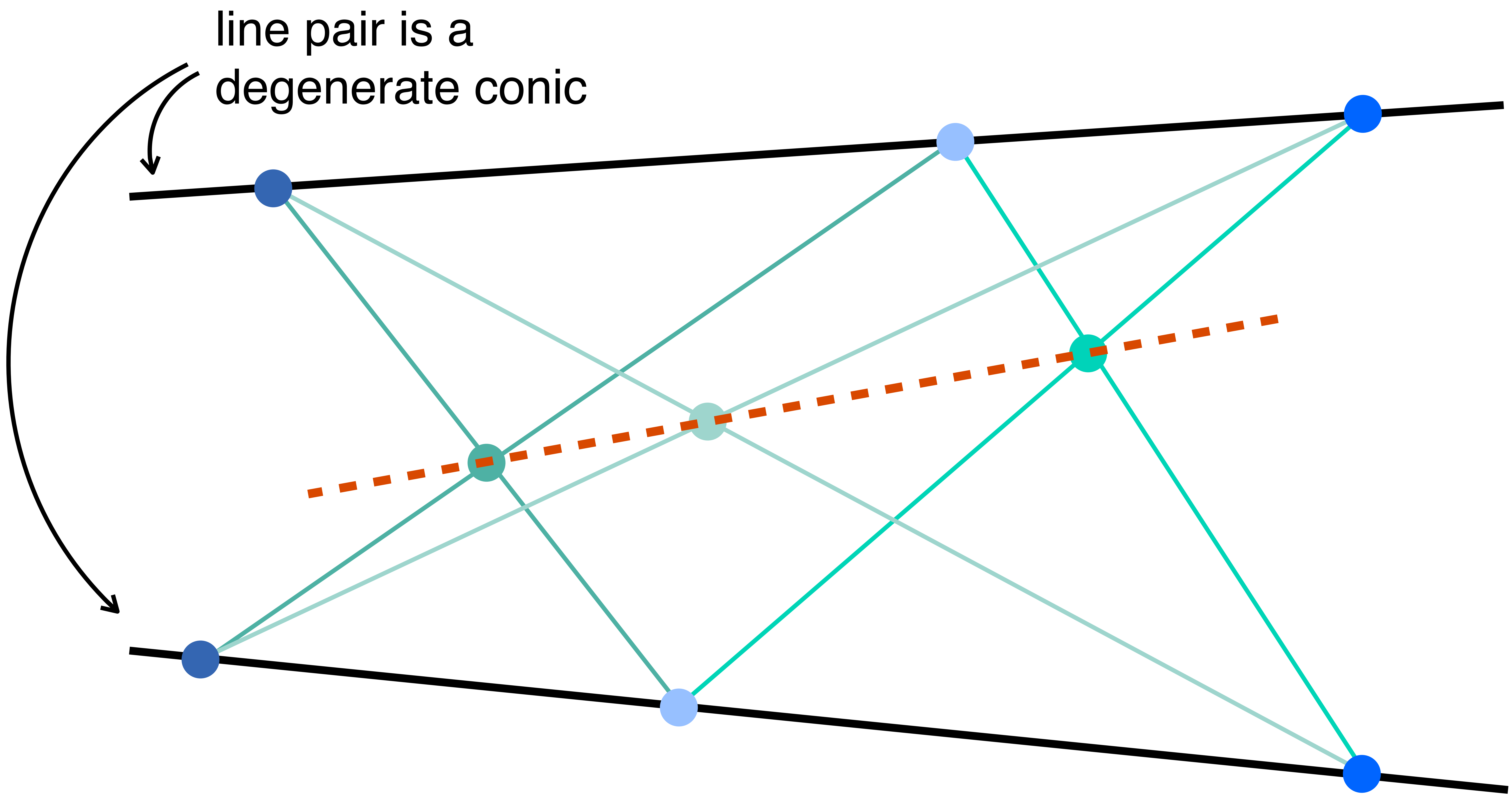
Dual Pappus' Theorem



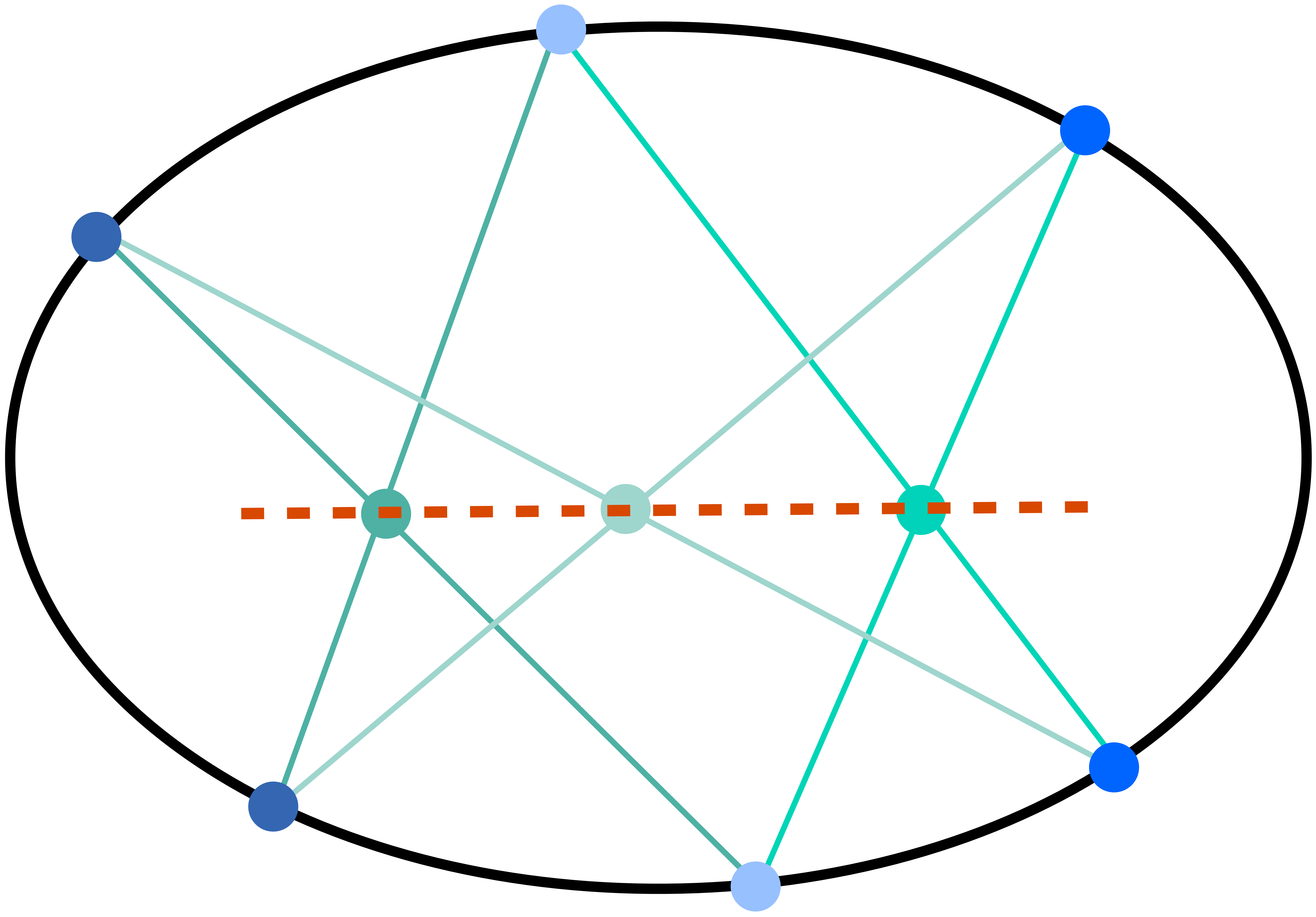




- Is there a more general theorem / pattern?
- Simple understanding of the general theorem?


line pair is a
degenerate conic

Pascal's Theorem (1640)



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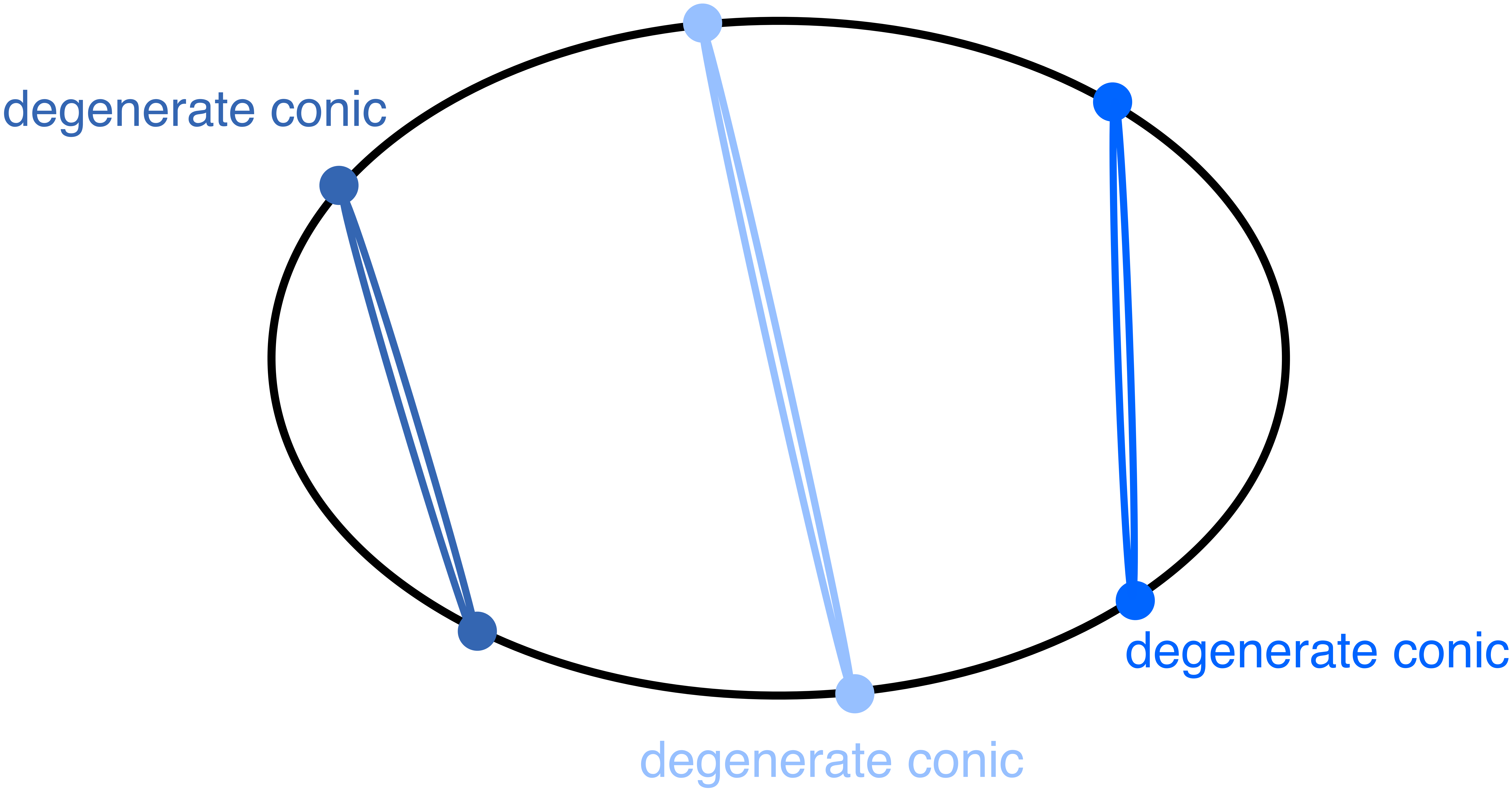
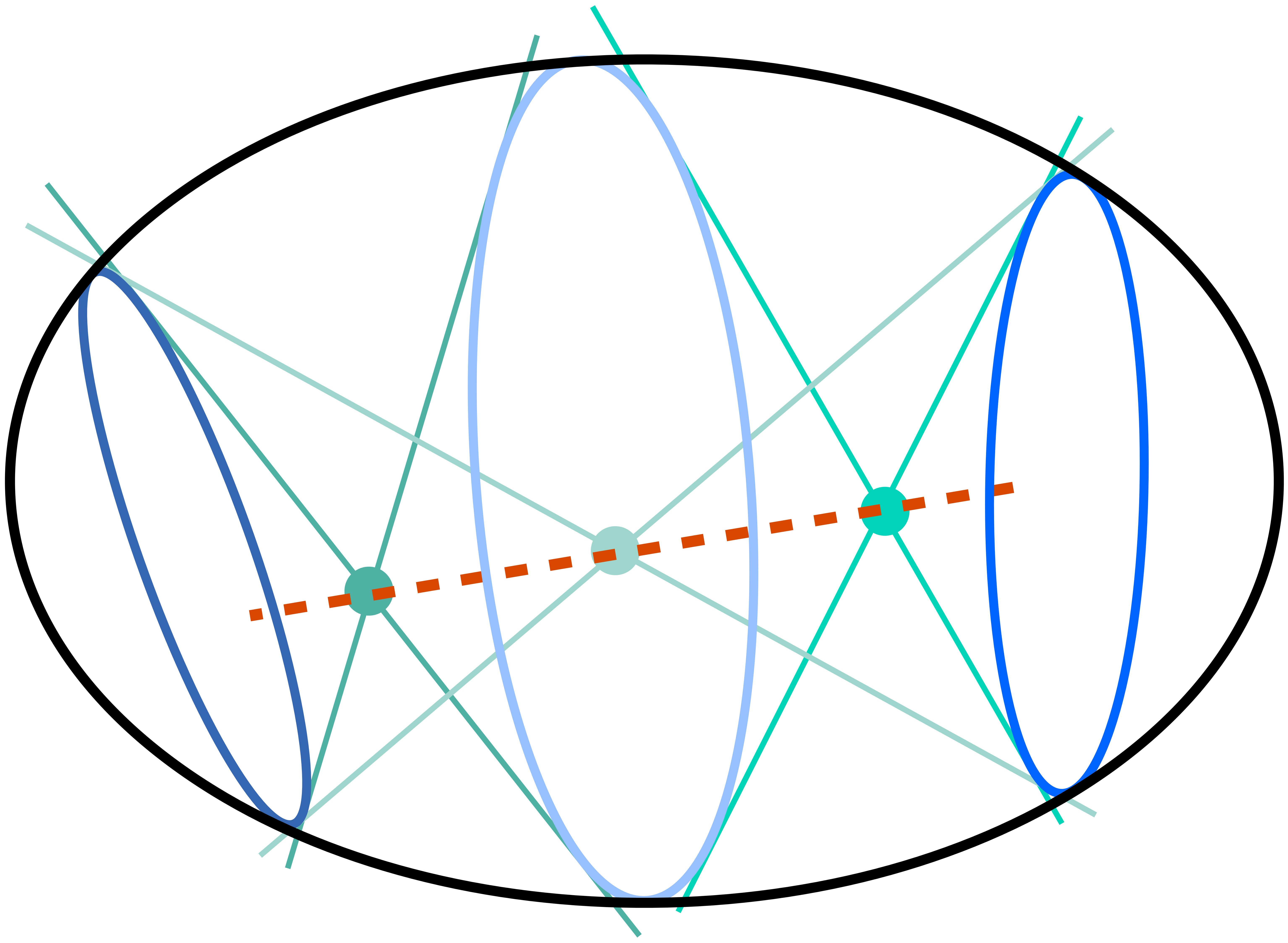
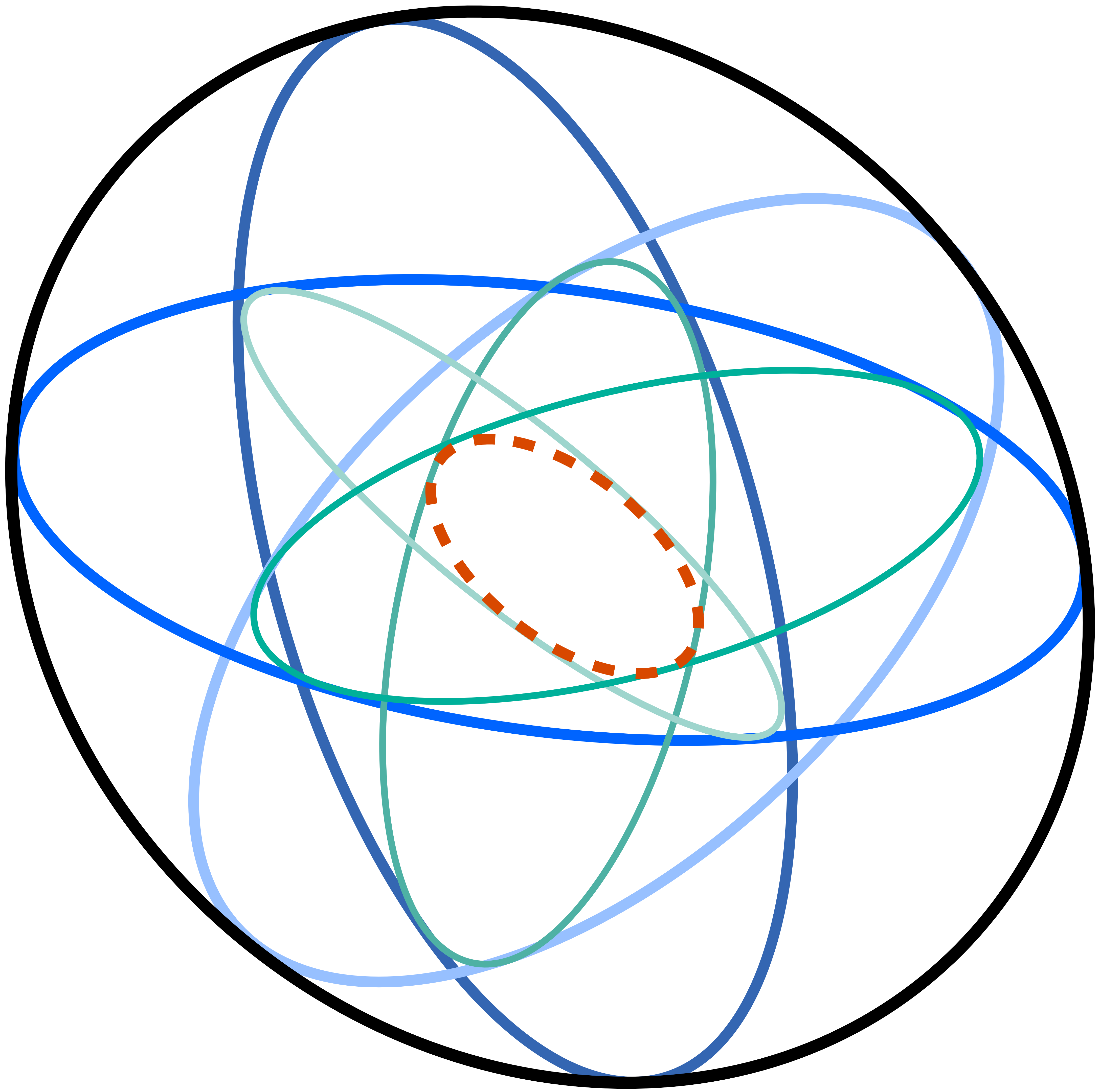
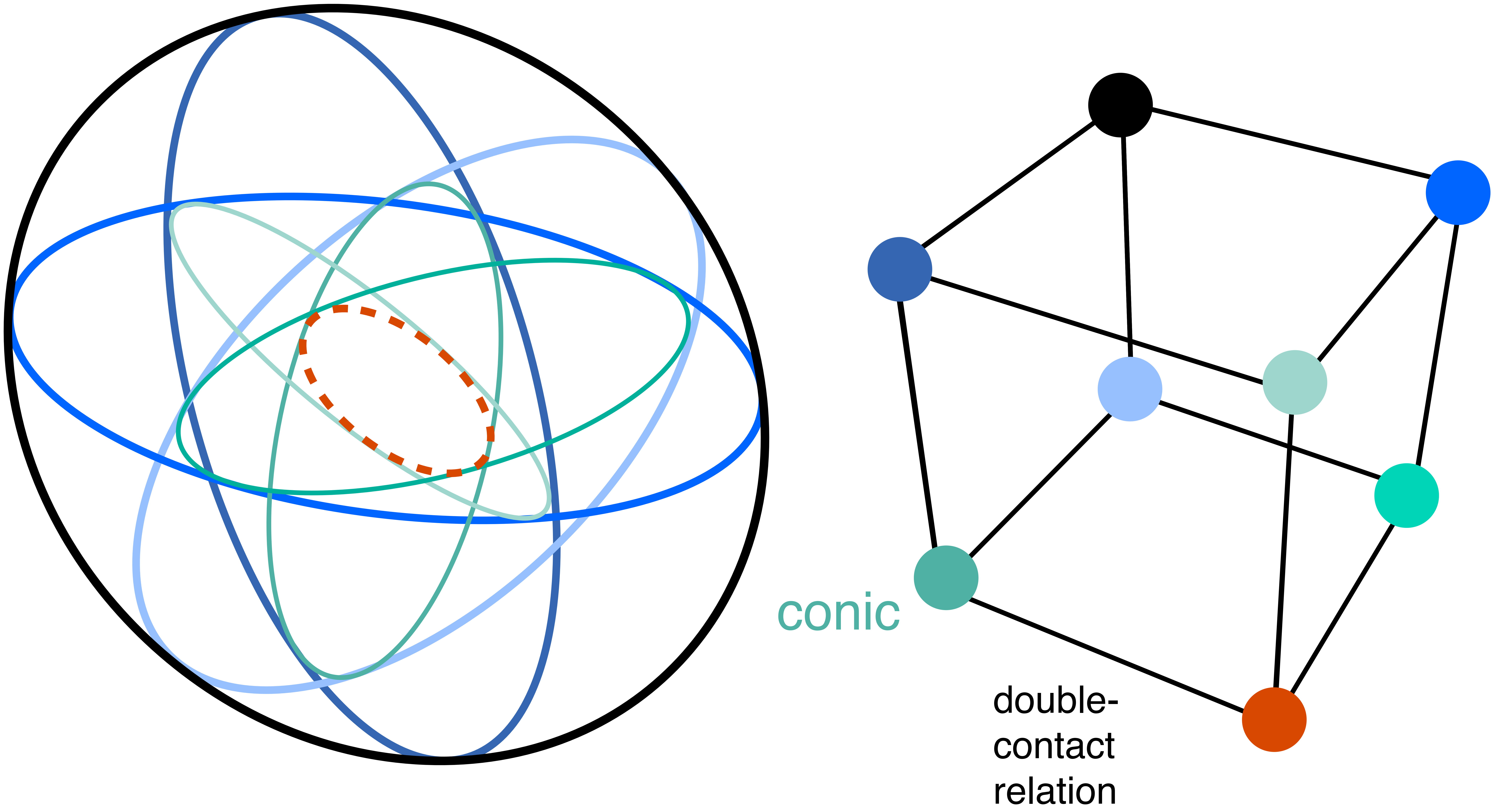


In G. Salmon's Treatise on Conics (1855)

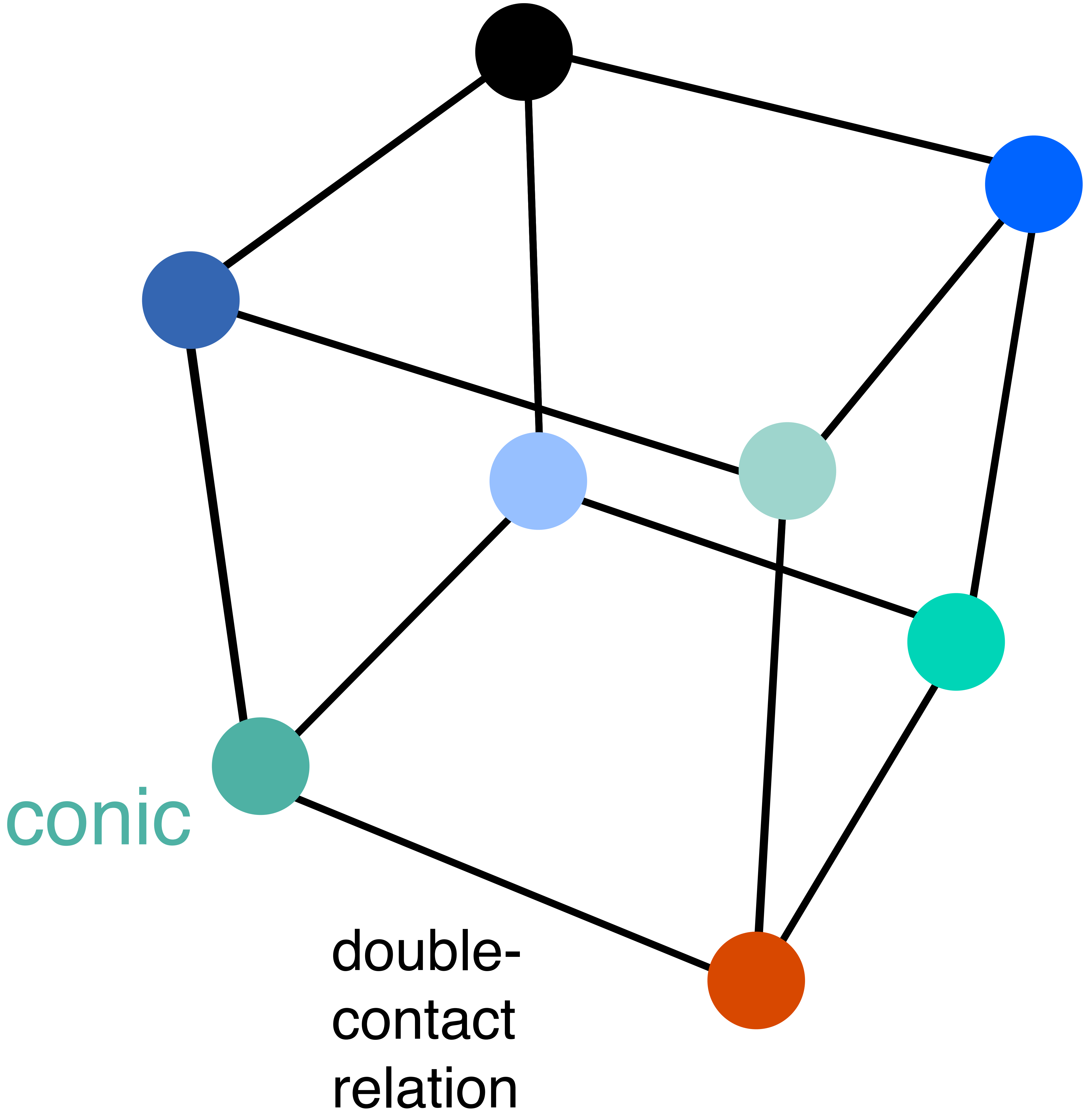





conic
double-
contact
relation

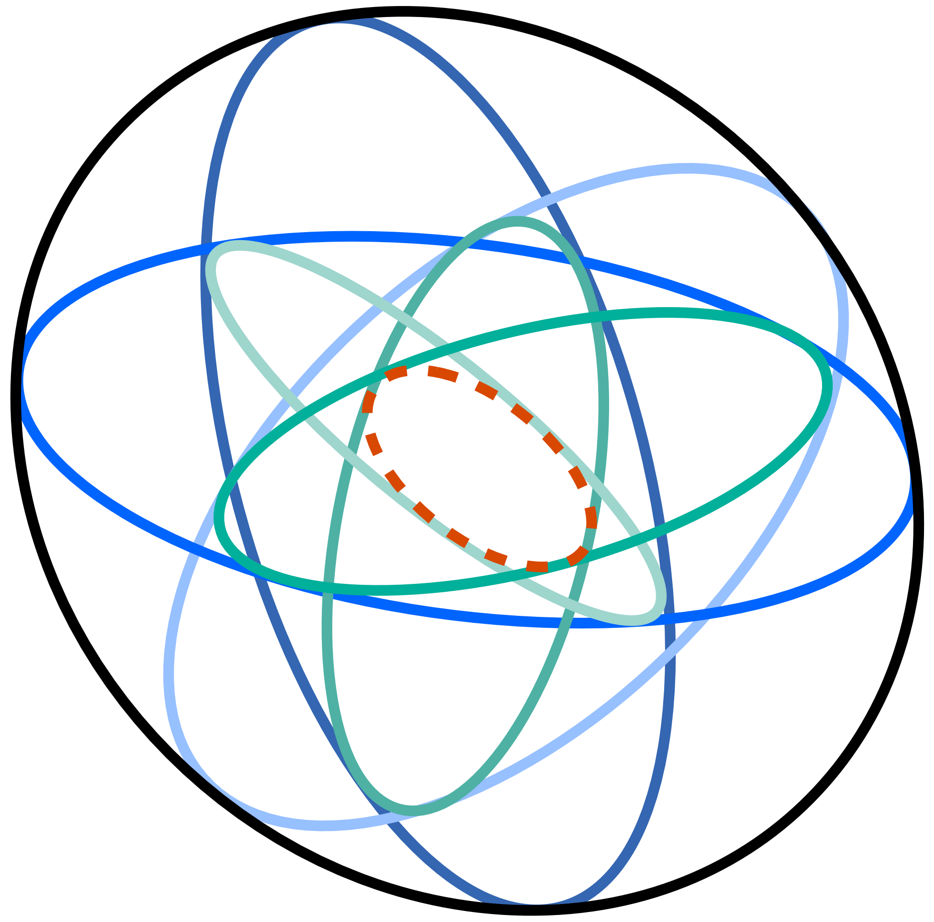
Eight-conic theorem

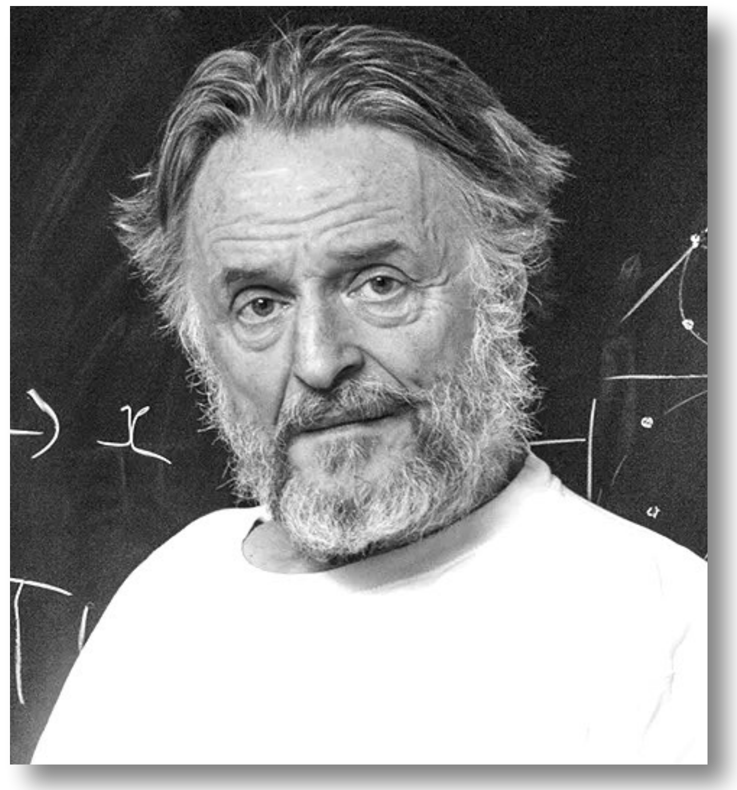
If seven out of the eight vertices of the cube is given, then the eighth one uniquely exists.



Eight-conic theorem

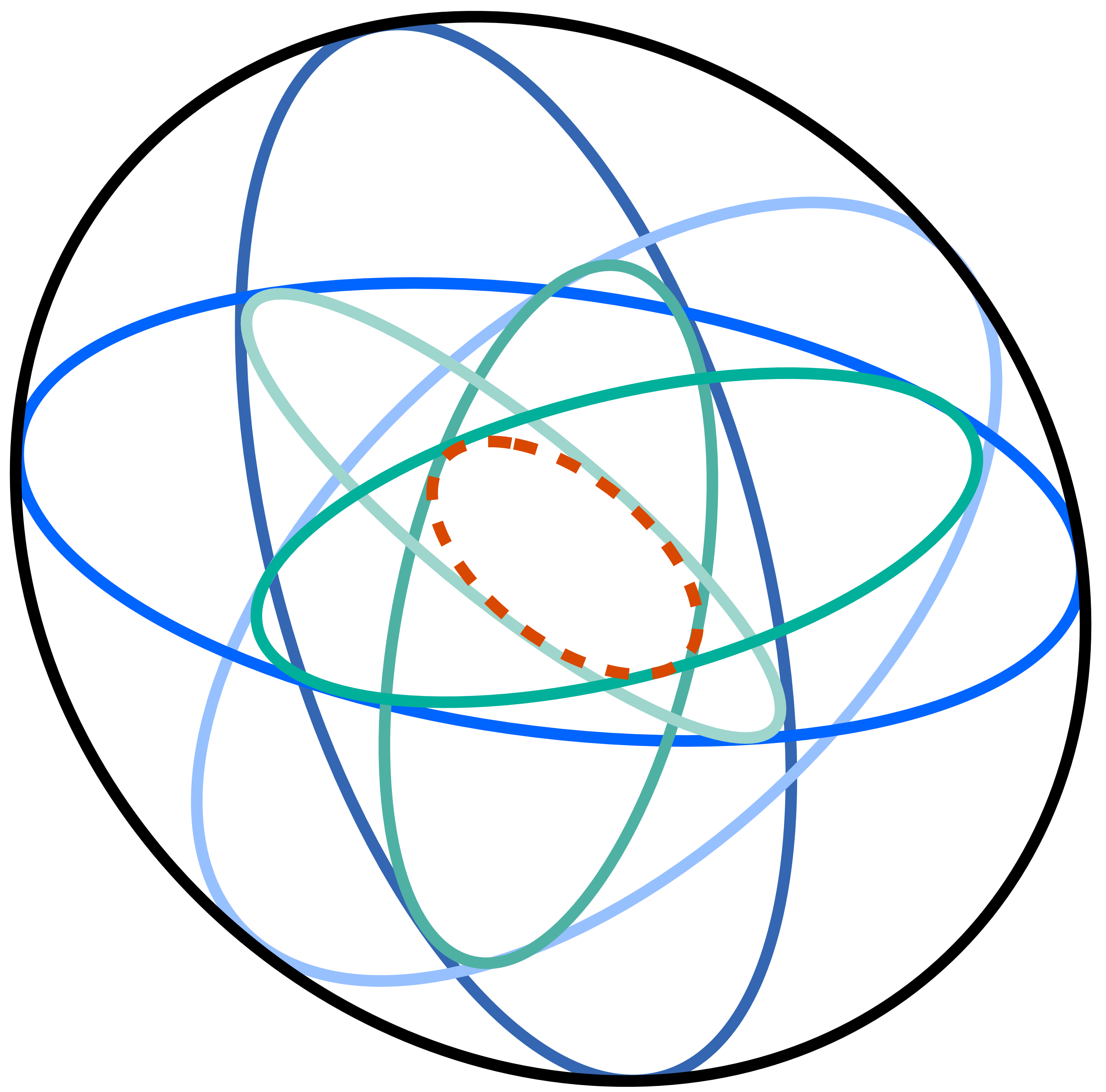
- 1950: Discovered by Roger Penrose as an undergrad.
- Never published.
- 1955: A simple proof was presented to his doctoral advisor Hodge, who found this geometric research too old-fashioned.
- The theorem was described to Conway, who loved the theorem.
- 2020: Penrose described the theorem in a Numberphile podcast.

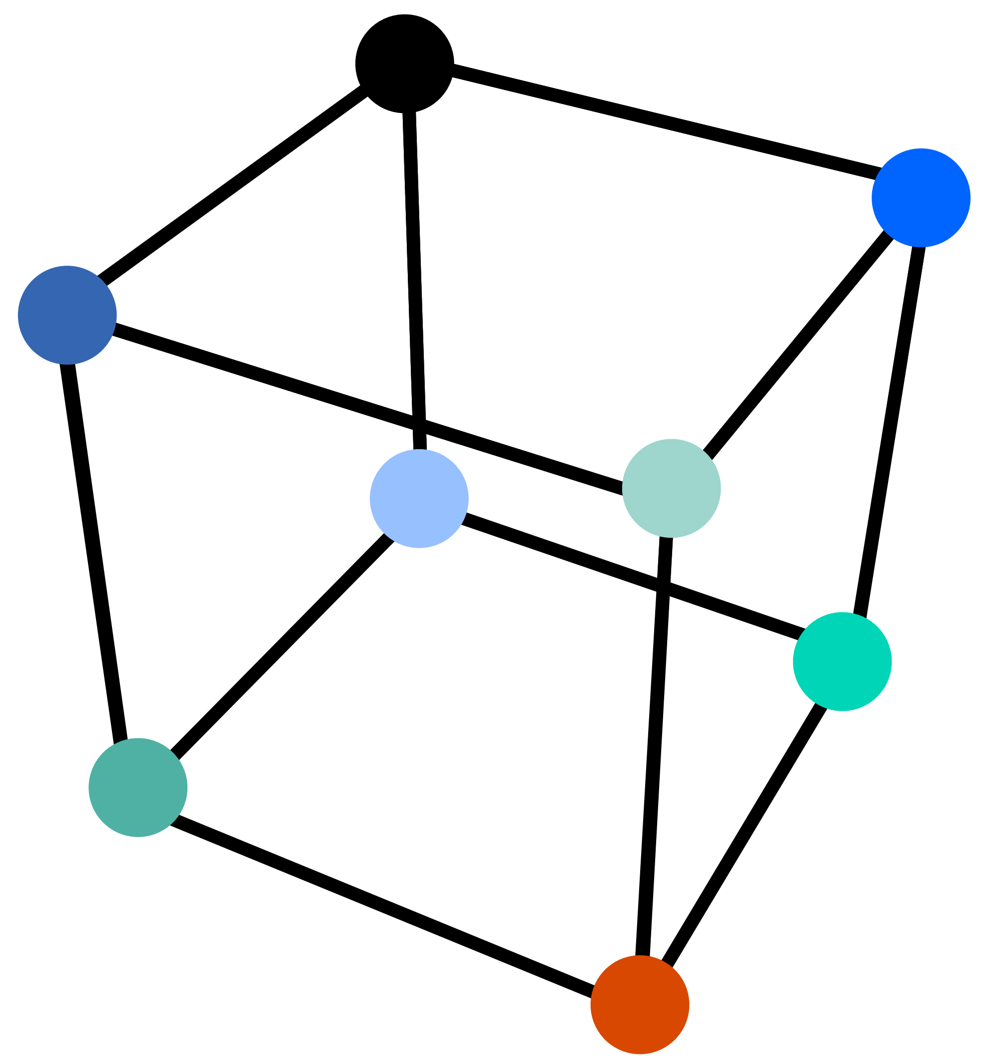




Overview

- Proof in the $\mathbb{P}^5$ space of conics
- Penrose's approach (undergrad)
- Penrose's 3D approach (Cambridge)





Overview

- Nice things about the eight-conic theorem
- Proof in the $\mathbb{P}^5$ space of conics
- Penrose's approach (undergrad)
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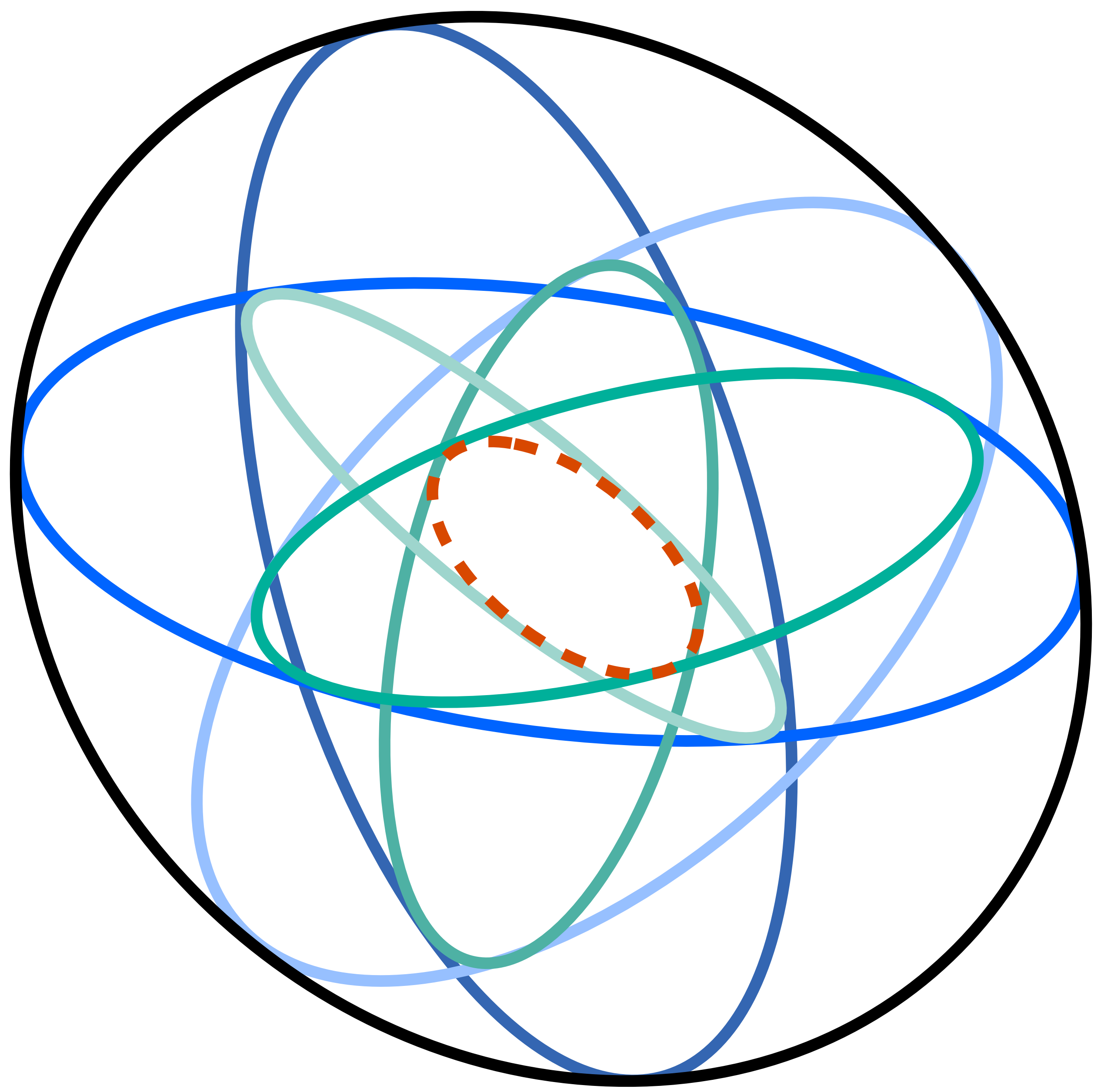
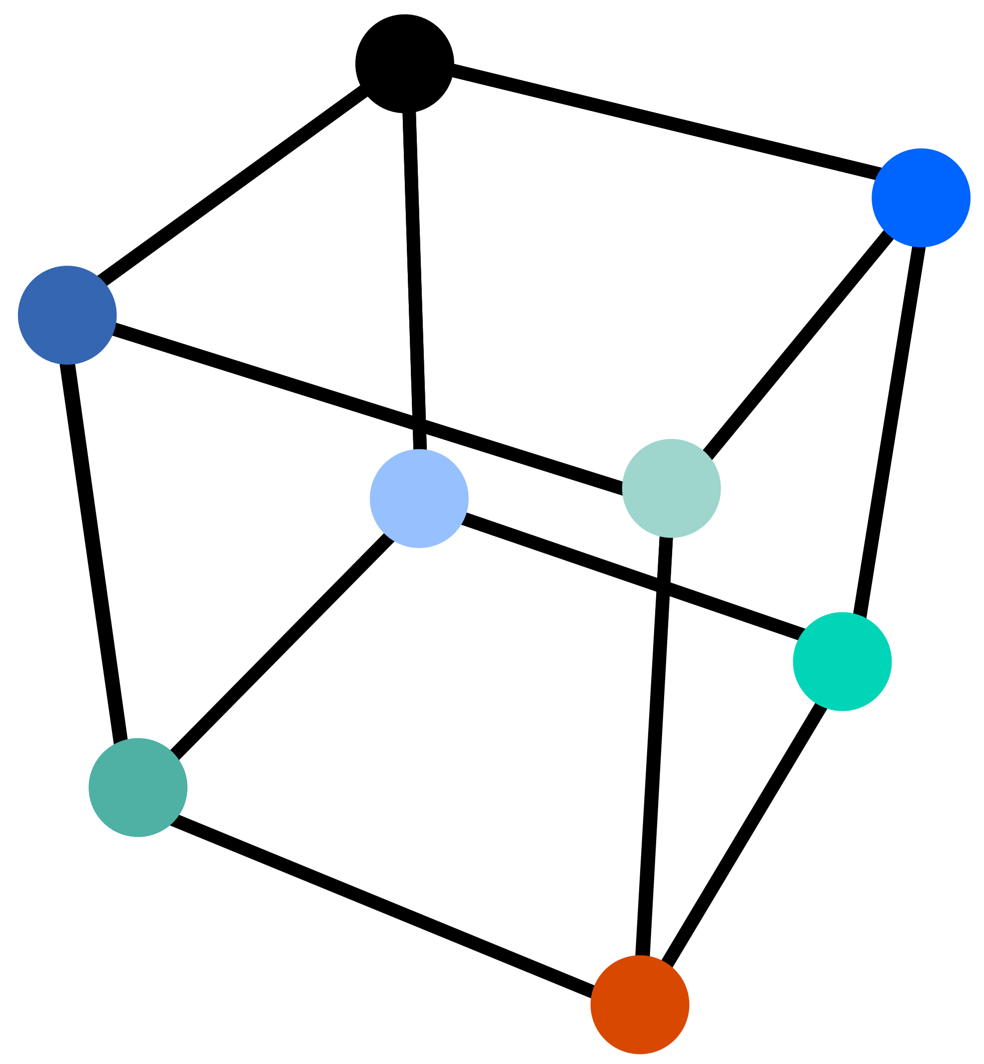
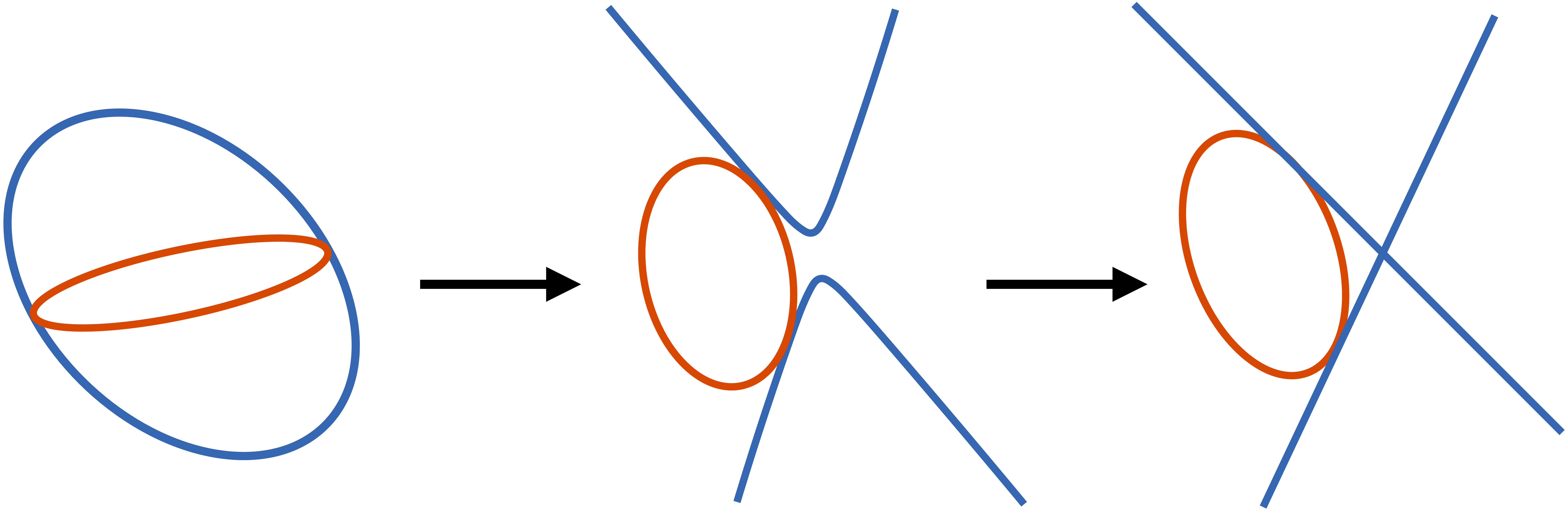




Degenerate conics



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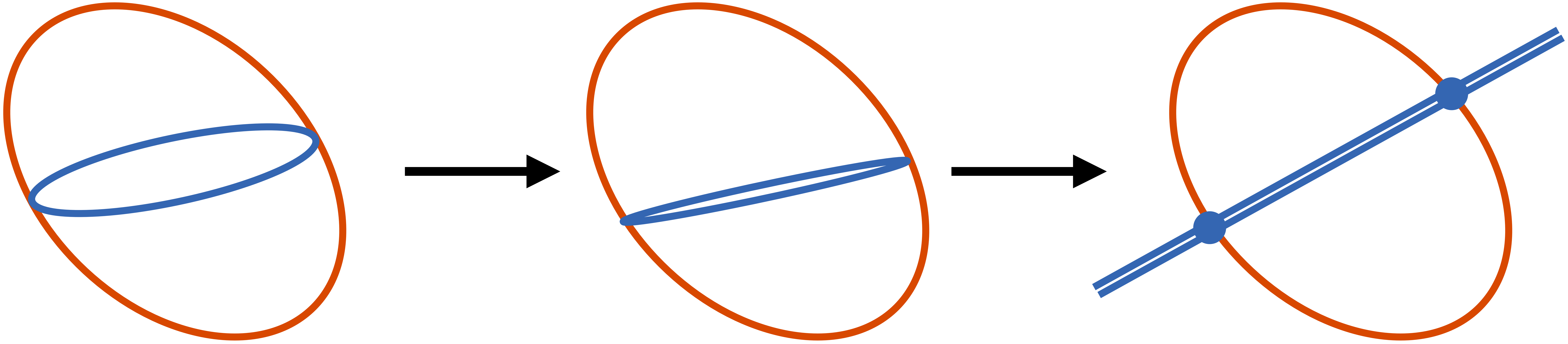

Conics

- Each conic can be represented by a 3x3 symmetric matrix up to a uniform scaling.

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$$

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$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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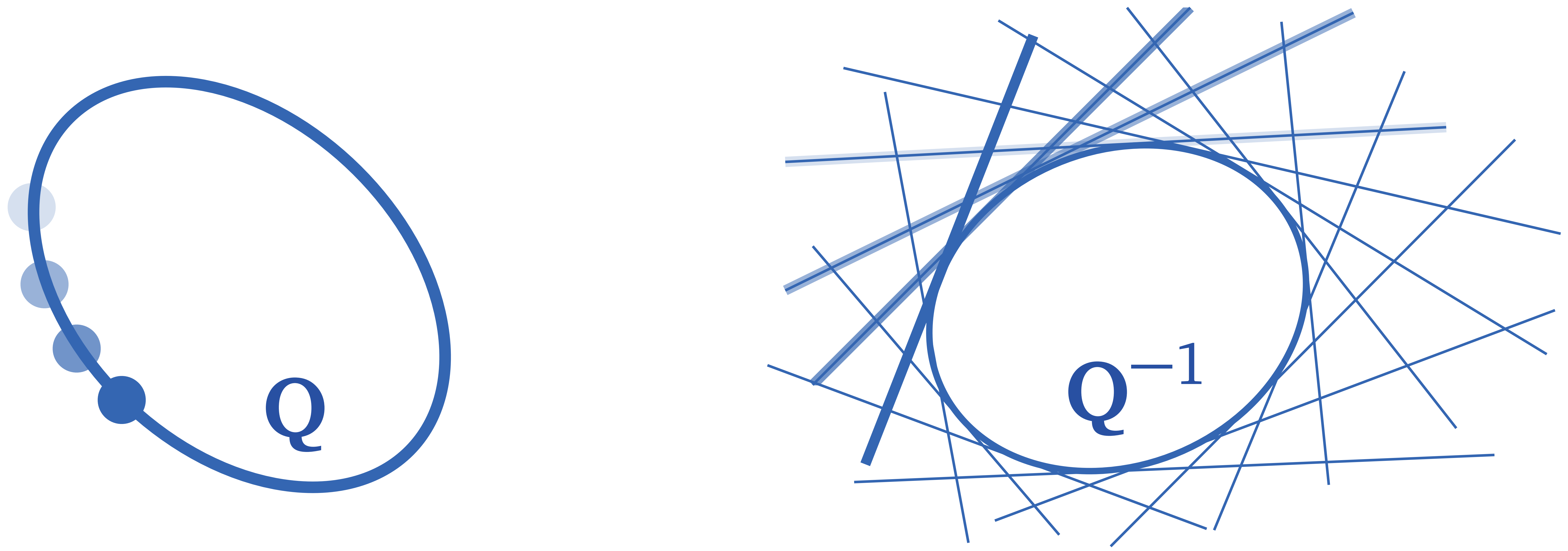
Conics

- Each conic can be represented by a 3x3 symmetric matrix up to a uniform scaling.
- The space of conics is $\mathbb{P}^5$
- Each regular ($\mathrm{rank} = 3$) conic $\mathbf{Q}$ has a dual conic $\mathbf{Q}^{-1}$



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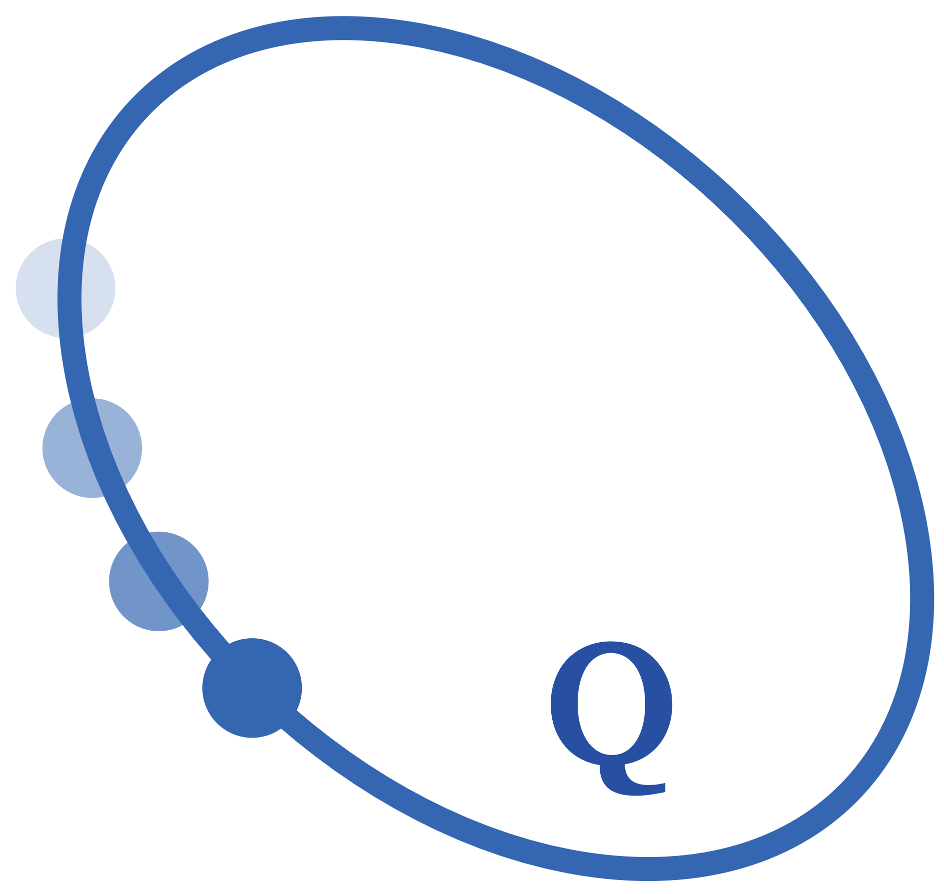
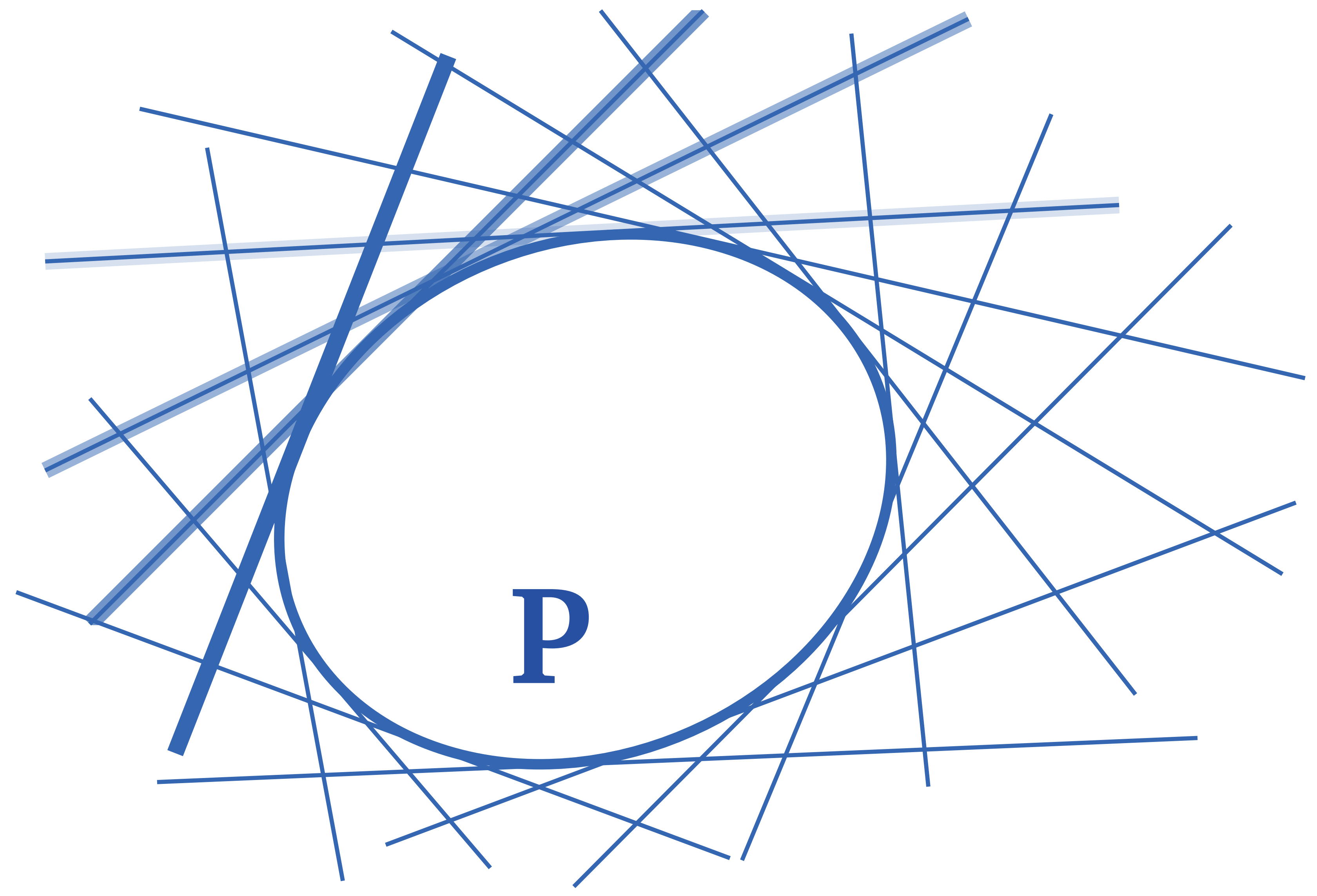
- A dual pair of conic is $\mathbf{Q} \in \mathbb{P}^5, \mathbf{P} \in \mathbb{P}^{5*}$ so that $\mathbf{PQ} = \lambda \mathbf{I}_{3\times 3}$





Conics

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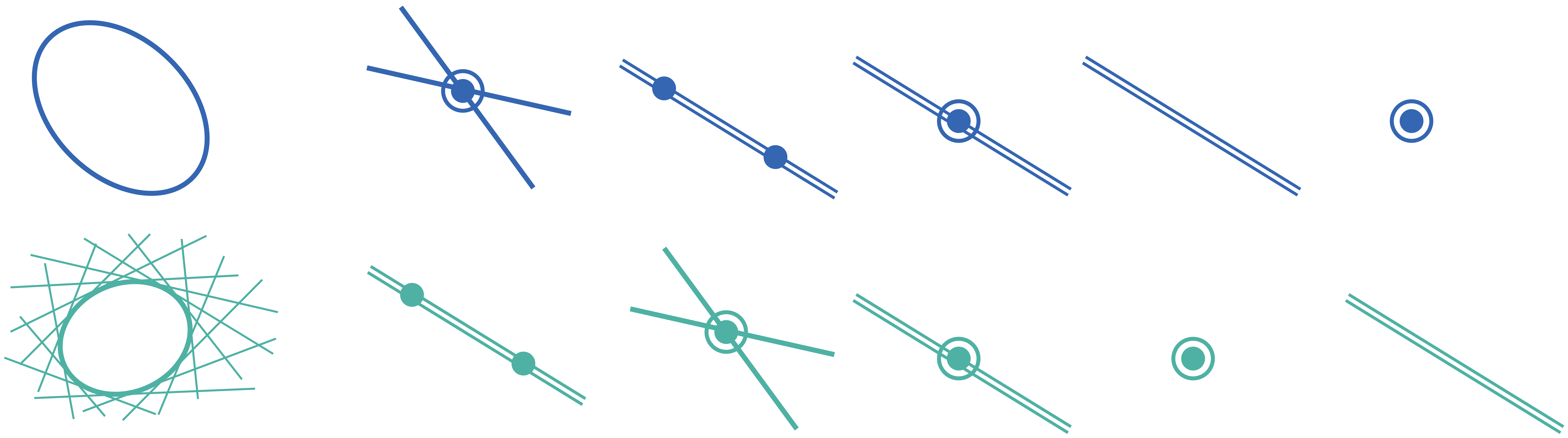
- The rank of $\mathbf{Q}, \mathbf{P}$ can be

$(3, 3)$ $(2, 1)$ $(1, 2)$ $(1, 1)$ $(1, 0)$ $(0, 1)$



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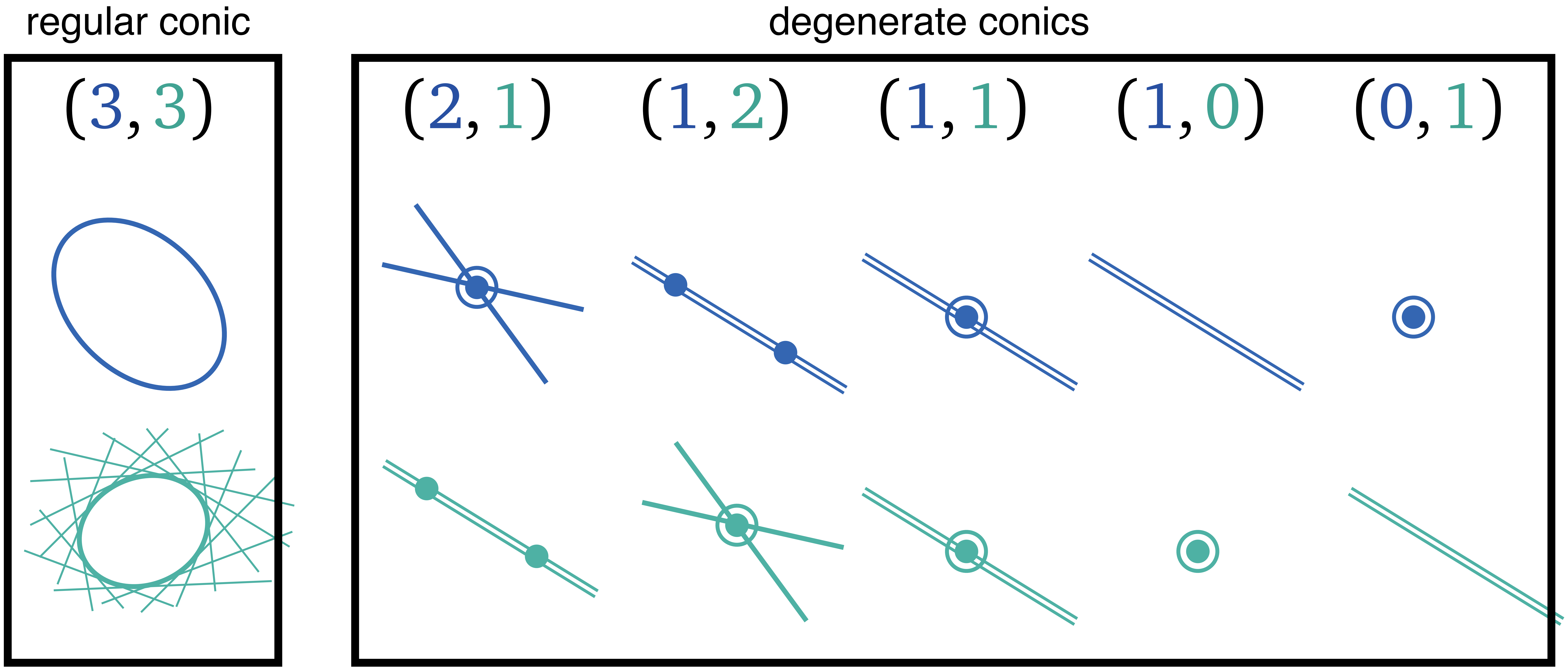
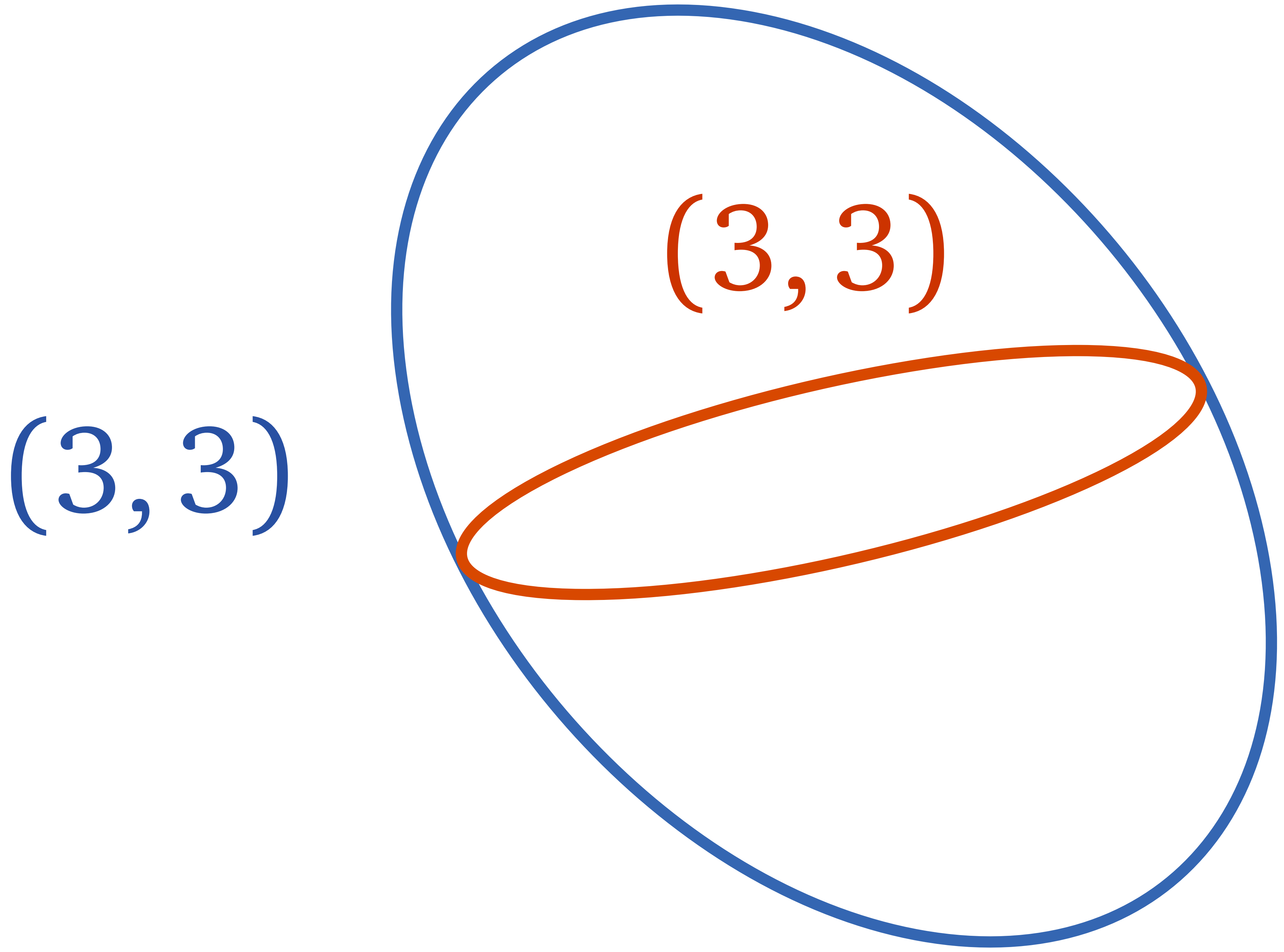
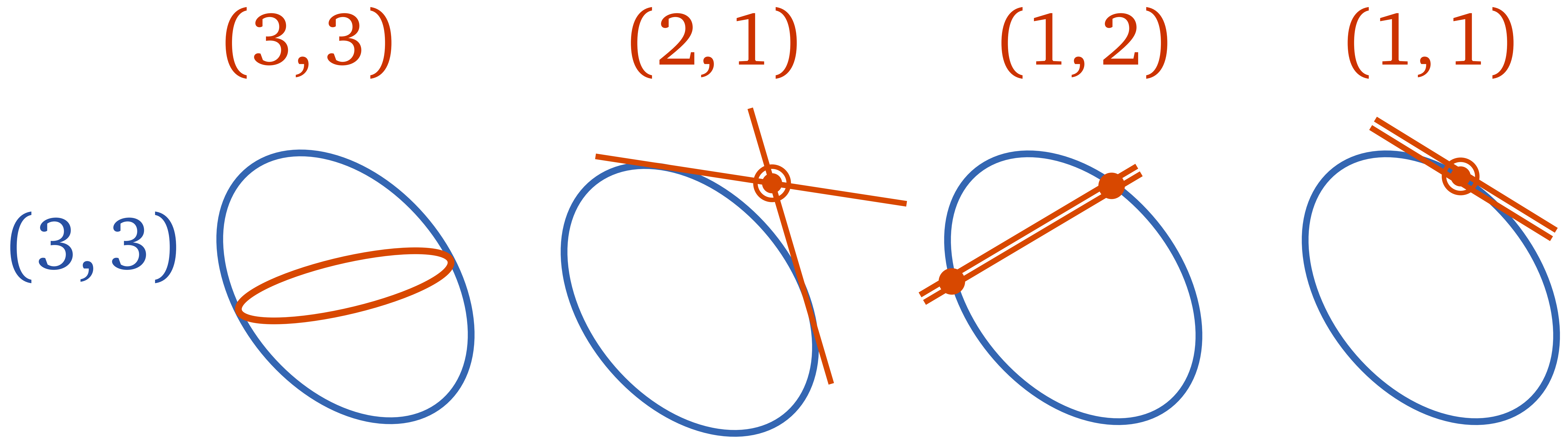


Conics in double contact



Take limits in the $\mathbb{P}^5 \times \mathbb{P}^{5*}$ topology

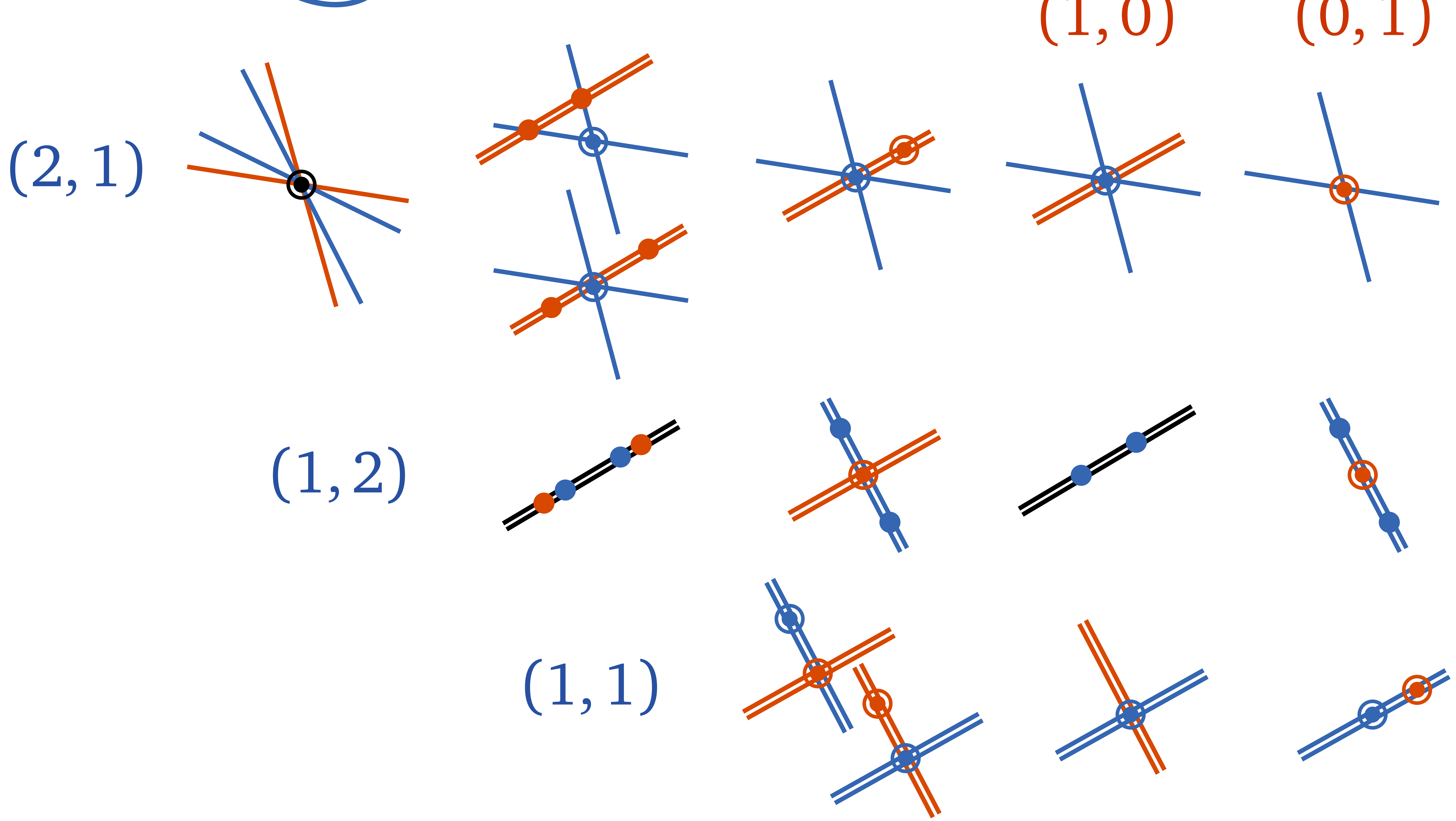
Conics in double contact



Conics in double contact

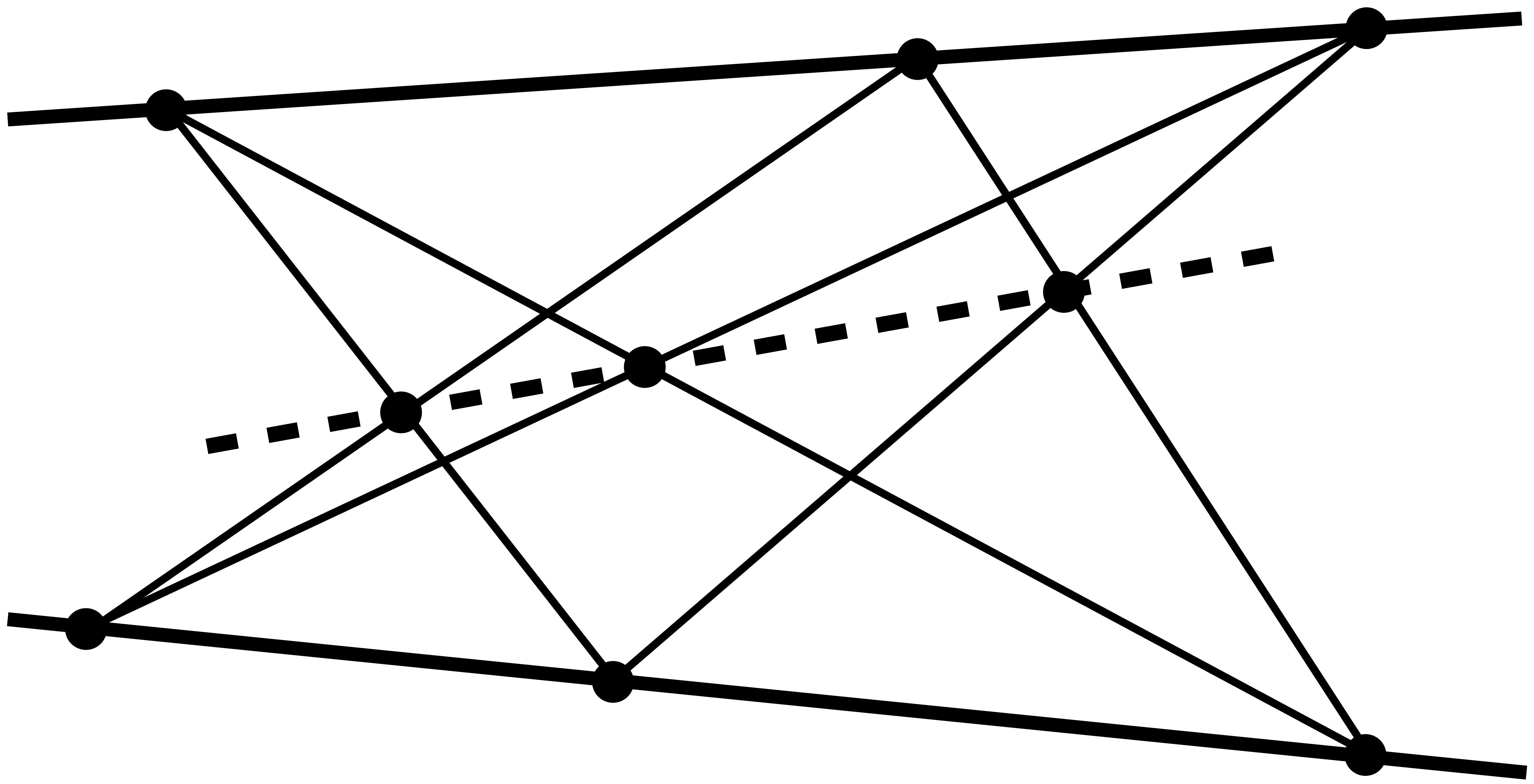

(3, 3)
(2, 1)
(1, 2)
(1, 1)
(1, 0)
(0, 1)
(3, 3)
(2, 1)
(1, 2)

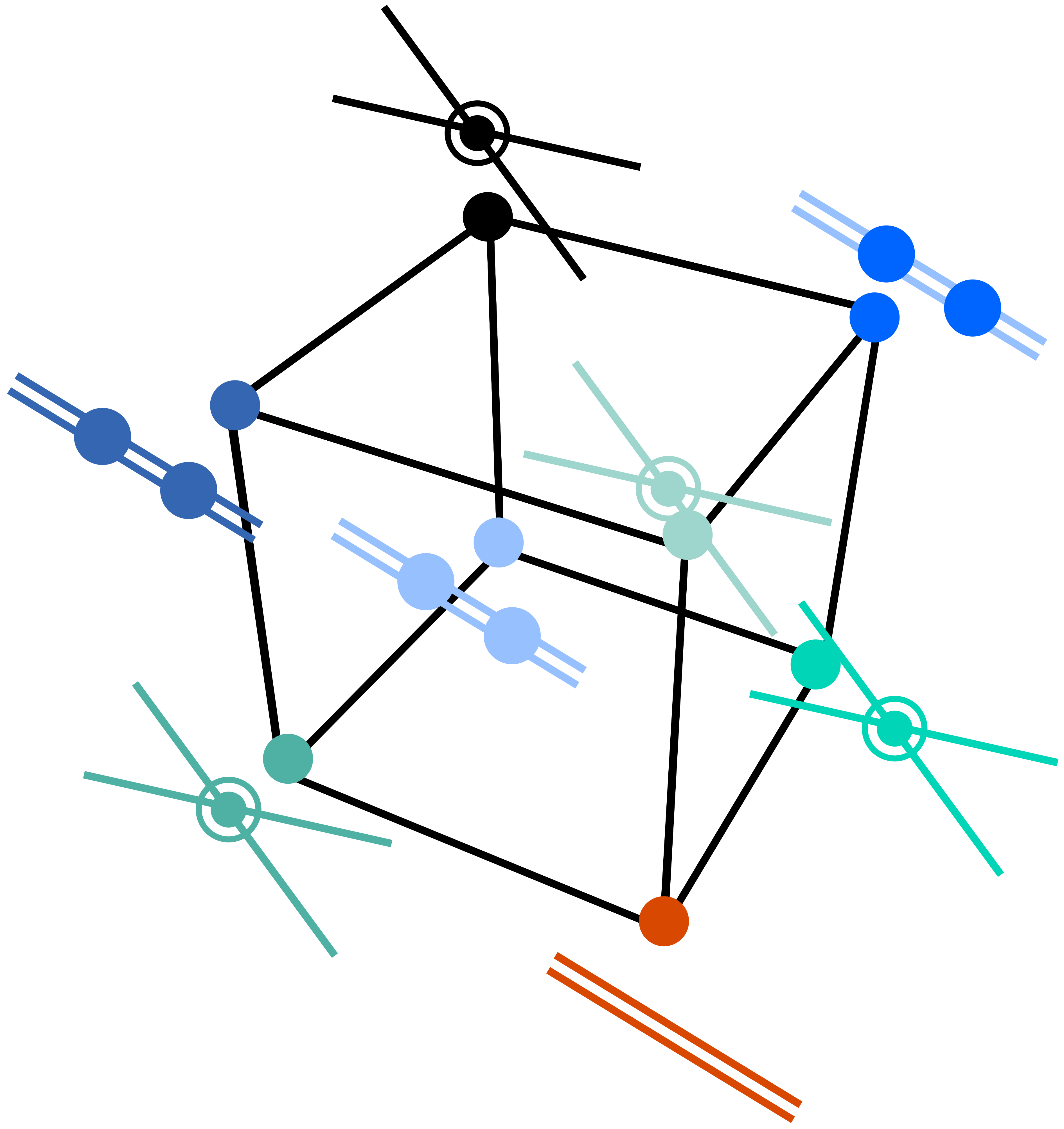

(1, 0)
(0, 1)
(2, 1)
(1, 2)
(1, 1)



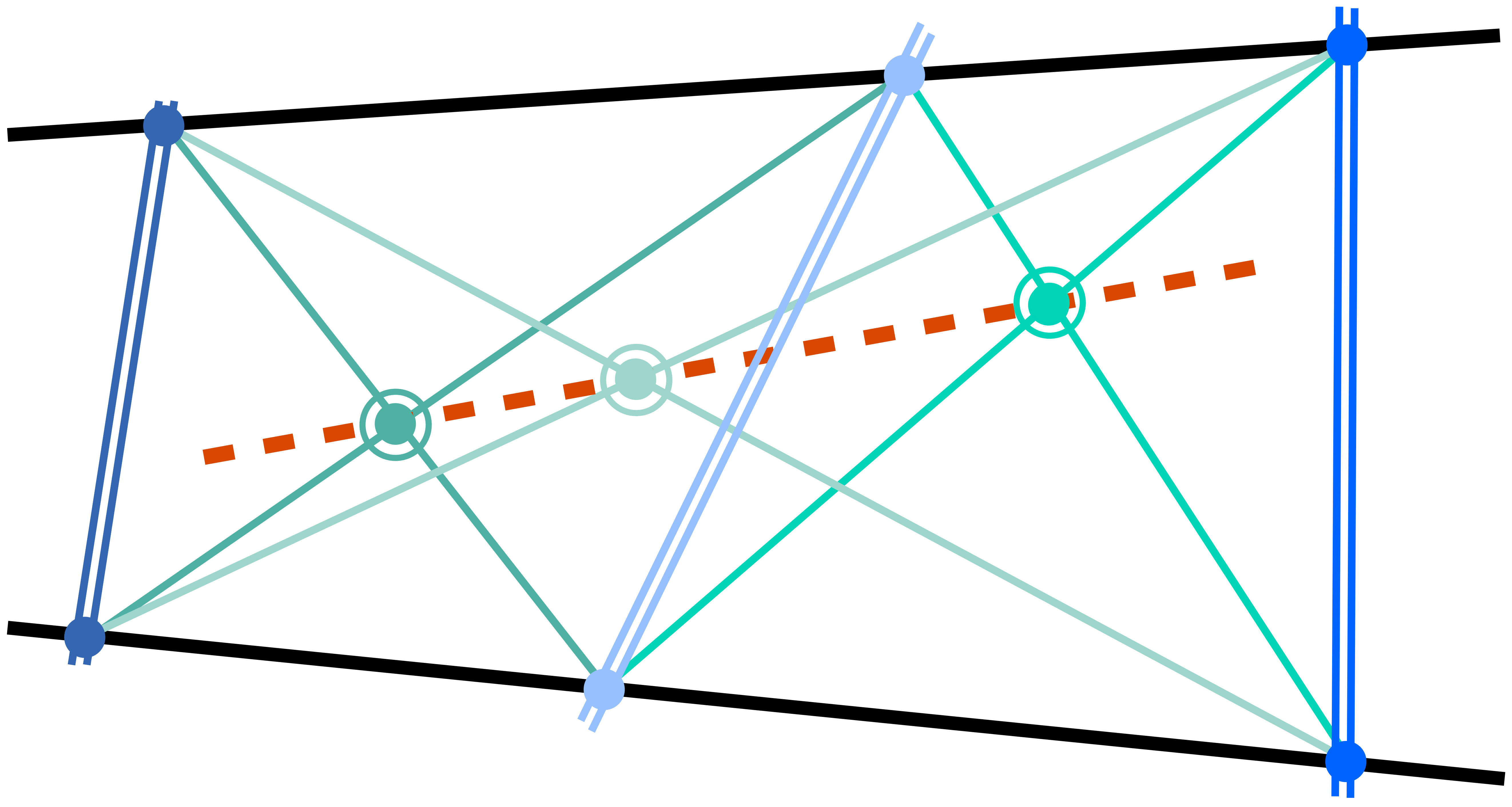
and so on

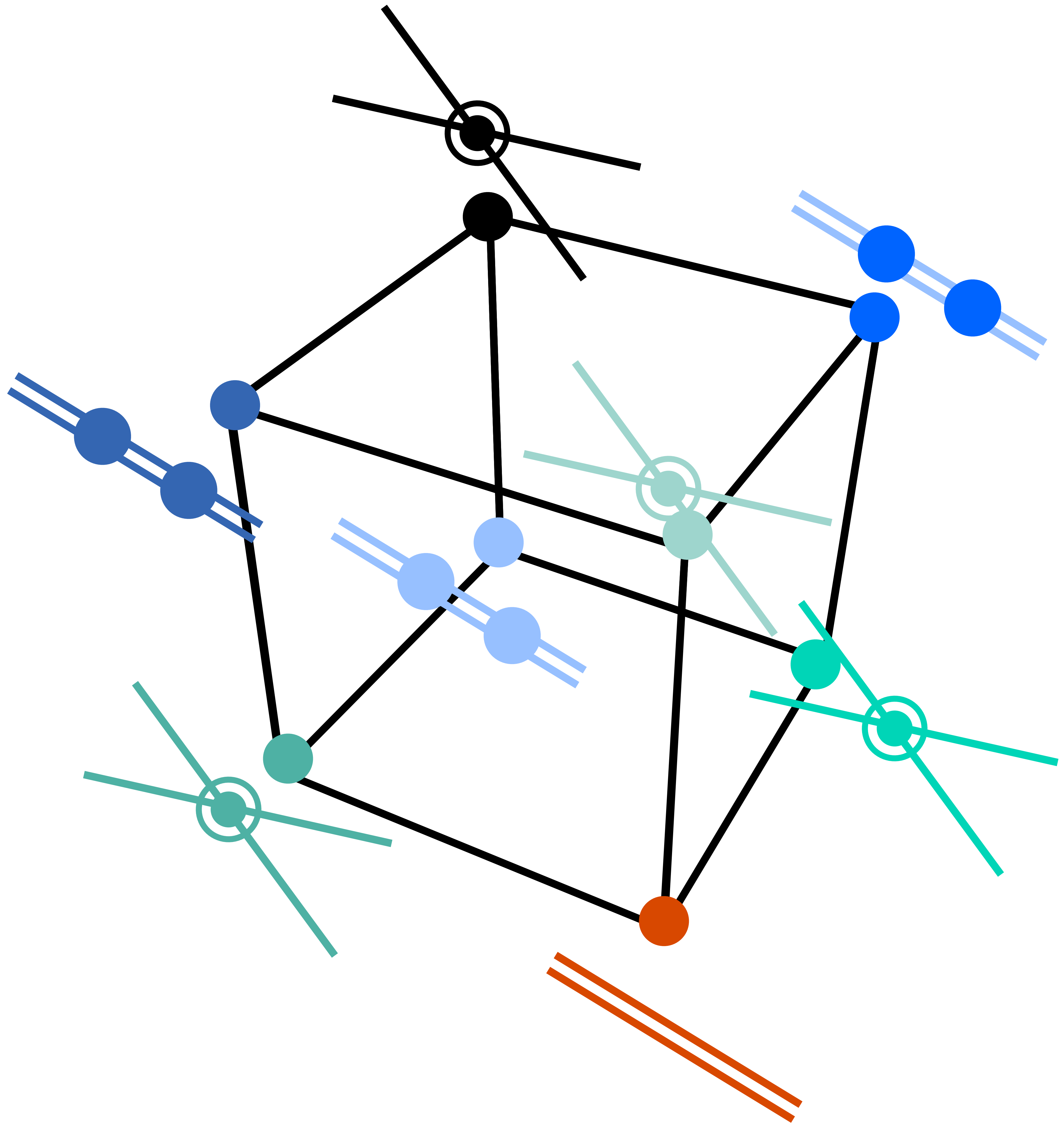
Pappus Theorem



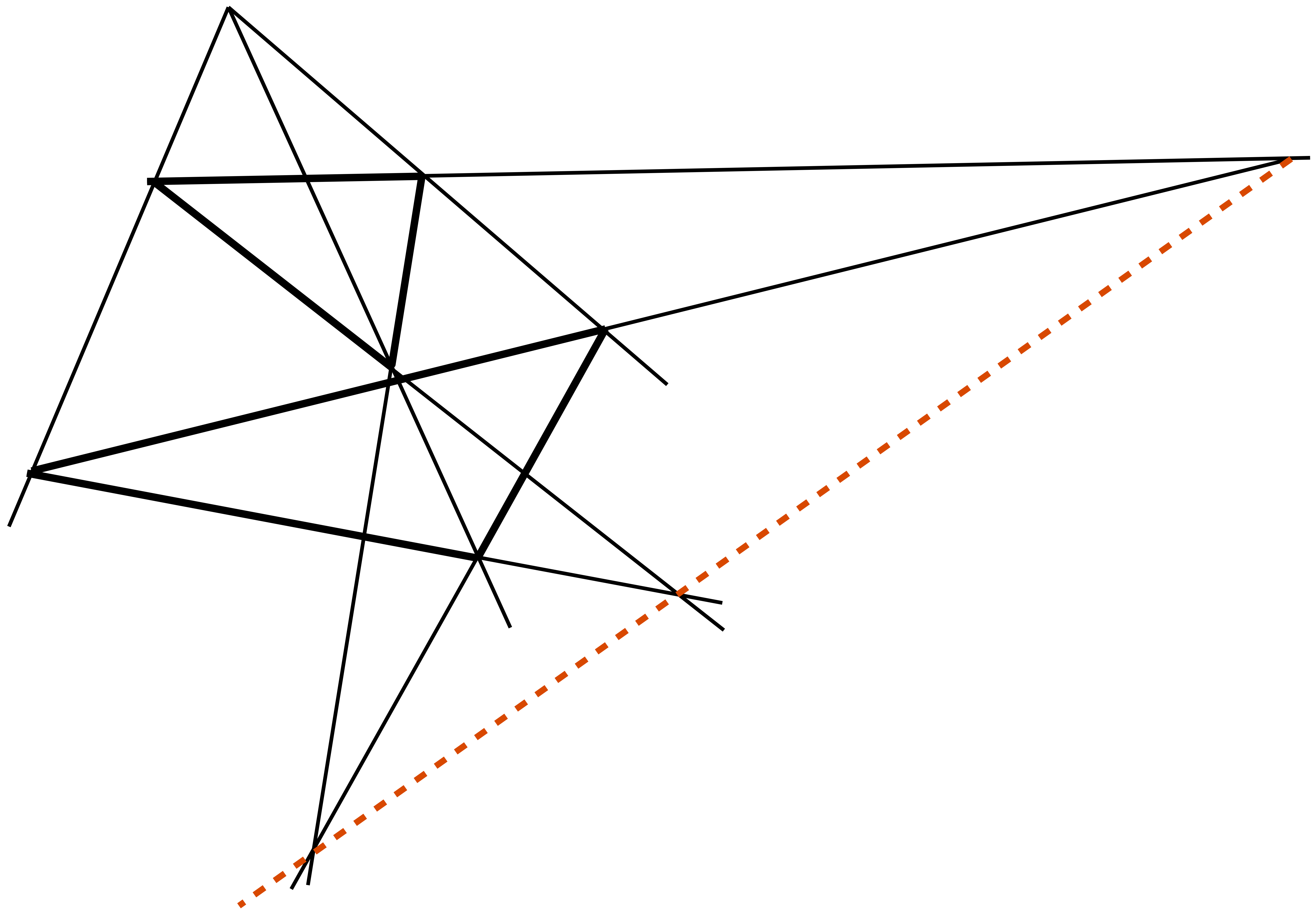


Pappus Theorem





Desargues theorem



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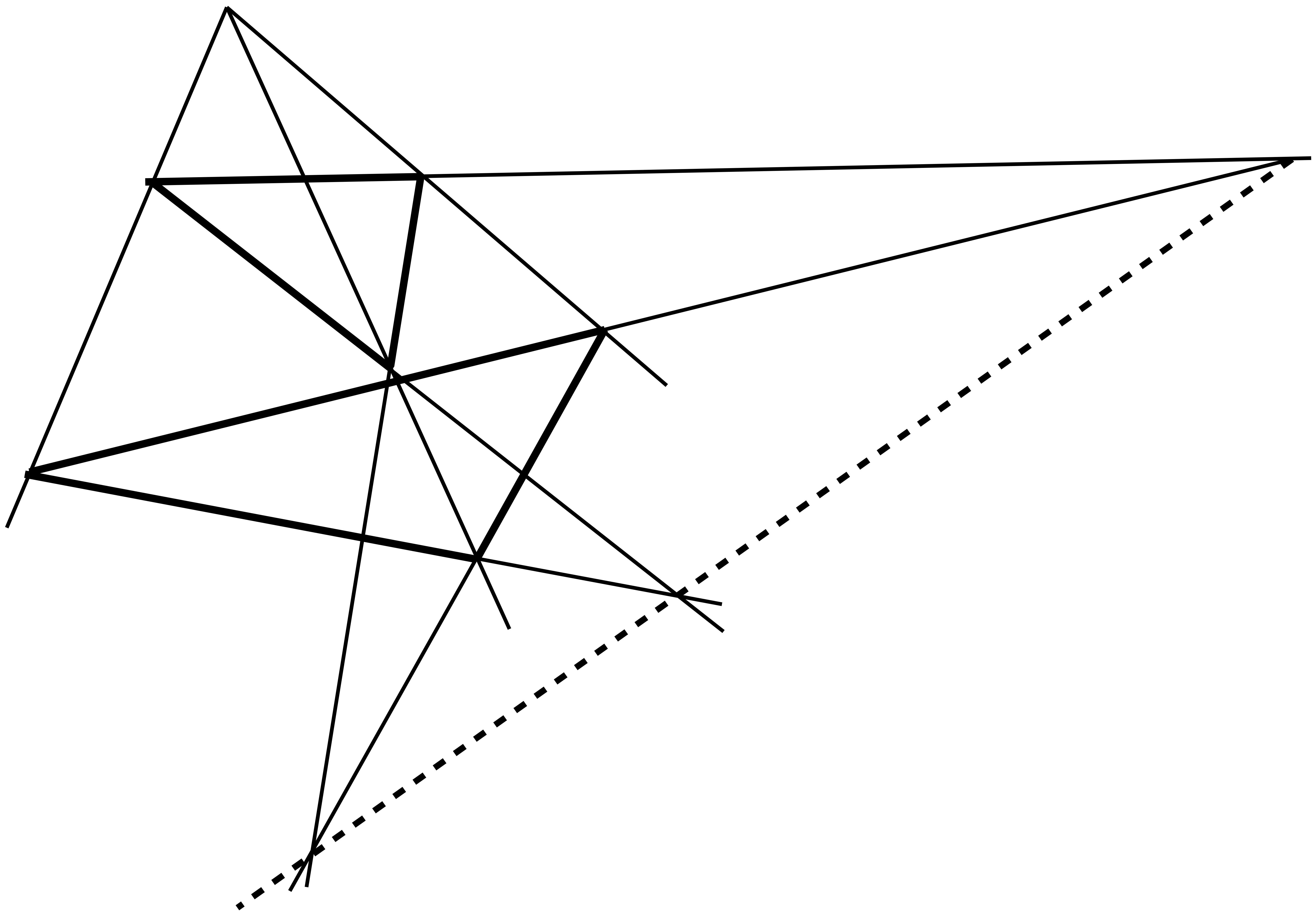

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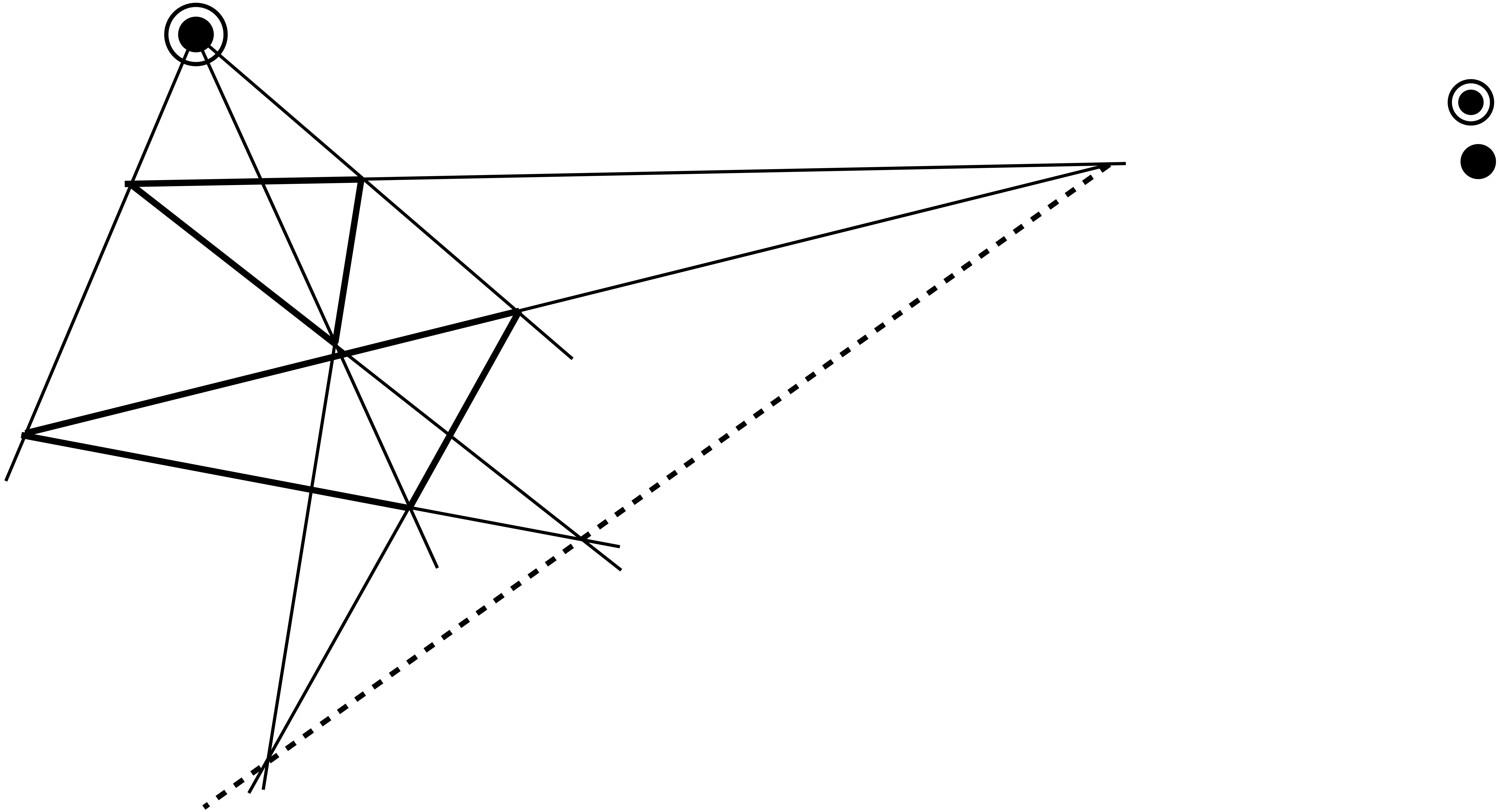

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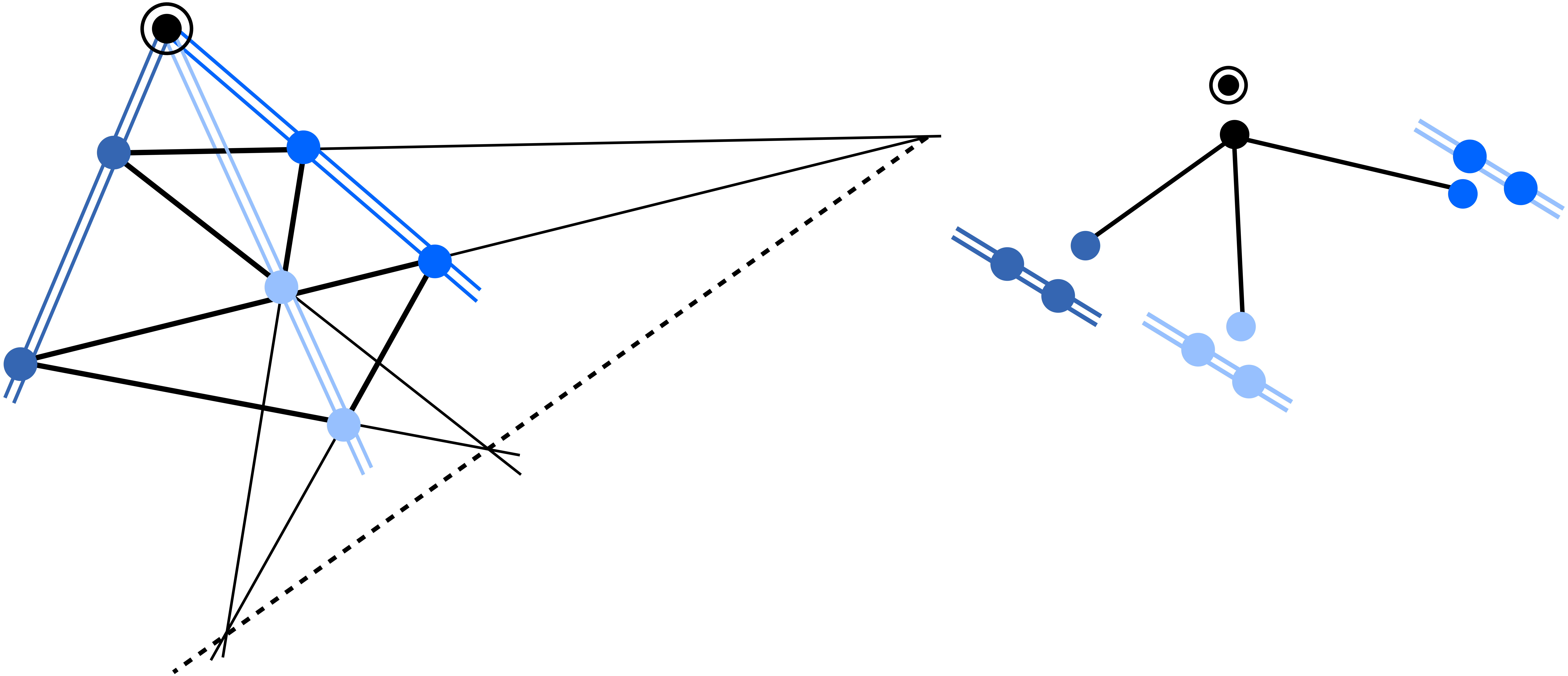
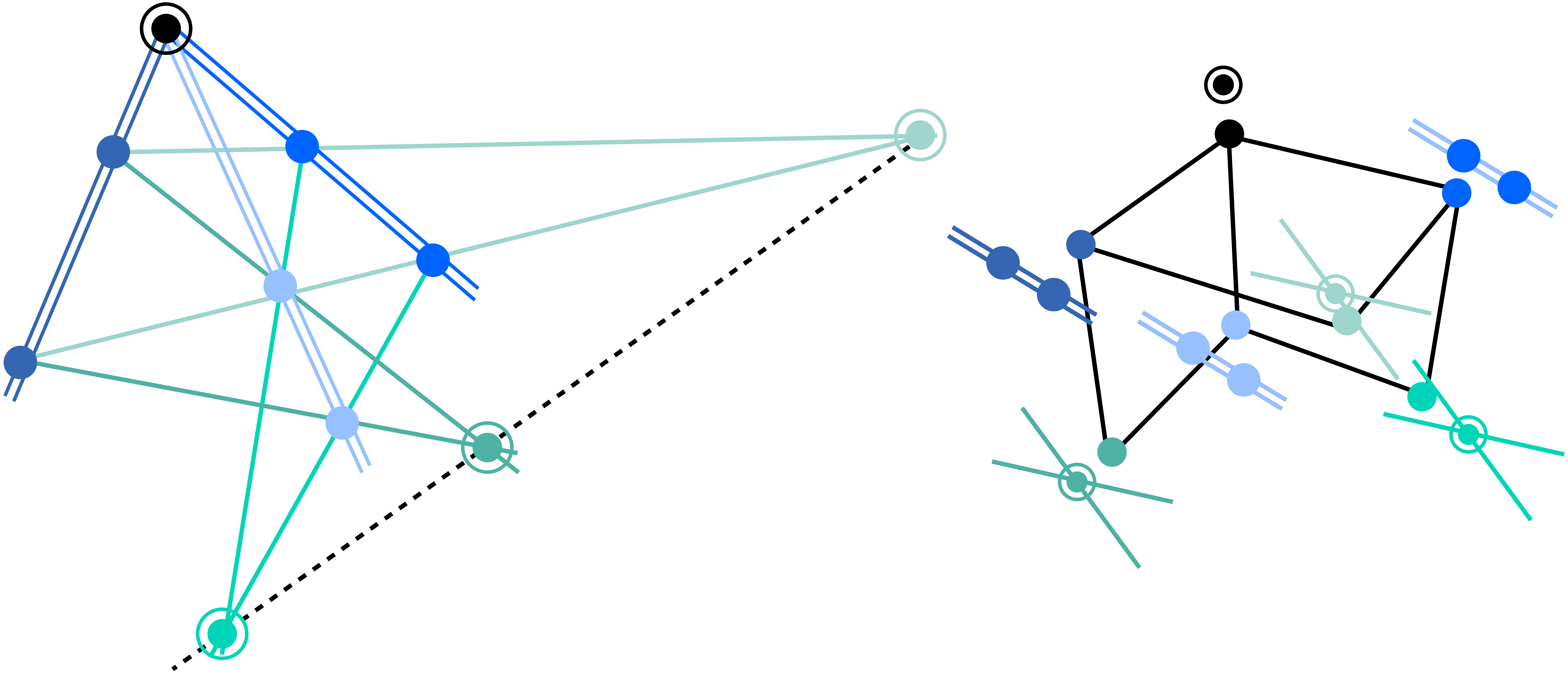


Desargues theorem



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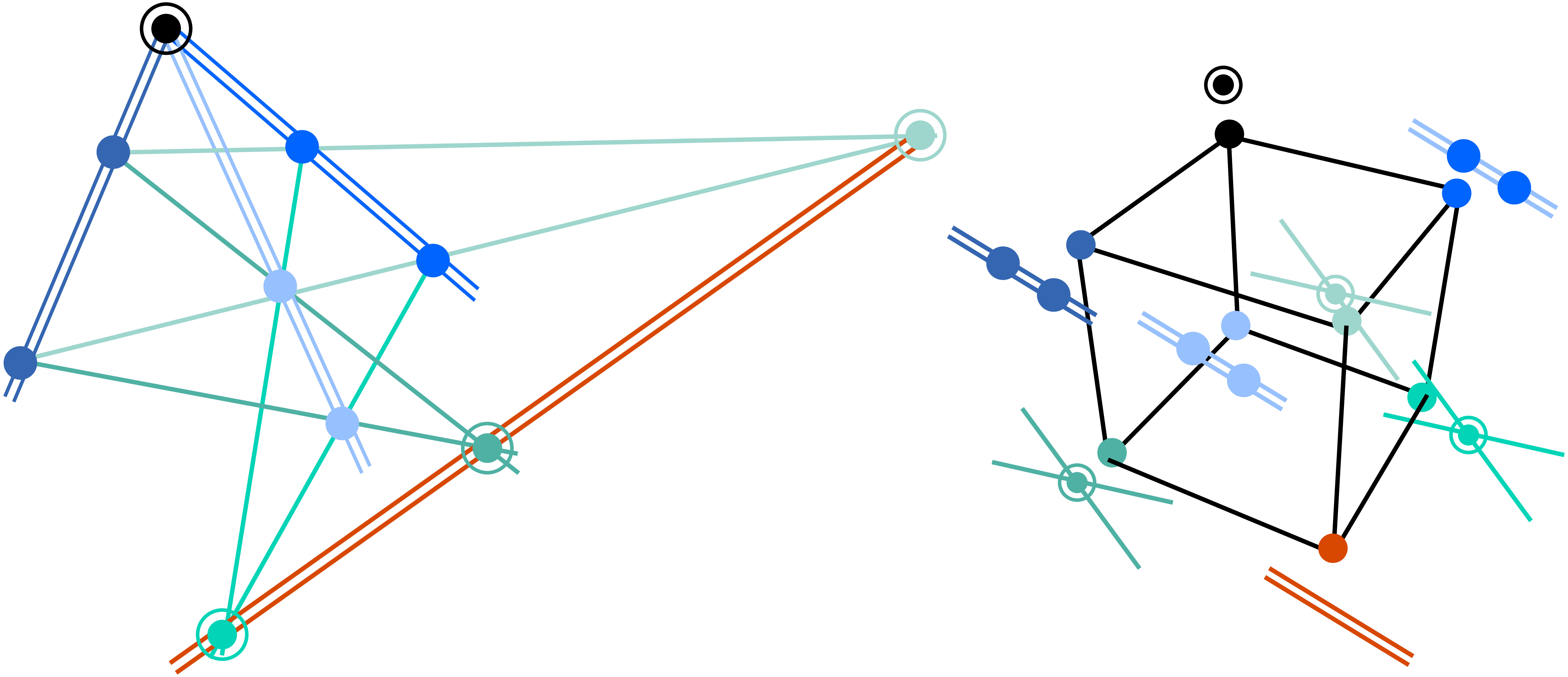
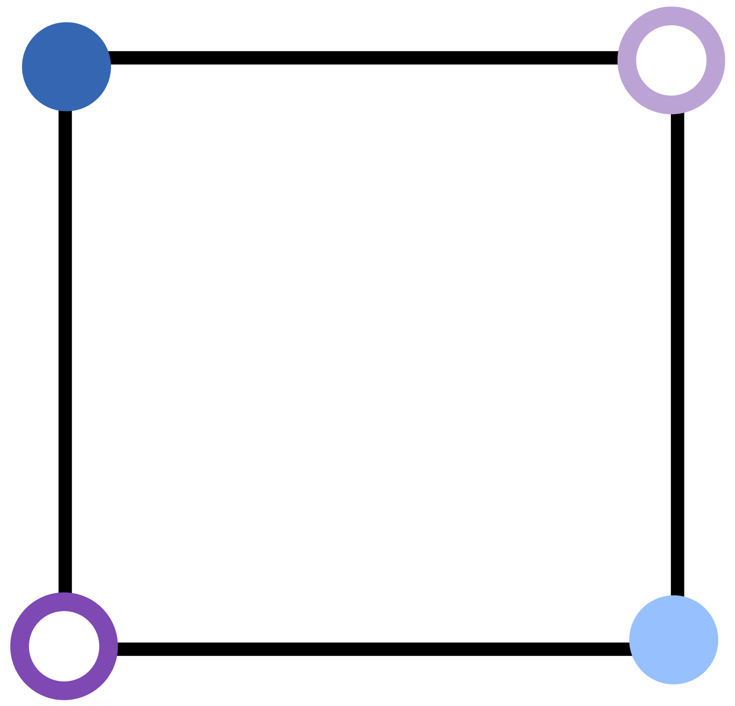
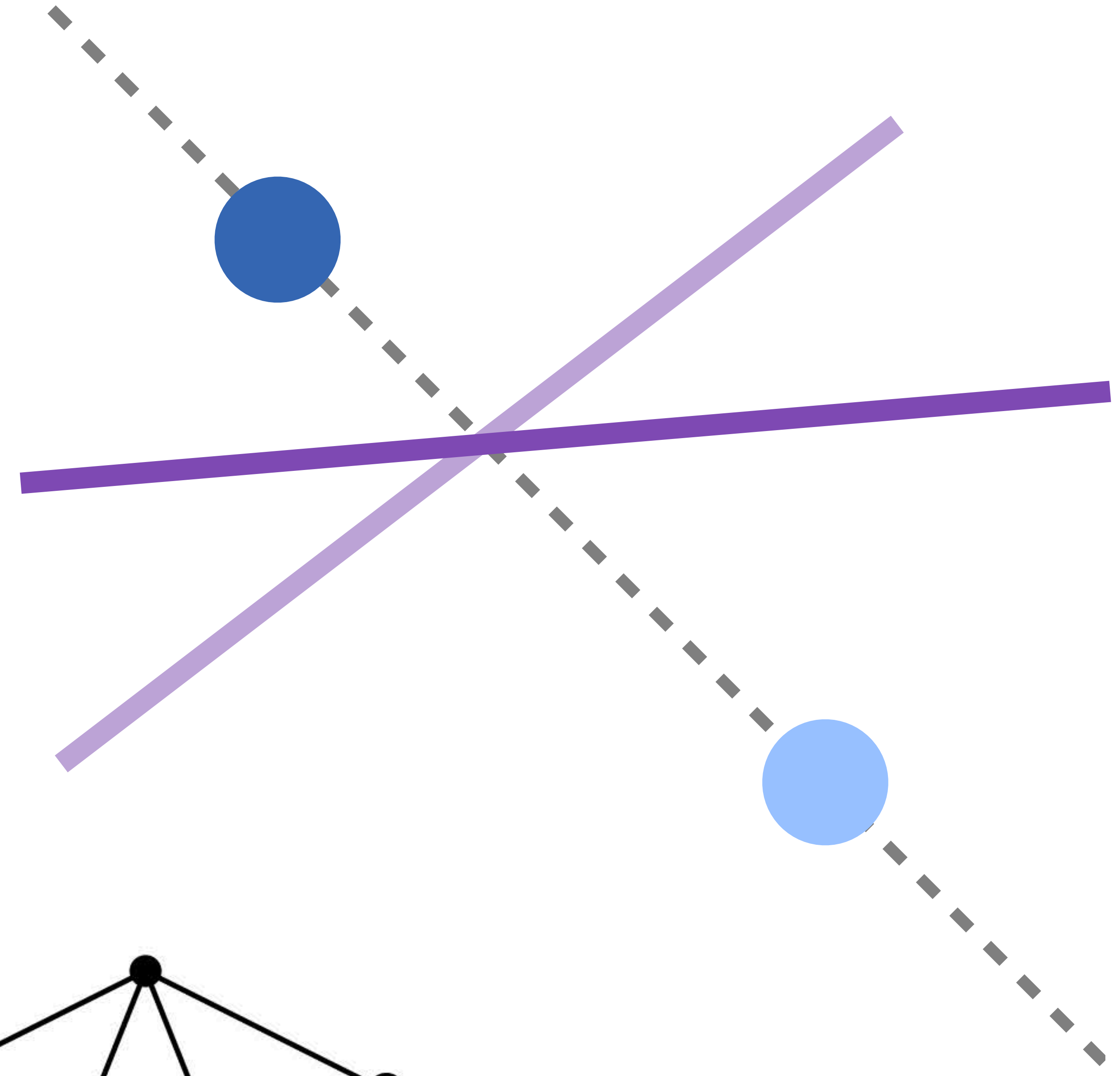
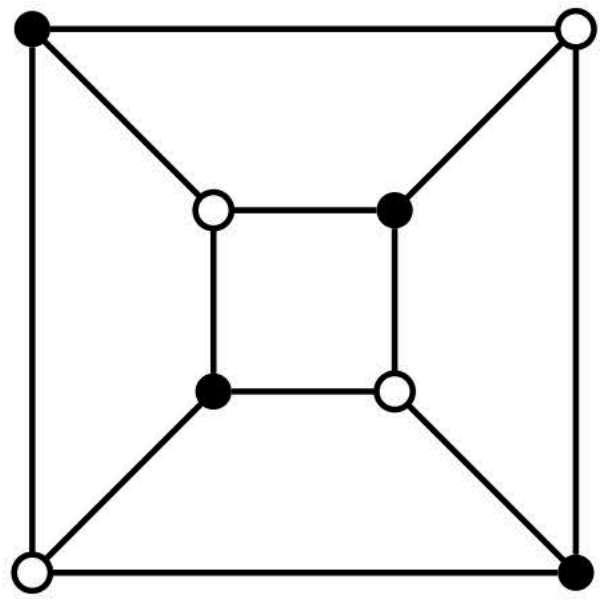


Related work

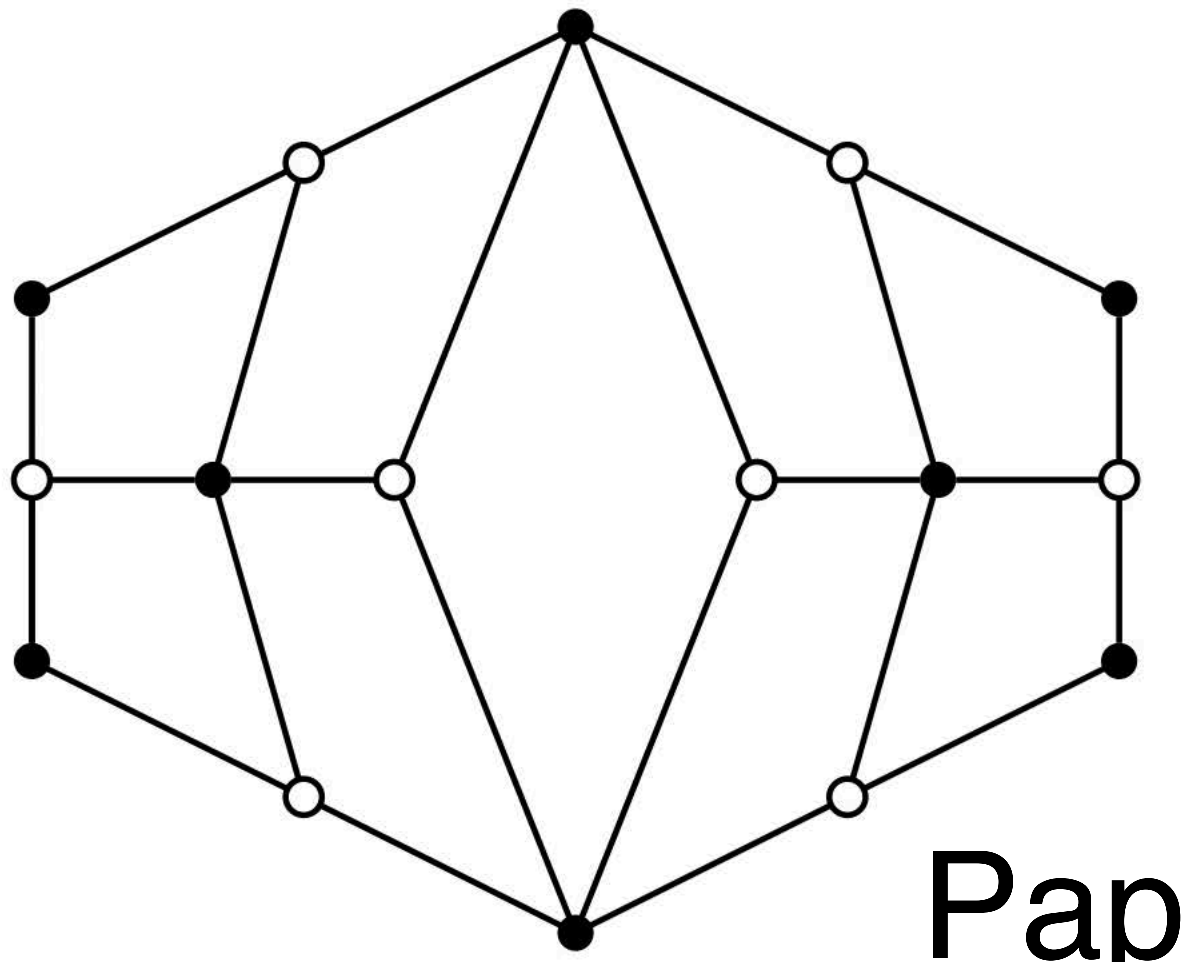
- Sergey Fomin & Pavlo Pylyavskyy “Incidences and Tilings” 2023





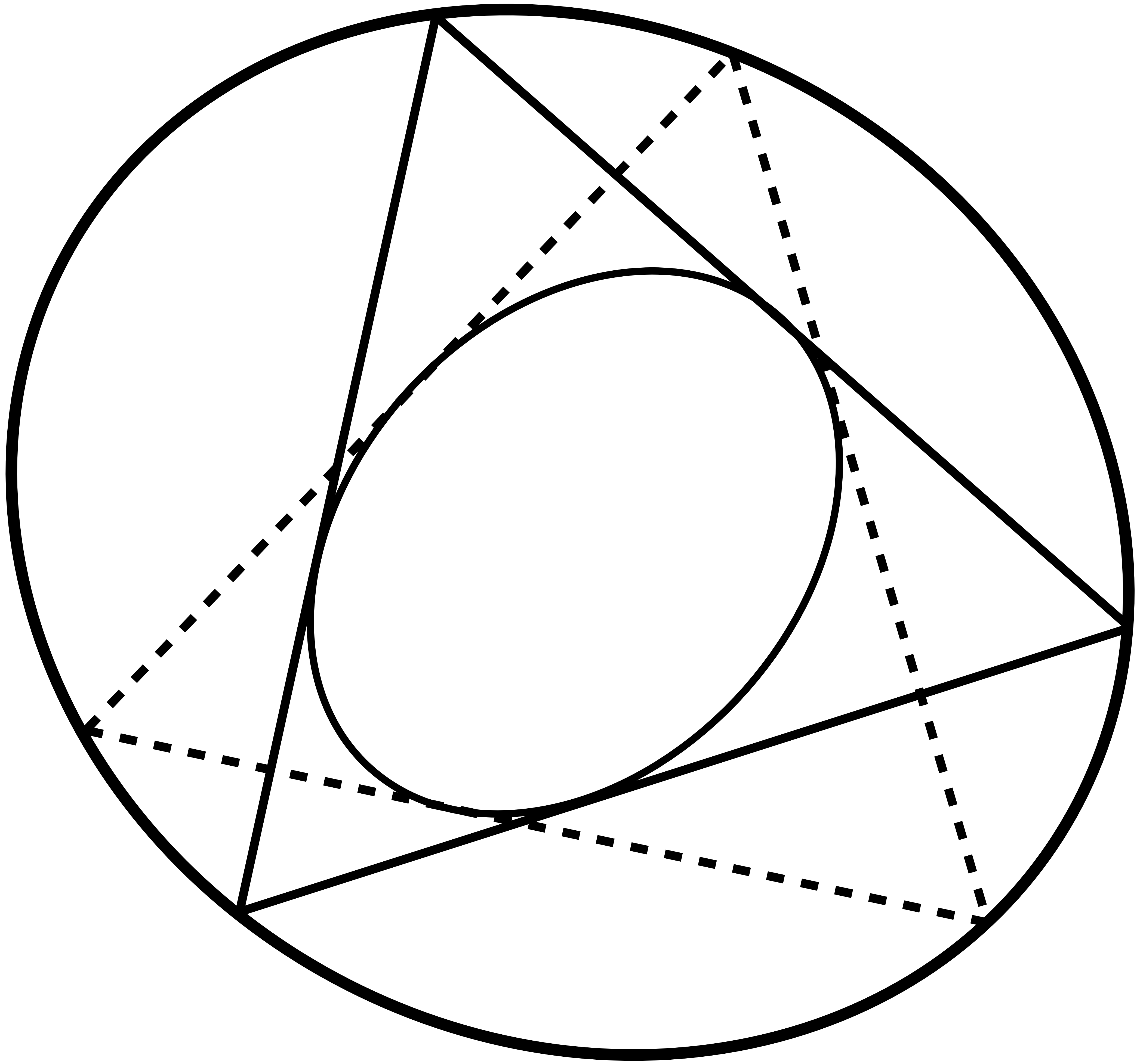


Desargues



Pappus

Poncelet Porism



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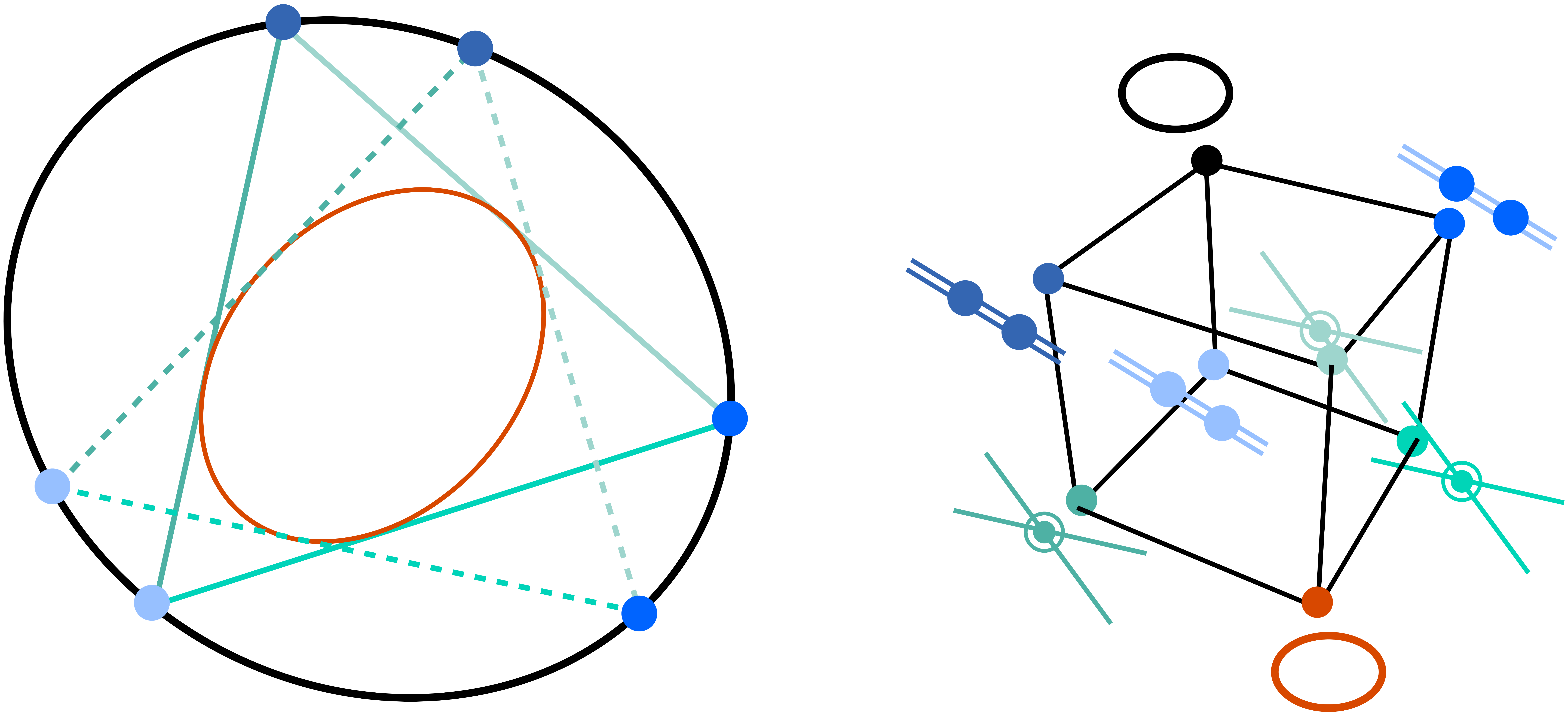
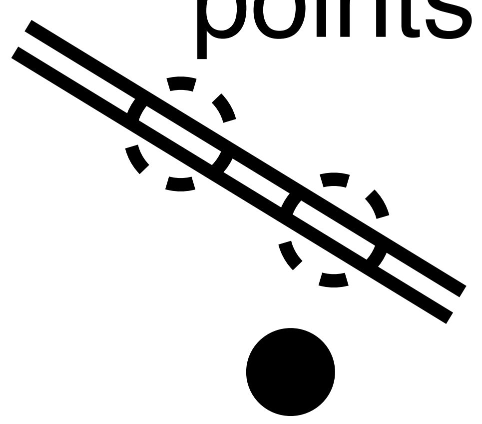
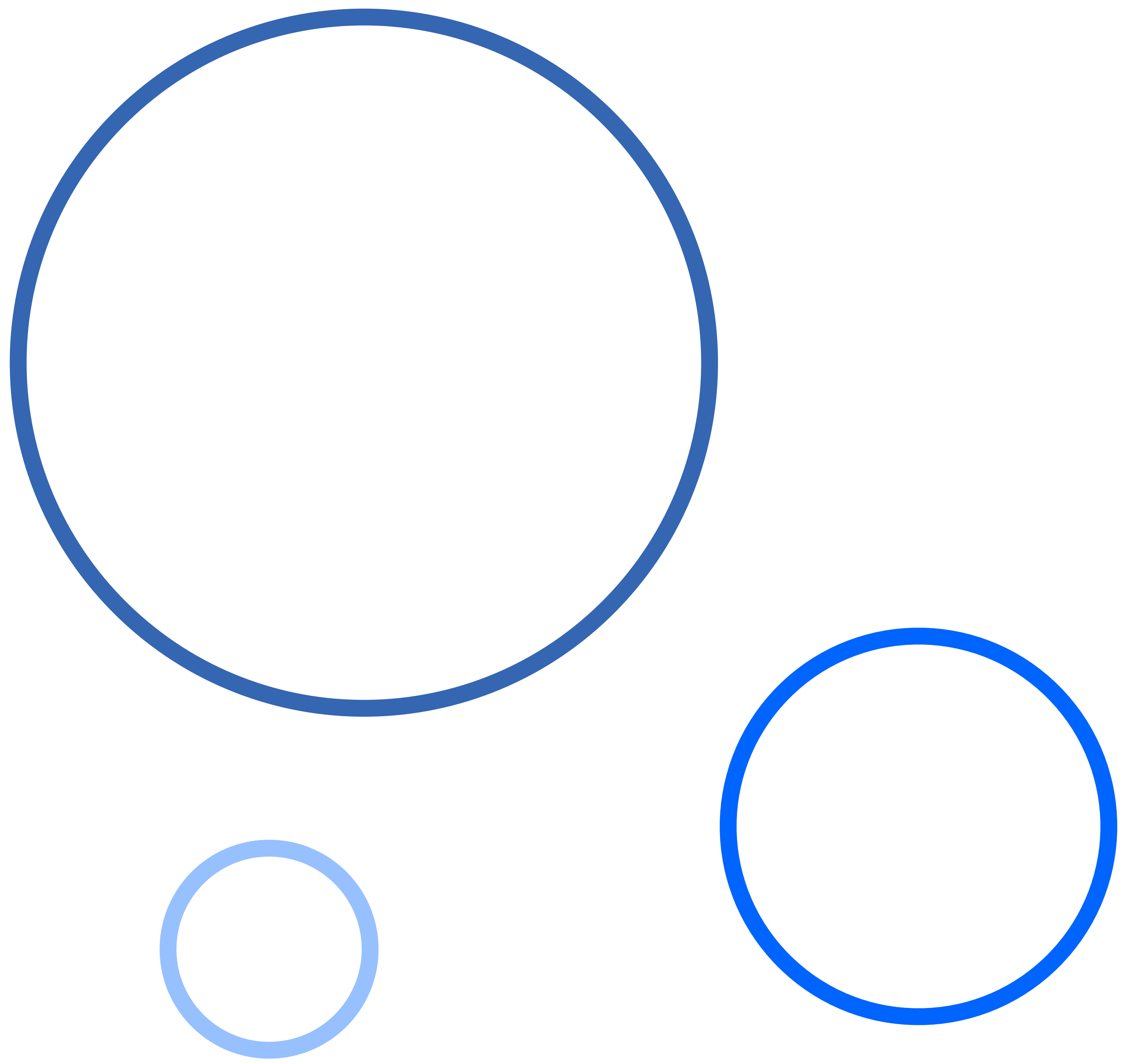
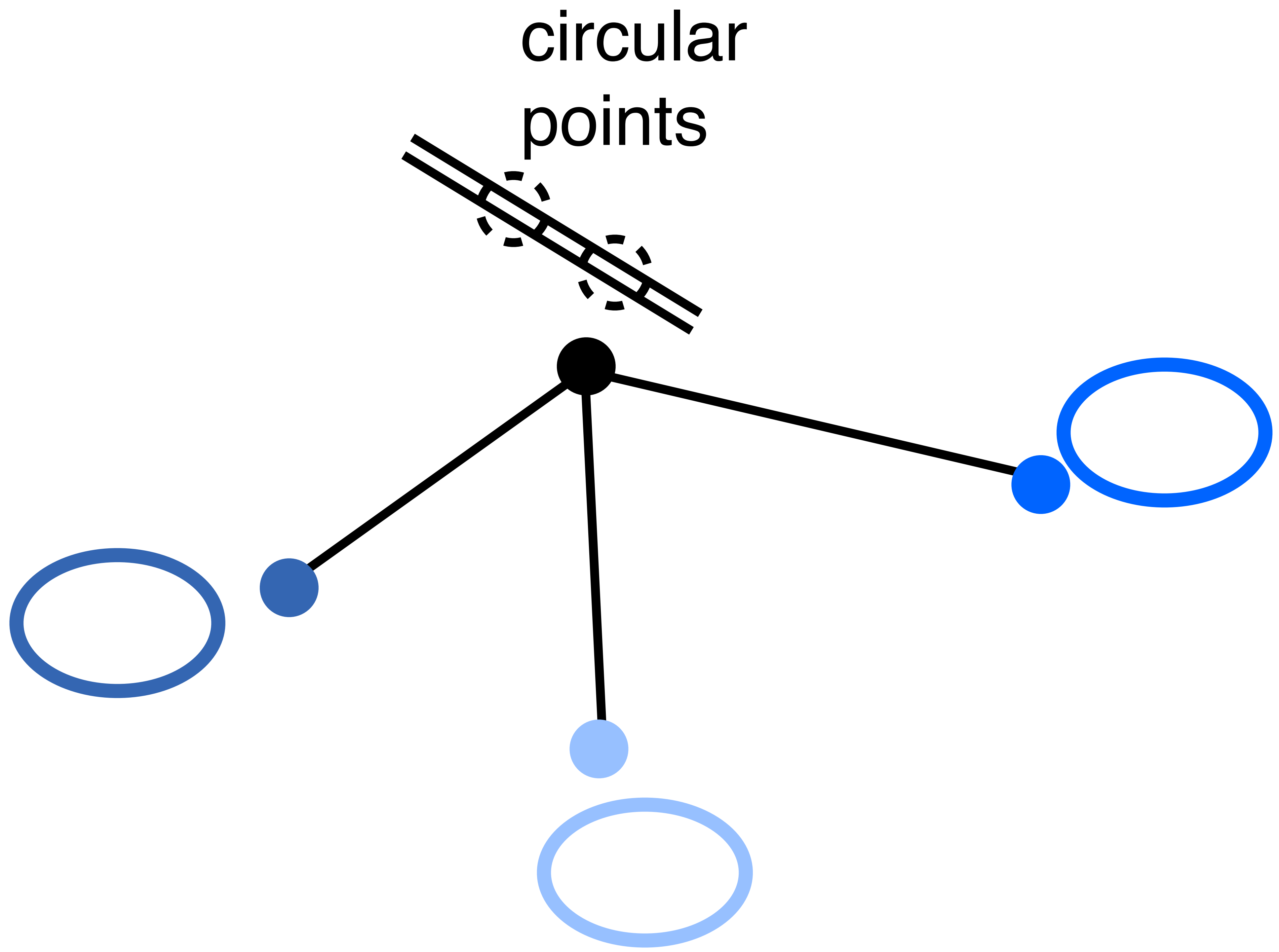


Monge Theorem



Monge Theorem





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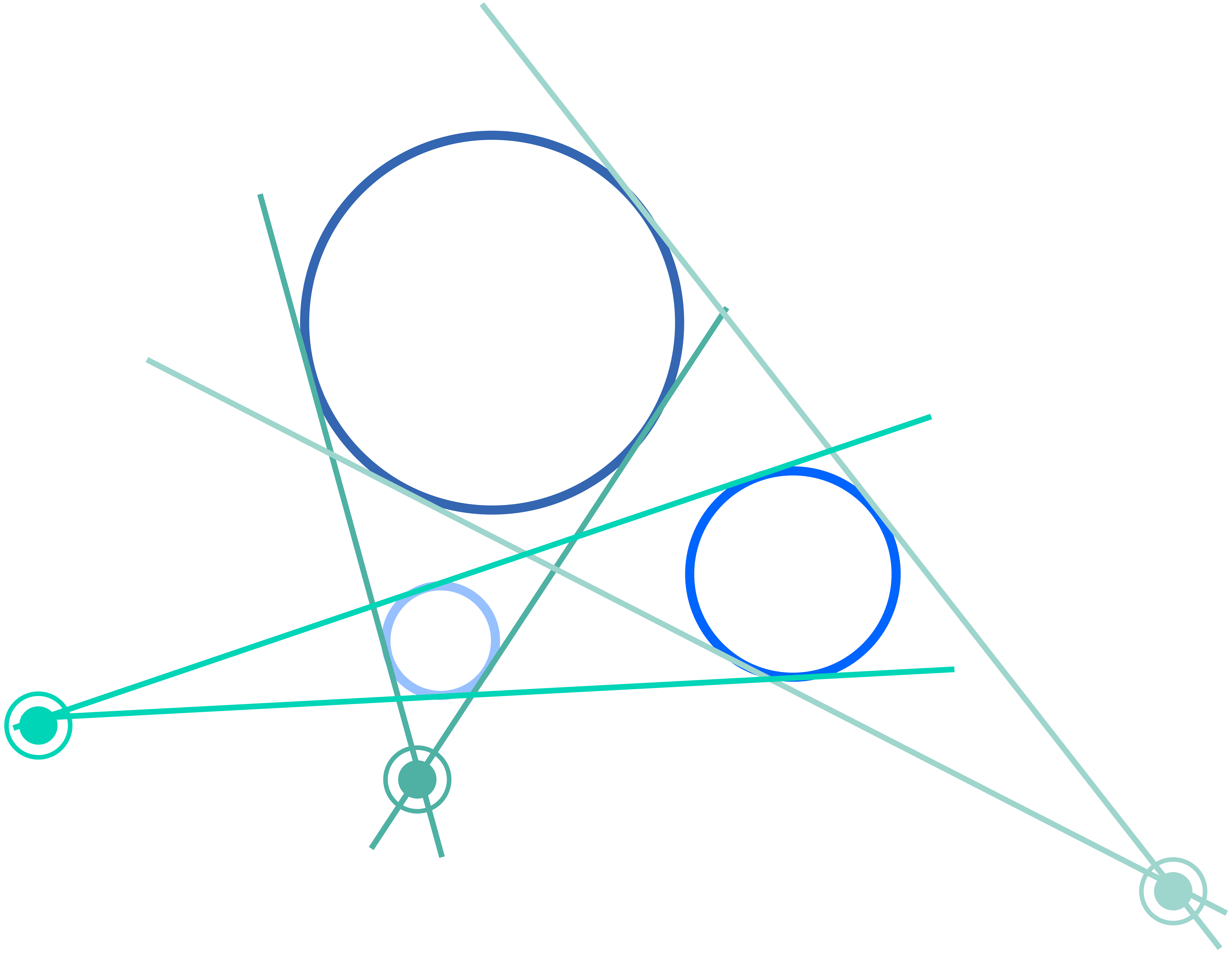

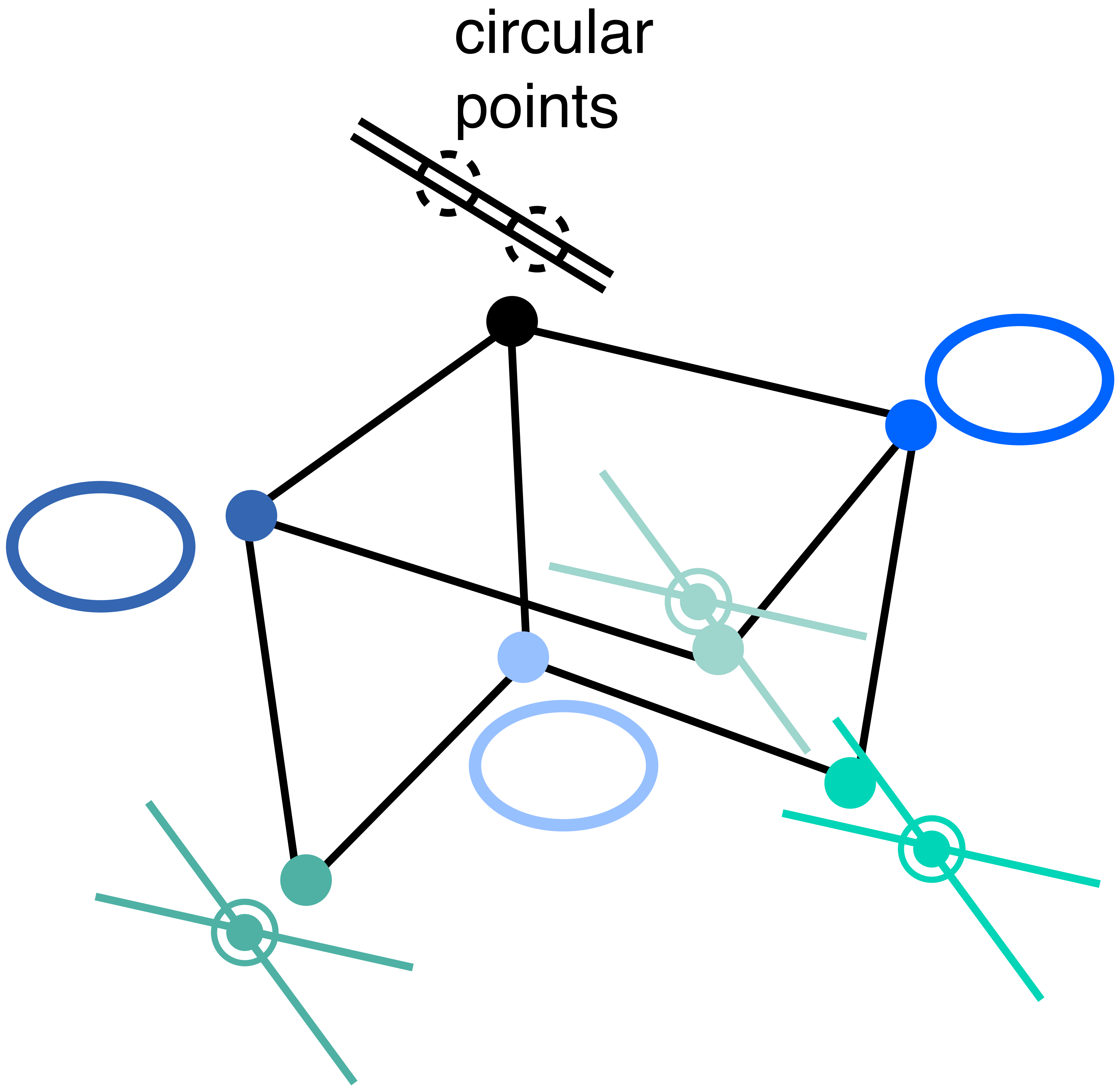

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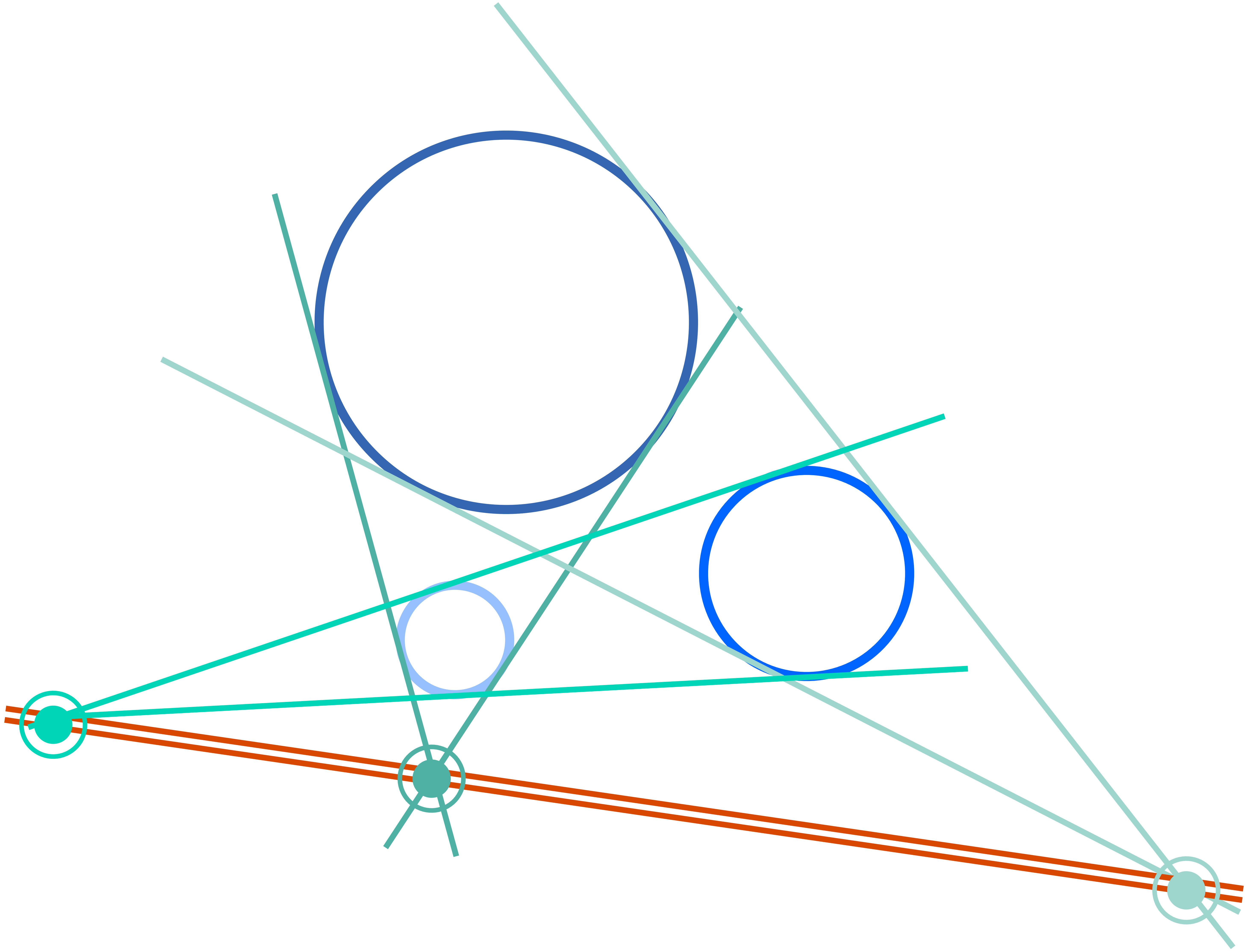
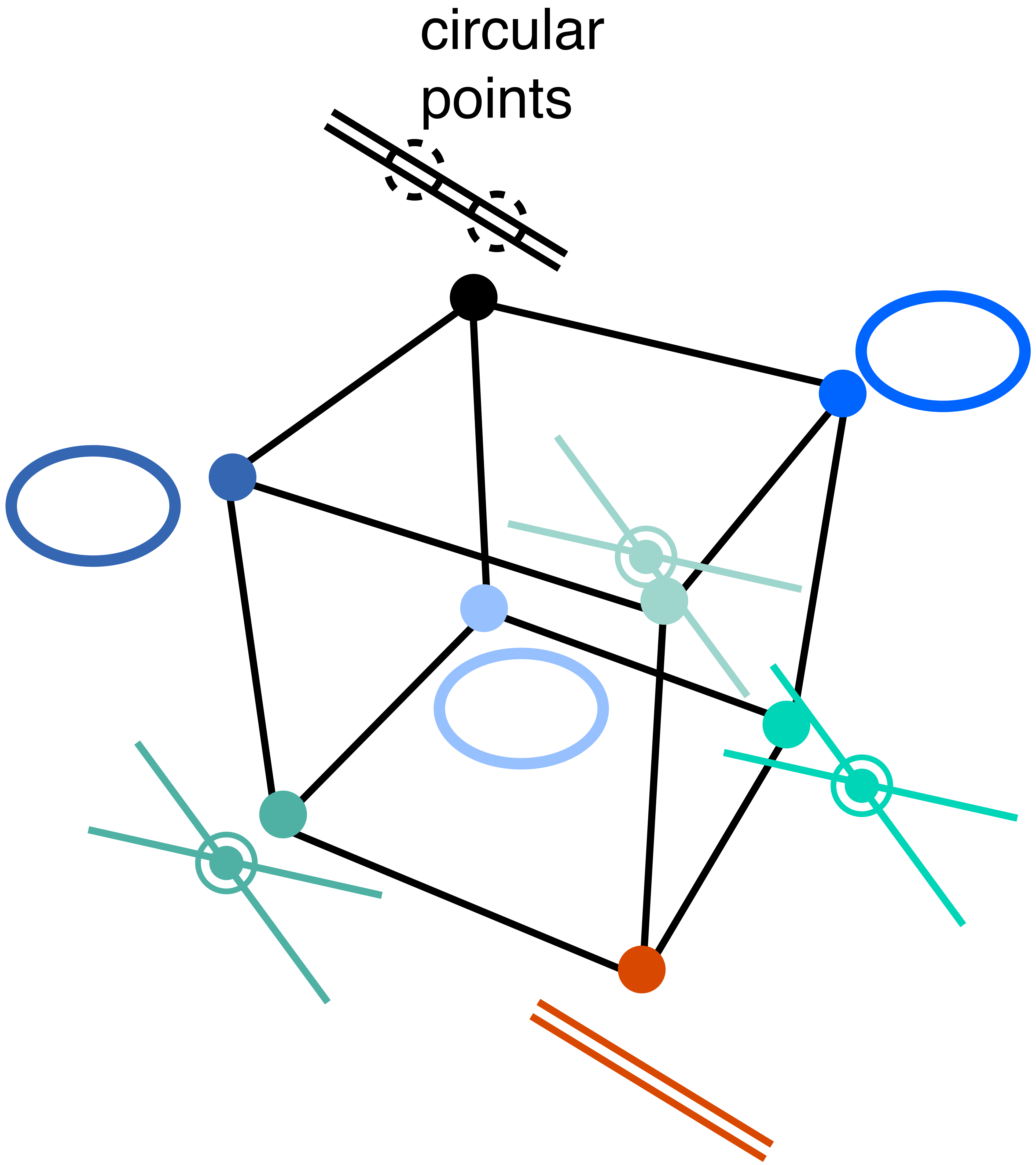
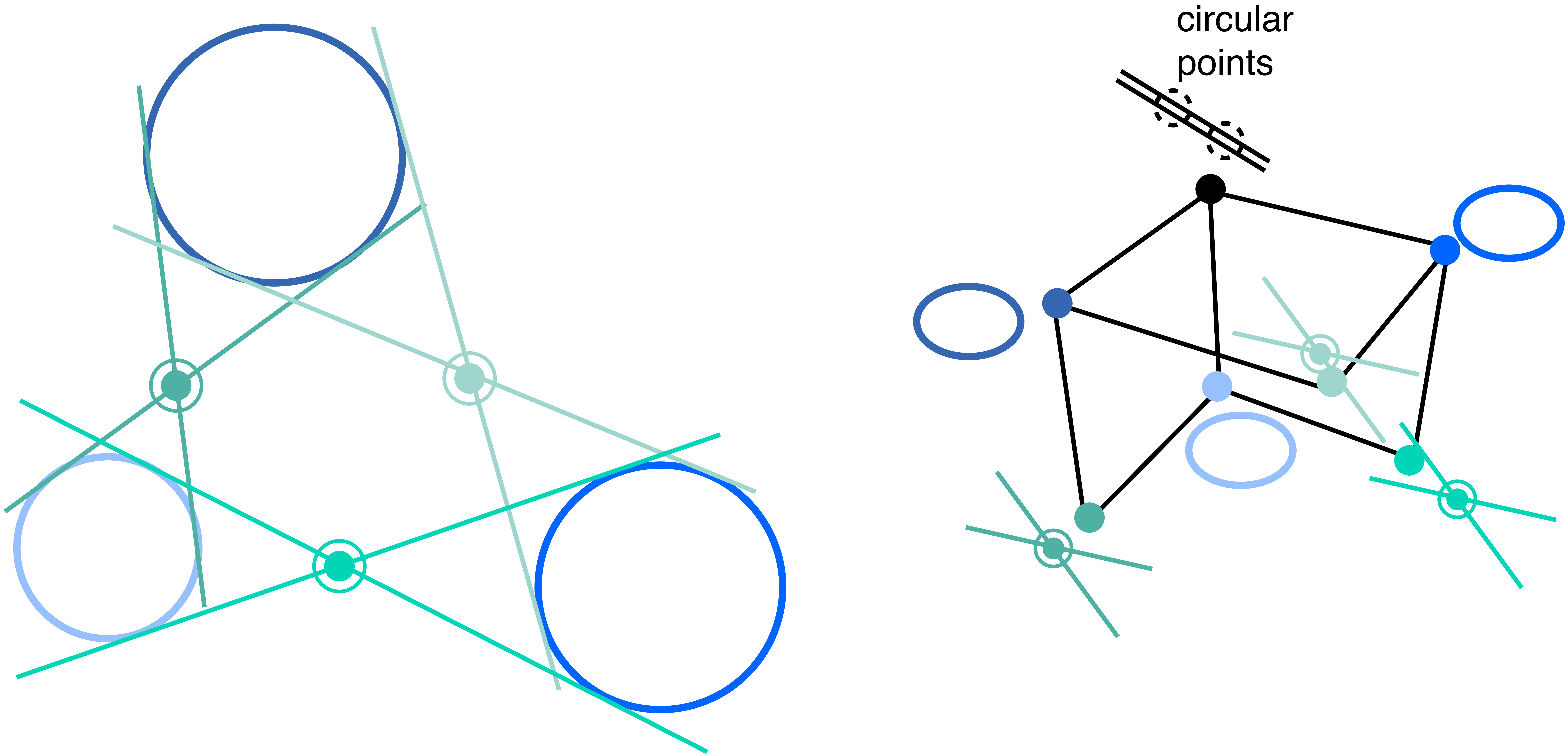
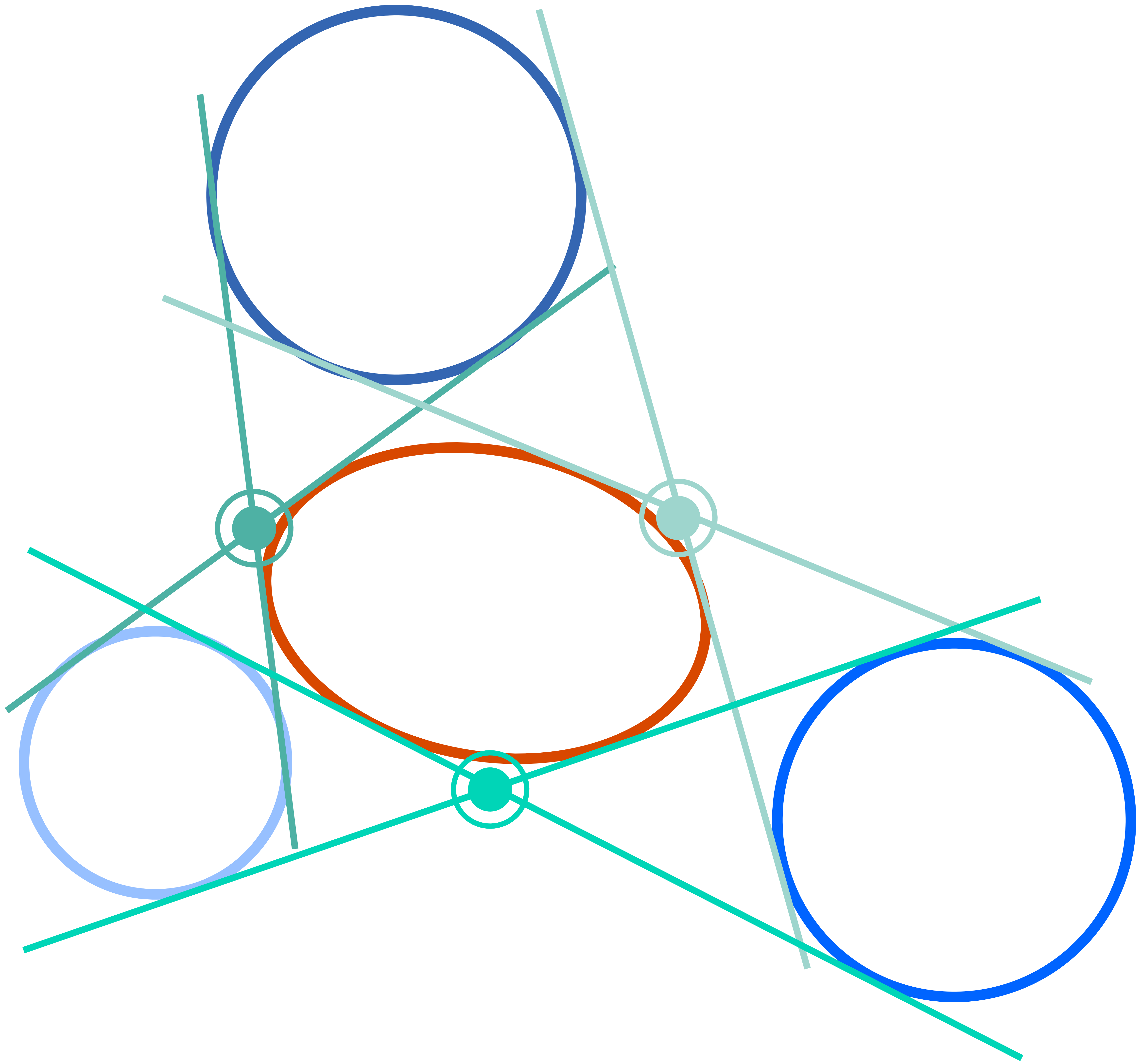
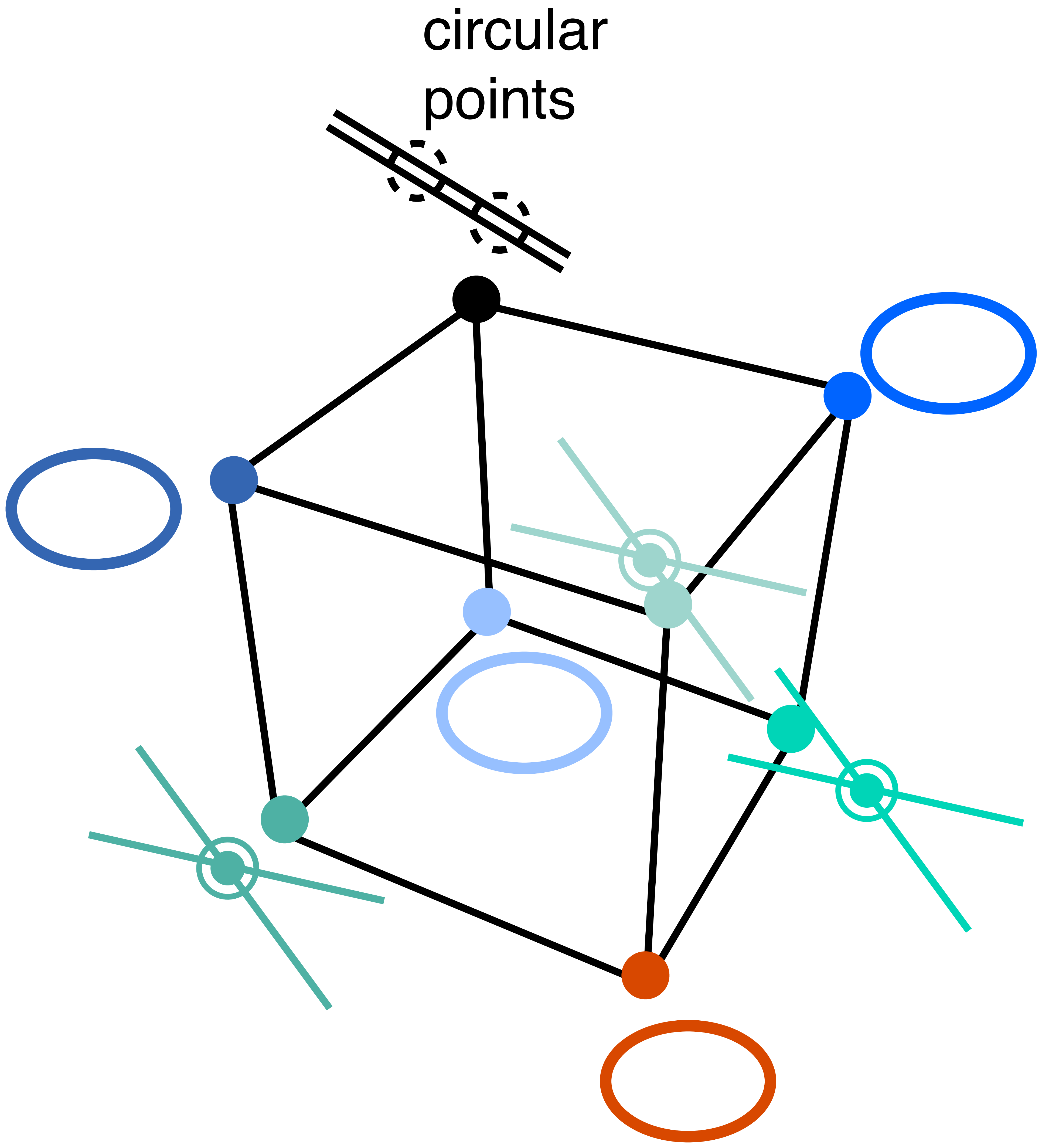


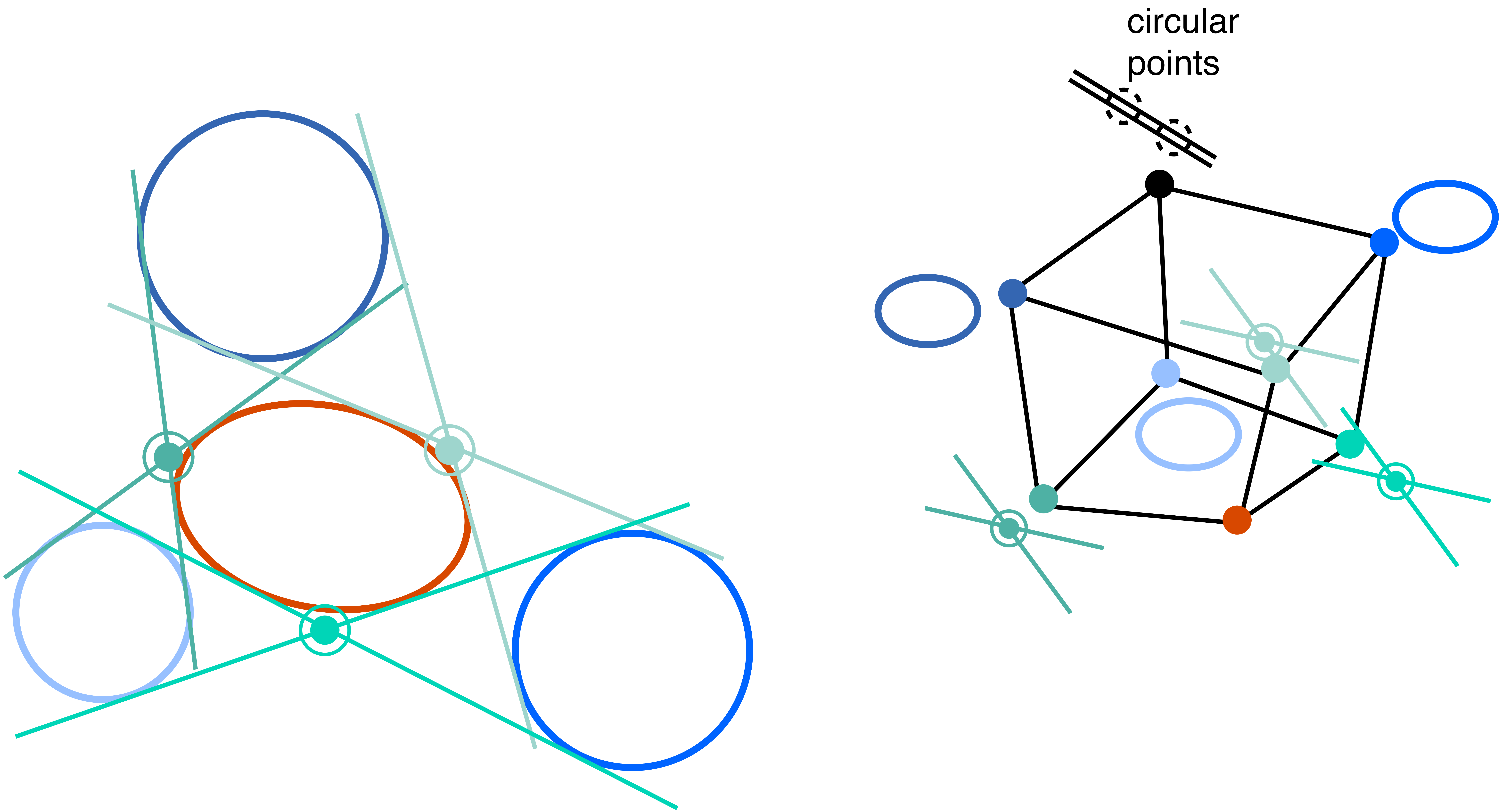



circular
points

Monge-like Theorem



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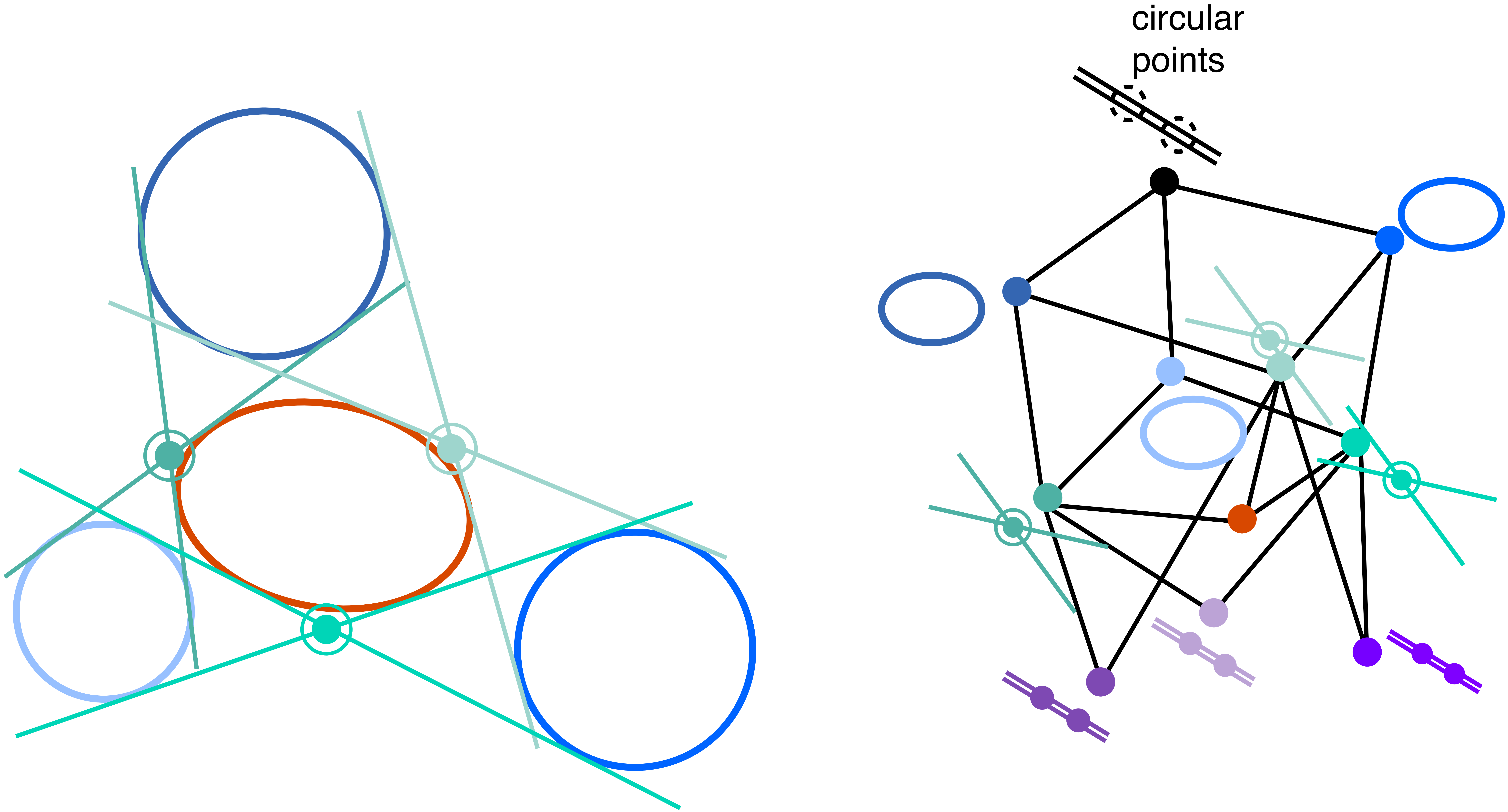
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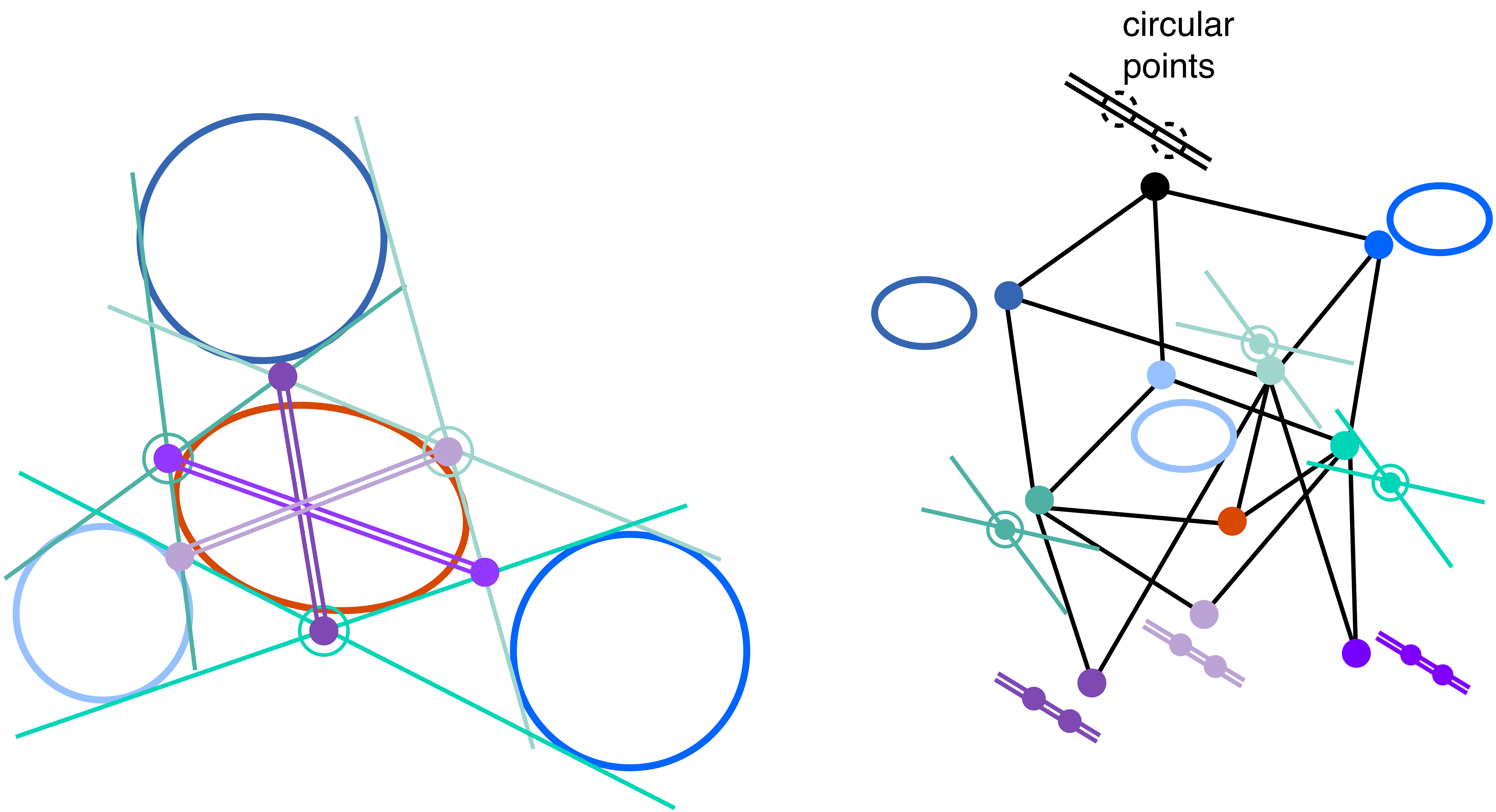
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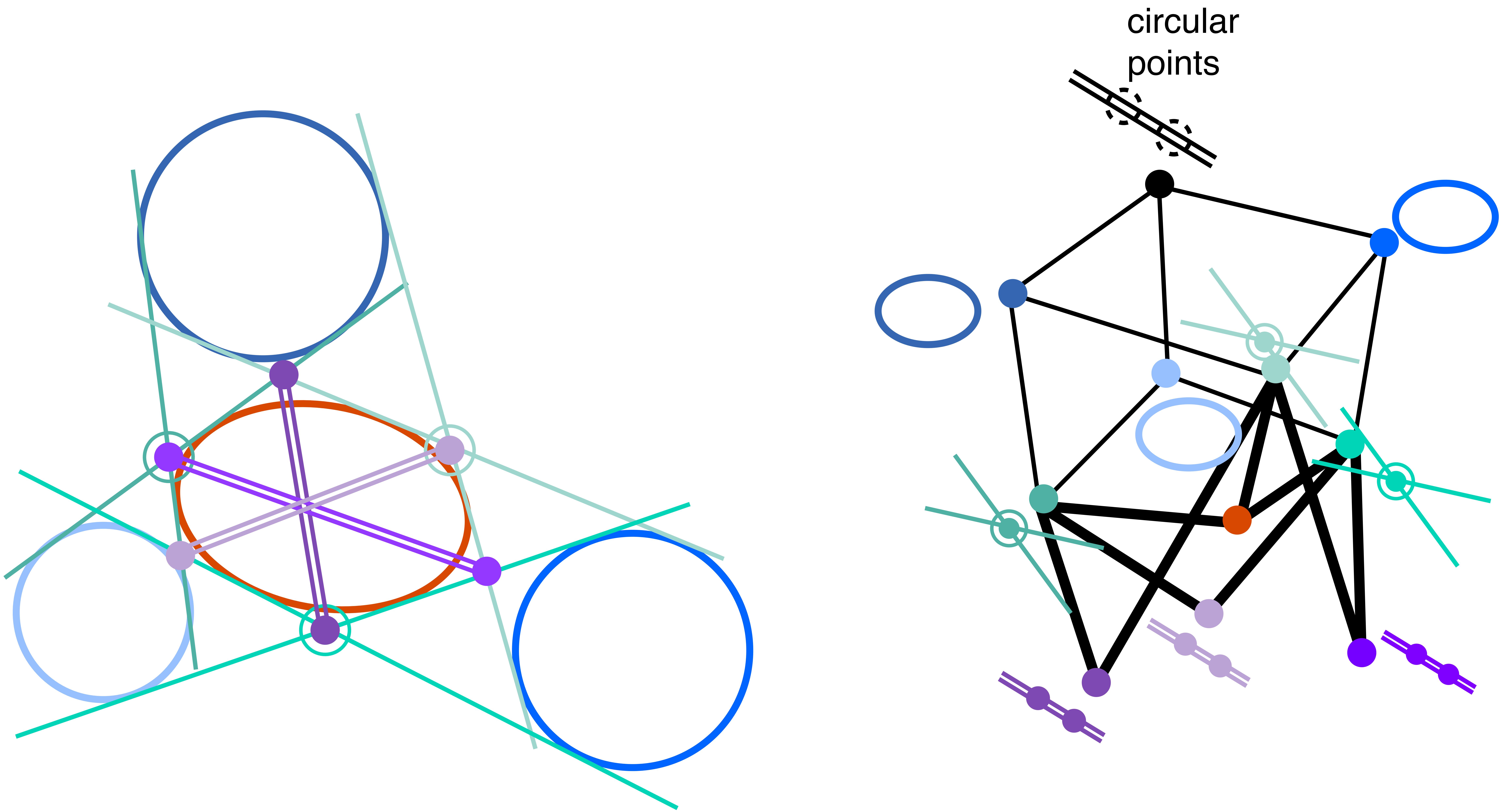
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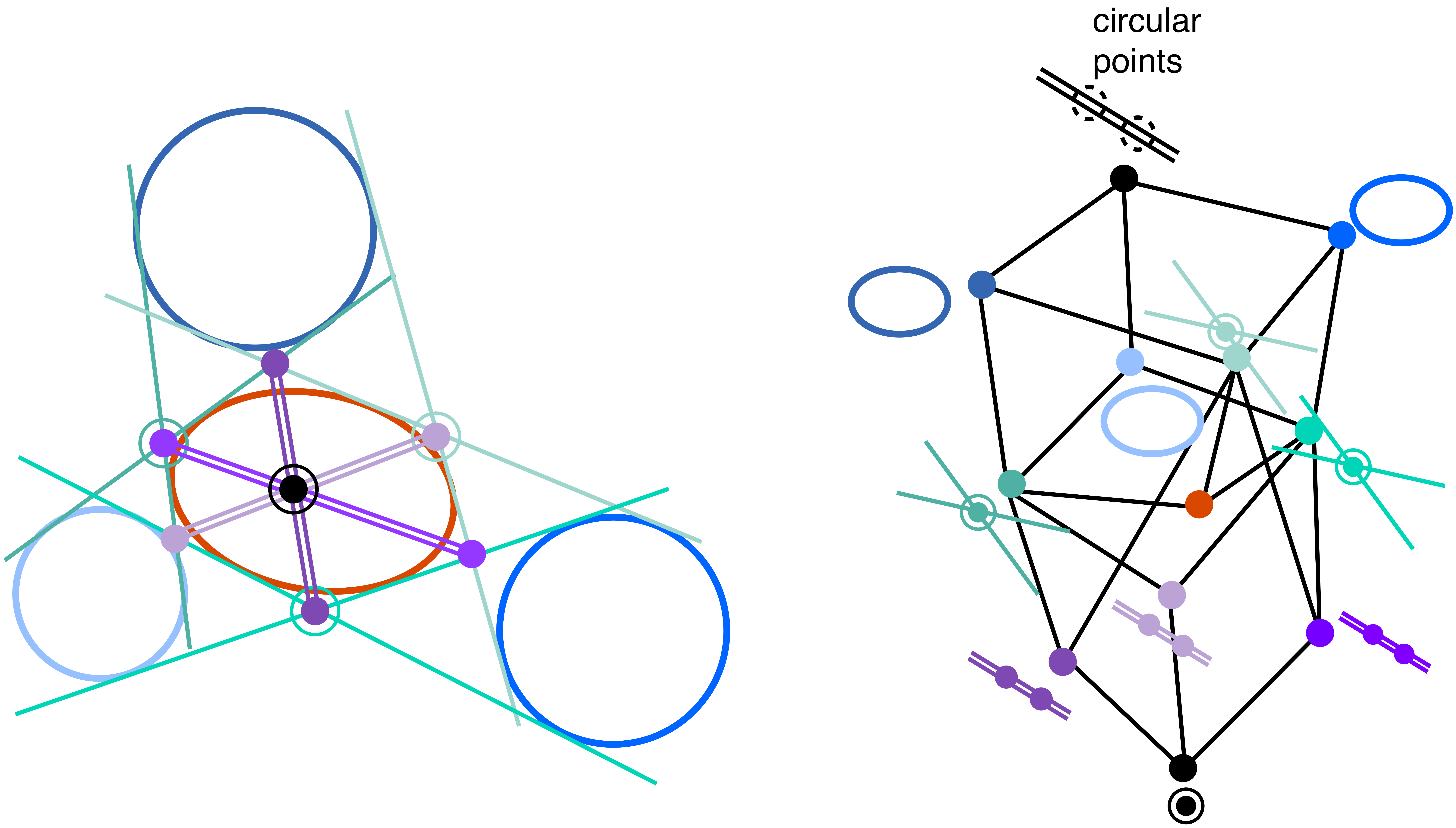
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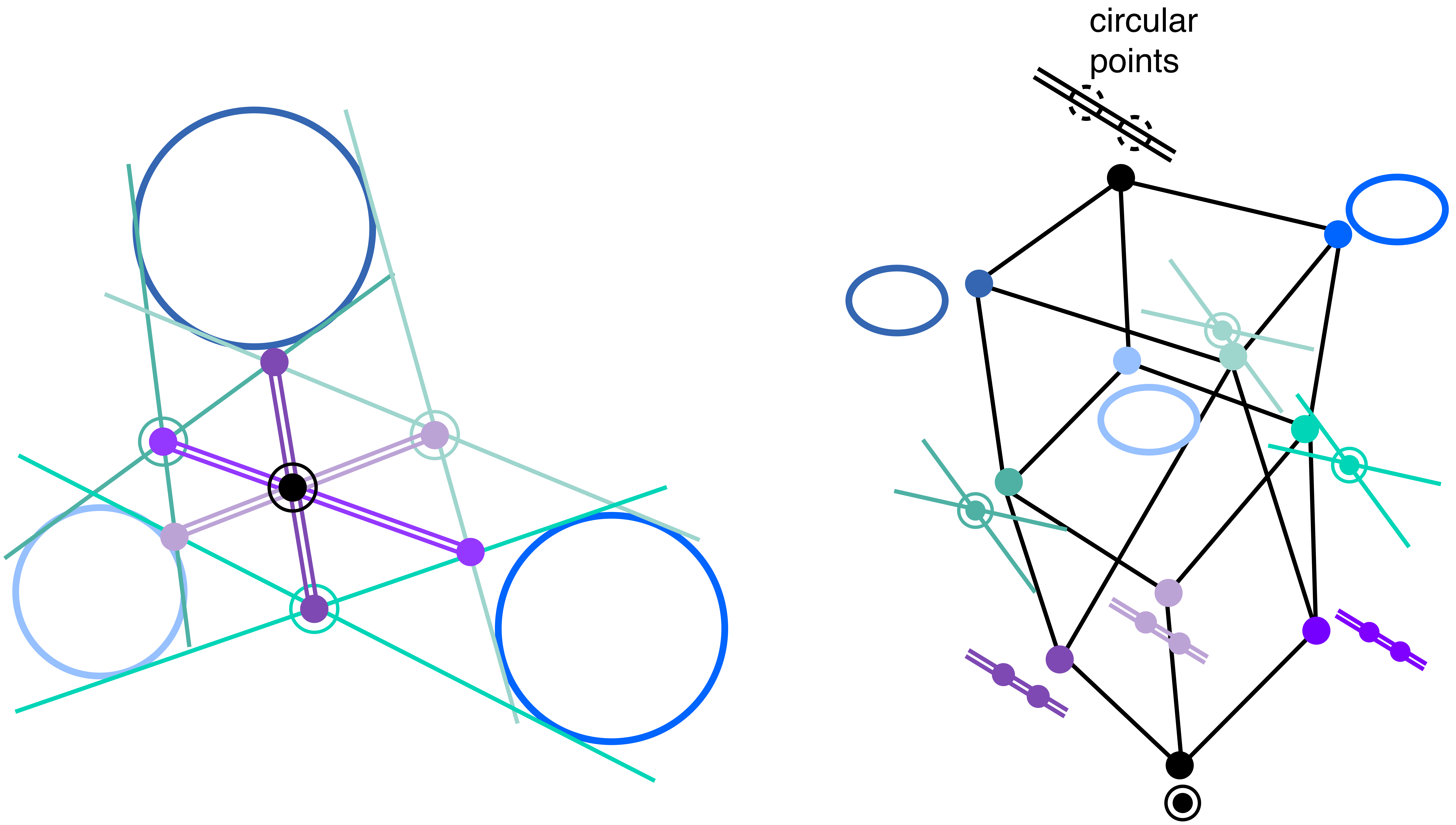
circular
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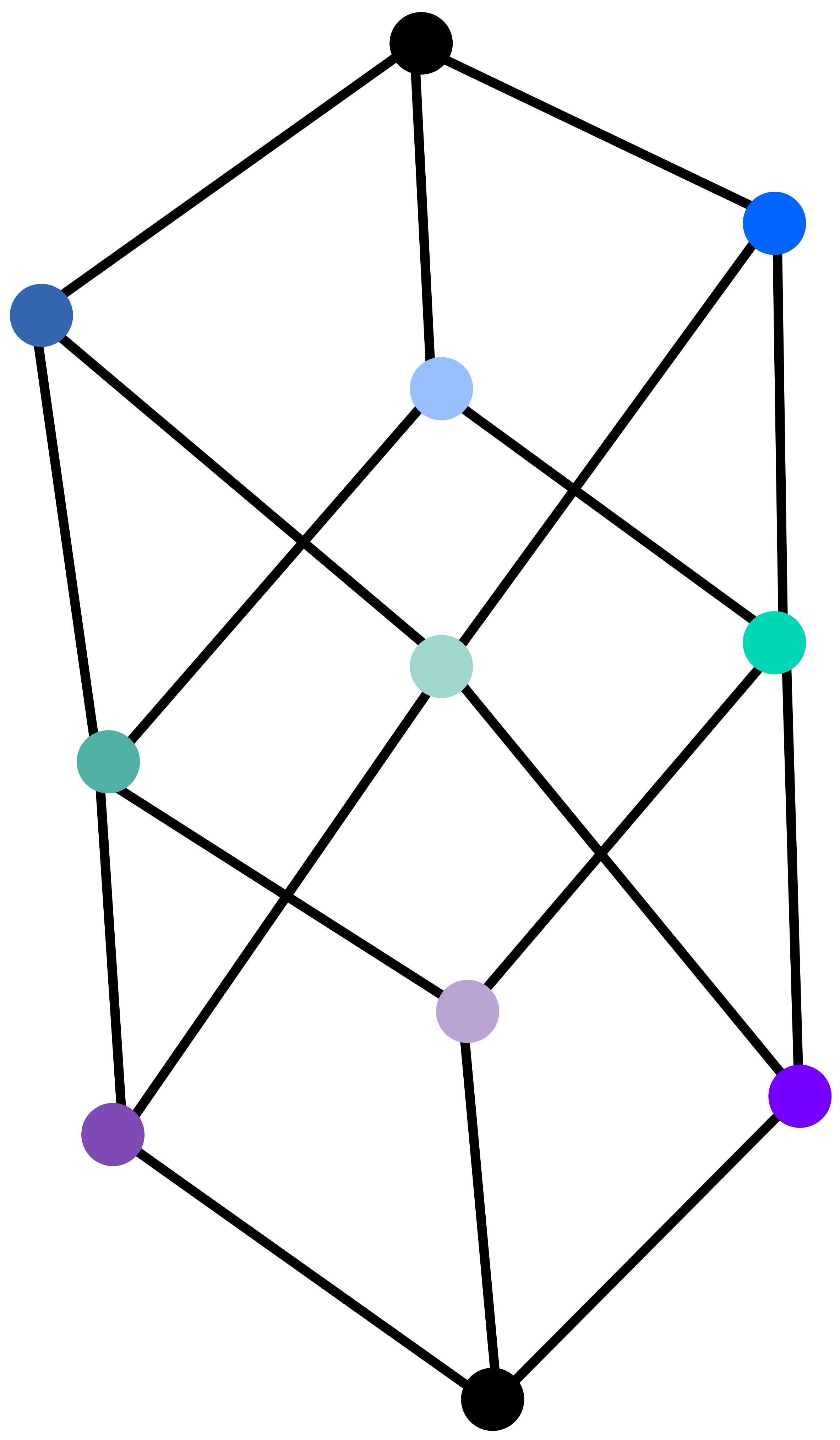
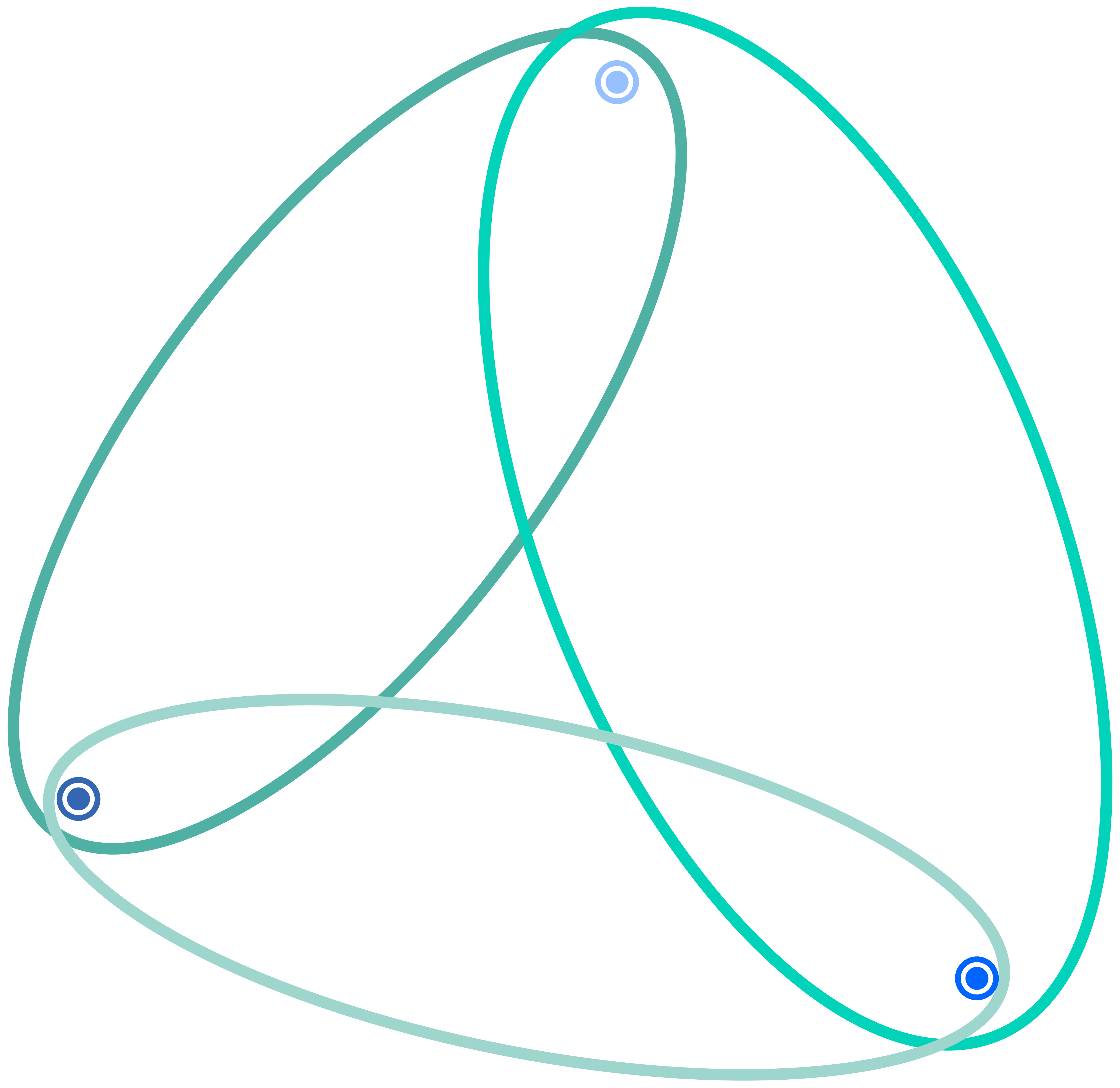
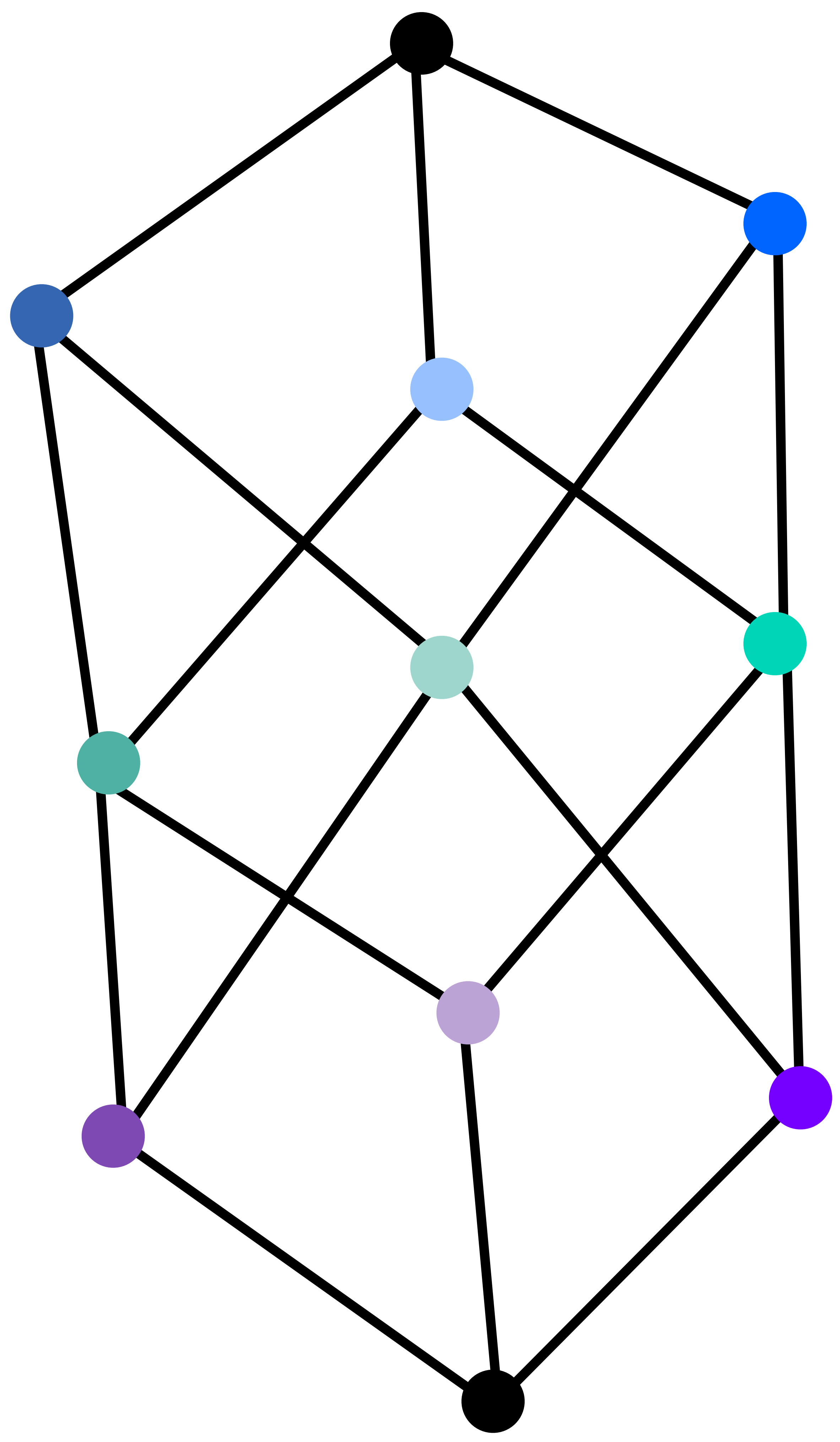
Eleven-conic Theorem



Neville's Theorem





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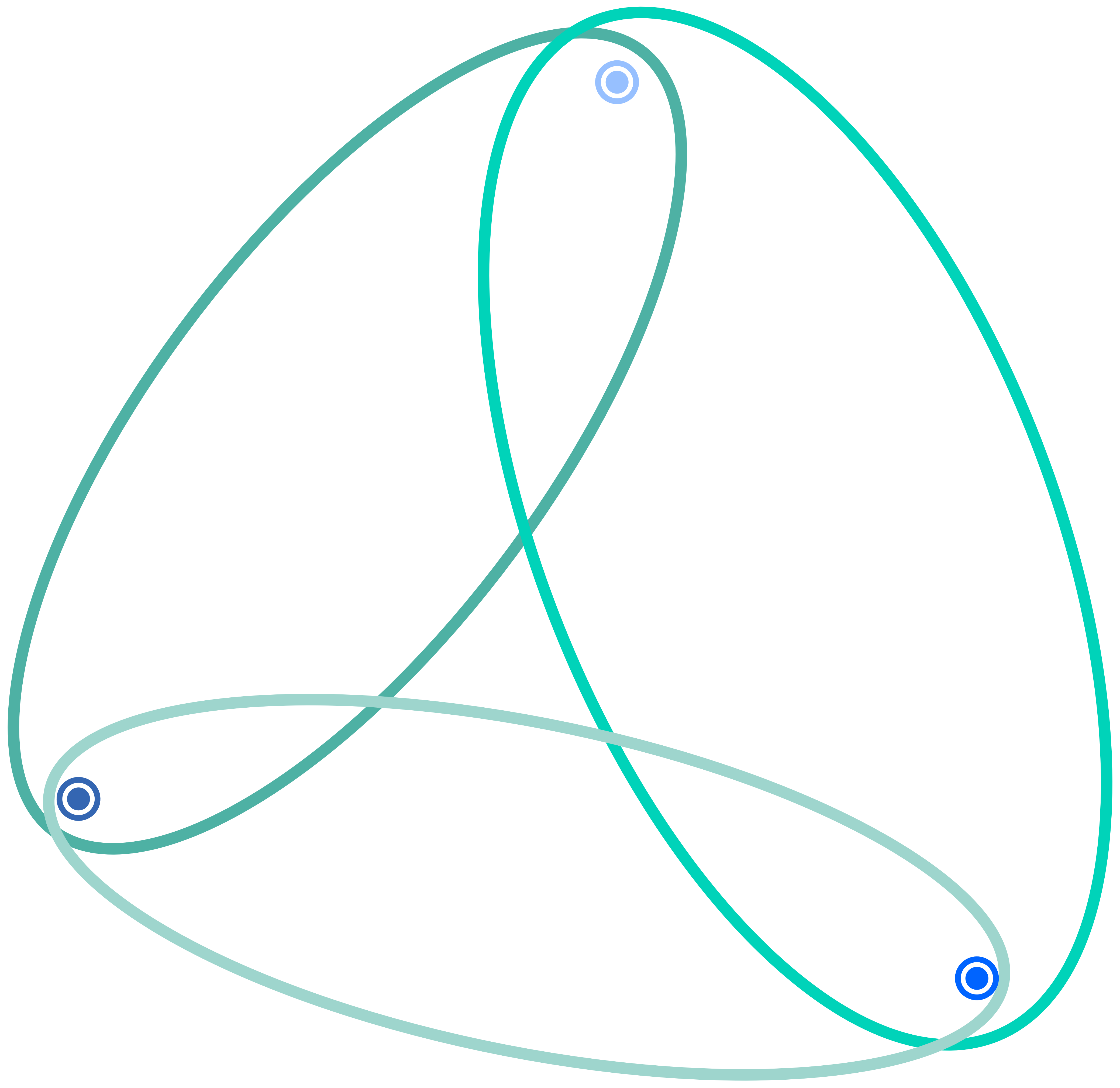

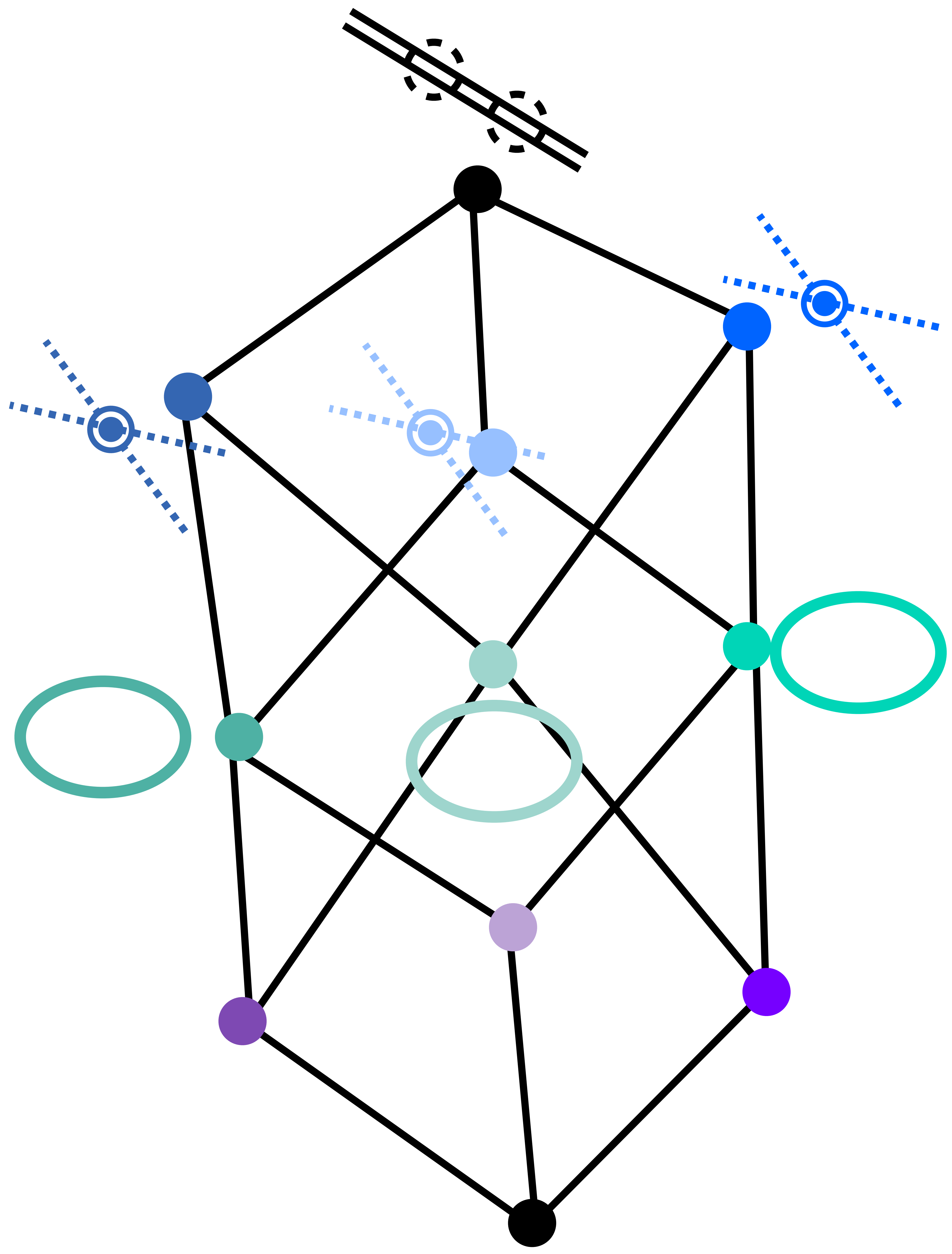

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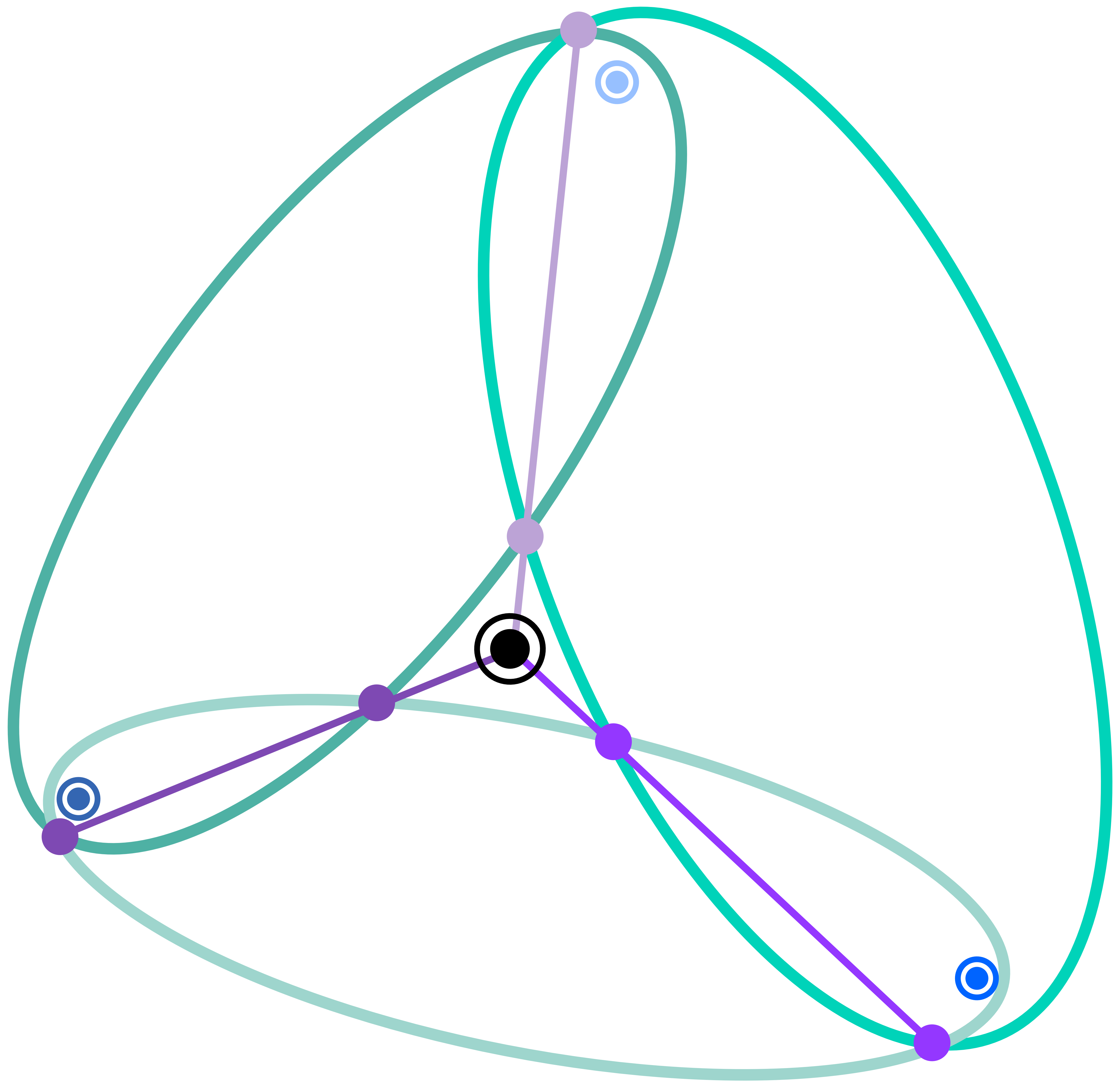

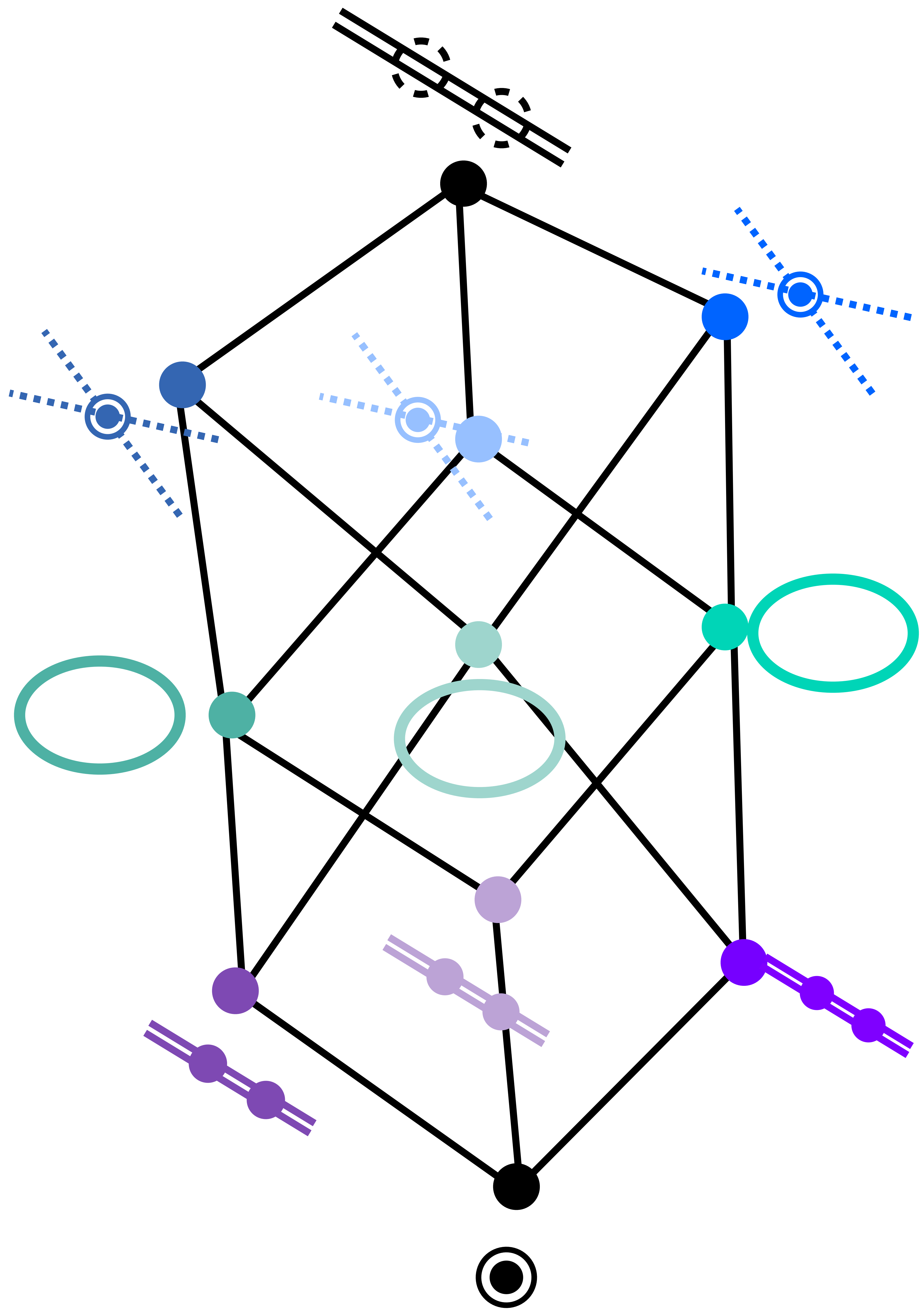

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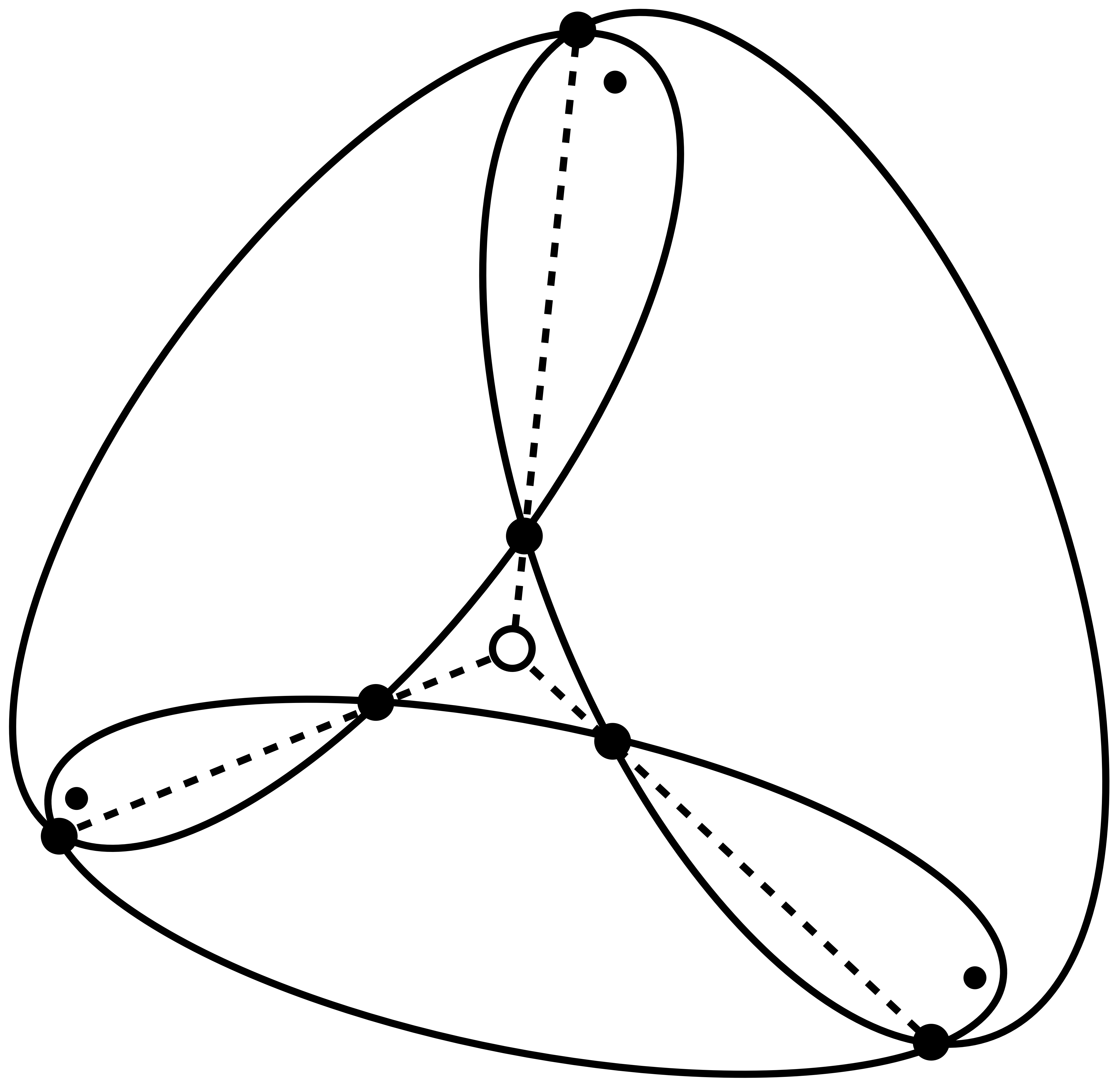
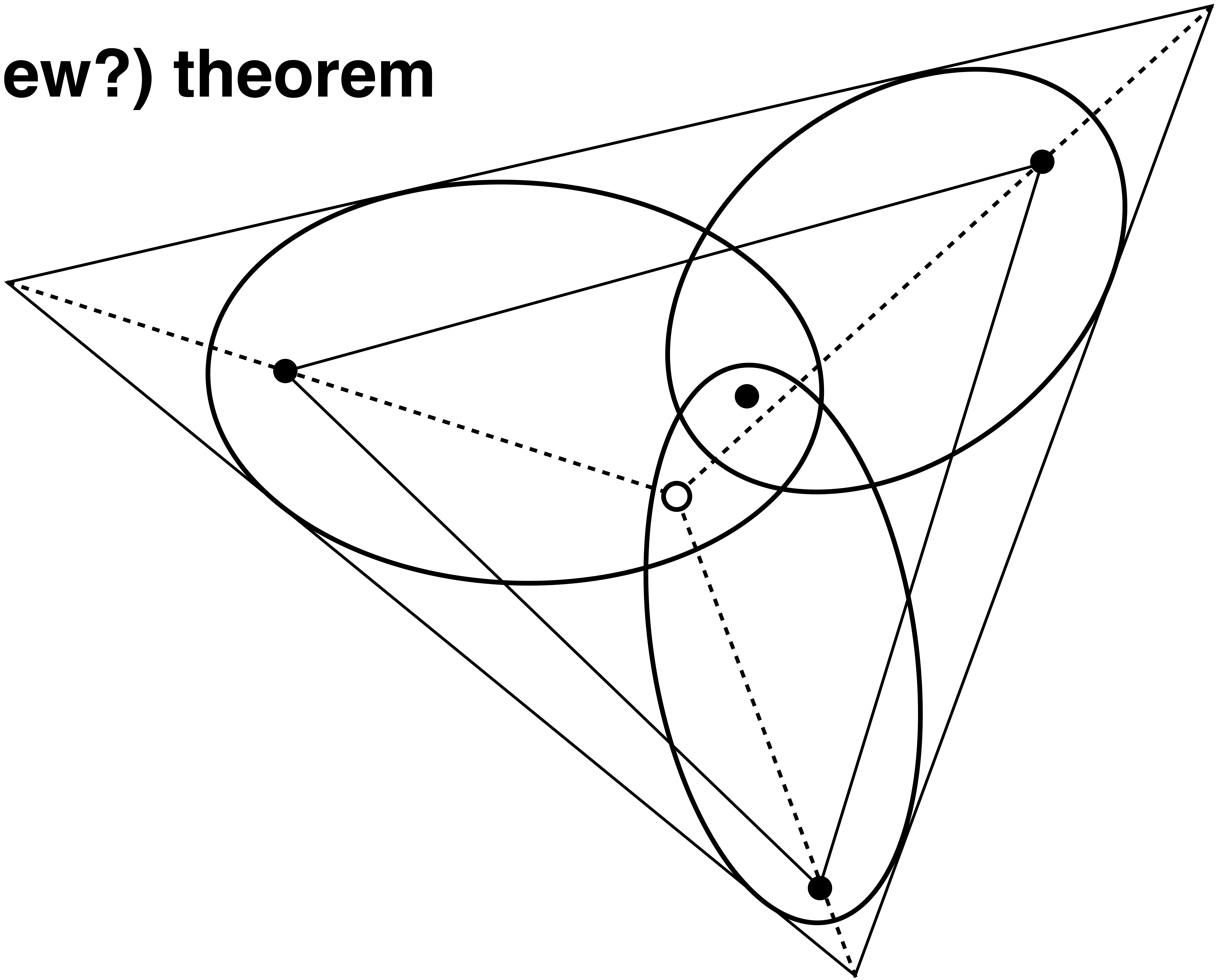
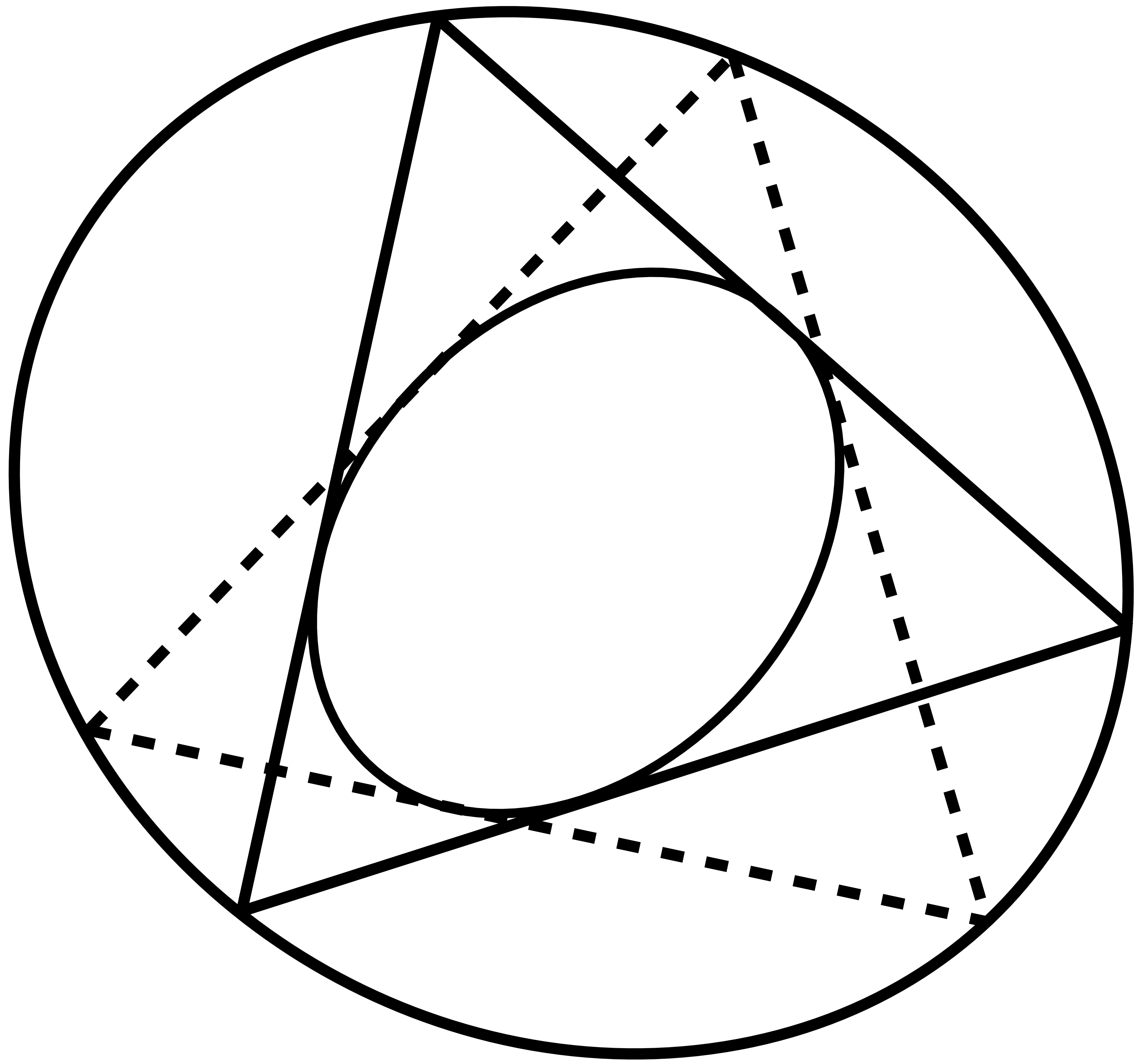


(New?) theorem



Poncelet porism for polygon



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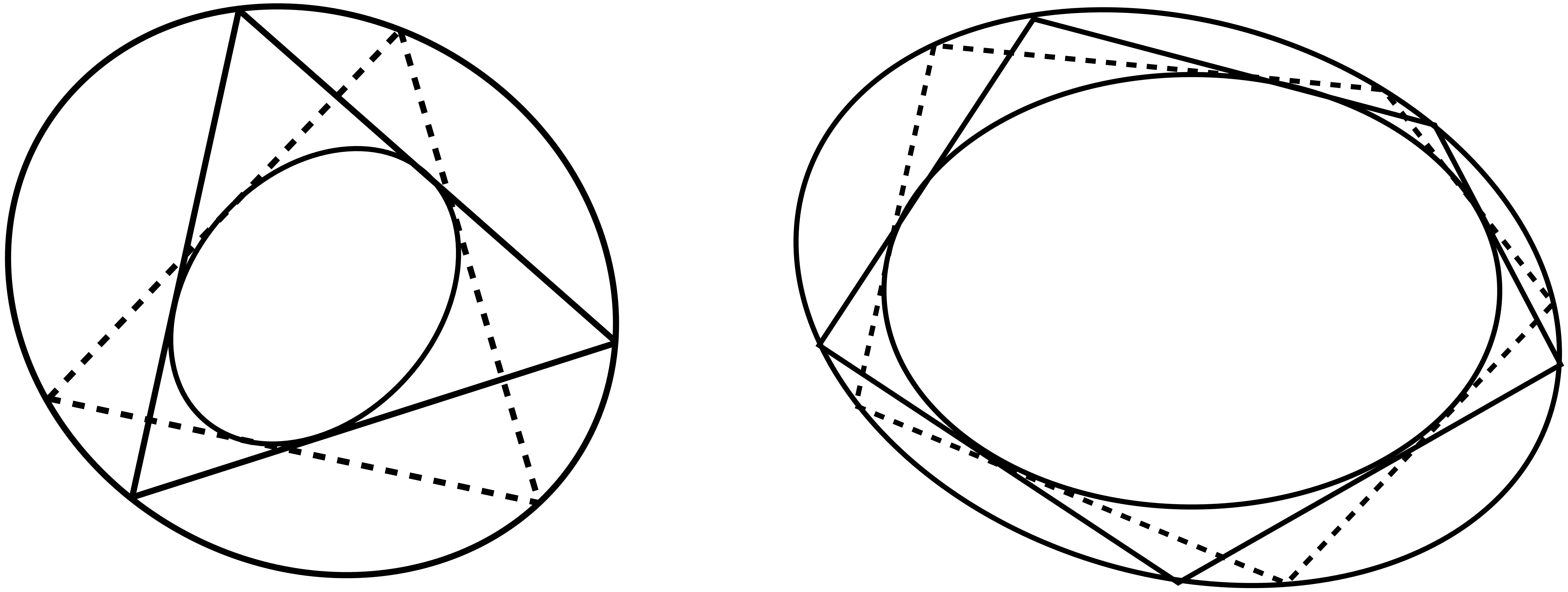
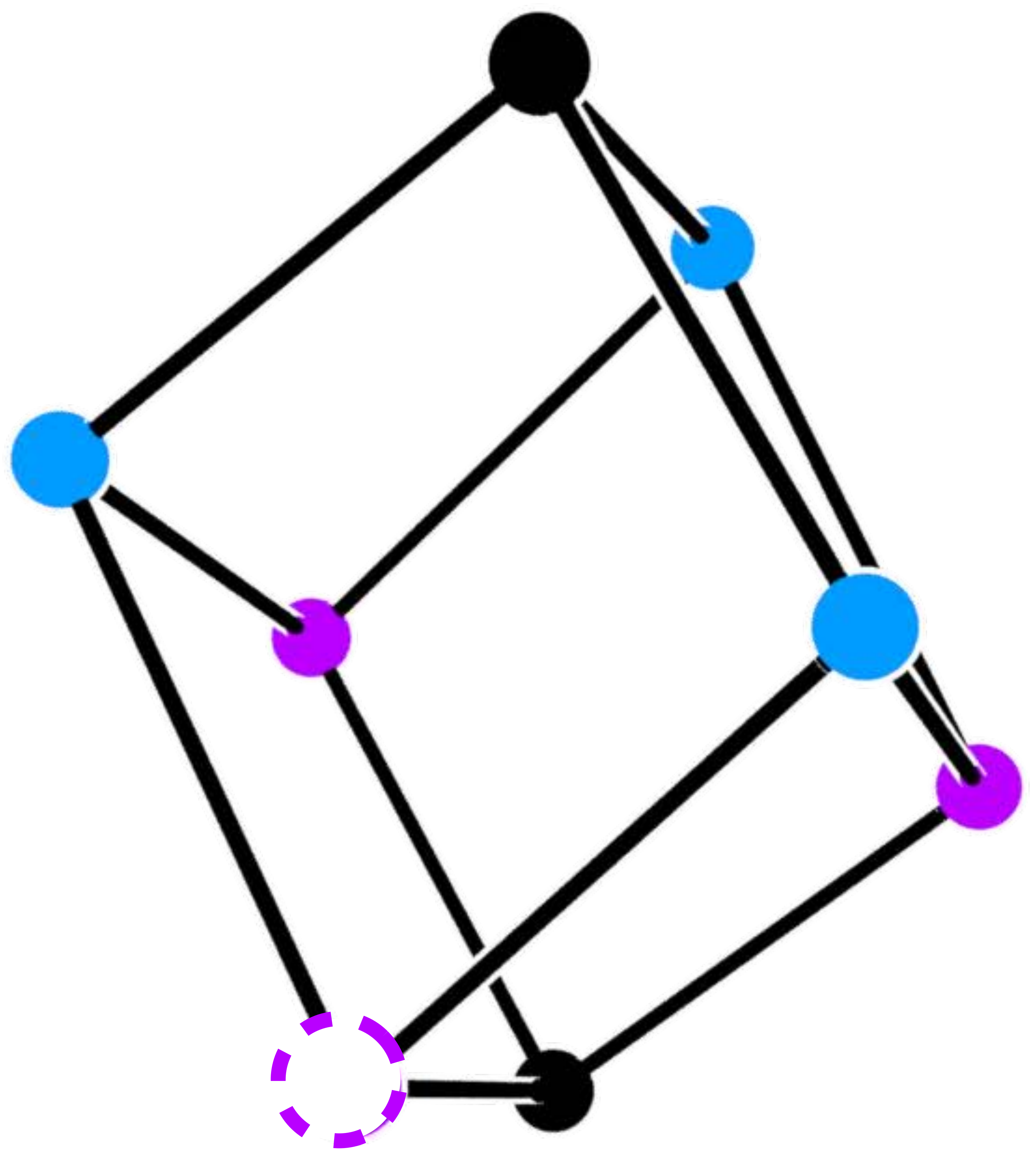
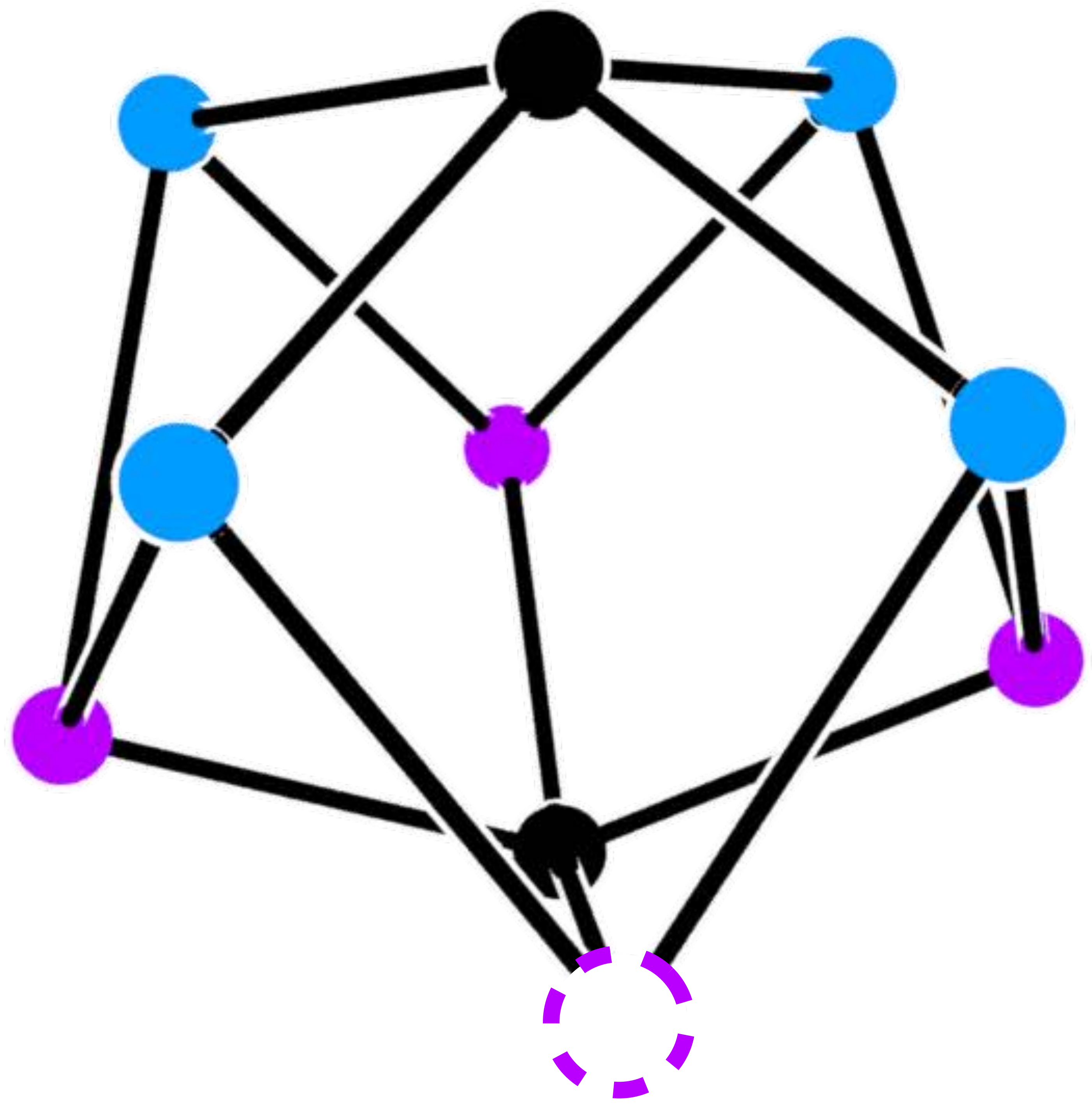


(2n + 2)-conic theorem



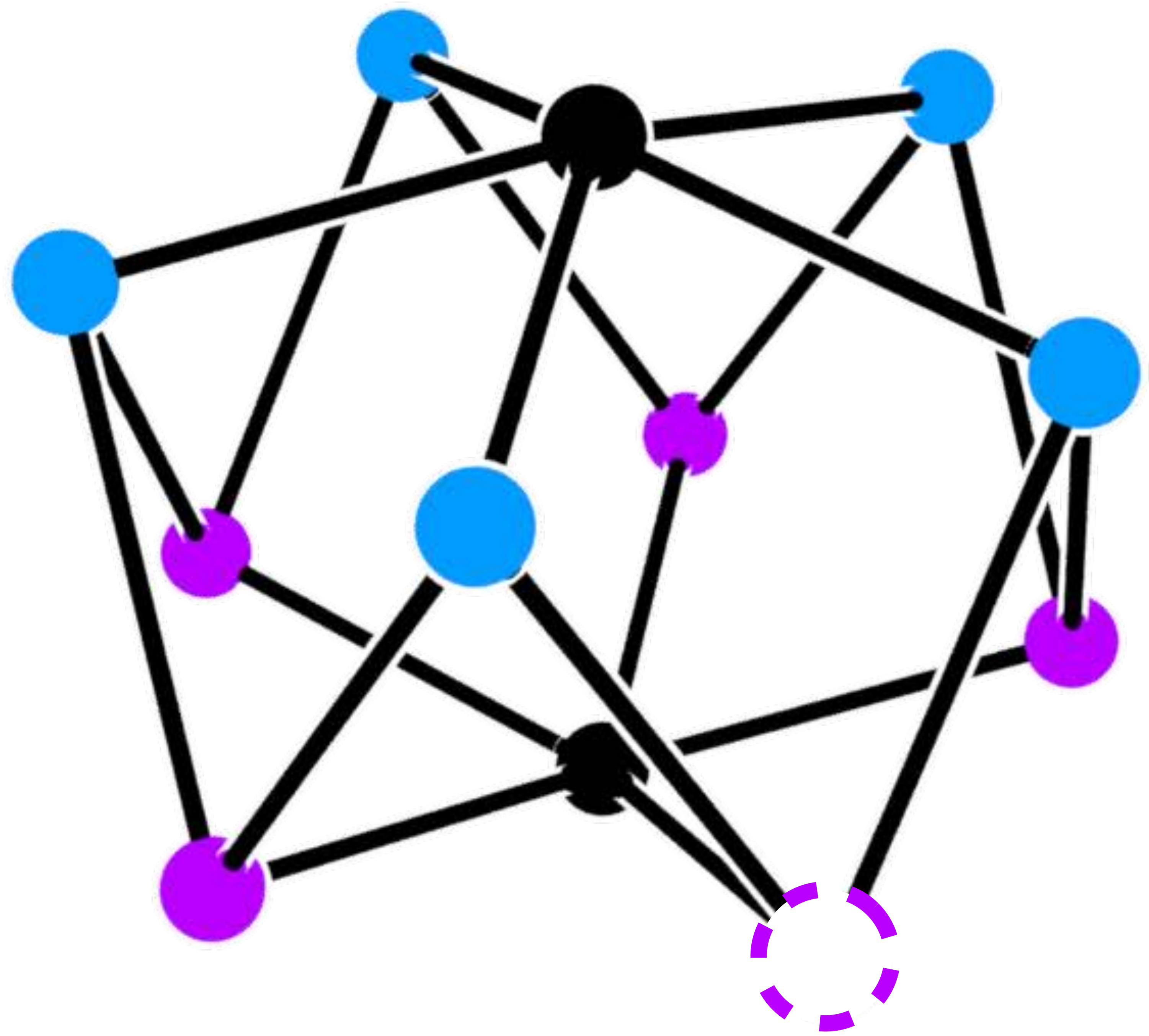
n = 3

(2n + 2)-conic theorem



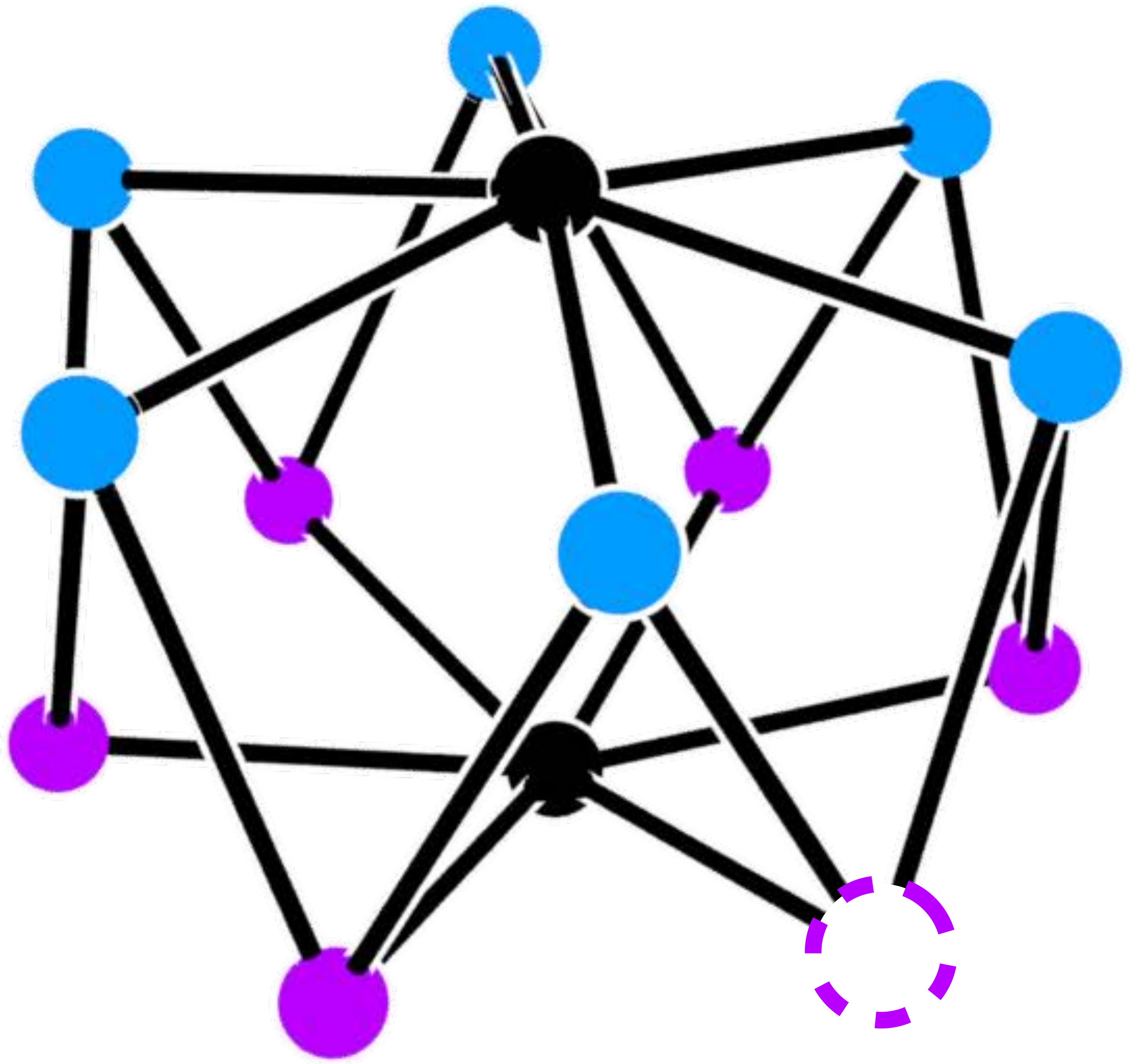
n = 4

(2n + 2)-conic theorem



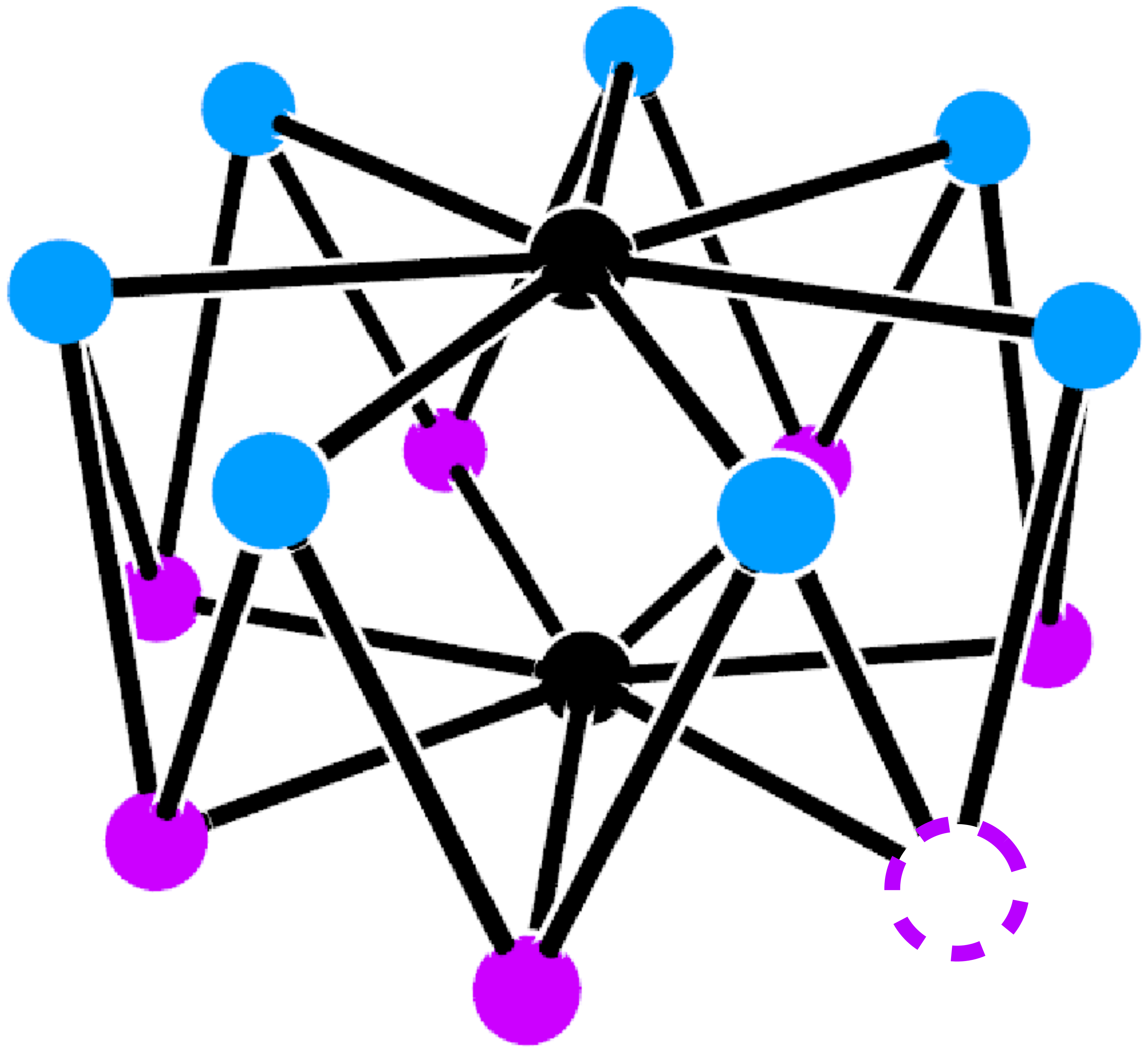
n = 5

(2n + 2)-conic theorem



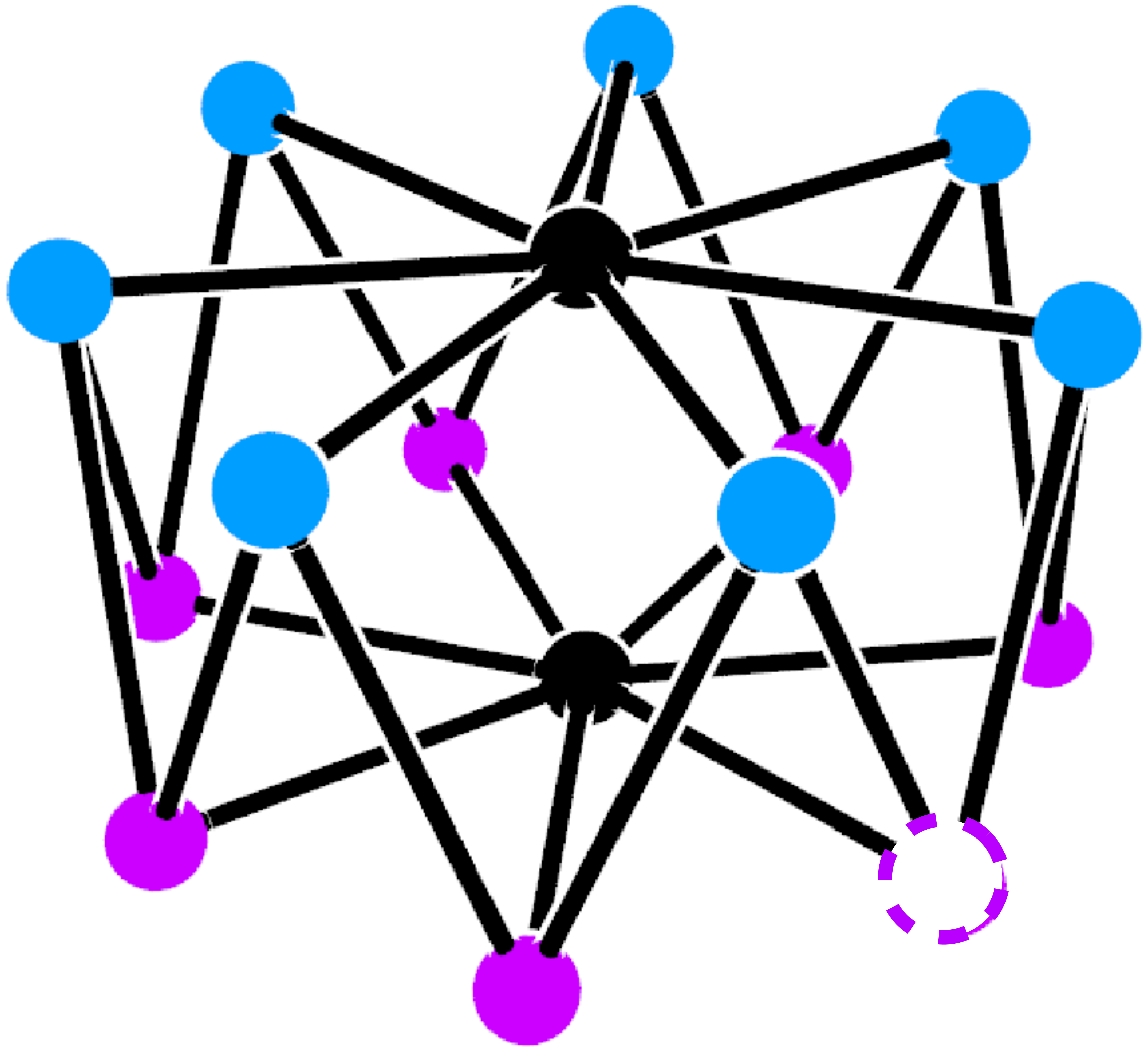
n = 6

(2n + 2)-conic theorem



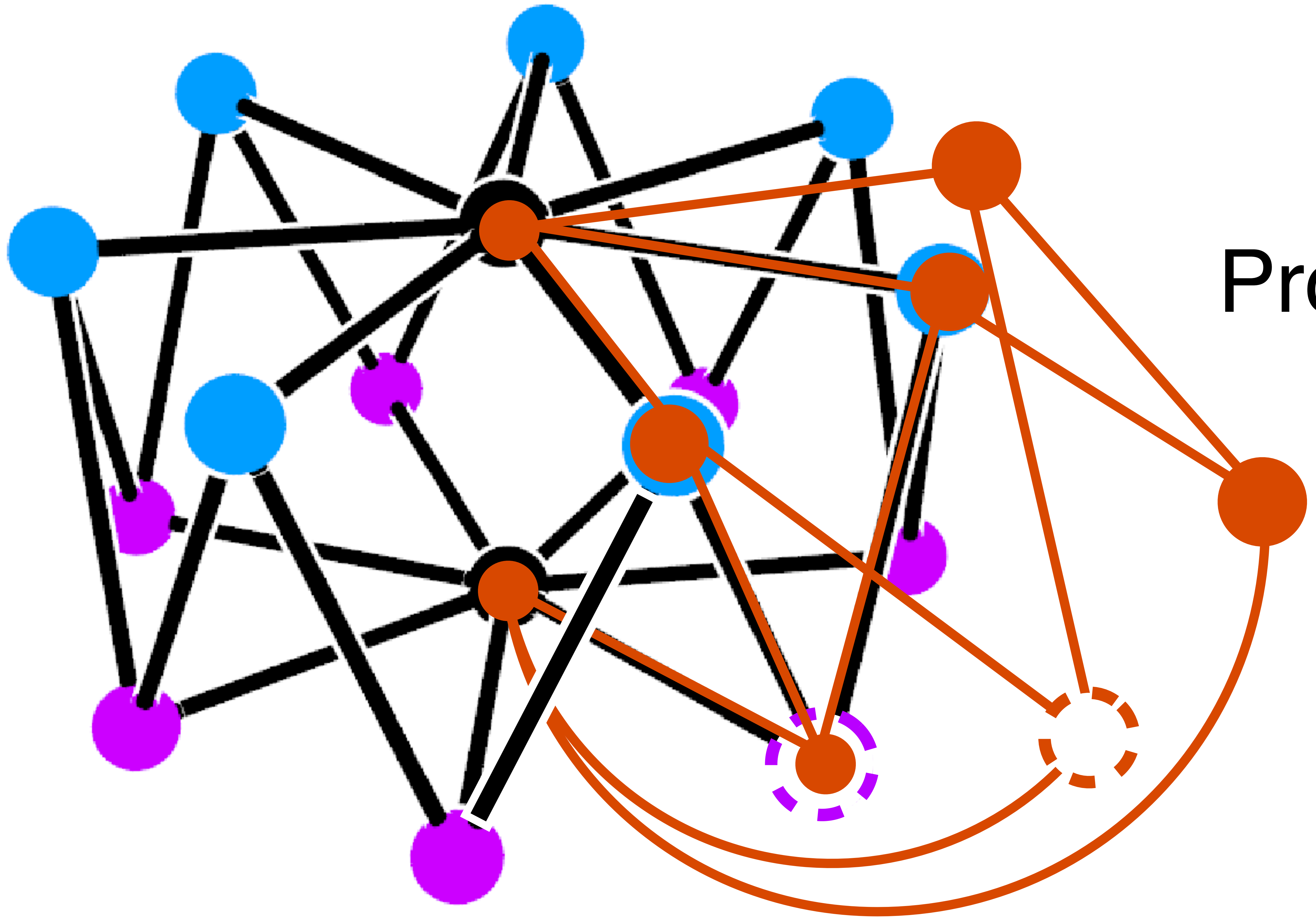
n = 7

(2n + 2)-conic theorem



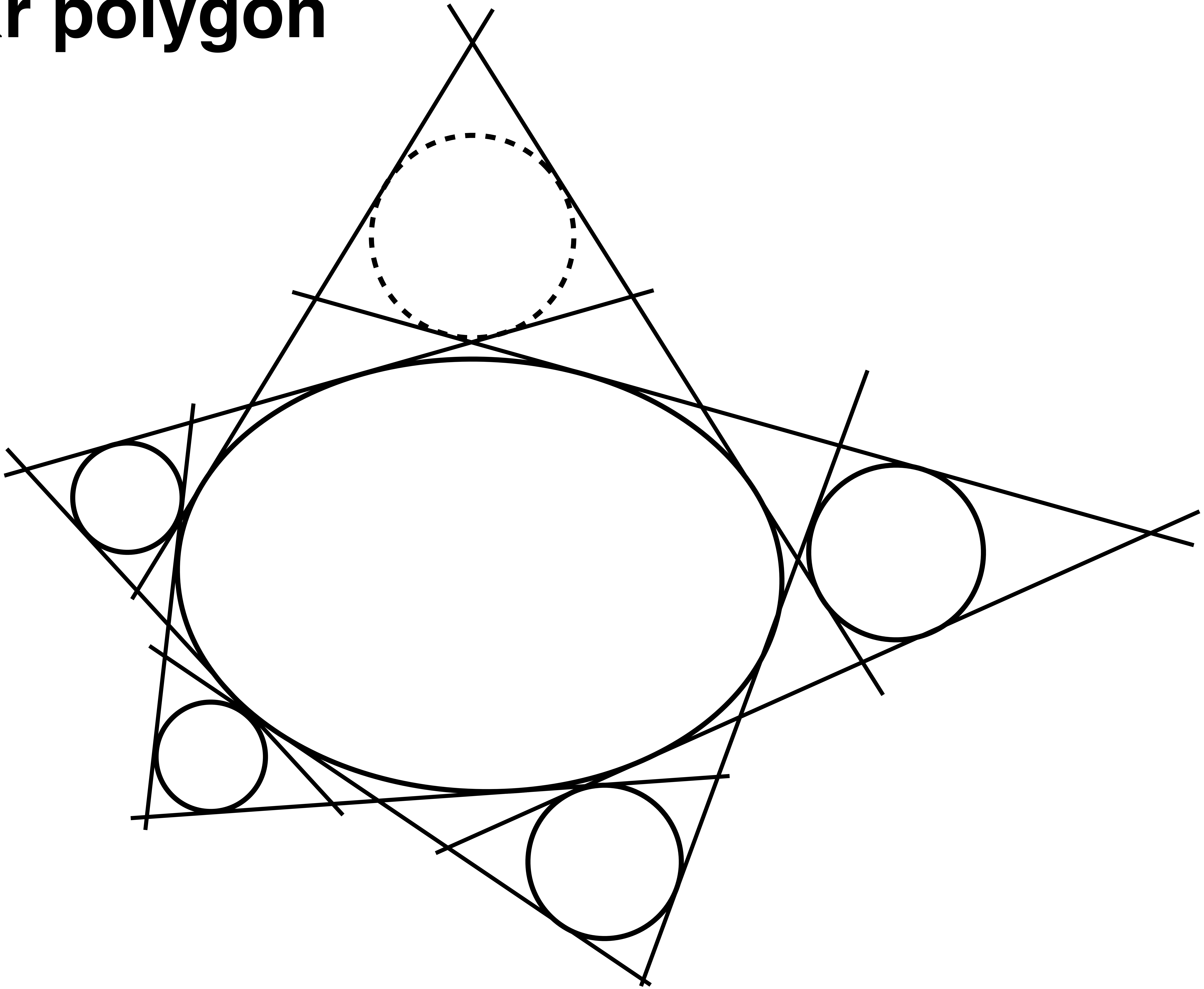
Proof by induction

(2n + 2)-conic theorem



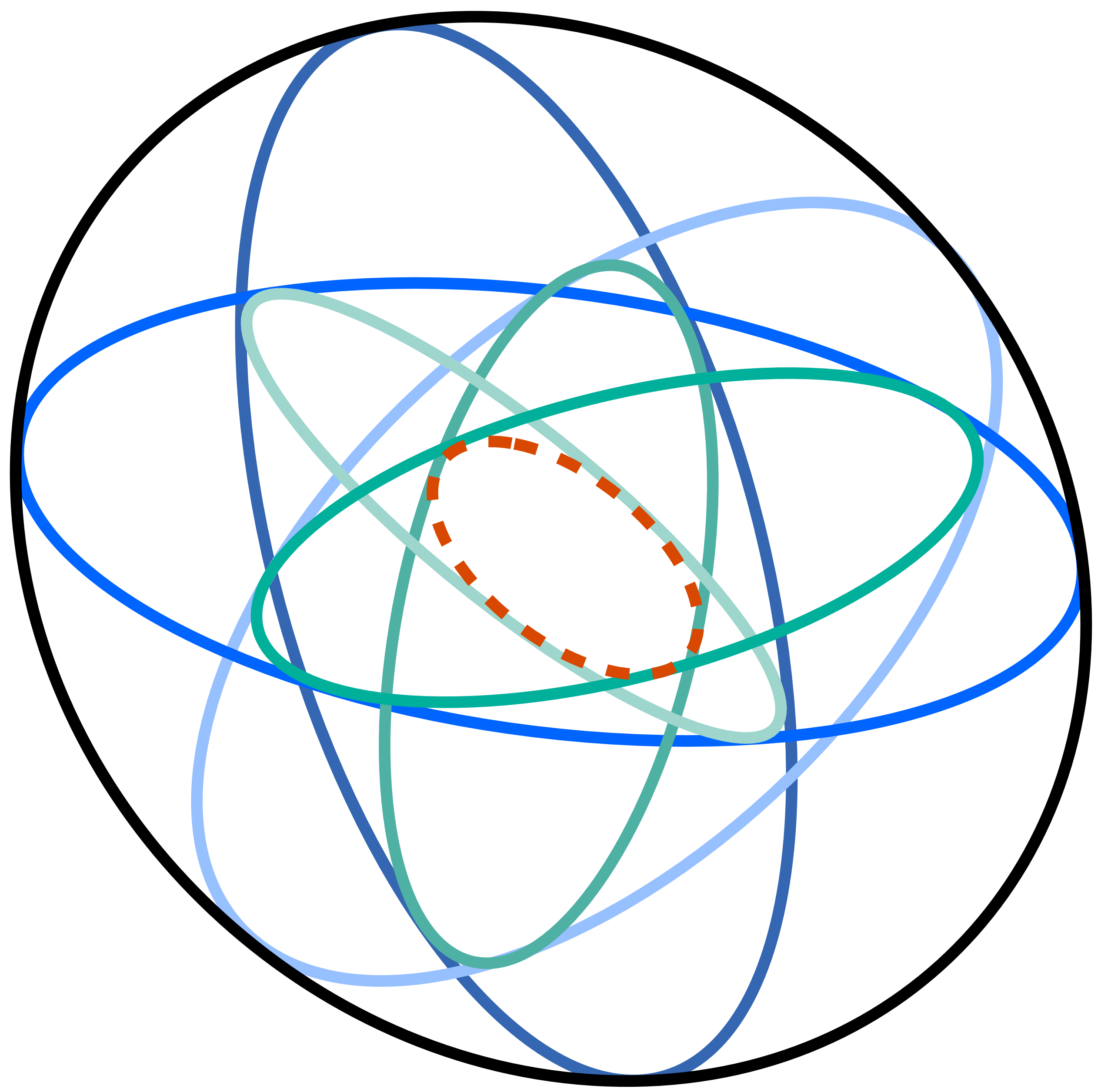
Proof by induction

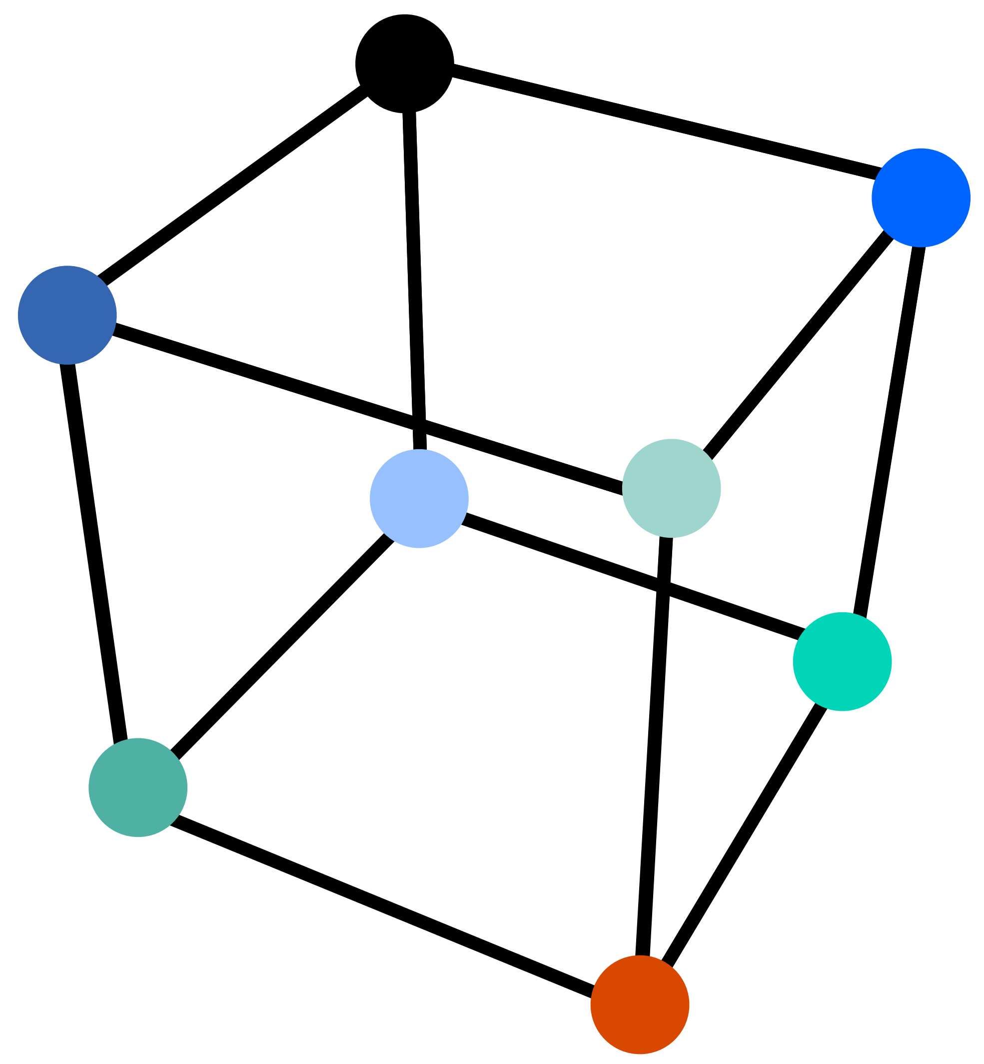
In-circular polygon



Overview

- Nice things about the eight-conic theorem
- Proof in the $\mathbb{P}^5$ space of conics
- Penrose's approach (undergrad)
- Penrose's 3D approach (Cambridge)





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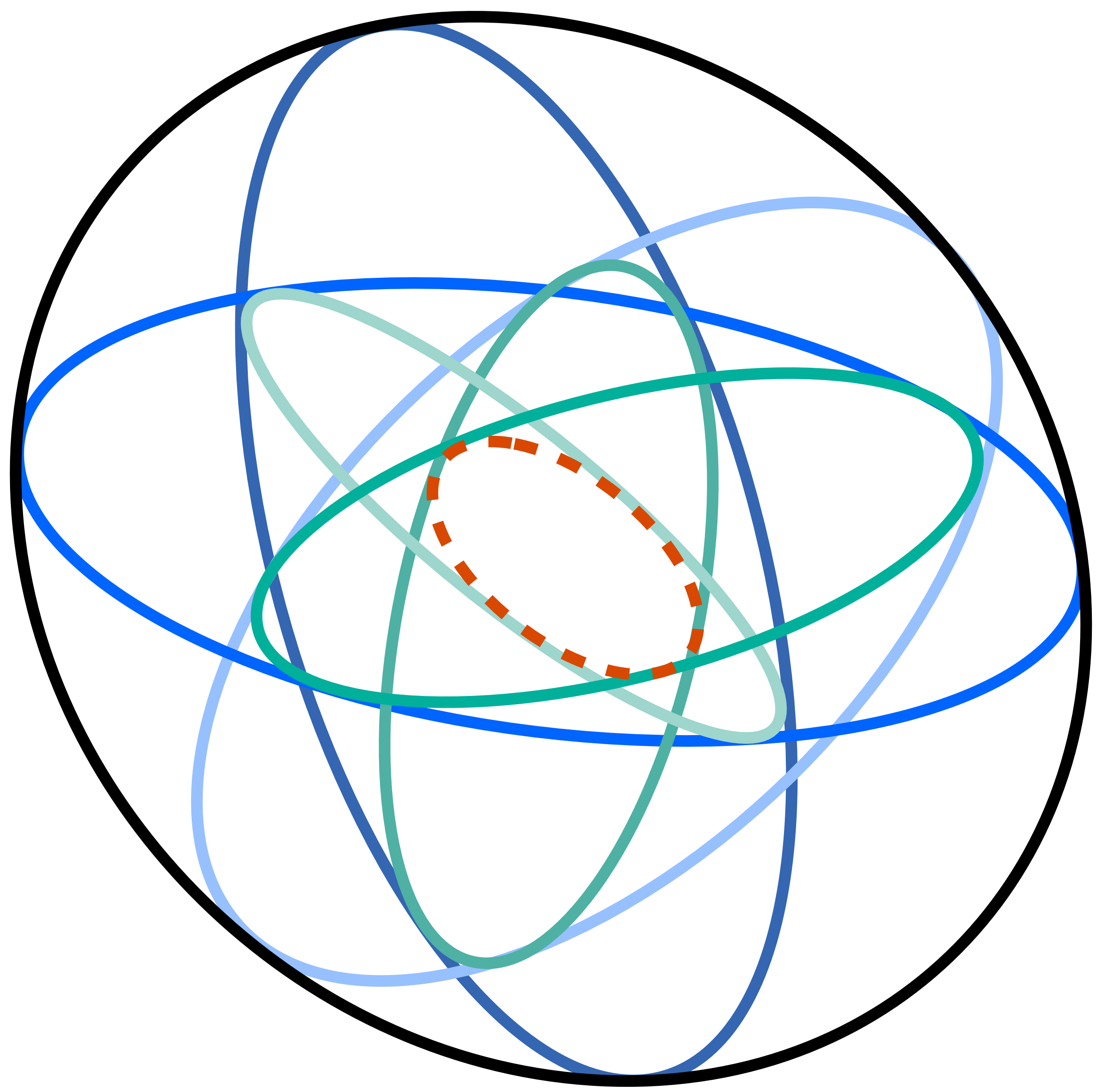
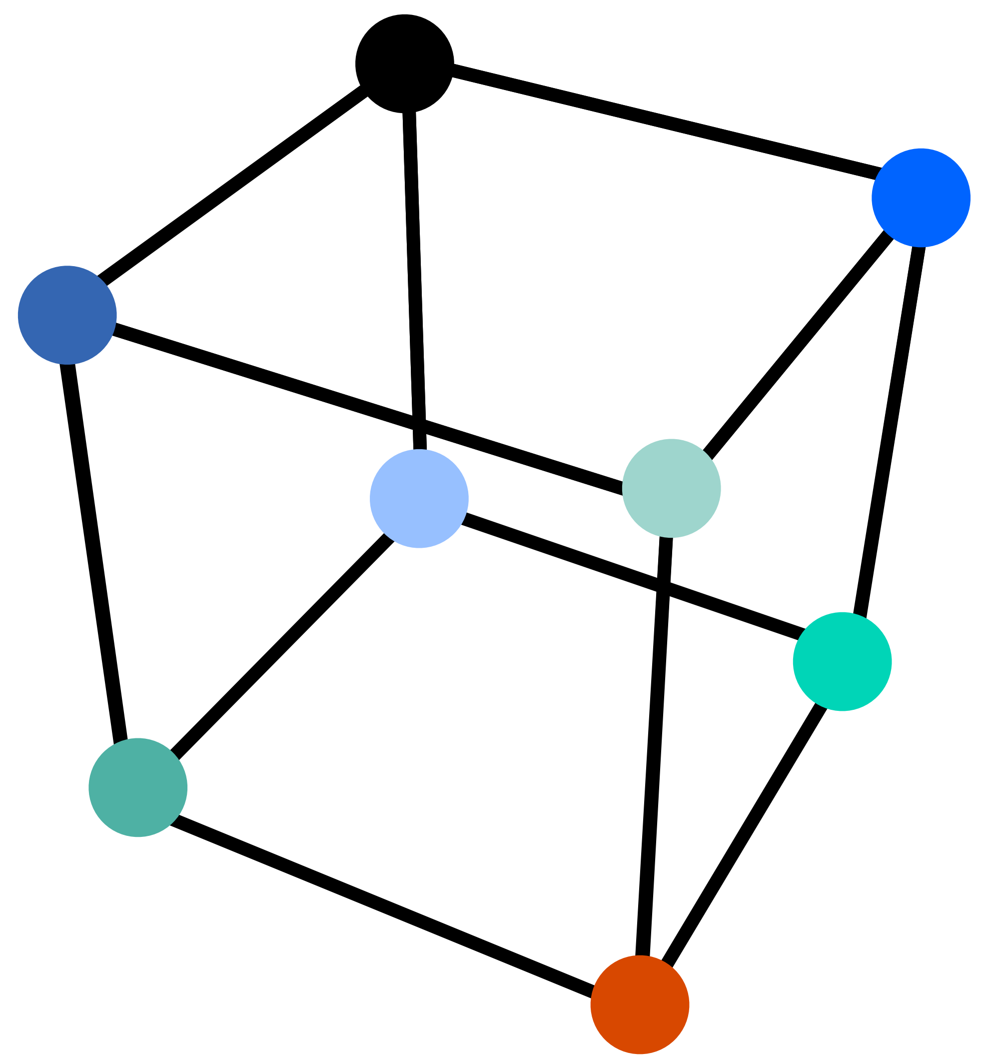
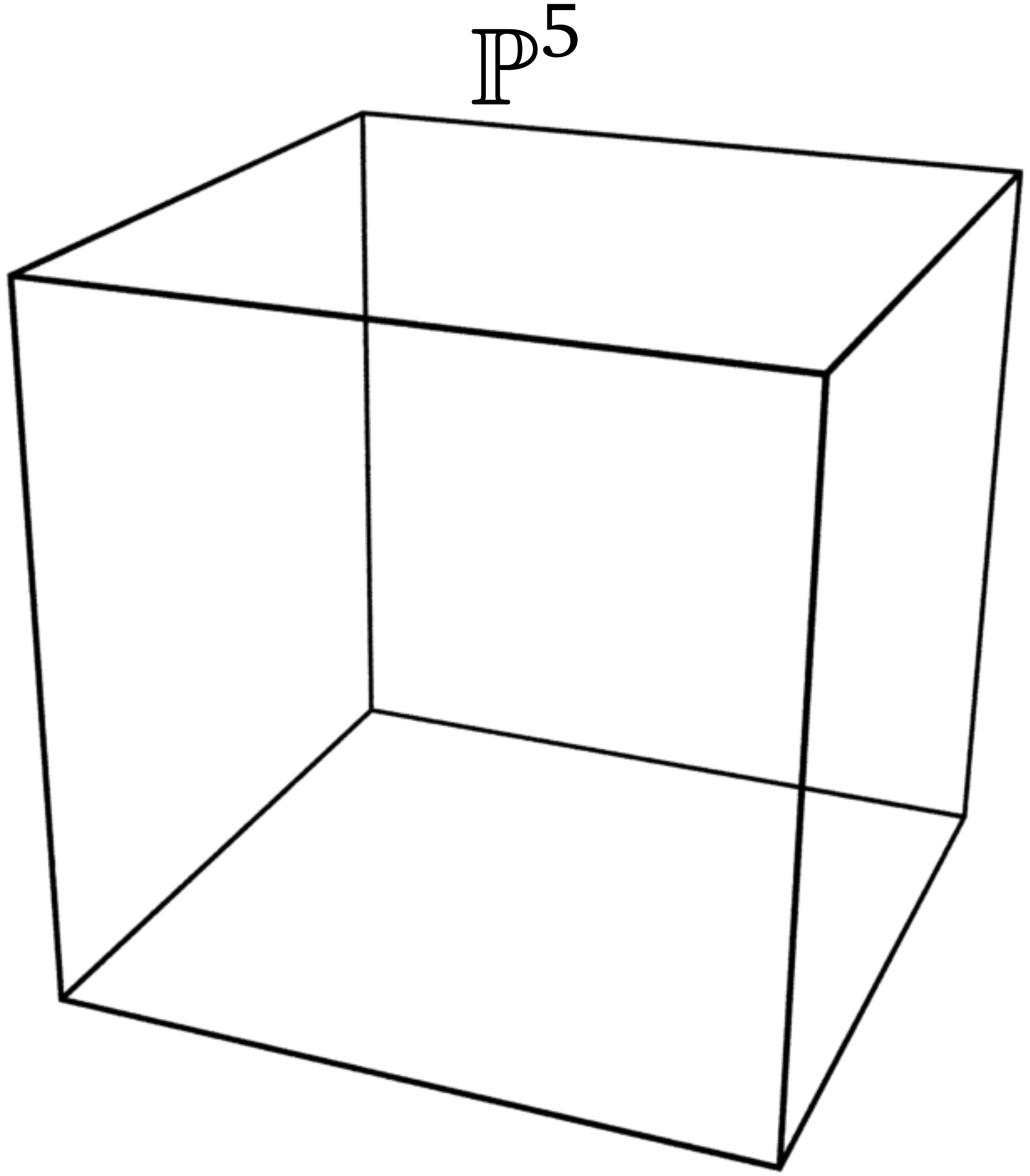




Space of conics

- The space of conics is $\mathbb{P}^5 = \{{}^{3\times 3 \text{ symm}}_{\text{matrices}}\}/\text{scaling}$



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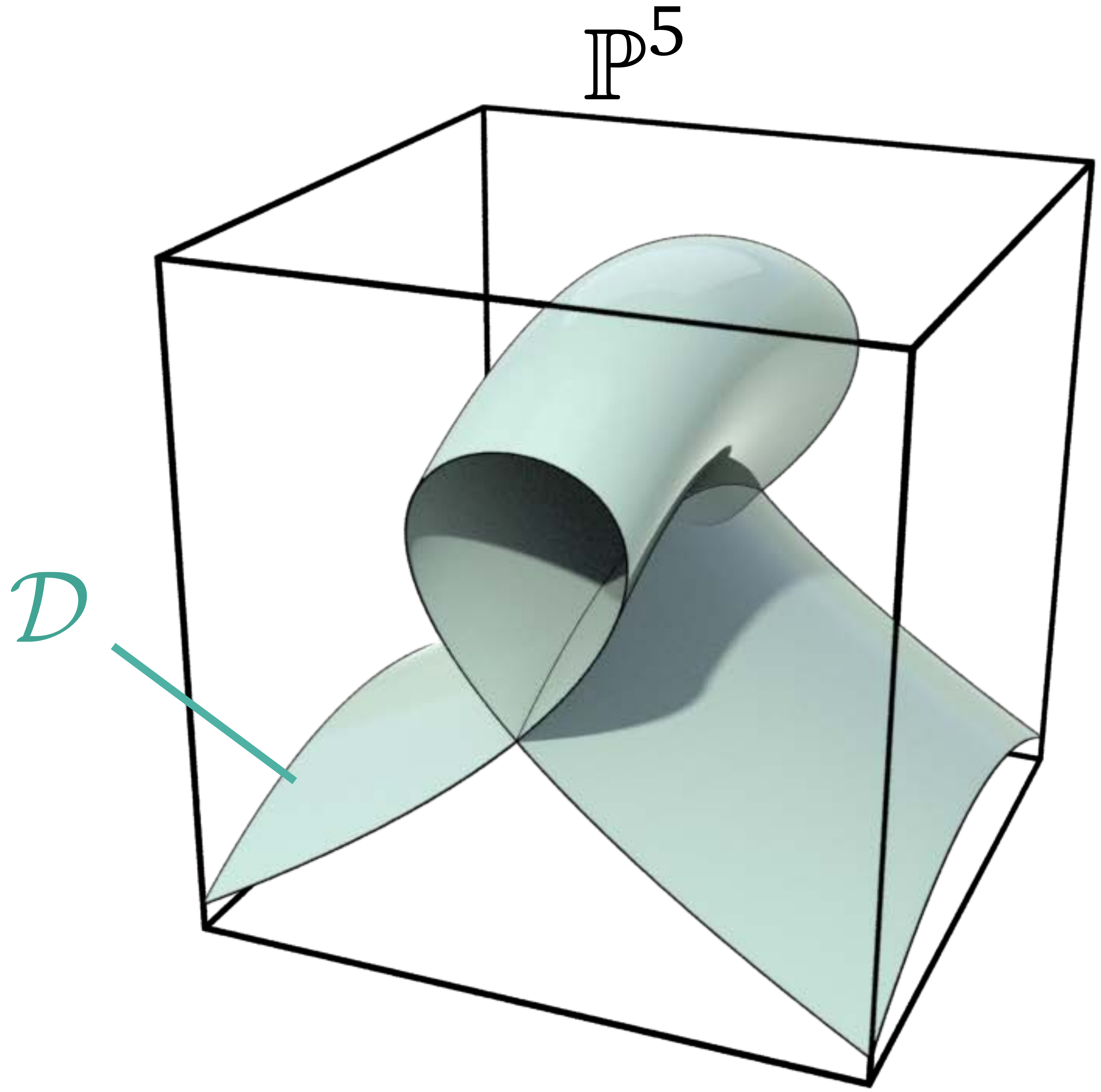
$$\mathcal{D} := \{{}^{\text{matrices with}}_{\text{rank} \leq 2}\}/\text{scaling} \subset \mathbb{P}^5$$



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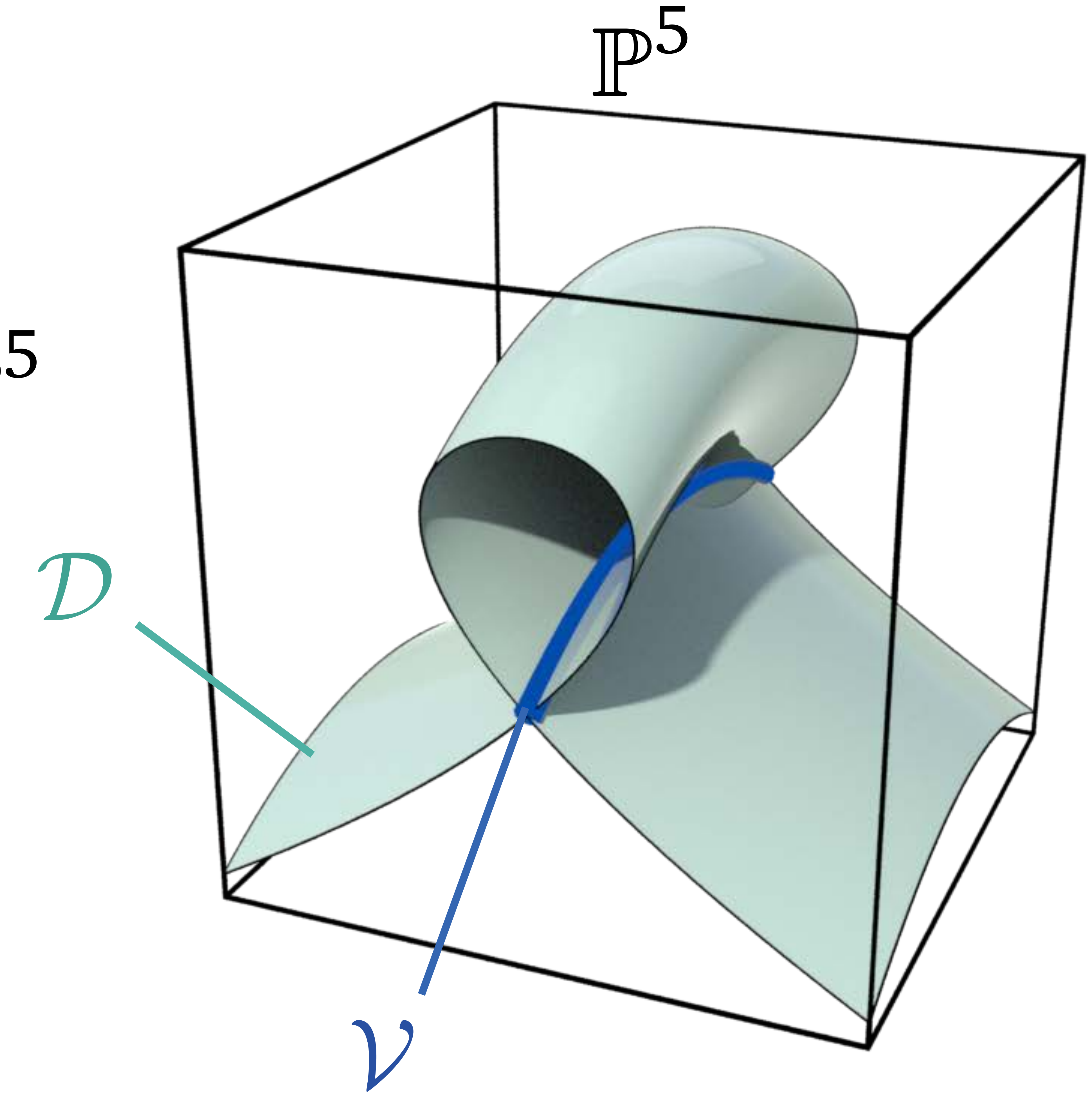
$$\mathcal{V} := \left\{ {\text{matrices with} \atop \text{rank} = 1} \right\} / \text{scaling} \subset \mathcal{D} \subset \mathbb{P}^5$$



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$$= \mathrm{P}\{\mathbf{A} \mid \det \mathbf{A} = 0\}$$

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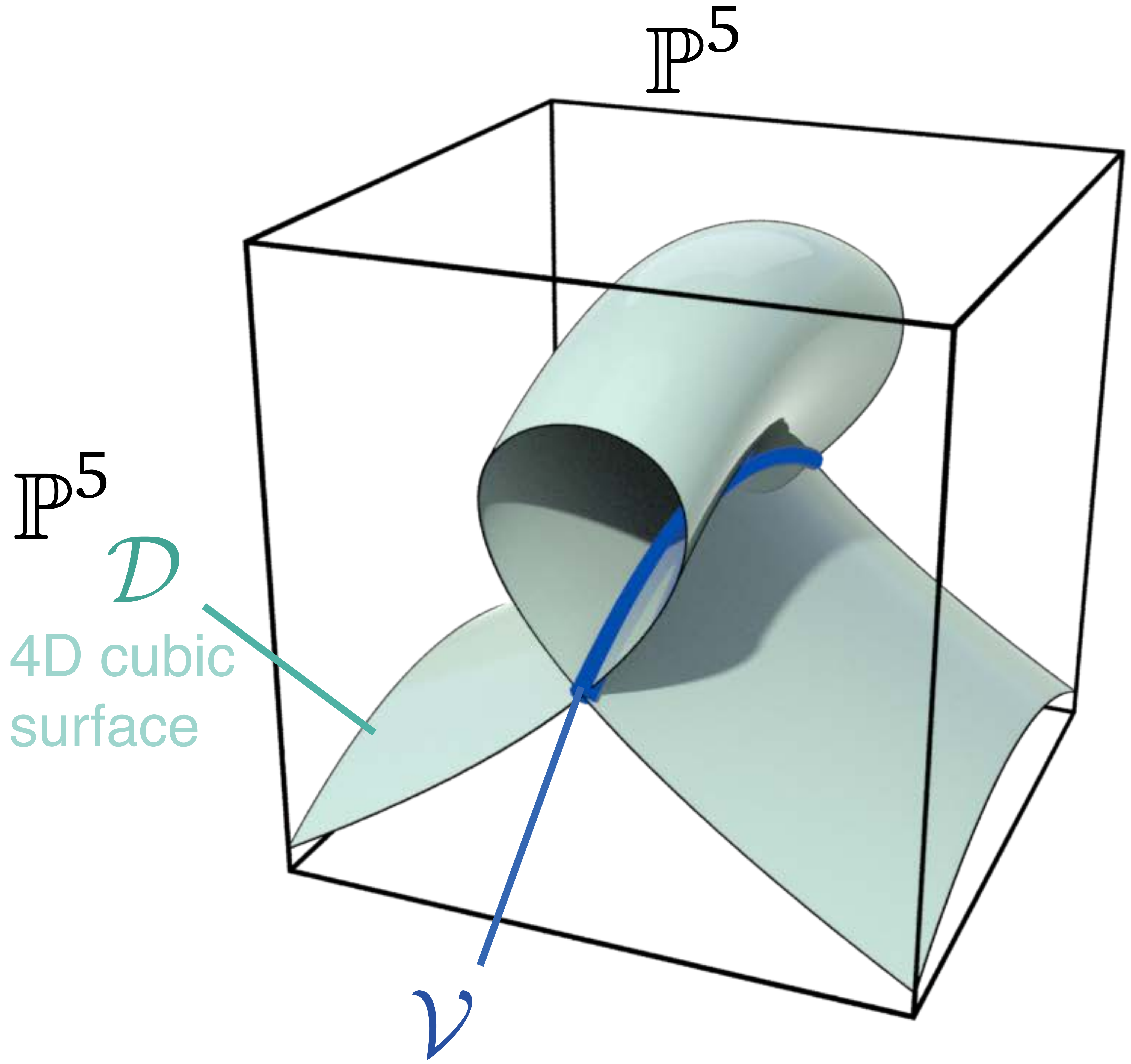

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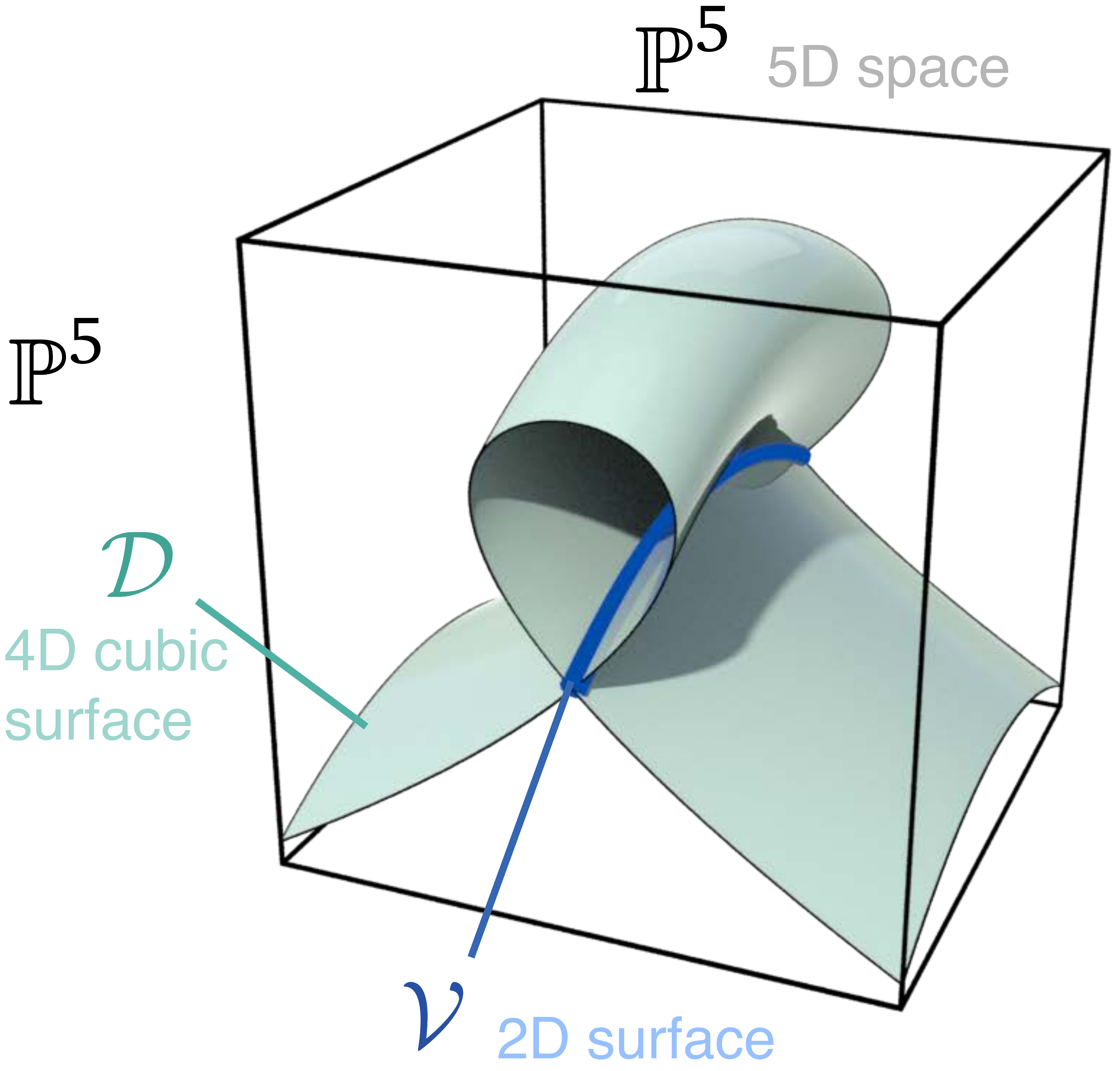
$$= \text{im}(\mathbf{x} \mapsto \mathbf{x}\mathbf{x}^\intercal)$$

Veronese map



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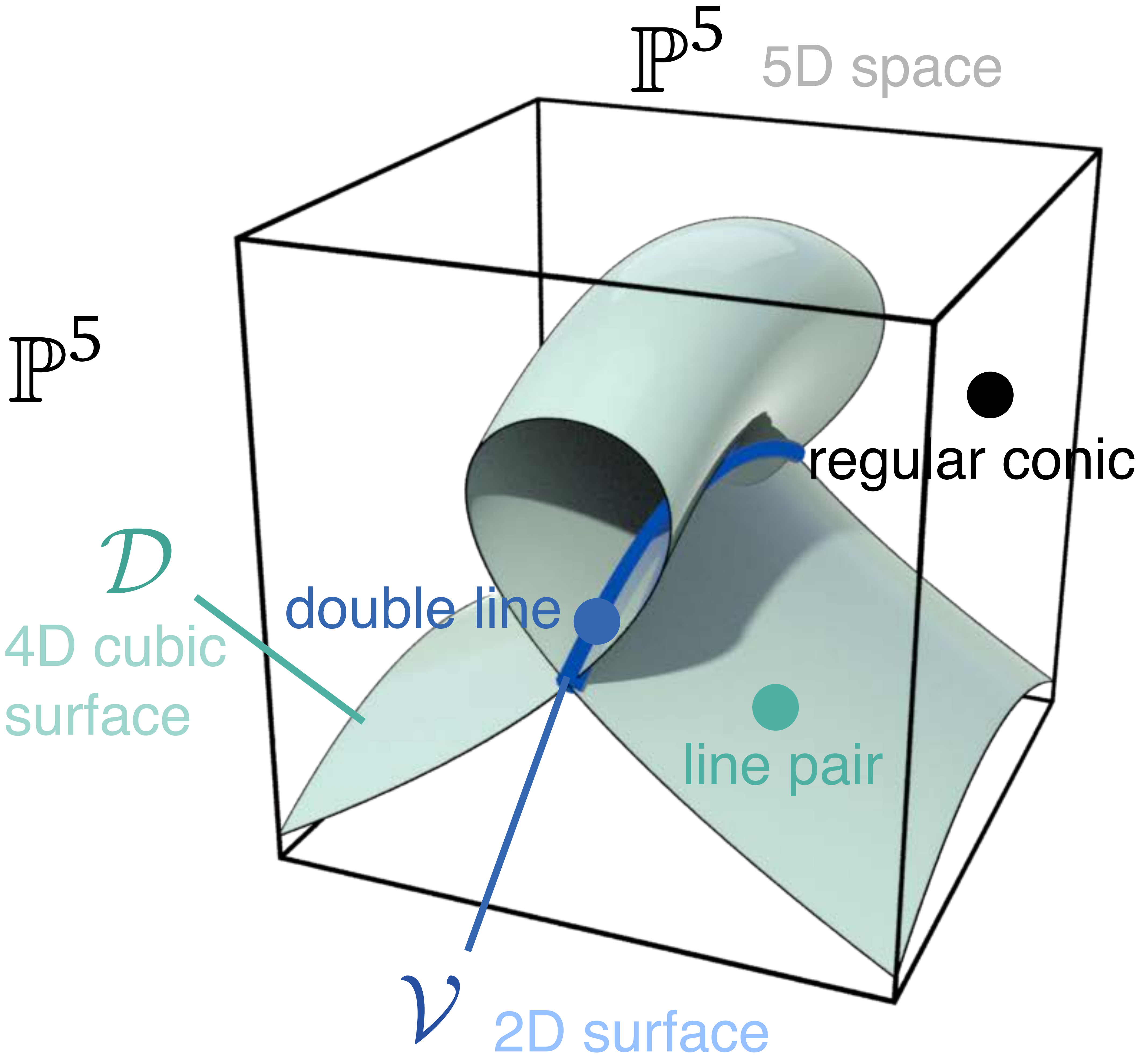
- $\mathbb{P}^5 \setminus \mathcal{D} = \left\{\begin{smallmatrix}\text{regular}\\ \text{conics}\end{smallmatrix}\right\}$

$$\mathcal{D} \setminus \mathcal{V} = \left\{\begin{smallmatrix}\text{line}\\ \text{pairs}\end{smallmatrix}\right\}$$

$$\mathcal{V} = \left\{\begin{smallmatrix}\text{double}\\ \text{lines}\end{smallmatrix}\right\}$$



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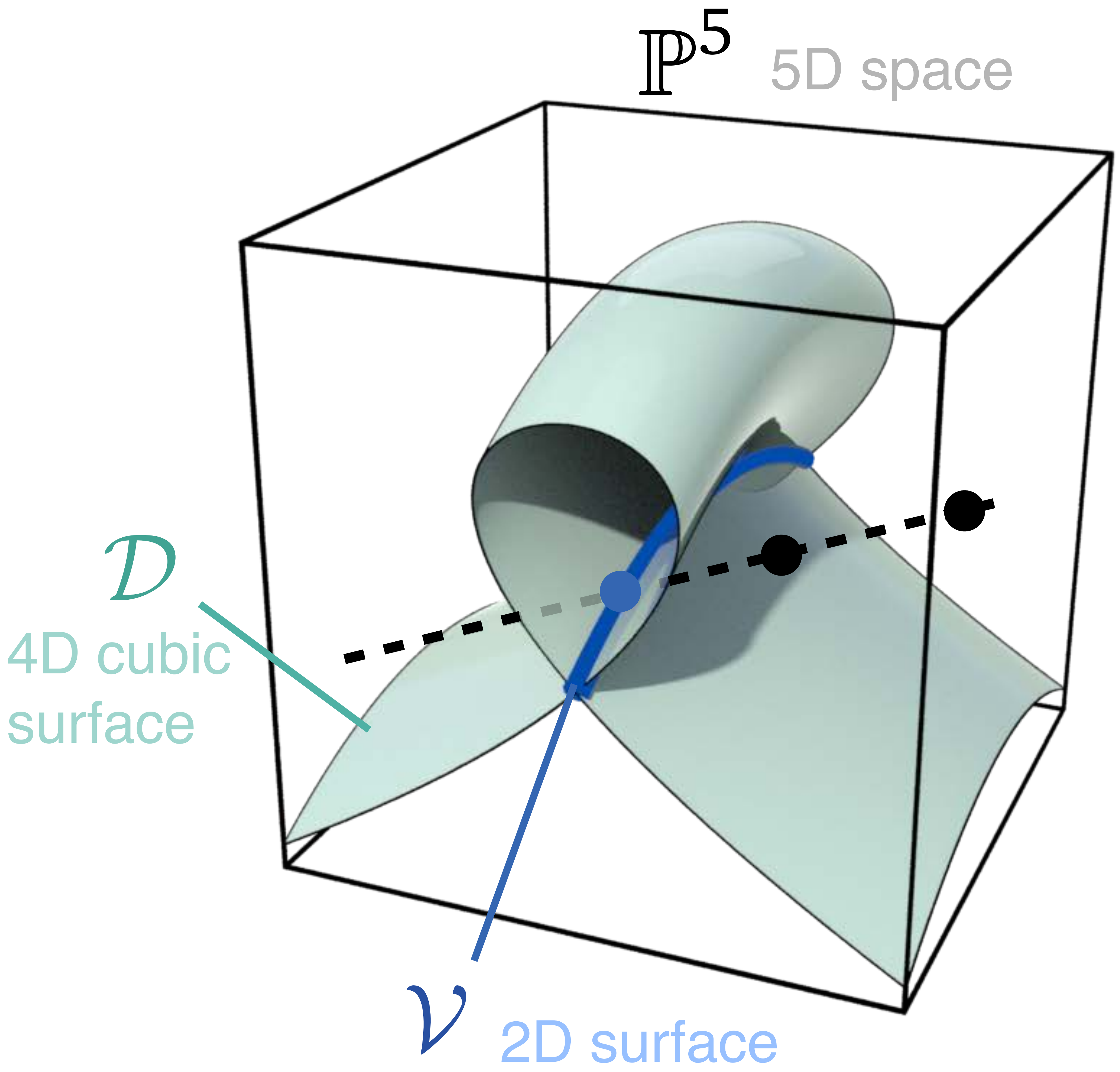
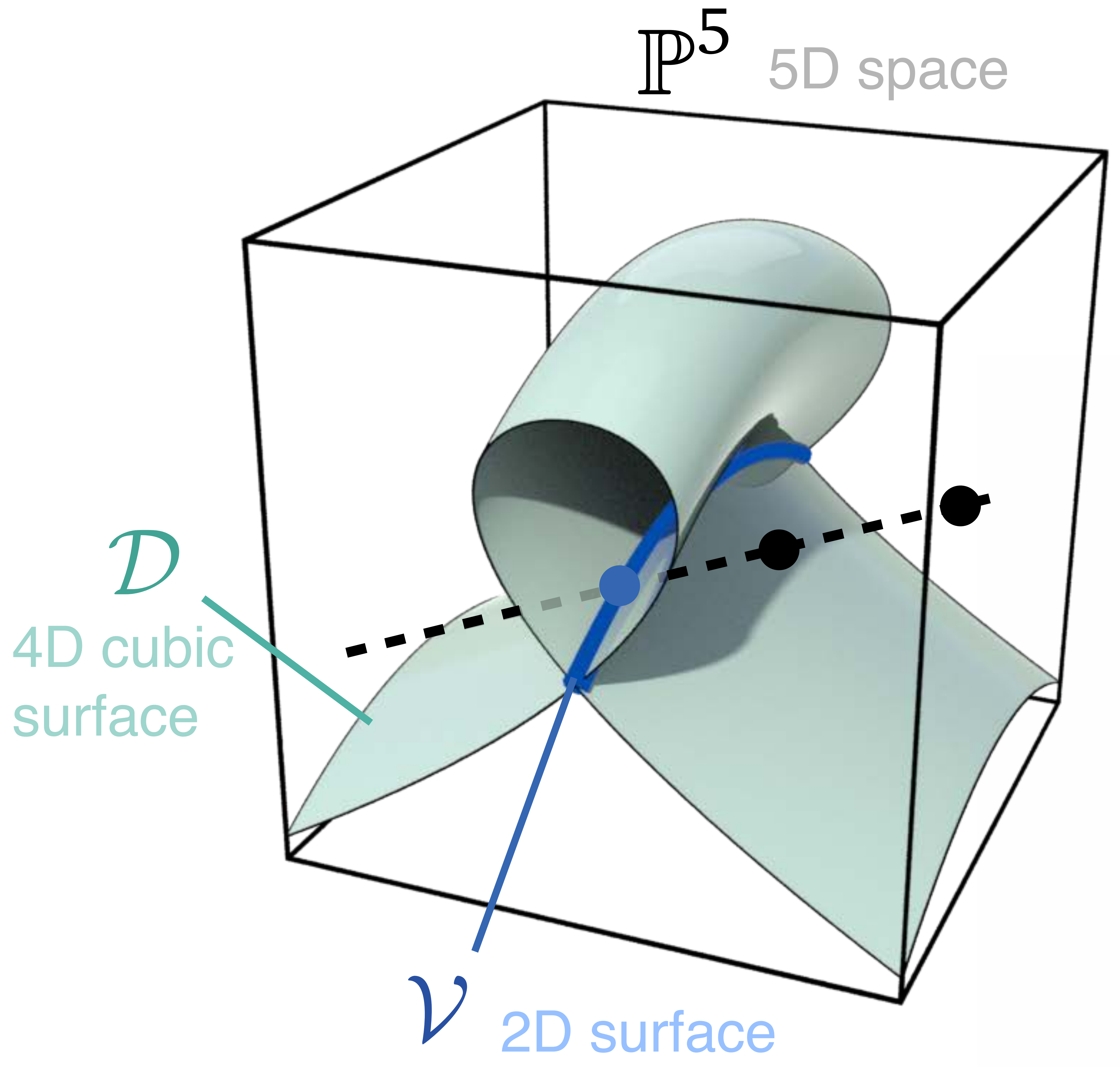


- *Two conics are in **double contact** if and only if their joining line meet $\mathcal{V}$*

Space of conics



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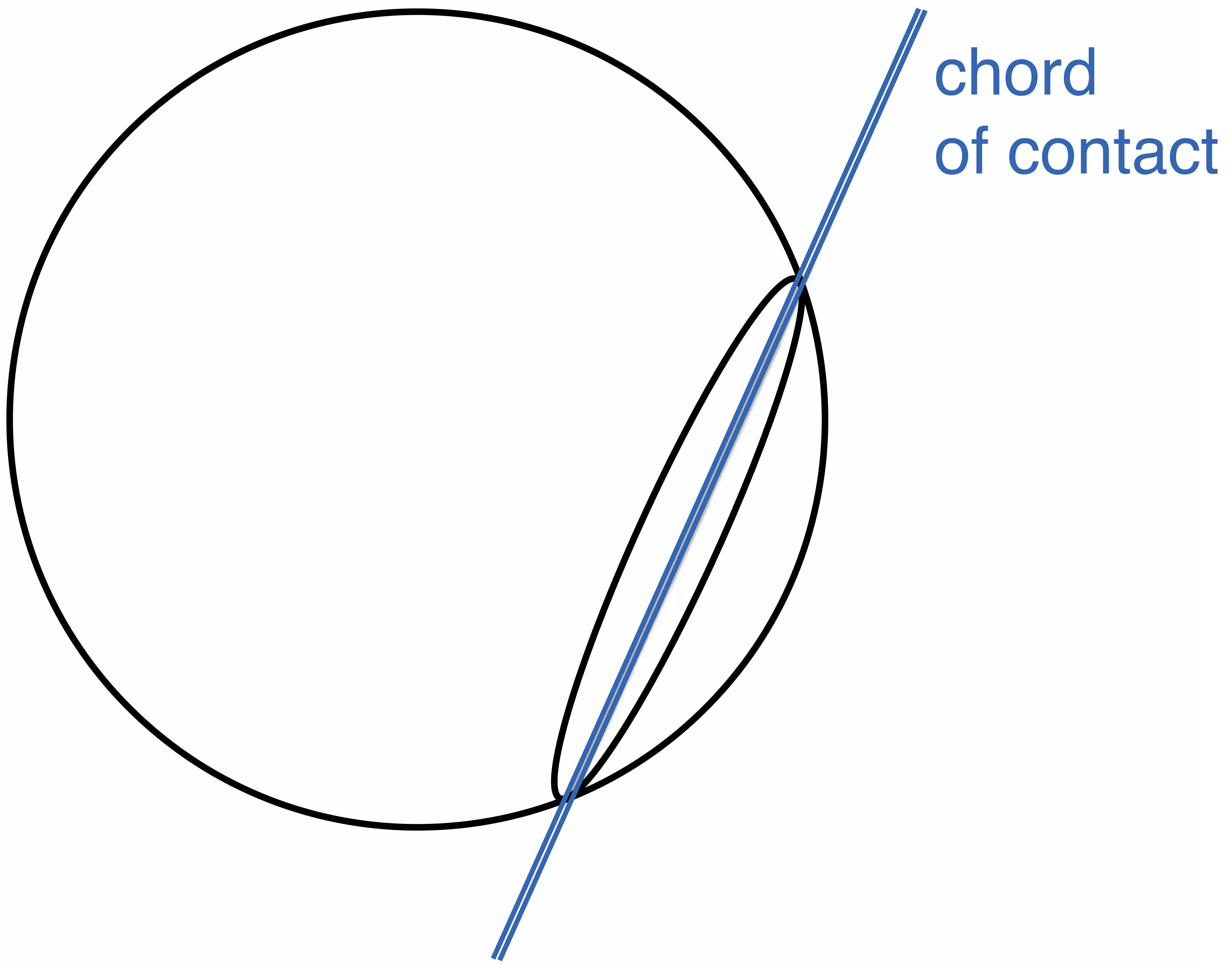
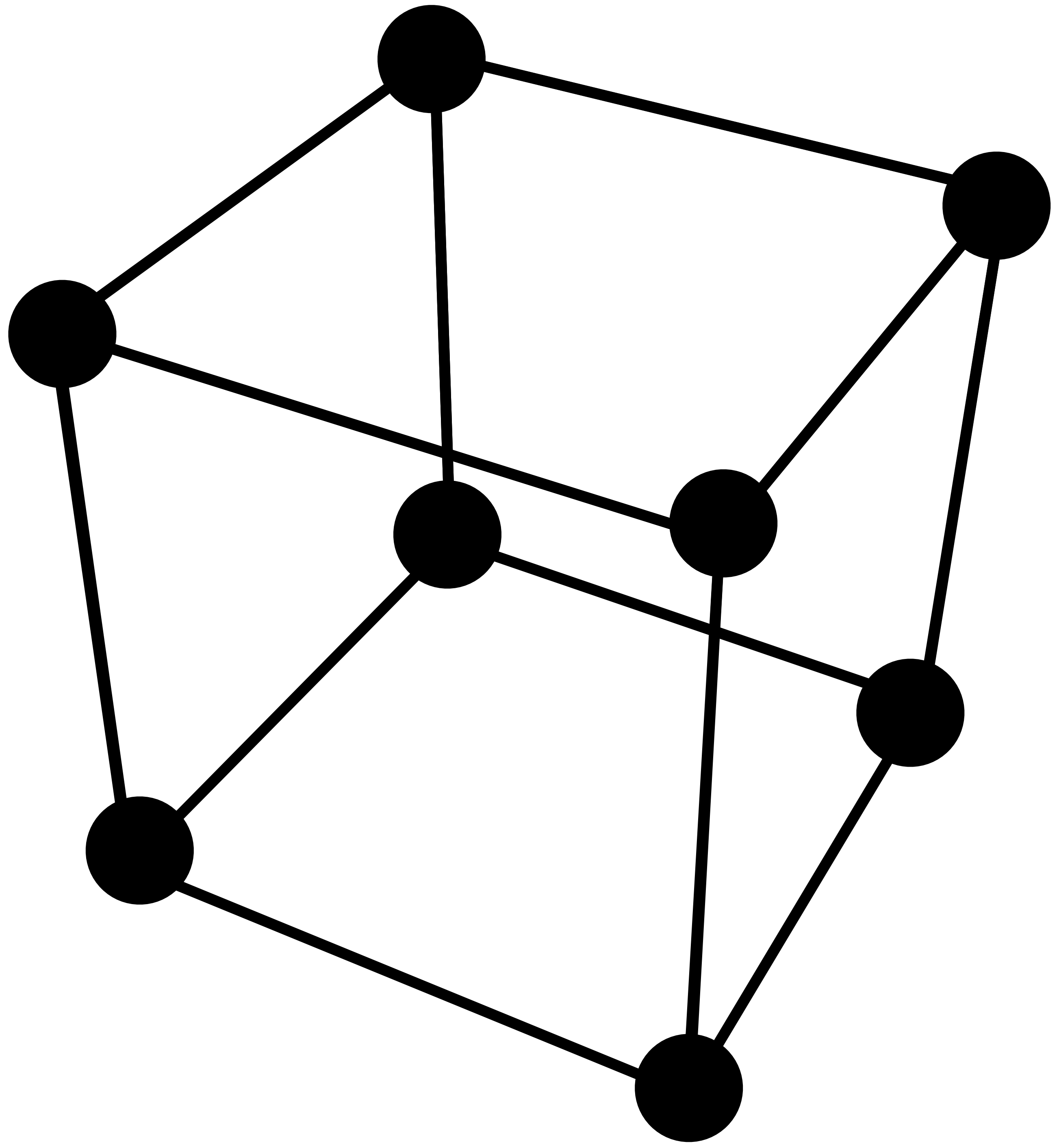


Eight-conic configuration

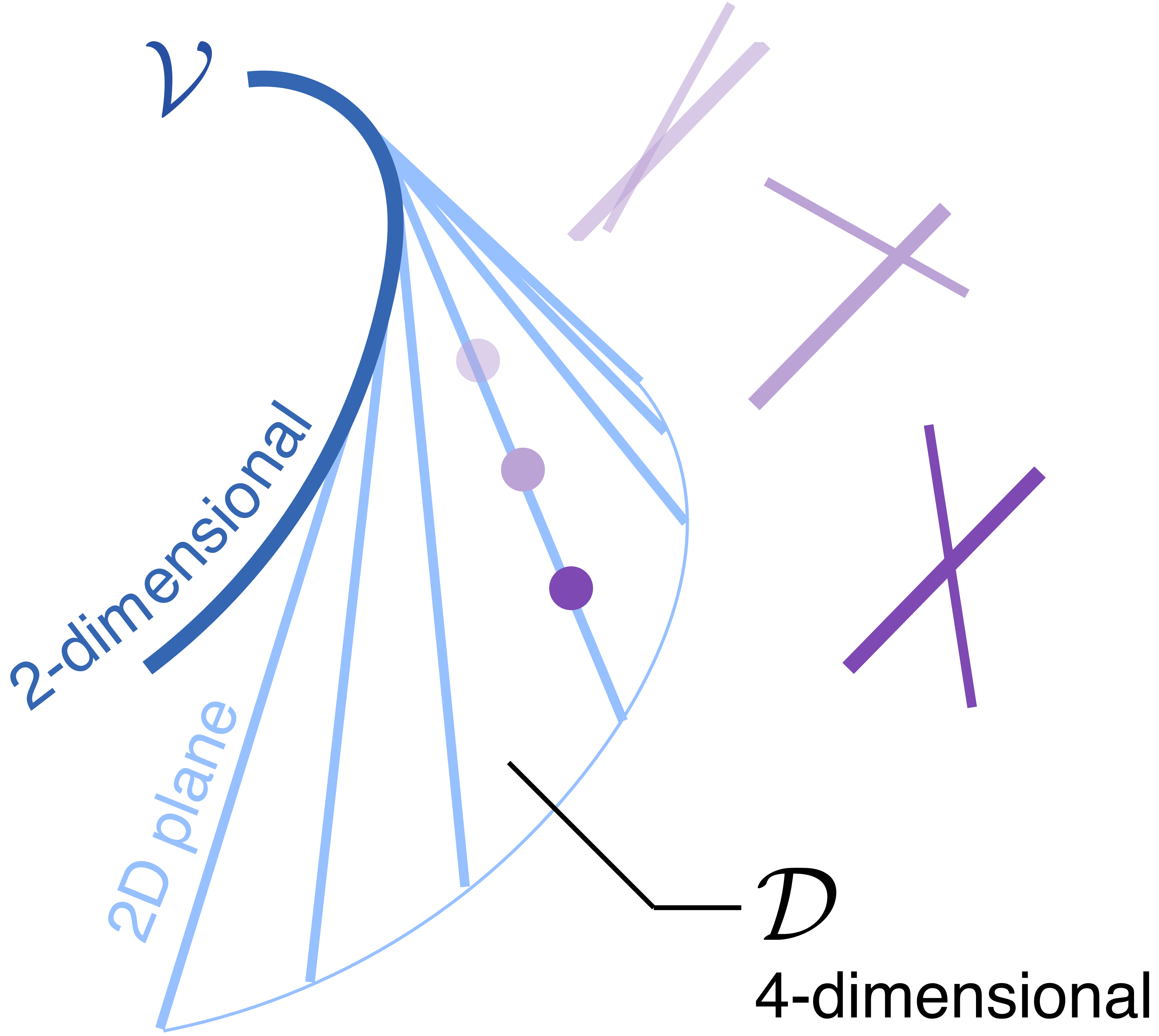


- It is a cube graph in $\mathbb{P}^5$
- Each edge line meets $\mathcal{V}$
- Penrose's theorem boils down to geometry of $\mathcal{V} \subset \mathcal{D} \subset \mathbb{P}^5$

Geometry of $\mathcal{D}$ and $\mathcal{V}$

- The cubic 4D surface $\mathcal{D} \subset \mathbb{P}^5$ contains many 2D planes.
  - $\mathcal{D} = \bigcup_{\mathbf{v}\in\mathcal{V}} \left(\text{tangent plane of } \mathcal{V} \text{ at } \mathbf{v}\right)$

“line planes”



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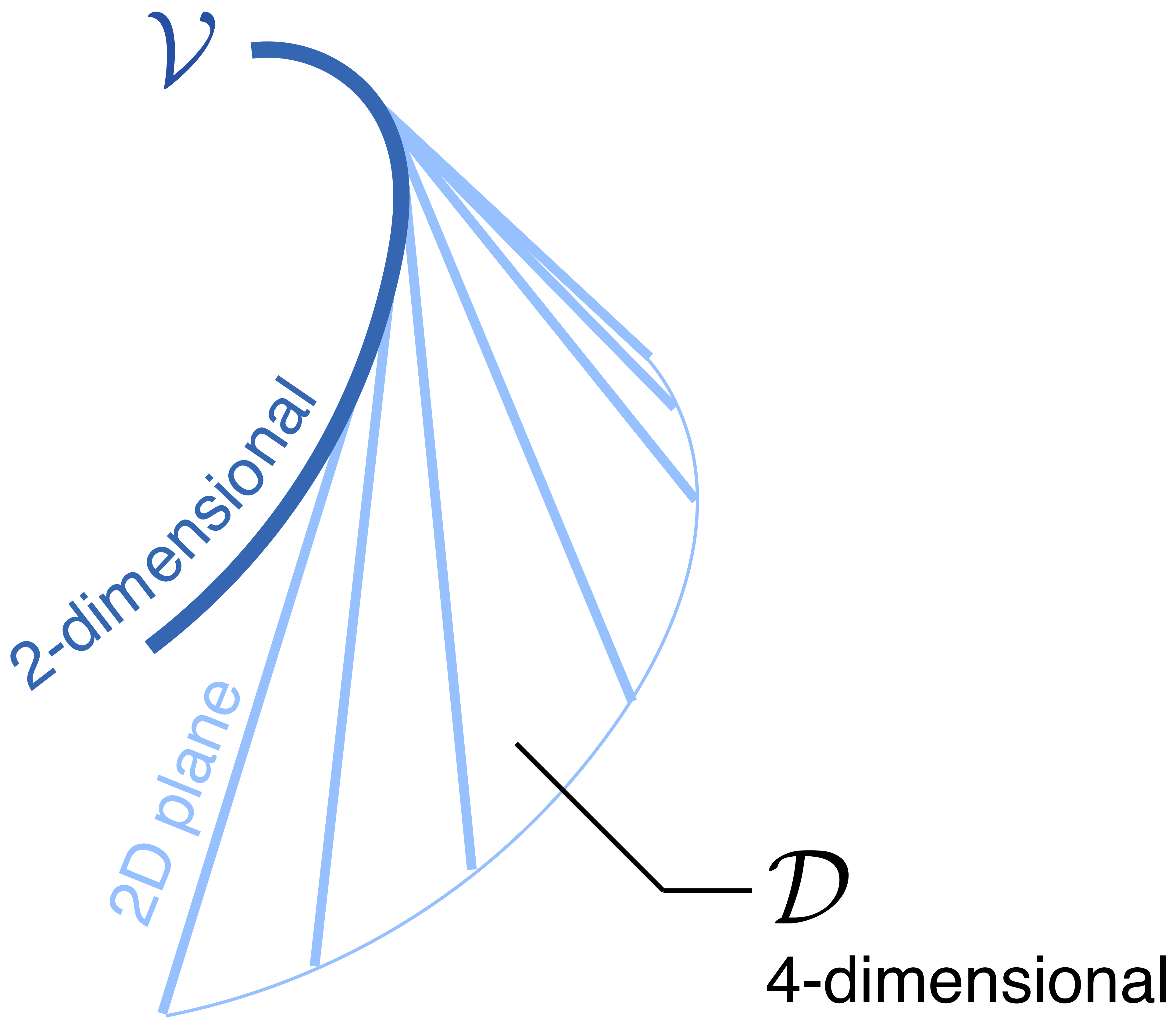
Each point plane consists of line pairs intersecting at a common point p



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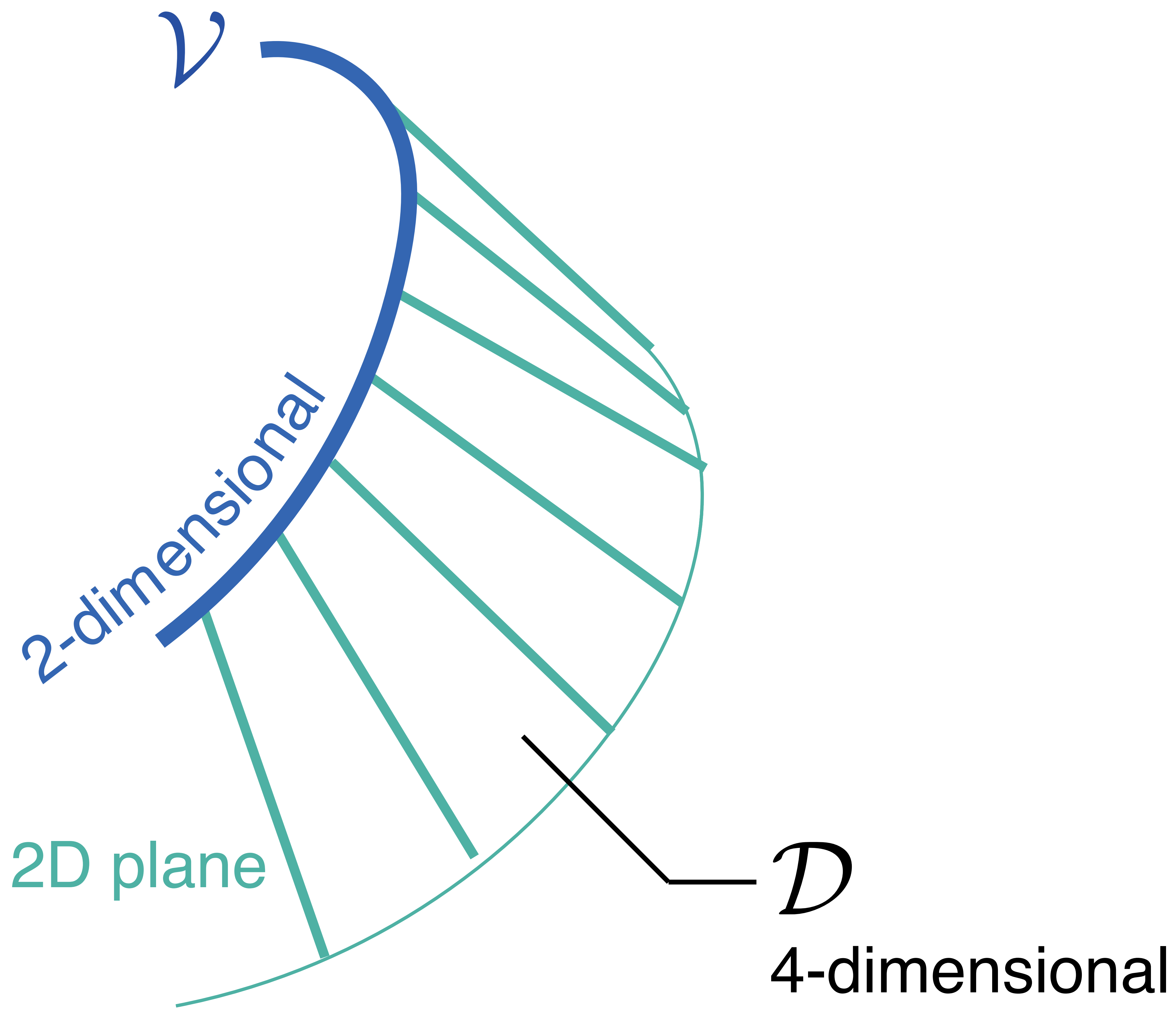

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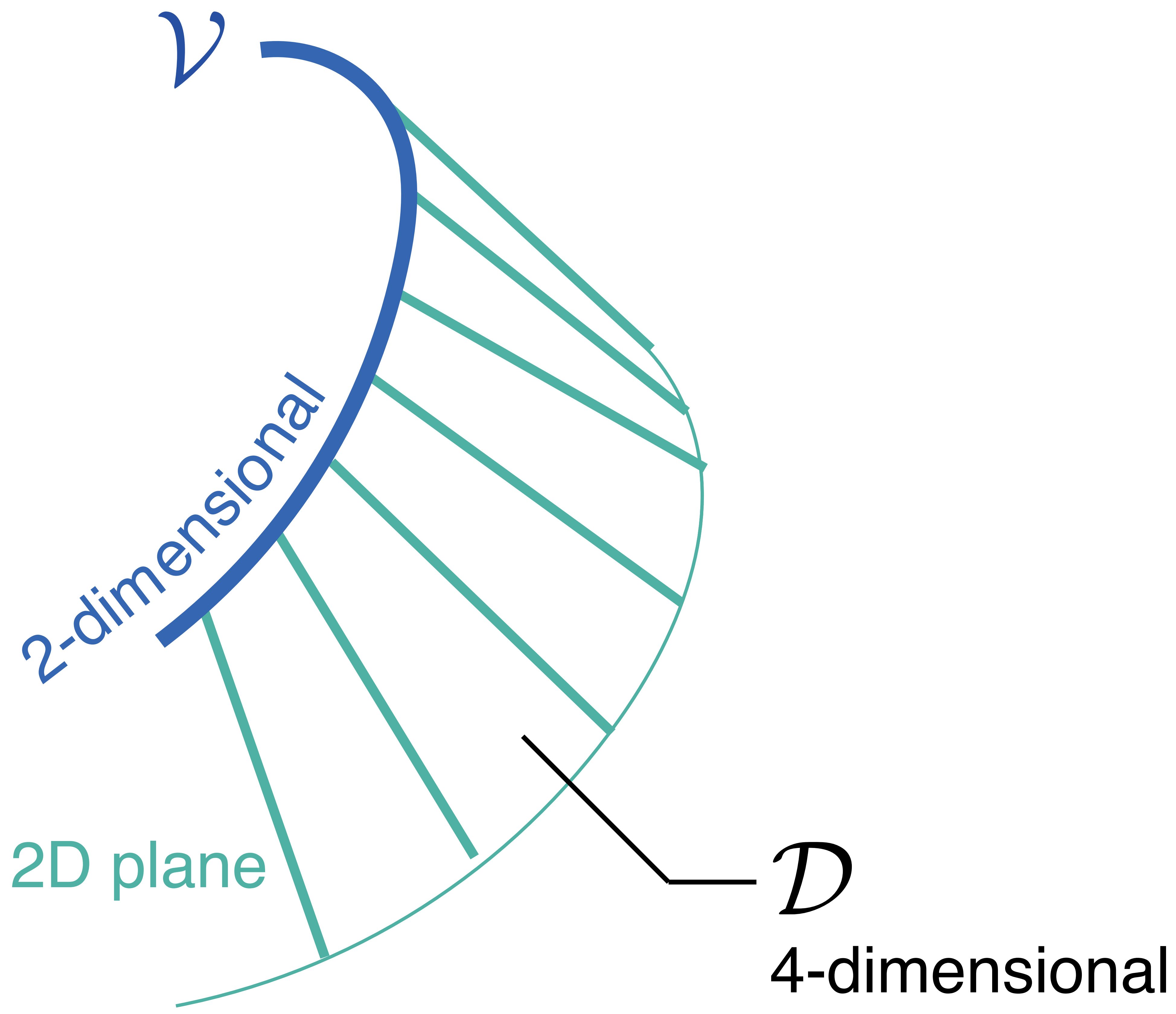

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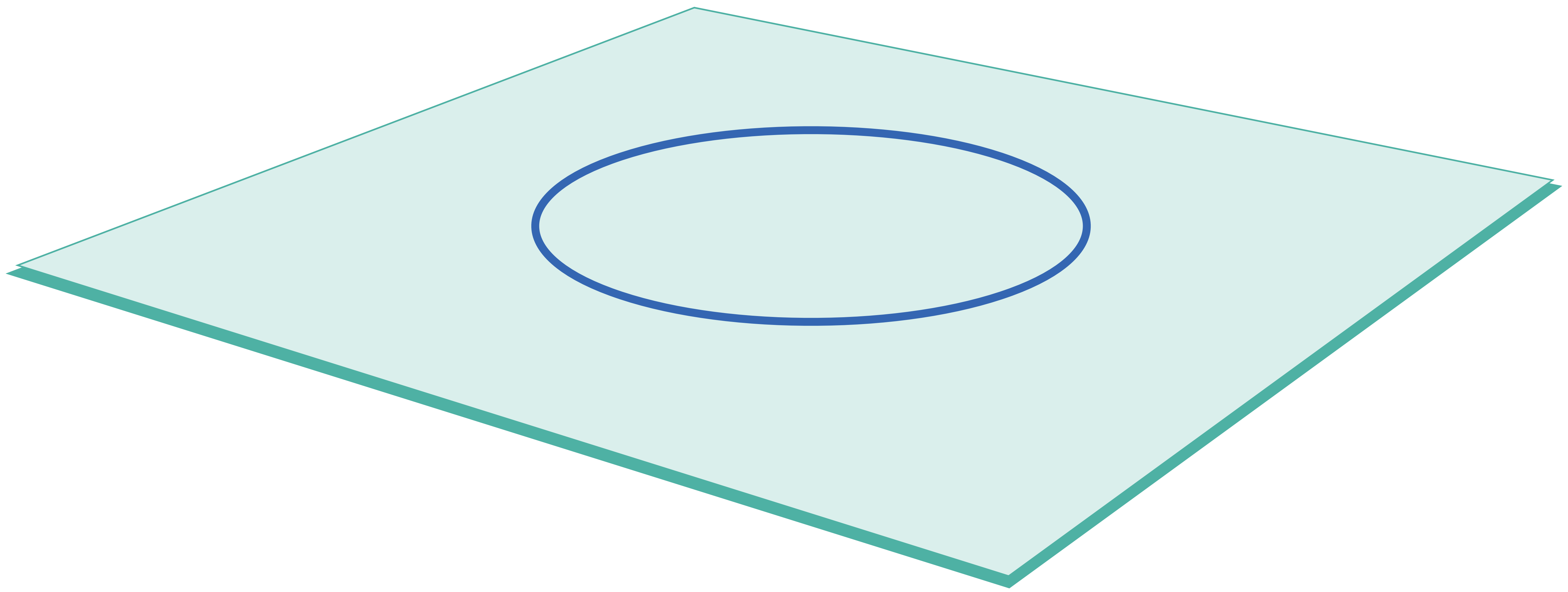
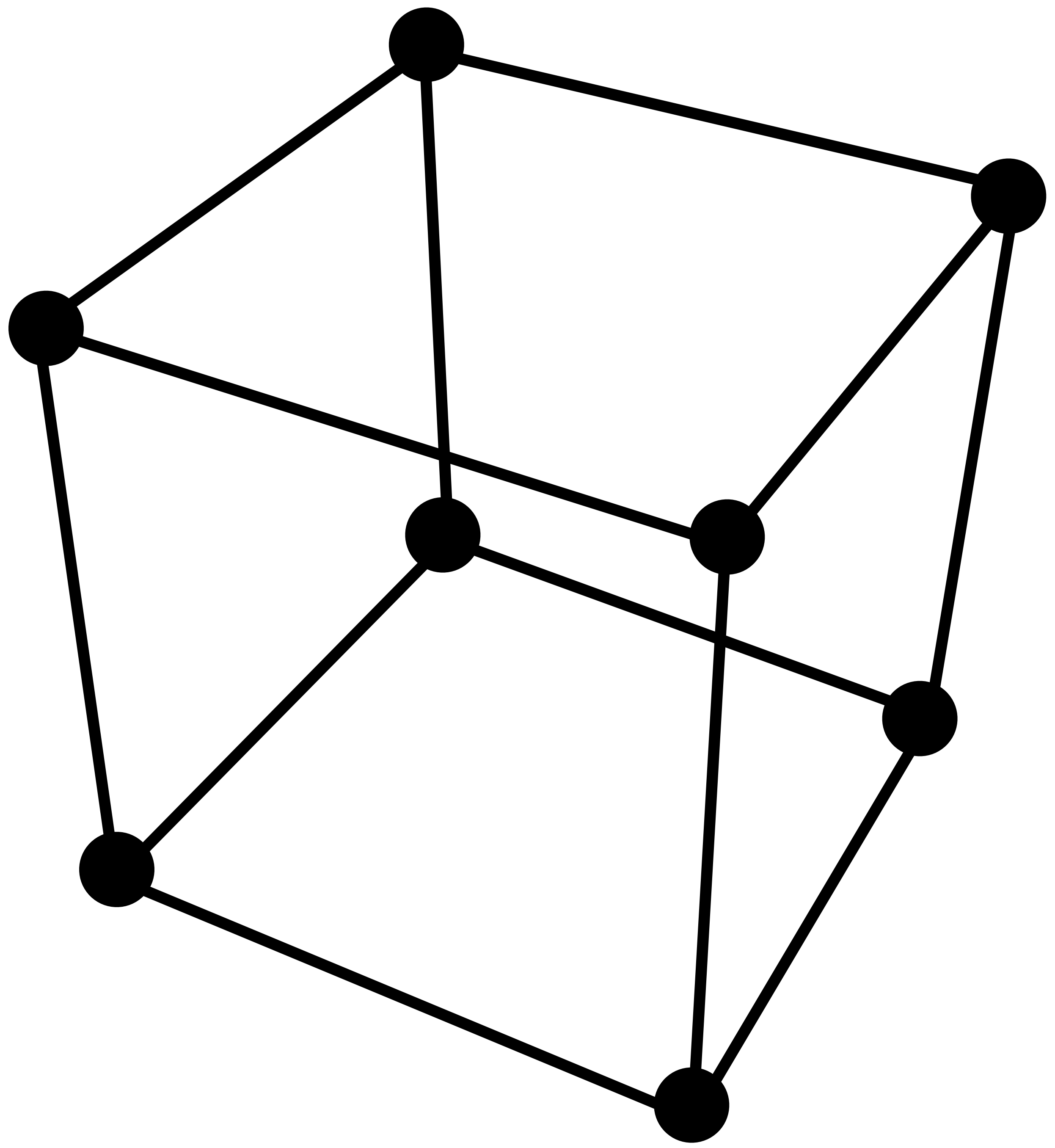
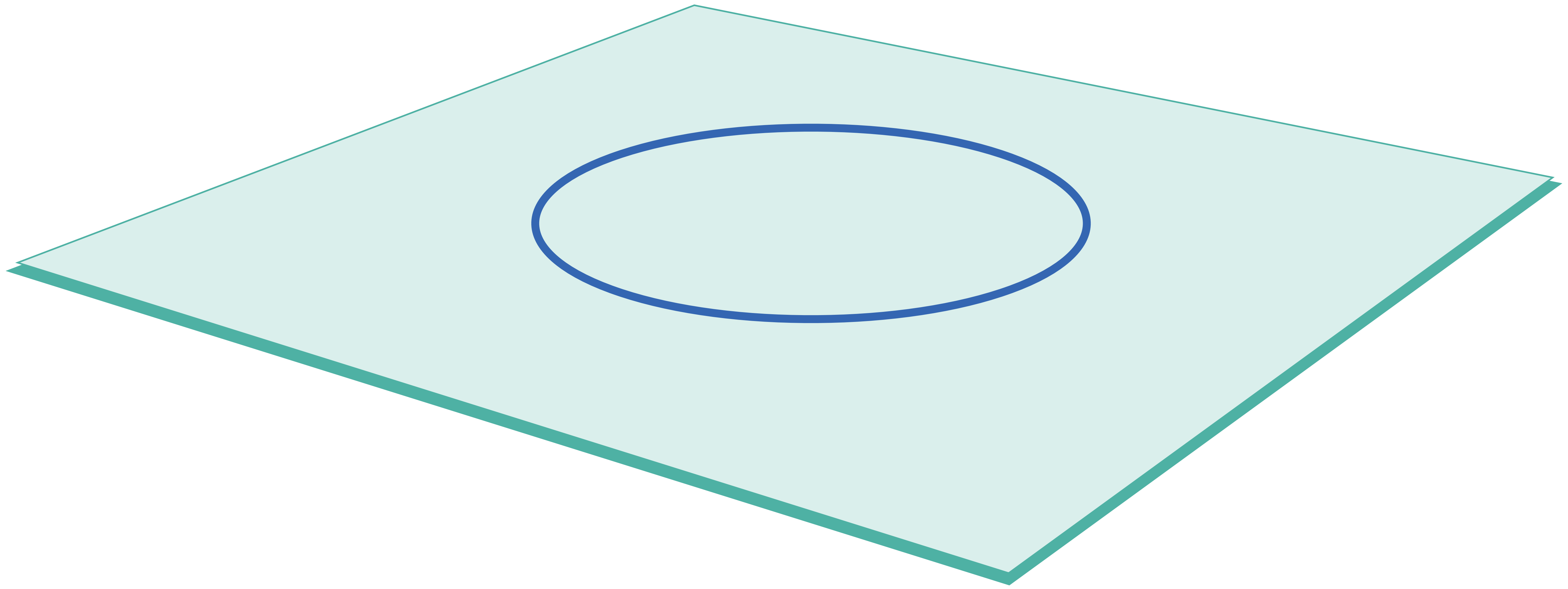


Face of a Penrose Cube





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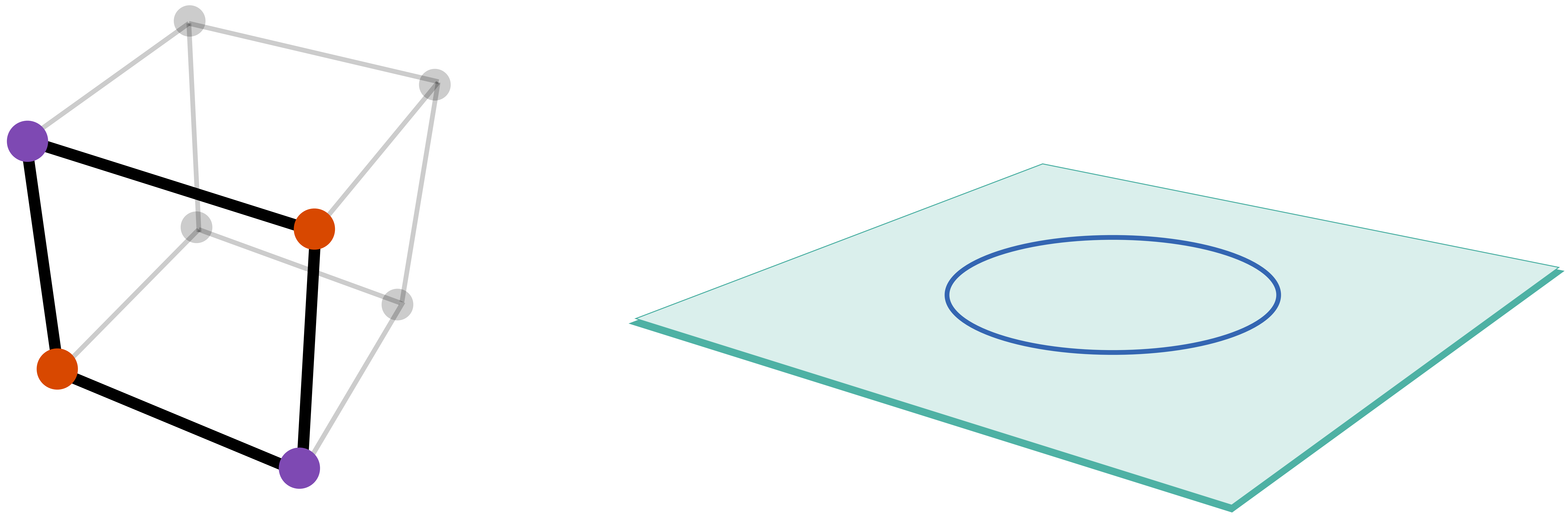

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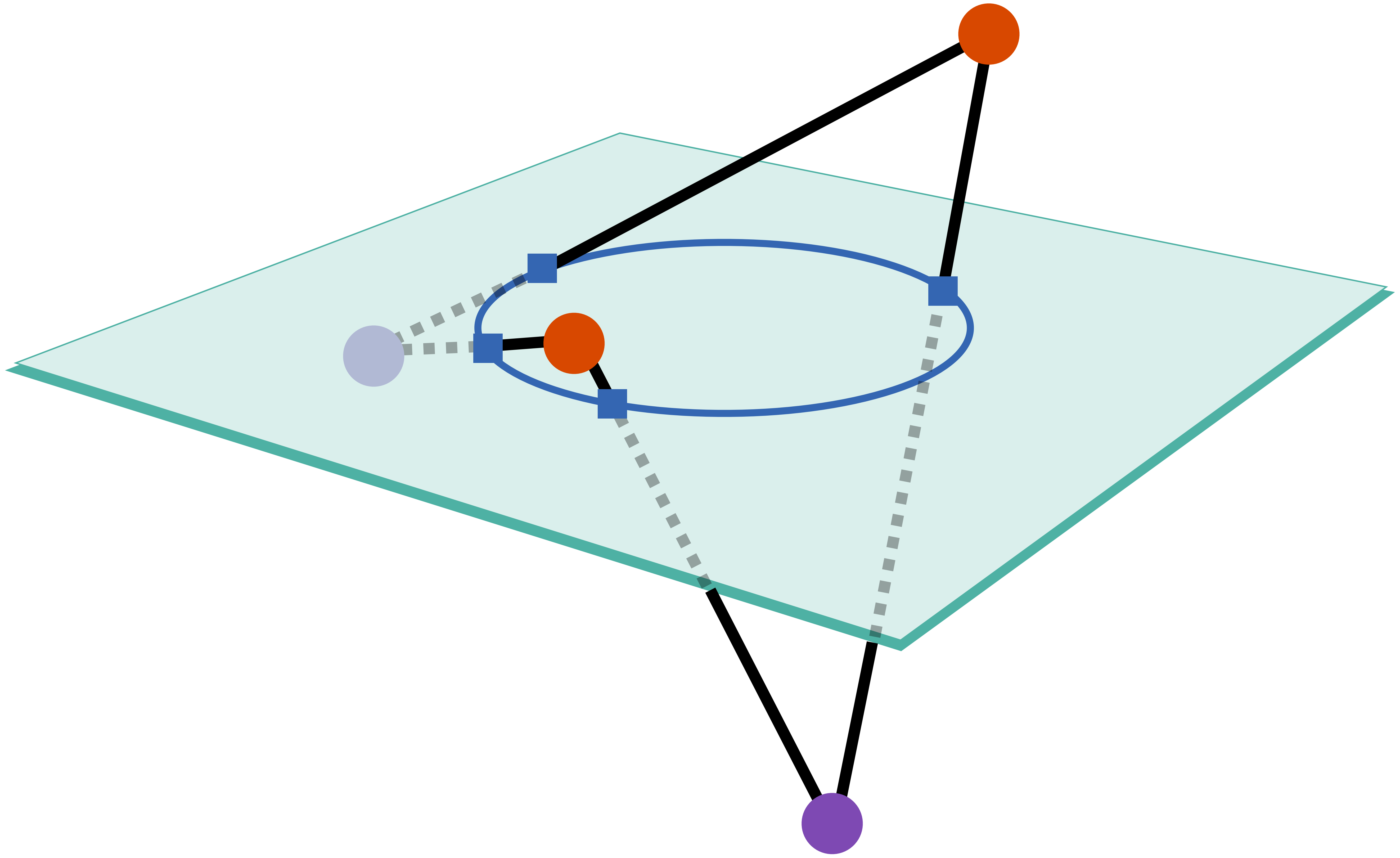
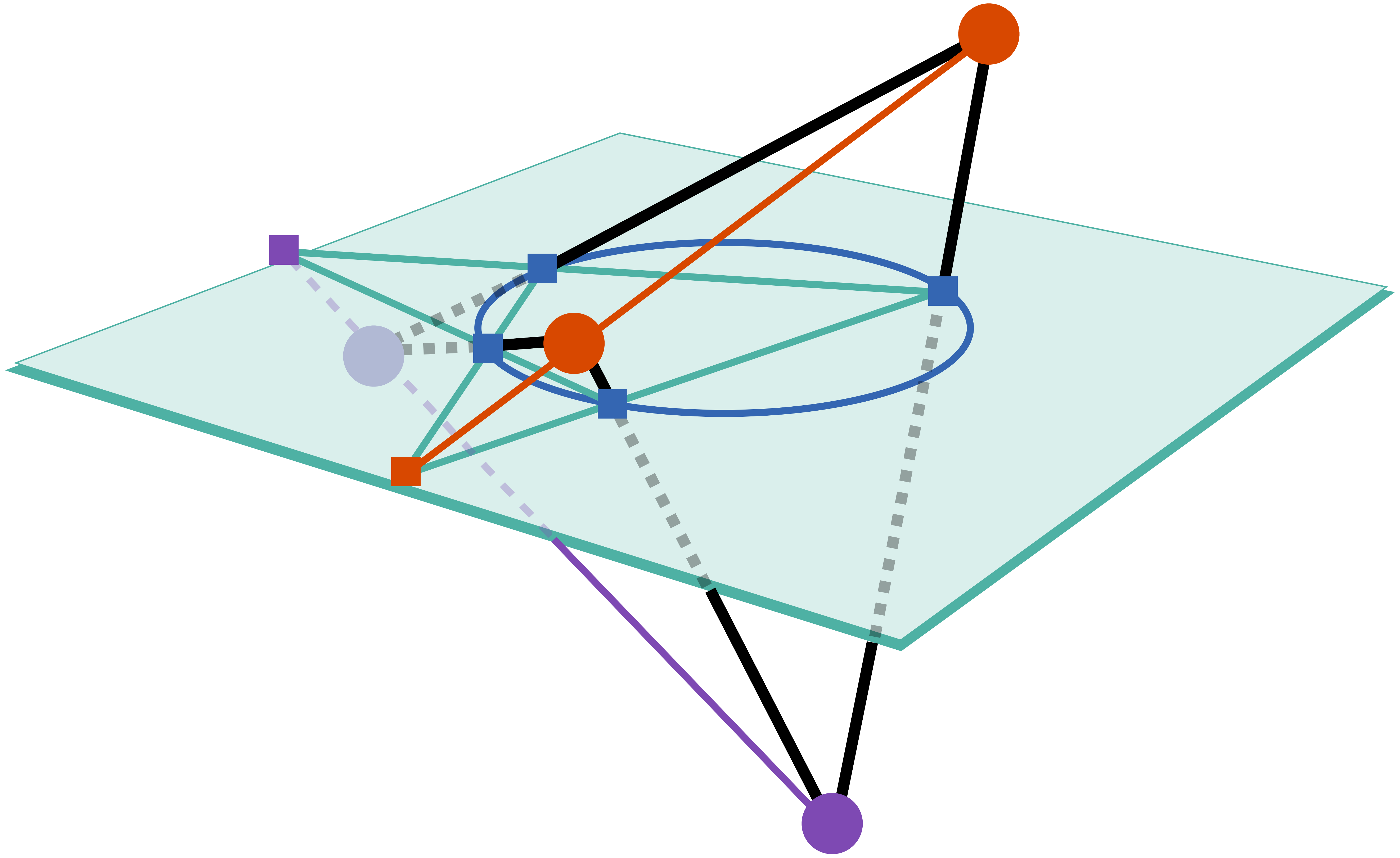


Face of a Penrose Cube

- *is a quadrilateral whose edges meet $\mathcal{V}$ in a point-plane.*



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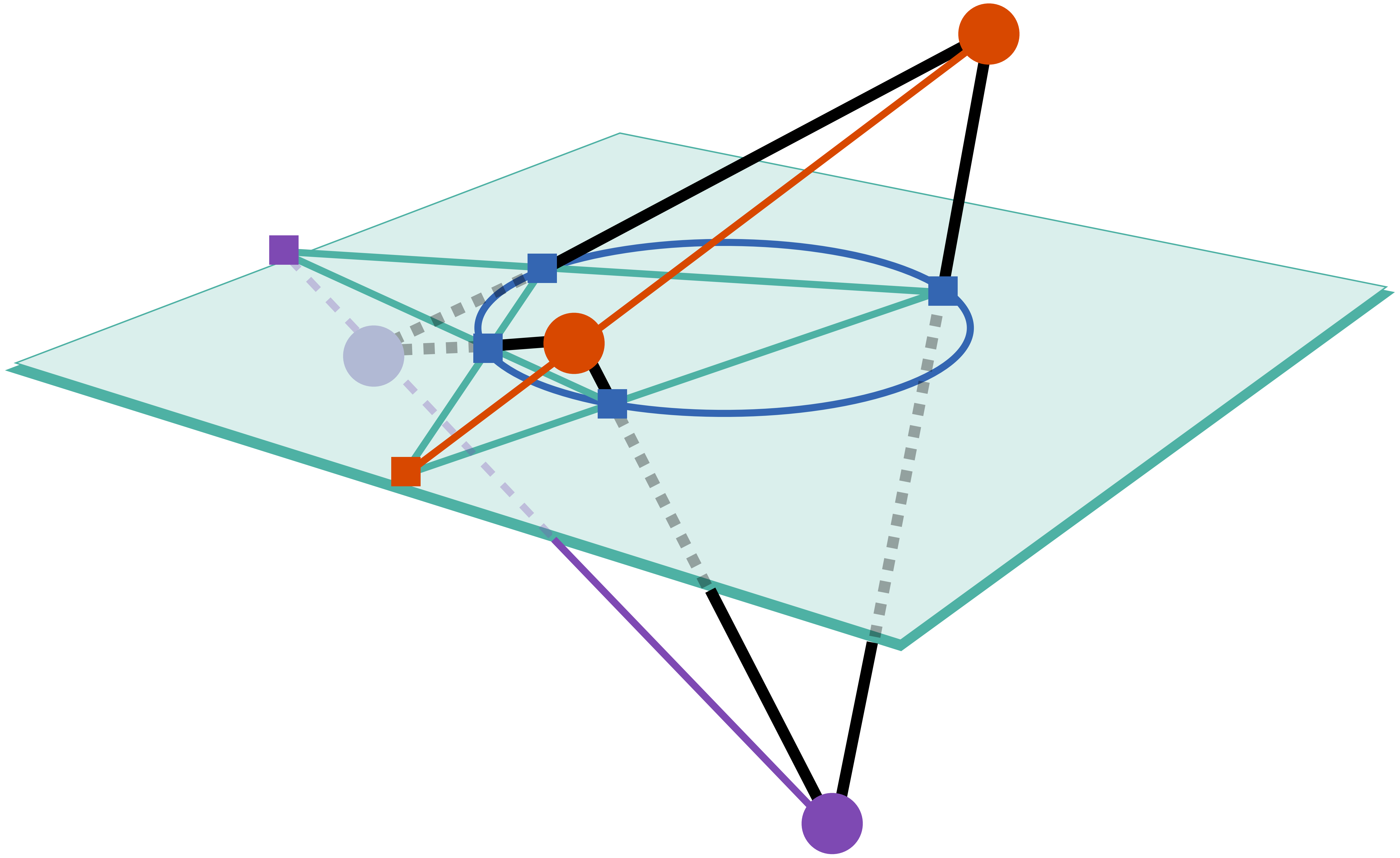
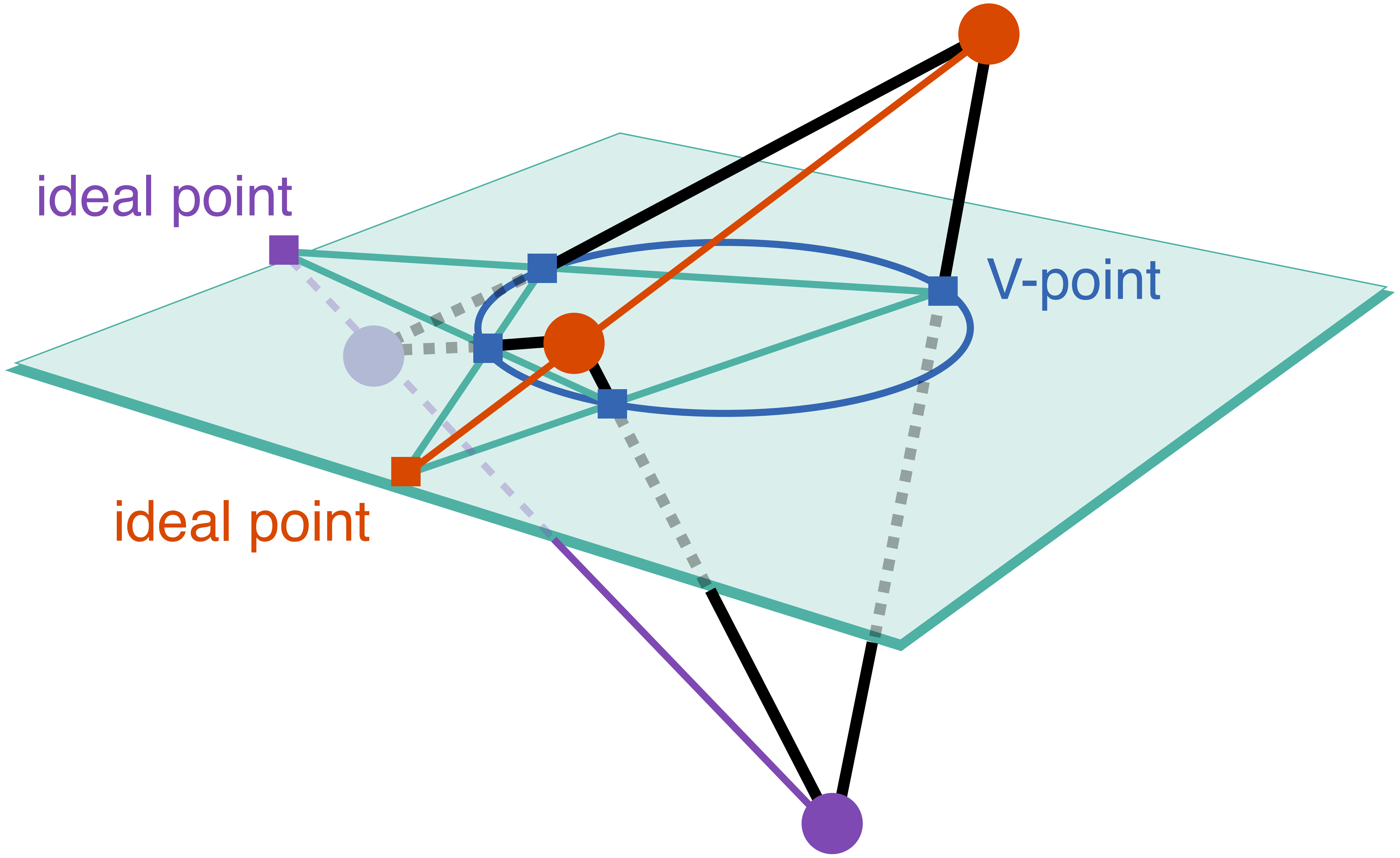
- *is a quadrilateral whose edges meet $\mathcal{V}$ in a point-plane.*
- *Completing the tetrahedron gives rise to two more special points on the point-plane.*



Face of a Penrose Cube

- *is a quadrilateral whose edges meet $\mathcal{V}$ in a point-plane.*
- *Completing the tetrahedron gives rise to two more special points on the point-plane.*
- *We call these two points "ideal points"*



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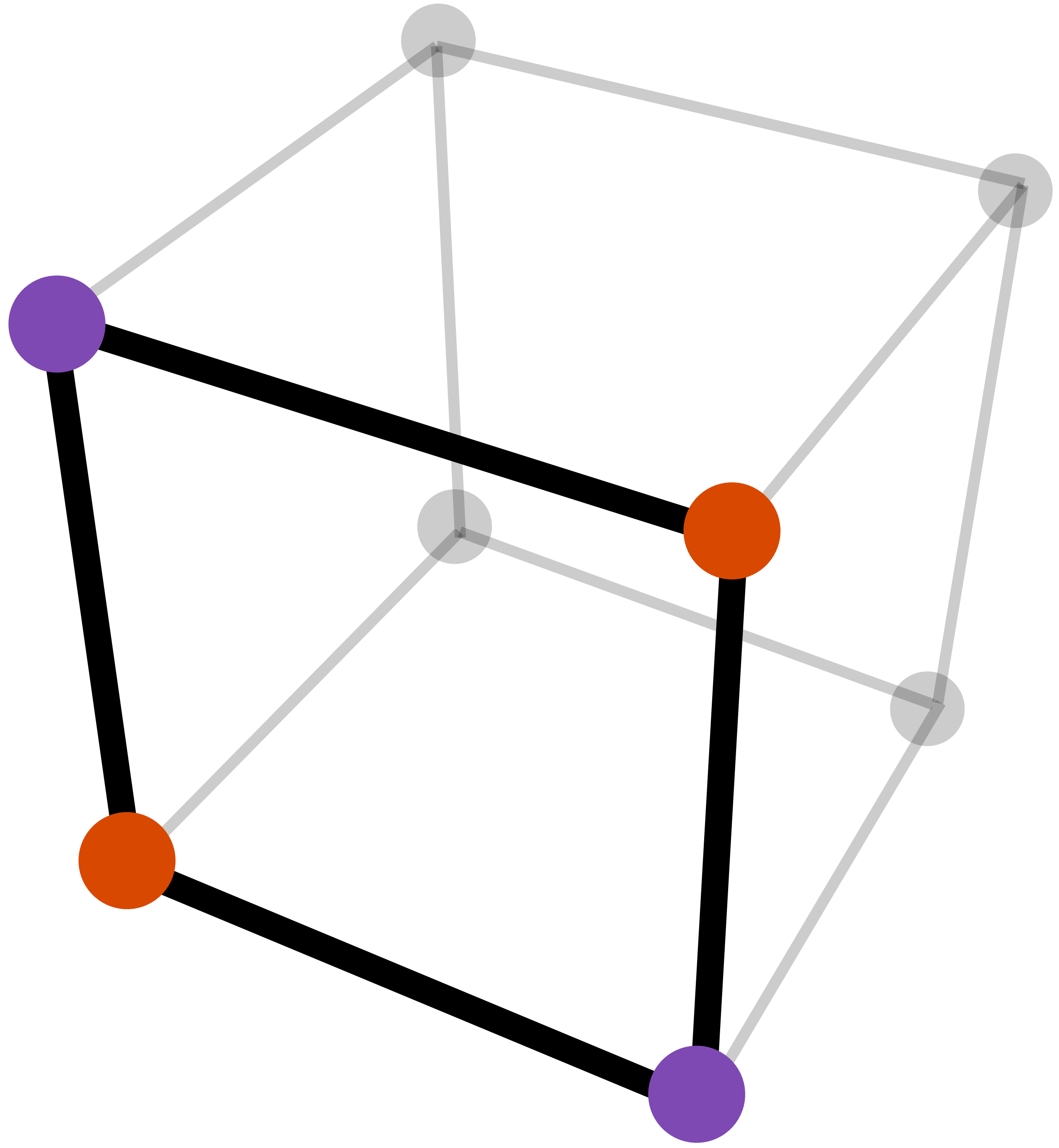

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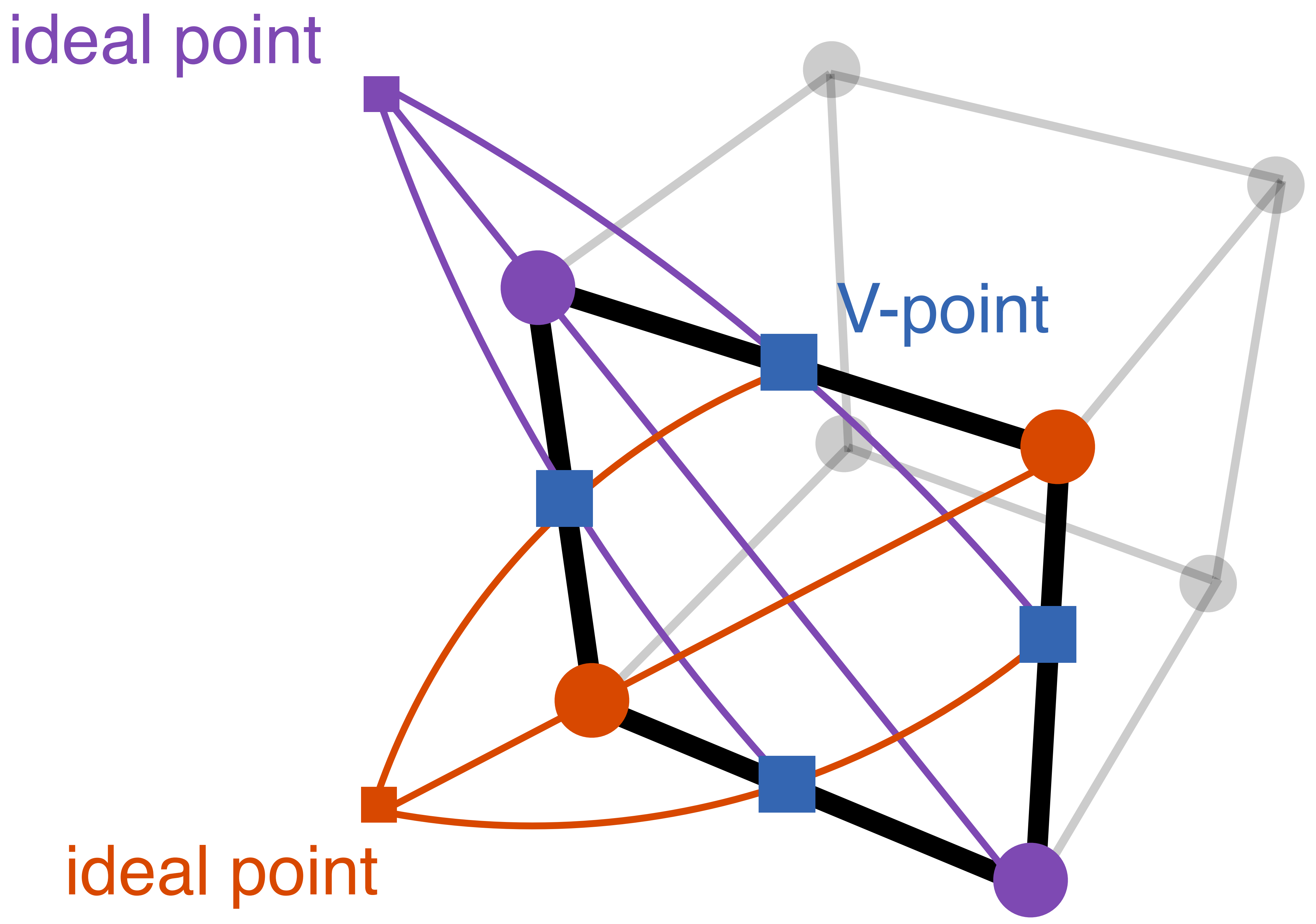

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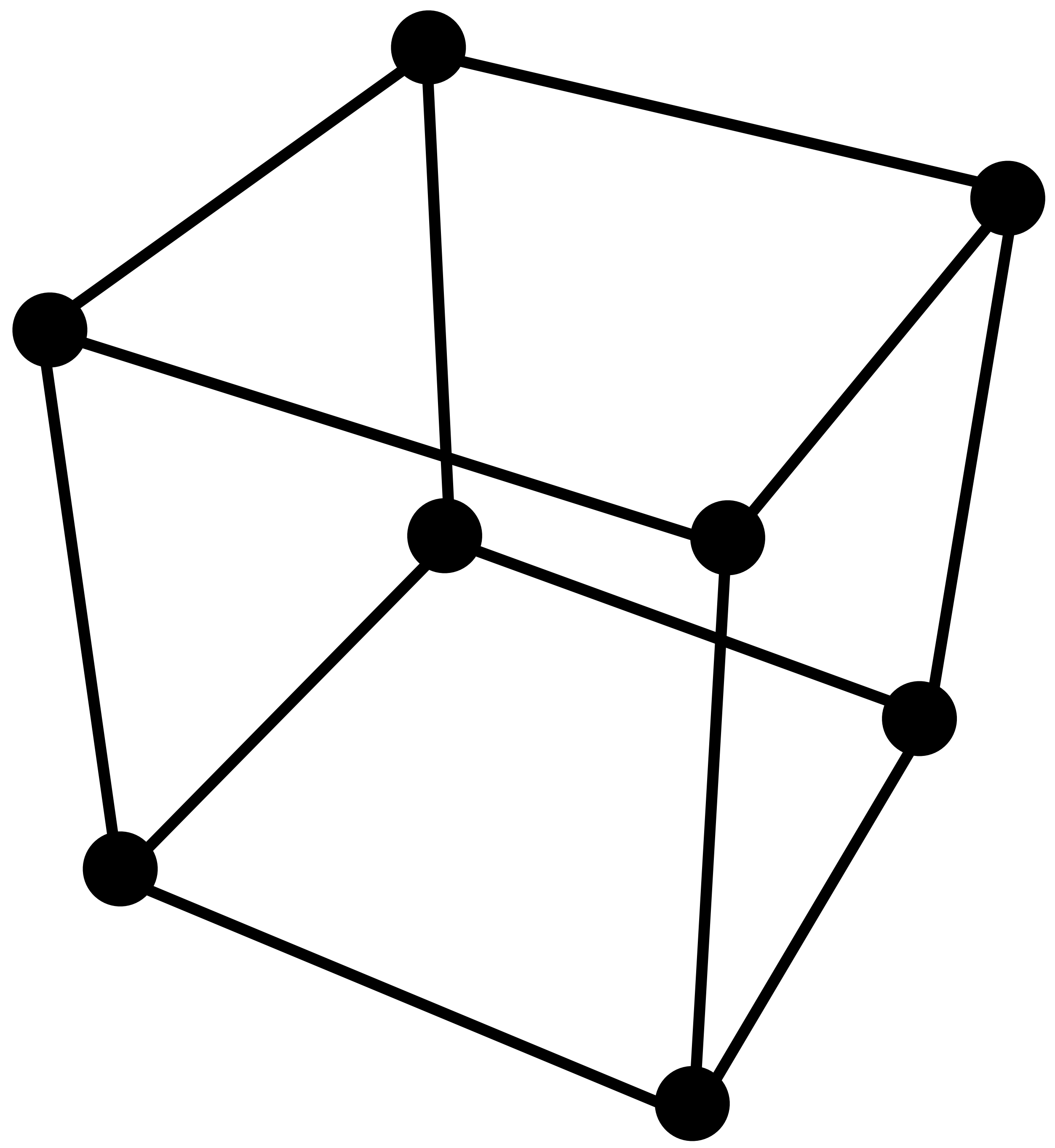
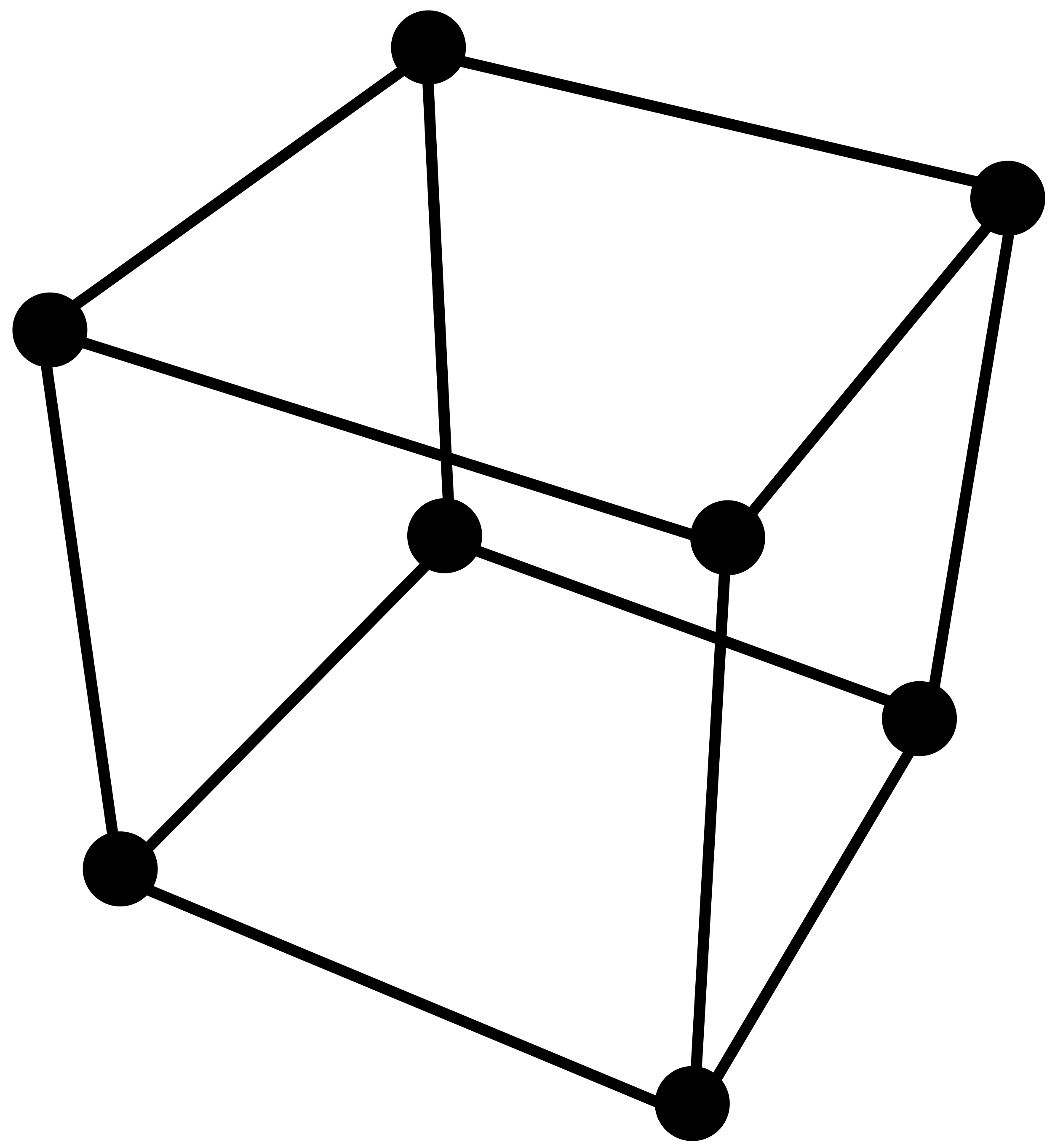


Corner of a Penrose Cube



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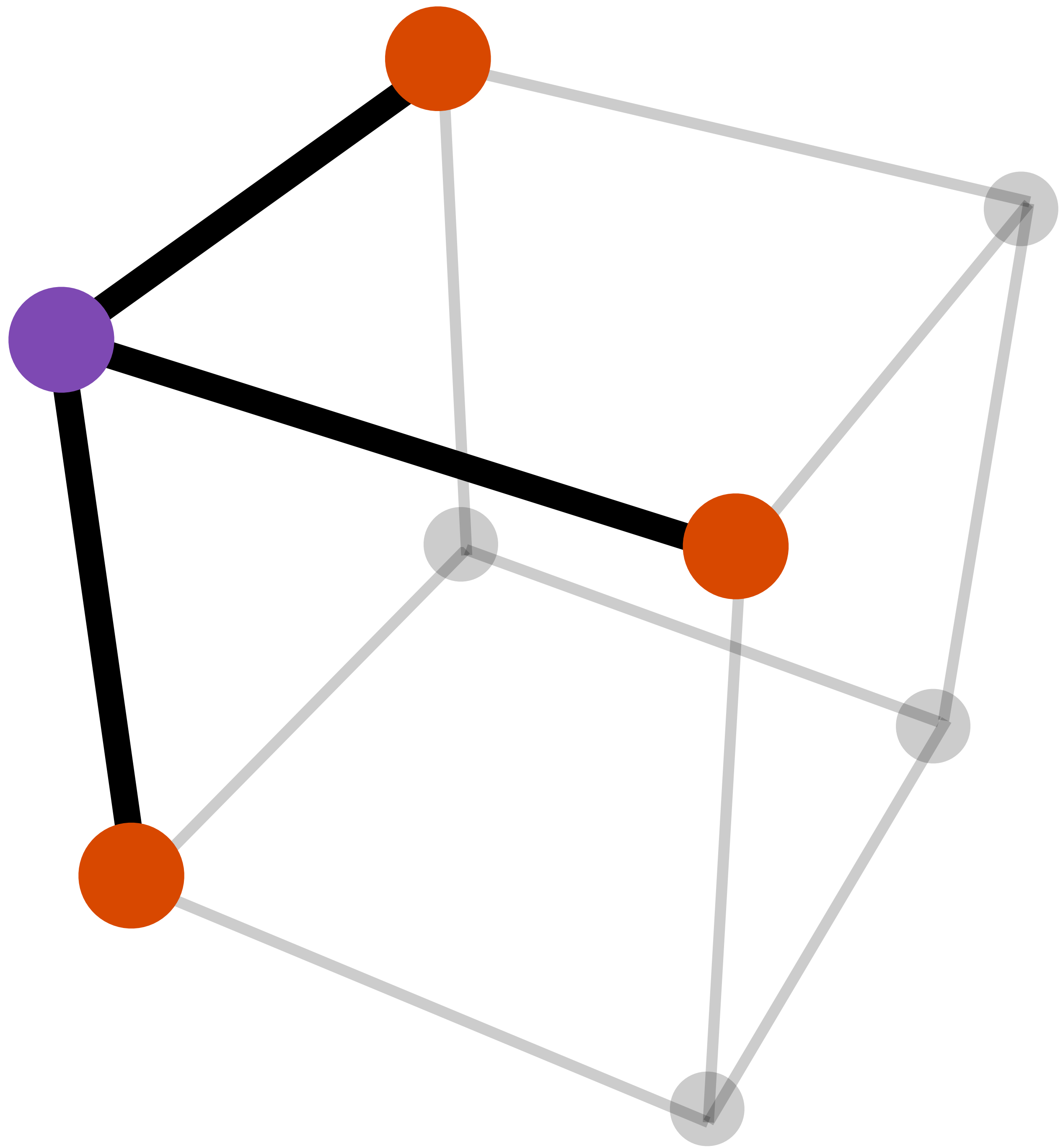

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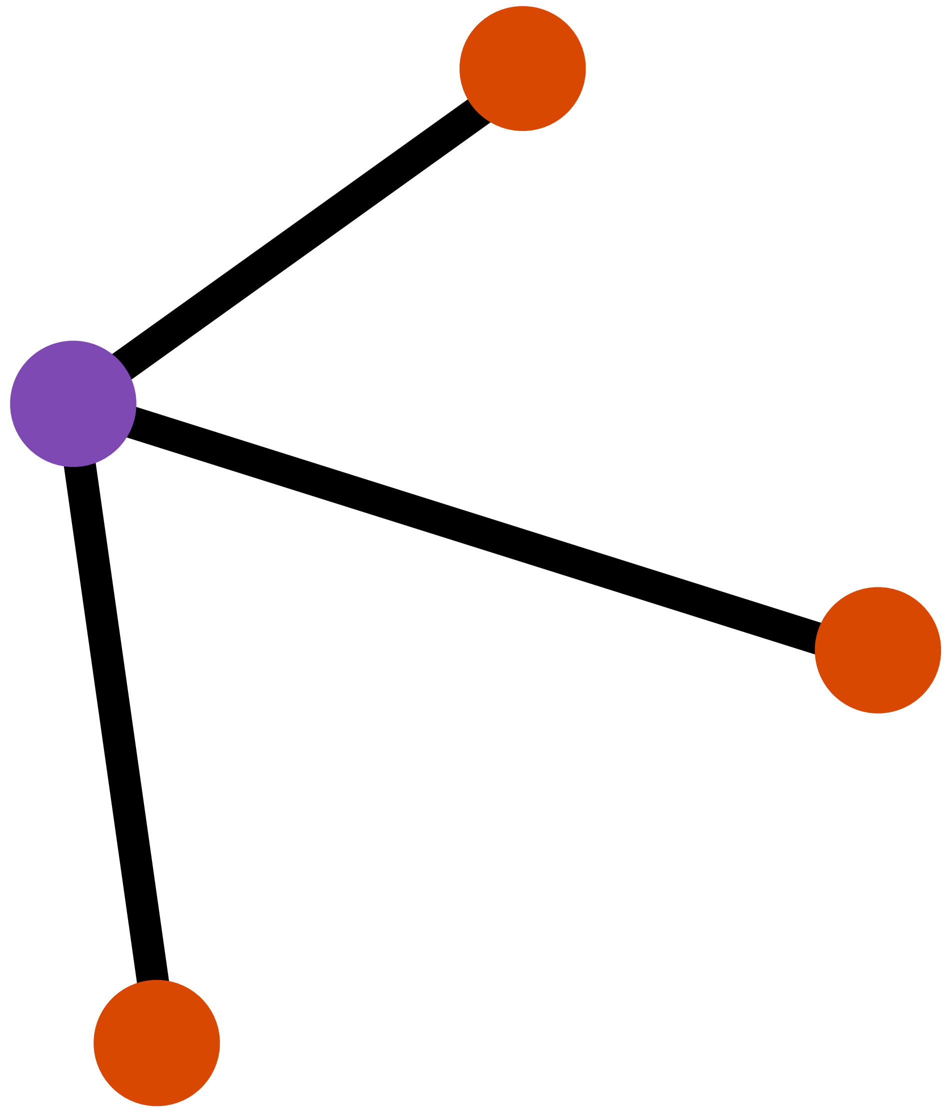

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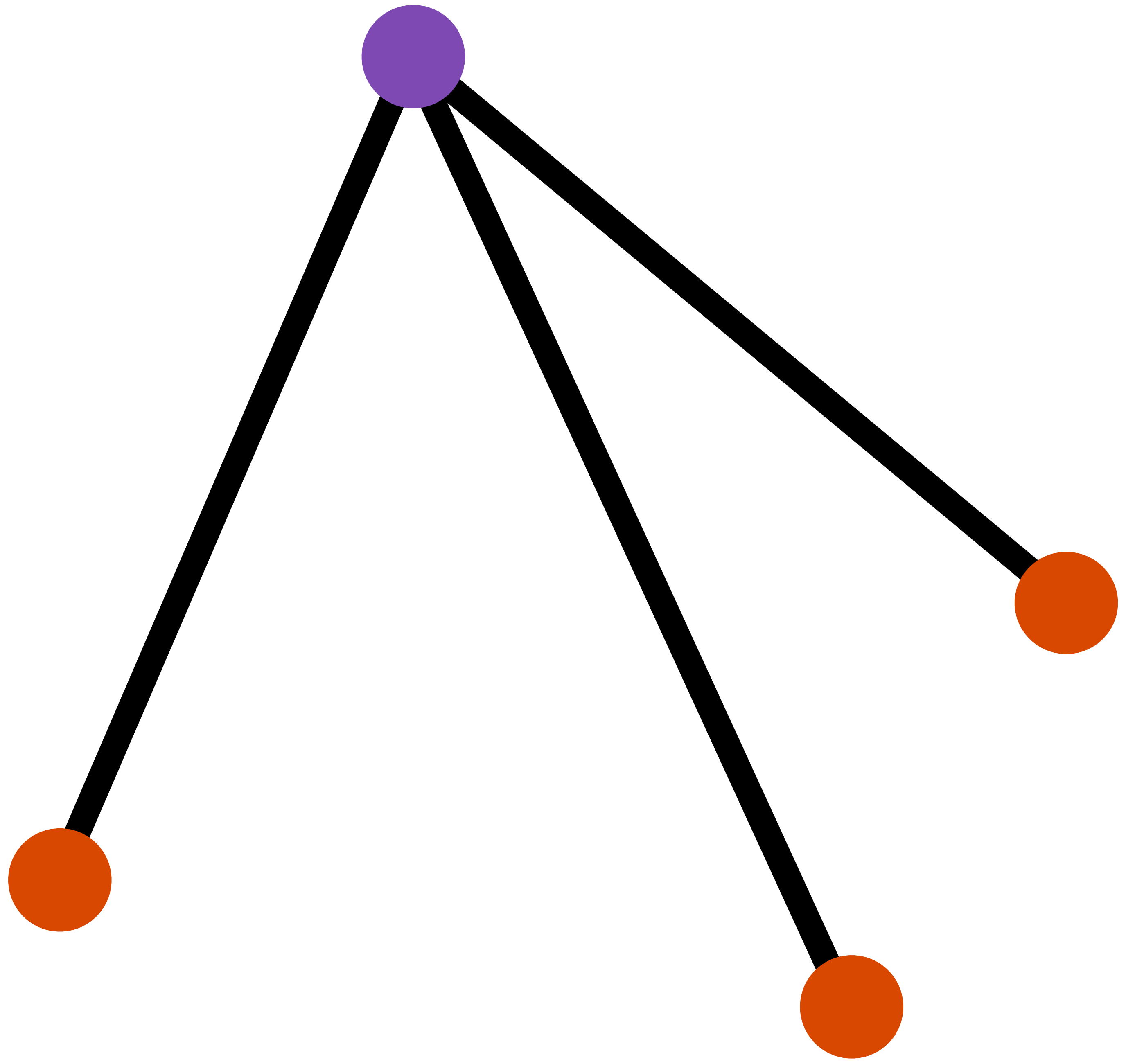

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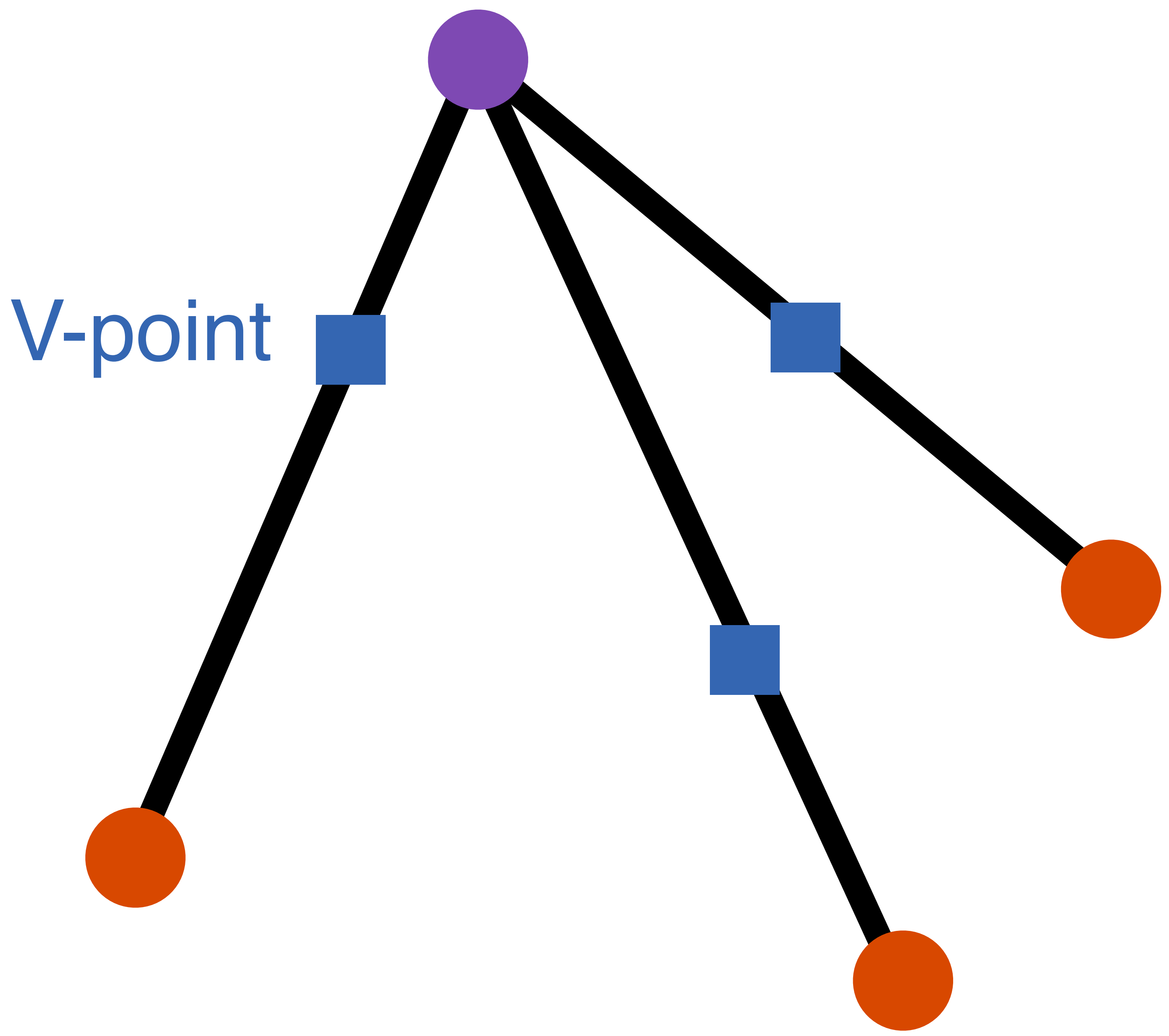

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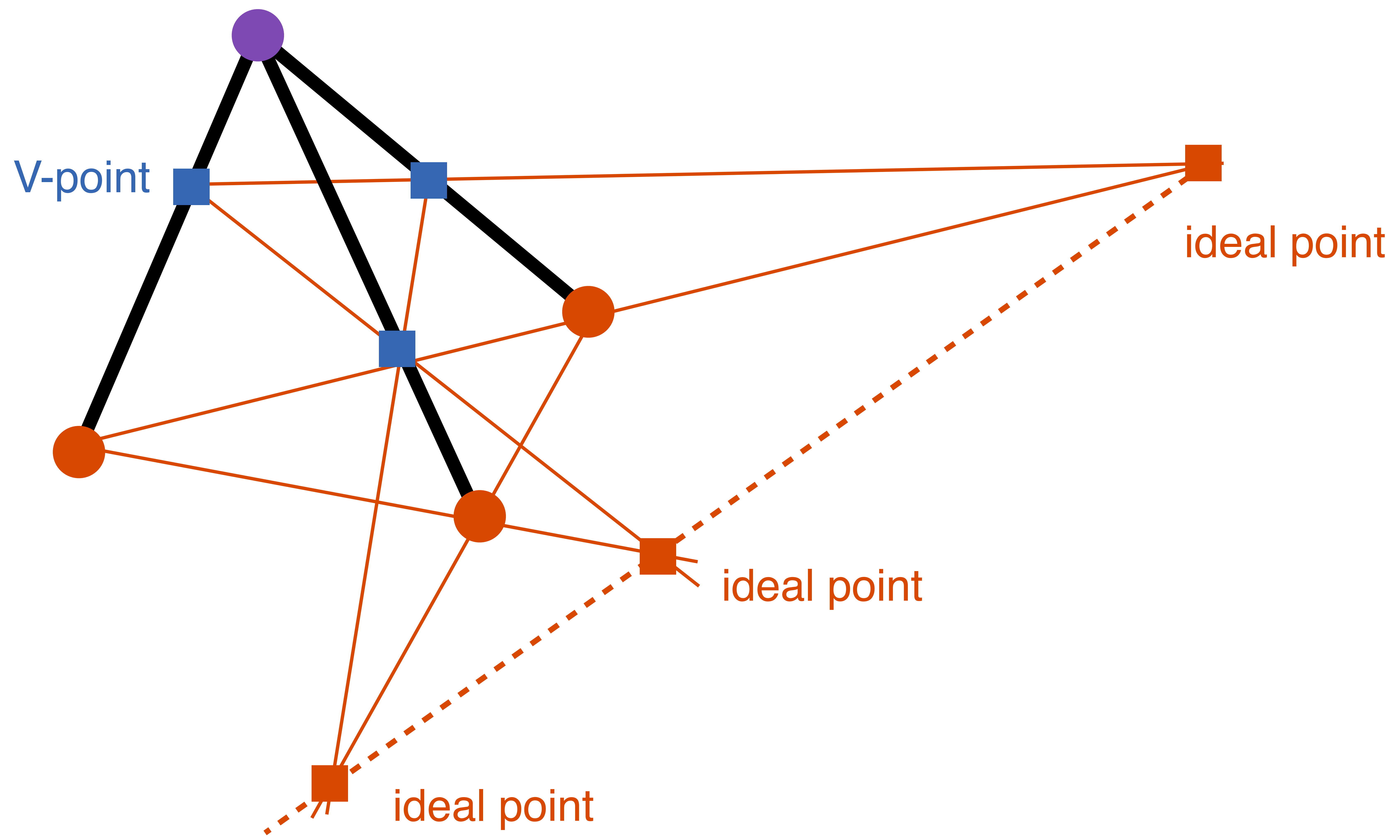
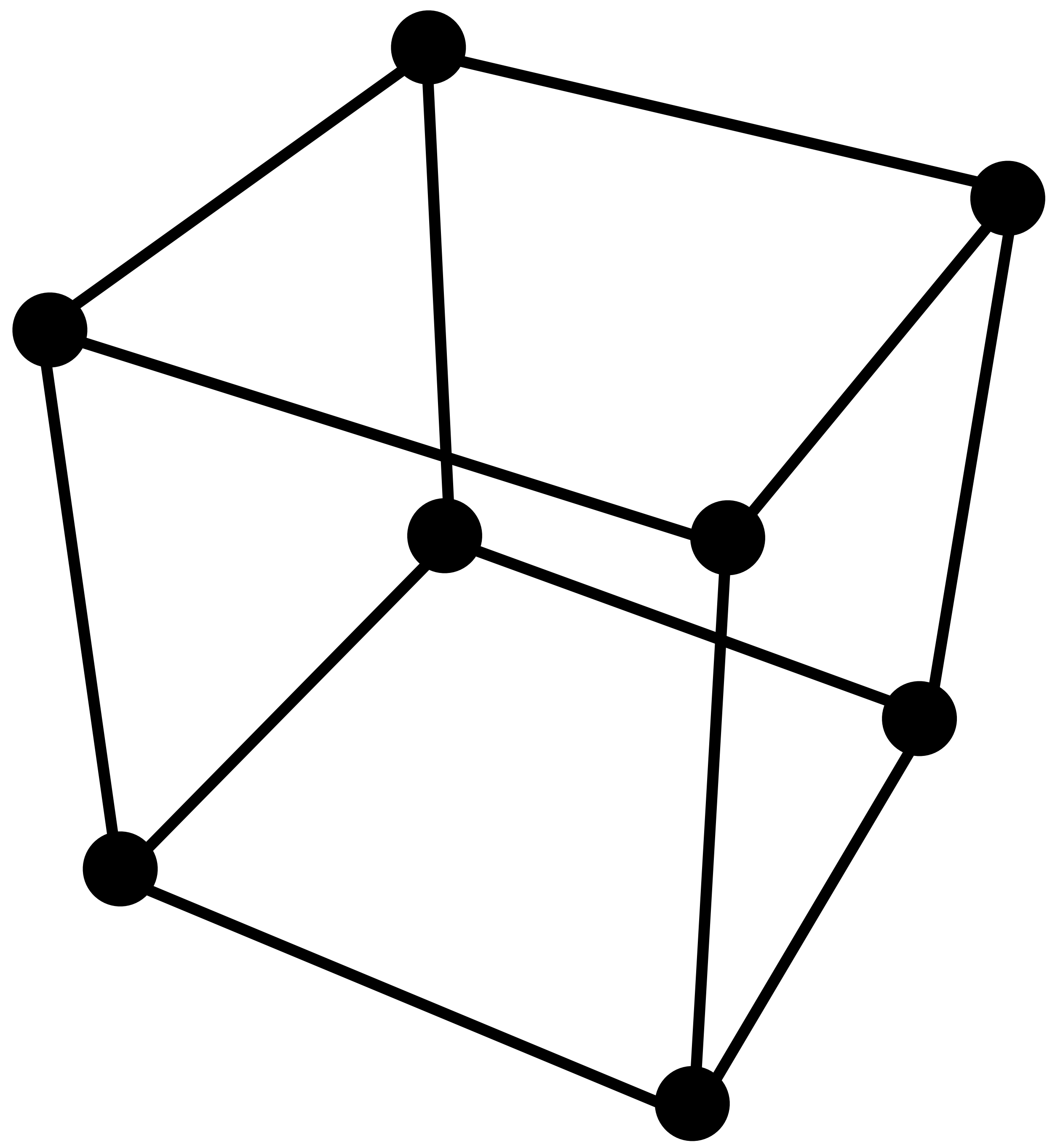
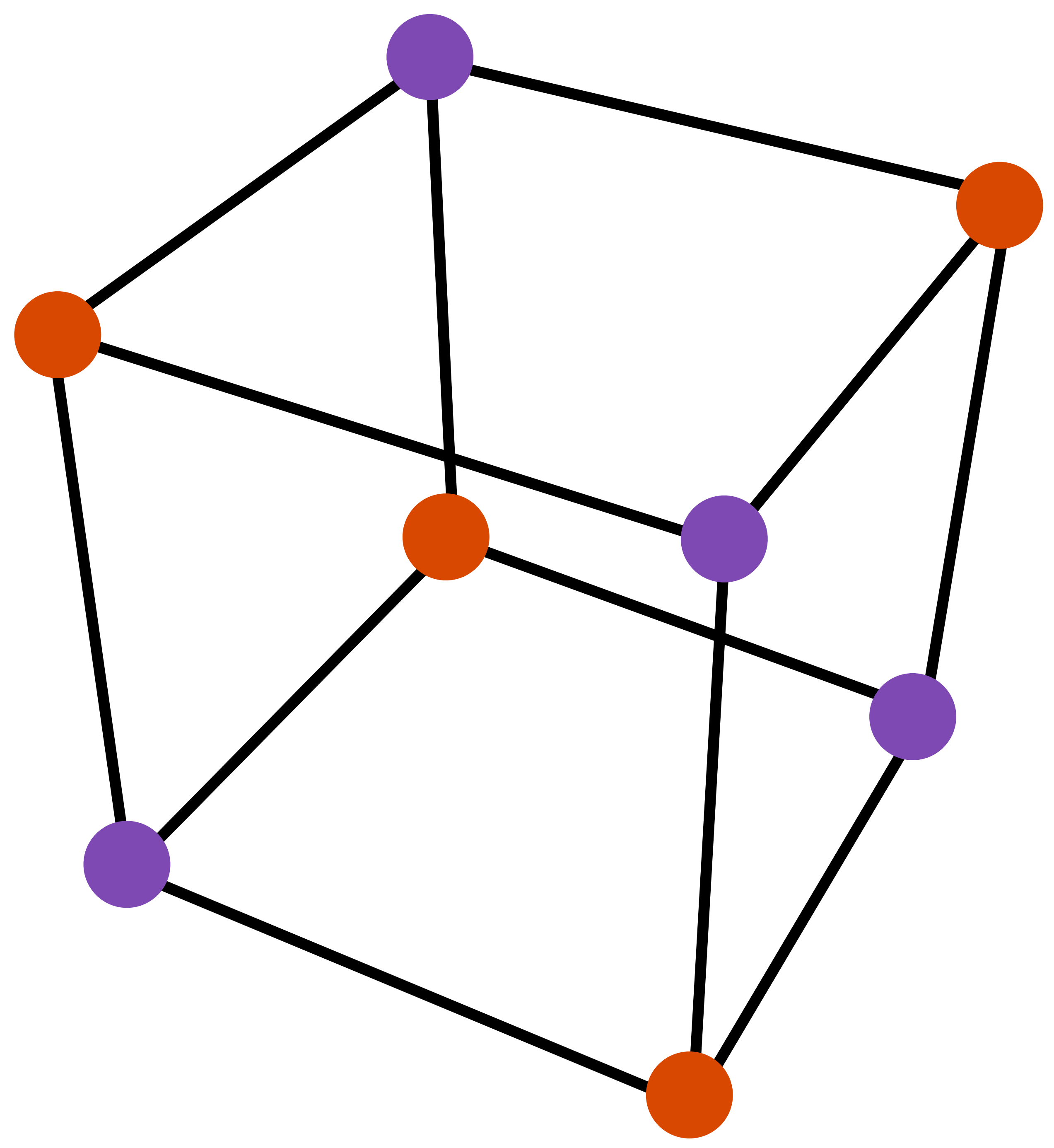


Structure of a Penrose Cube



Structure of a Penrose Cube



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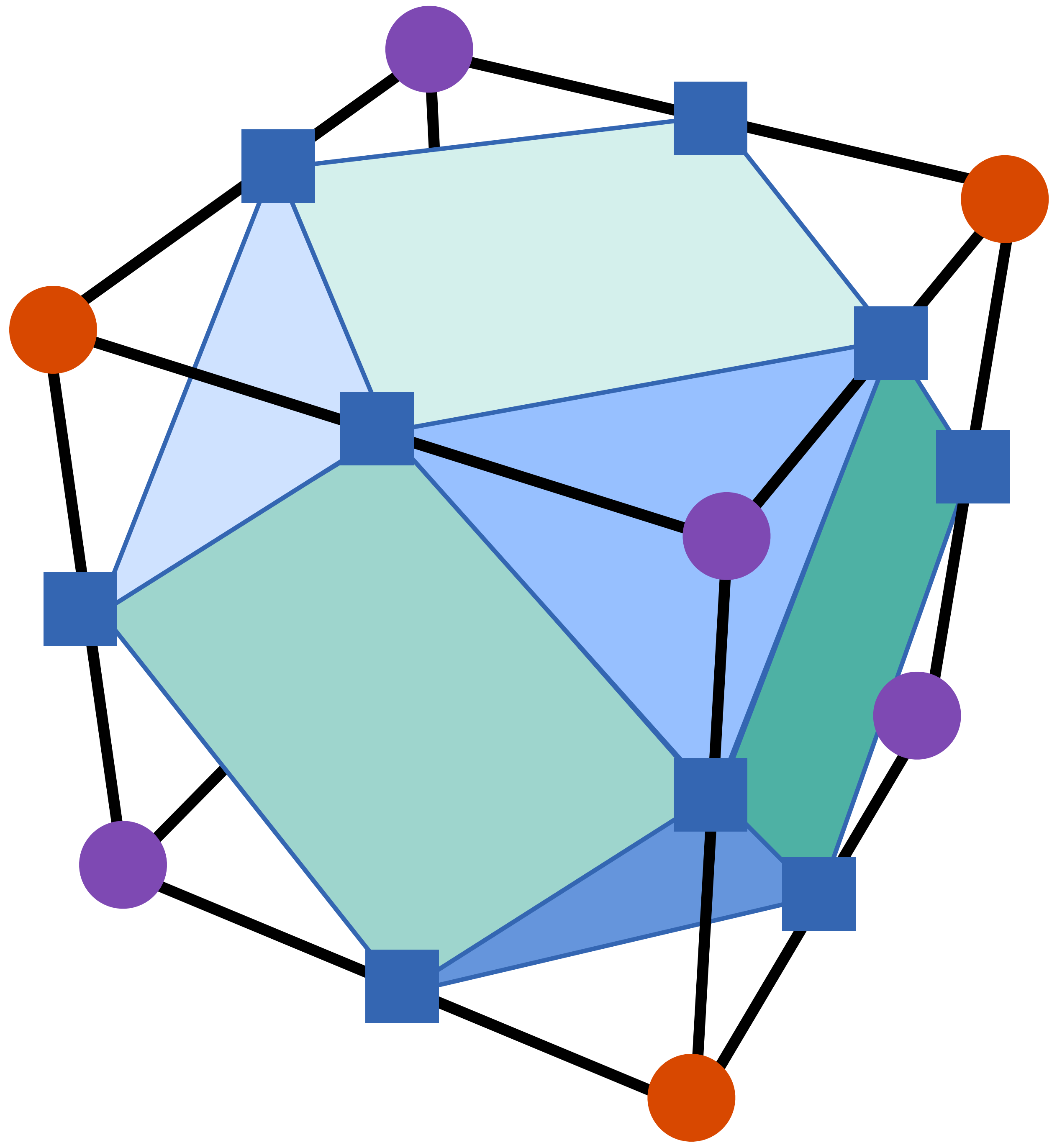

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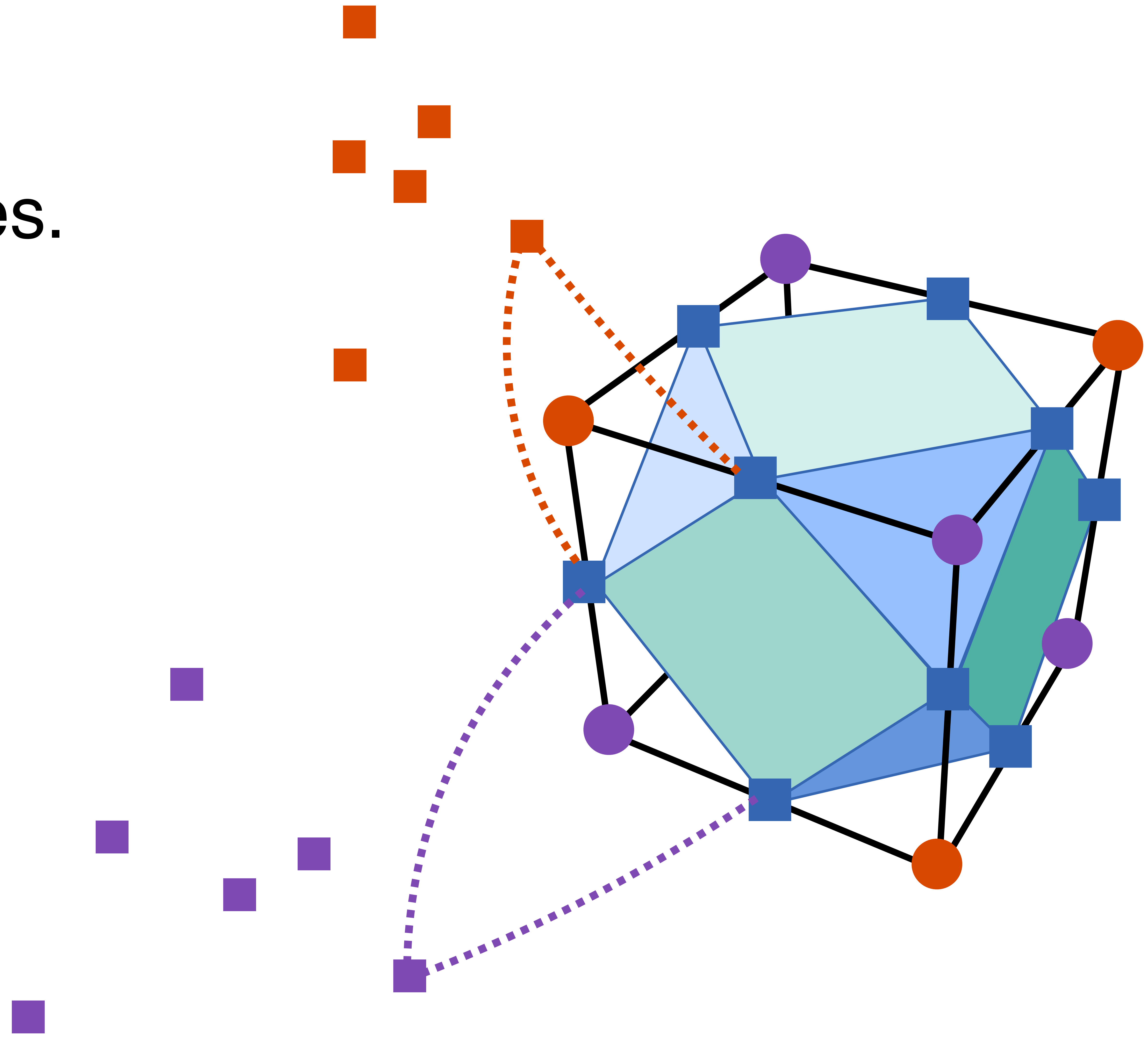

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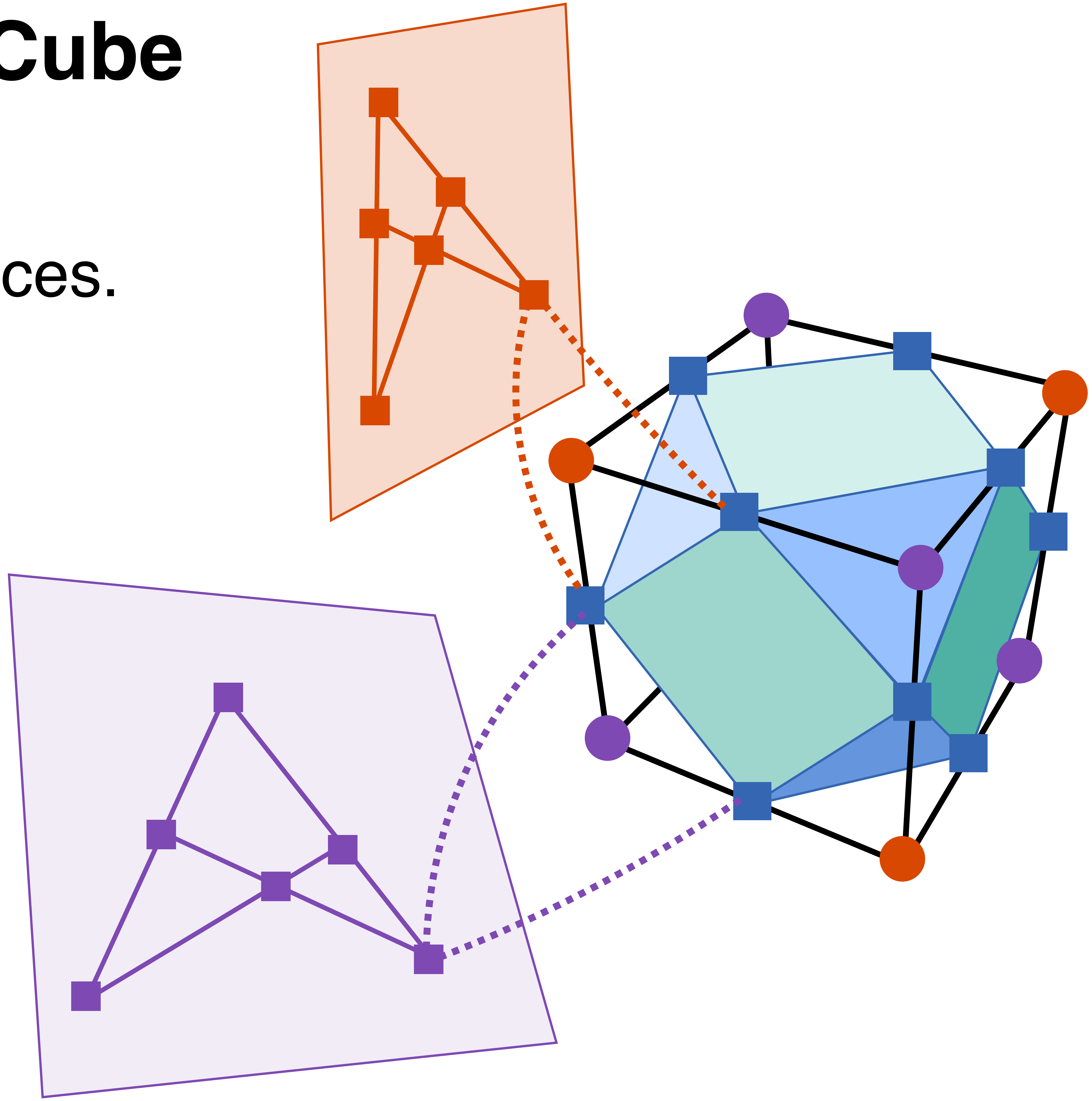

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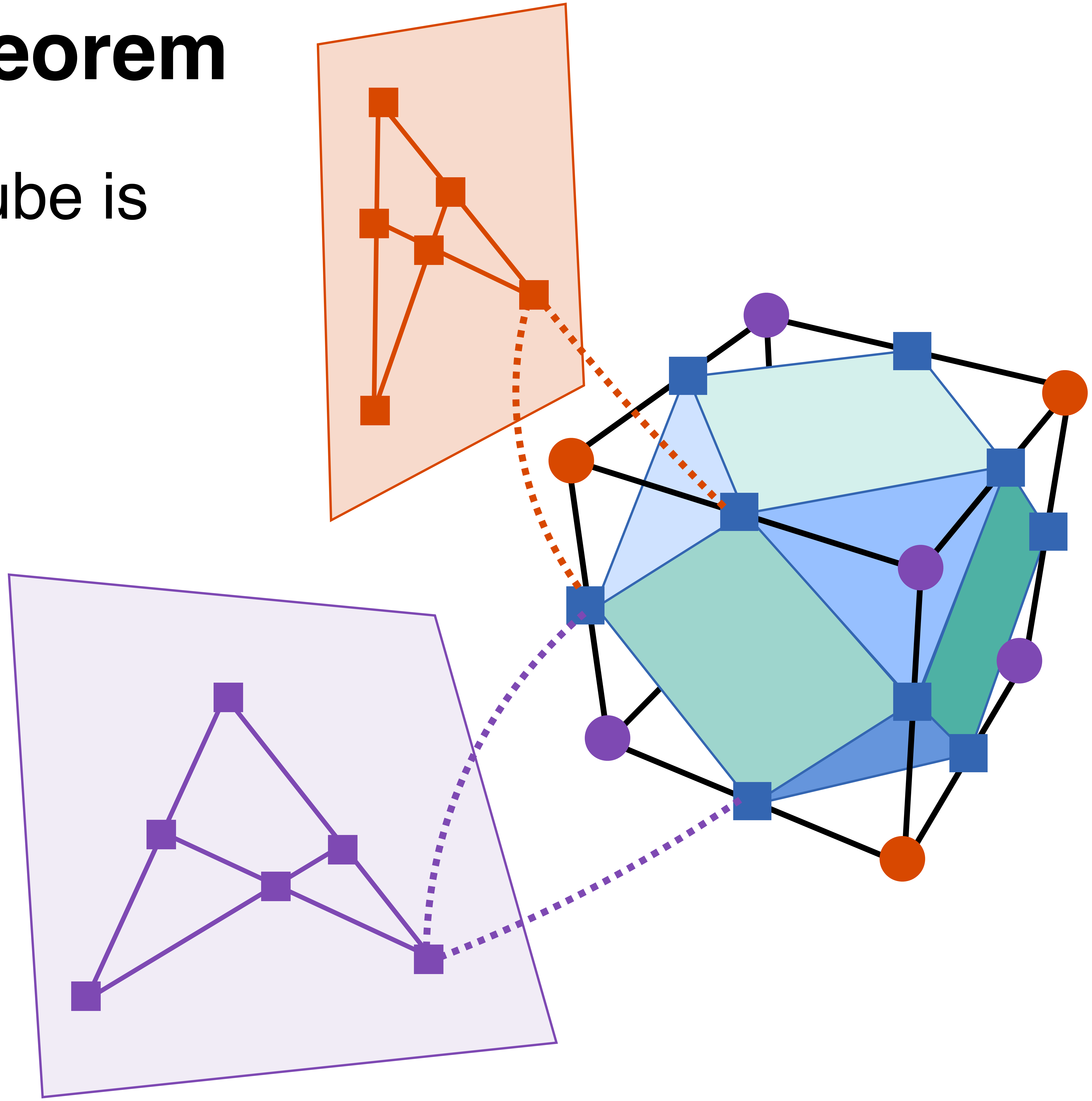

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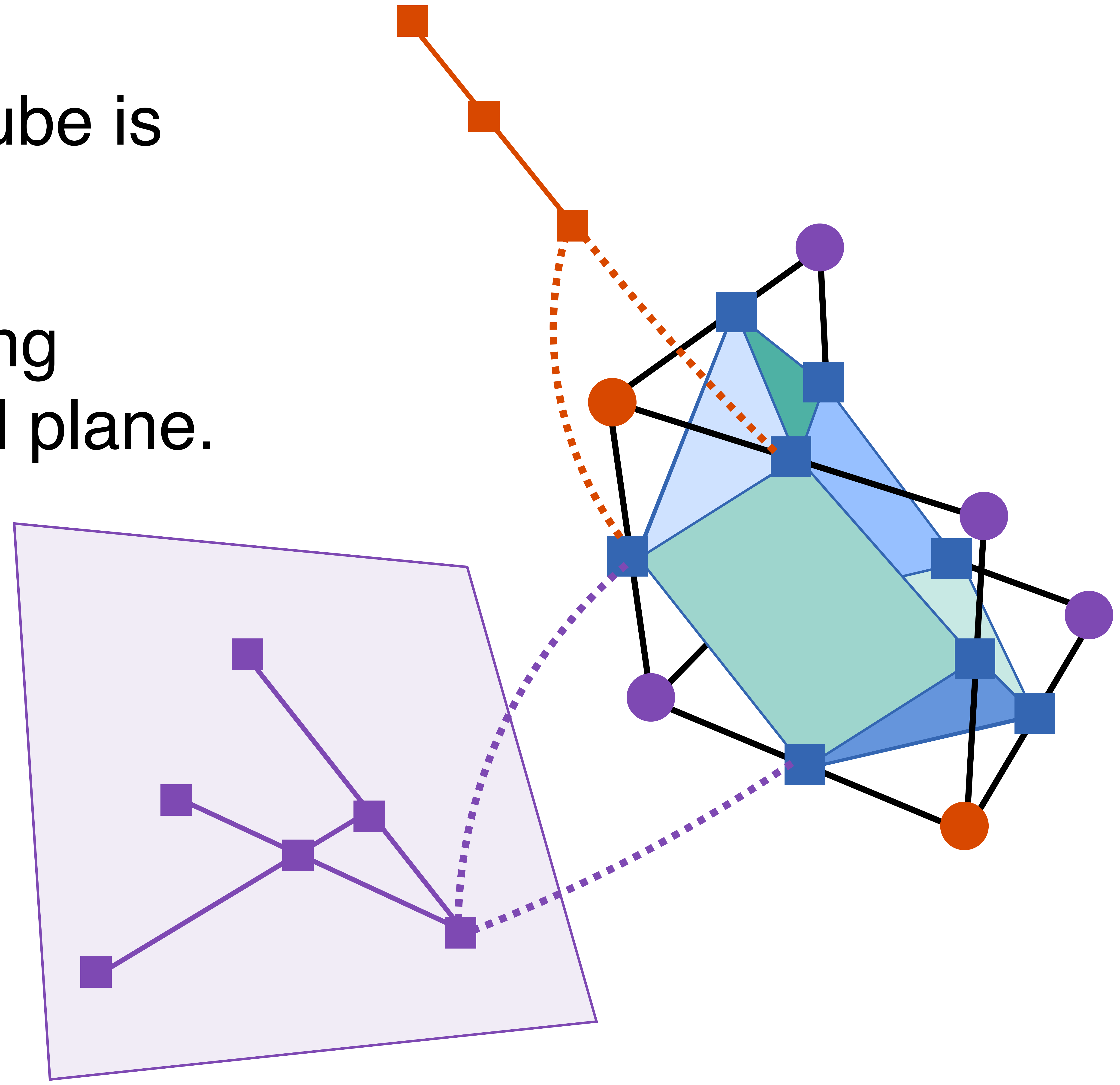

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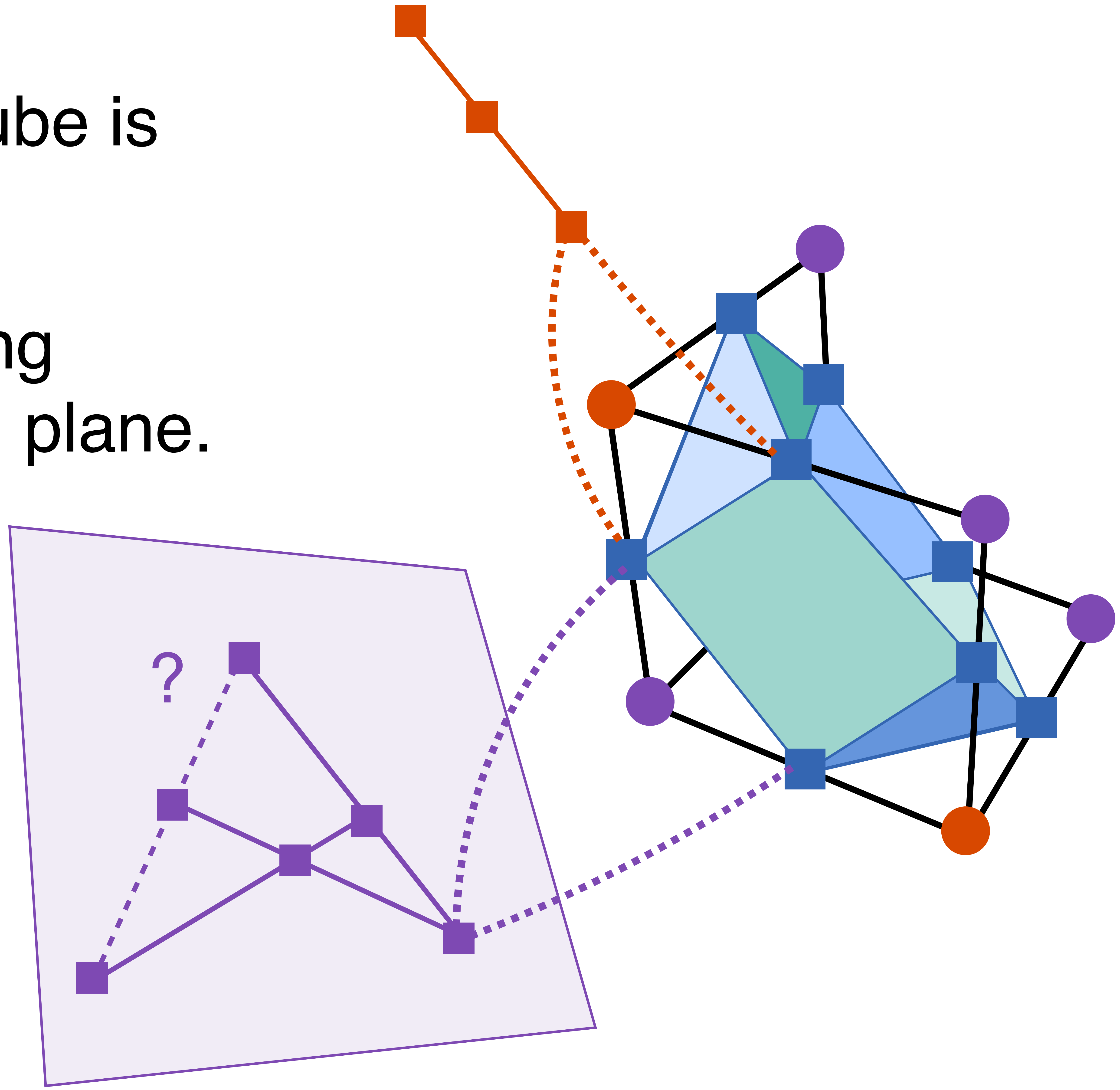

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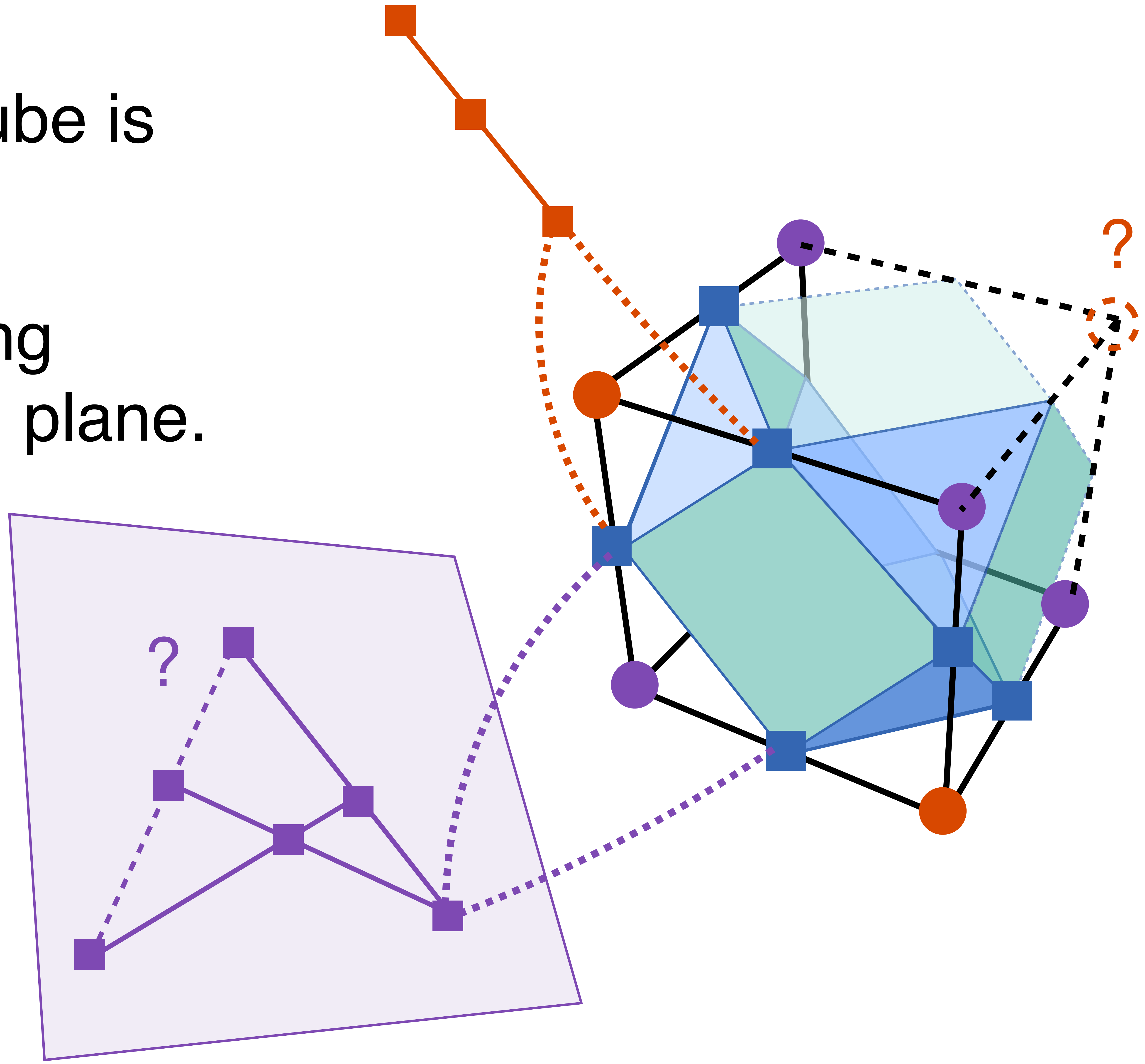

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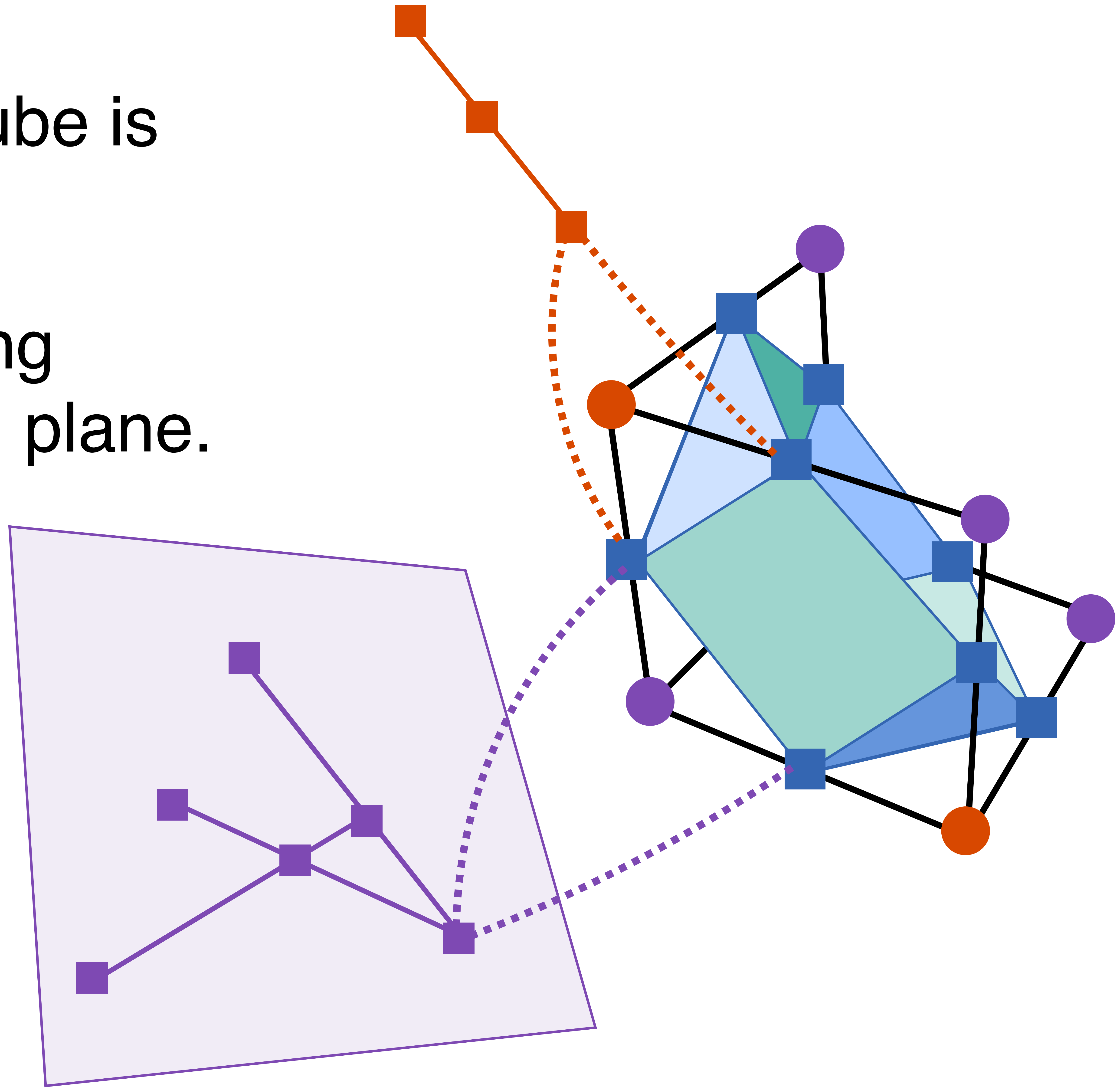

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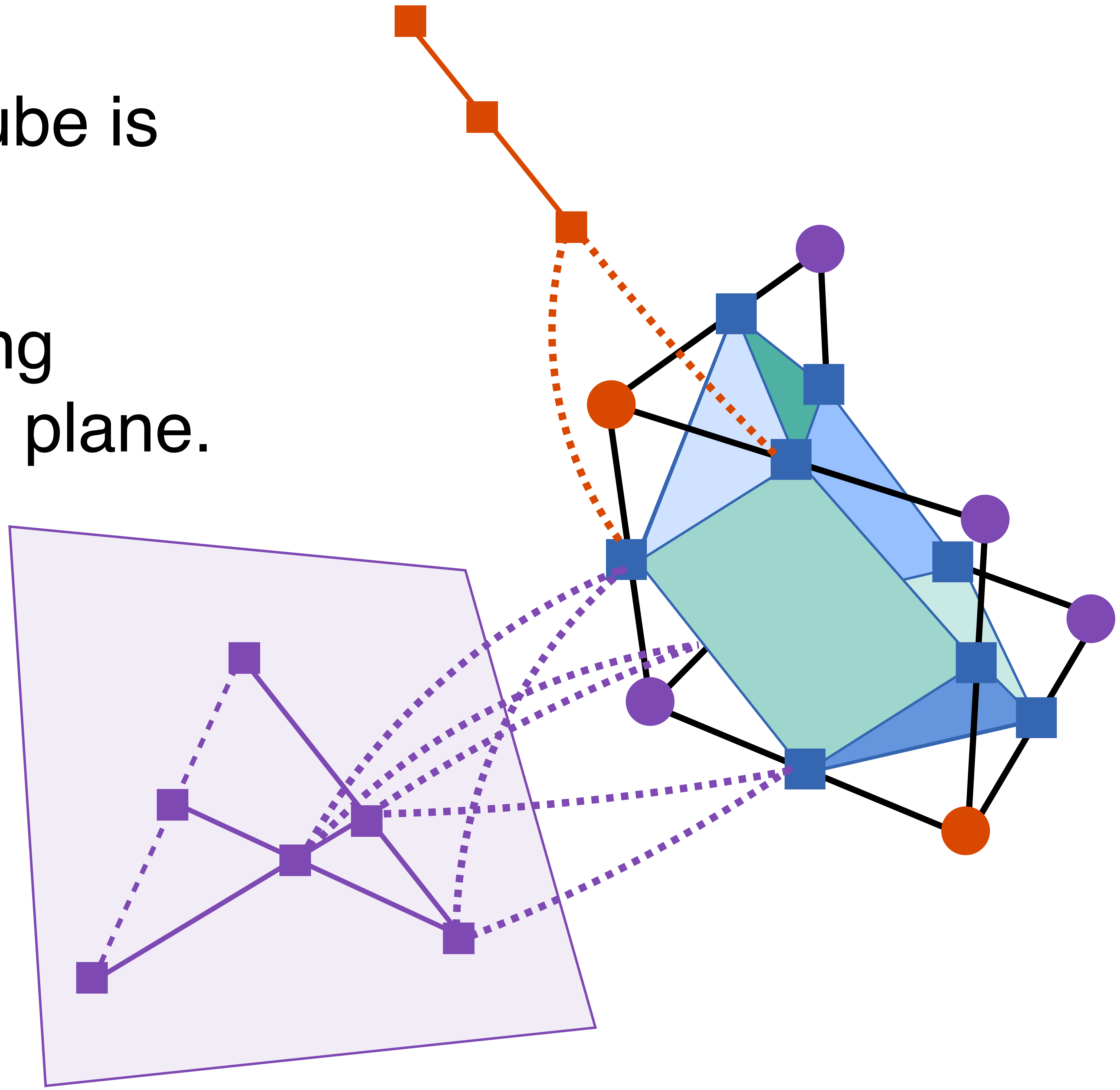

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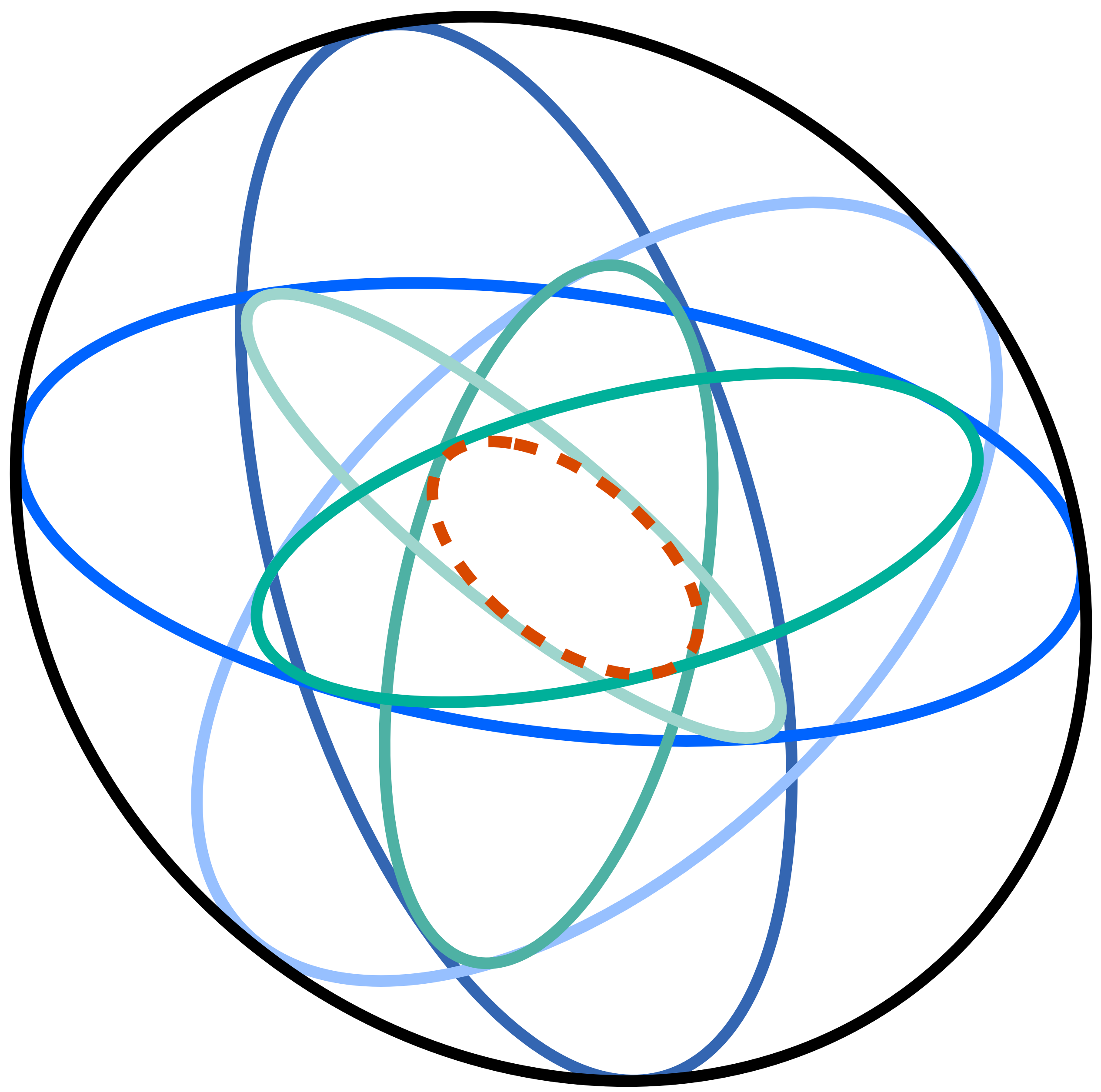
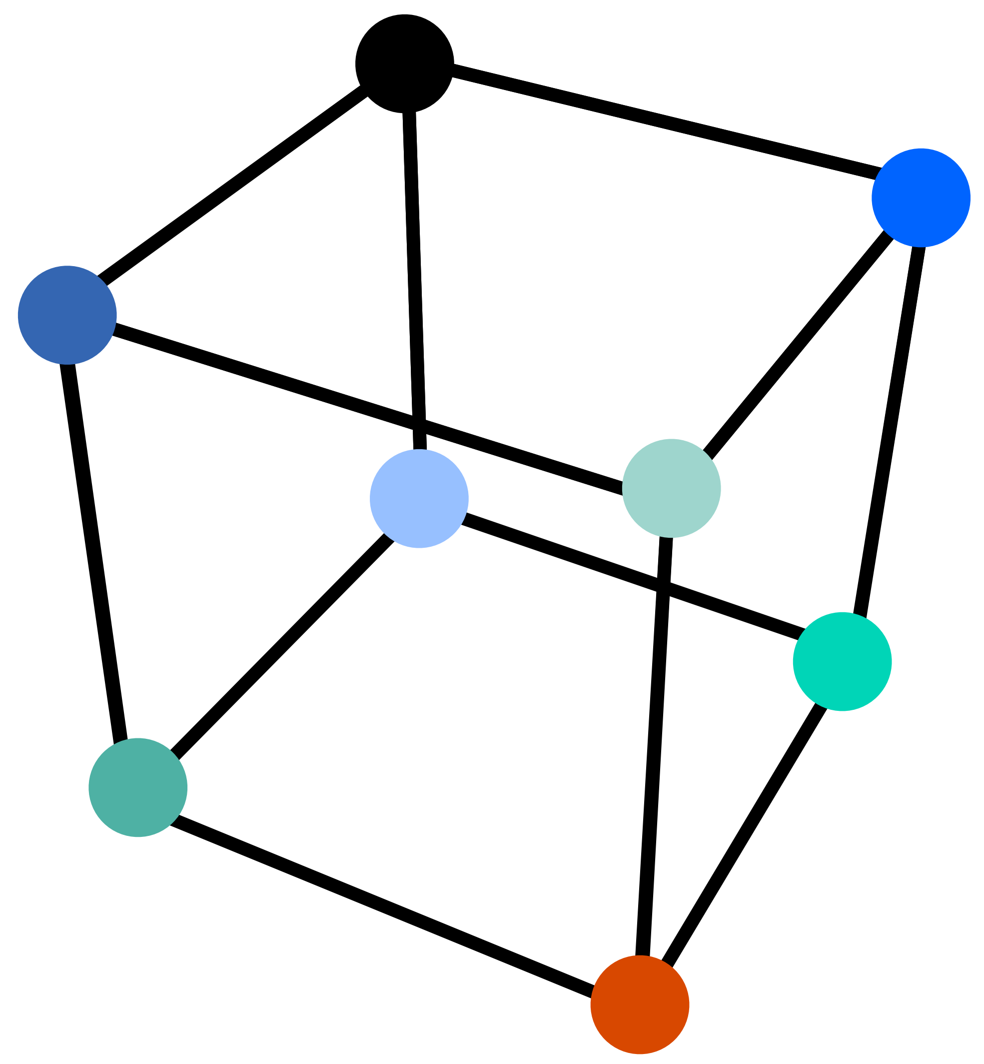
- Proof in the $\mathbb{P}^5$ space of conics
- Penrose's approach (undergrad)
- Penrose's 3D approach (Cambridge)





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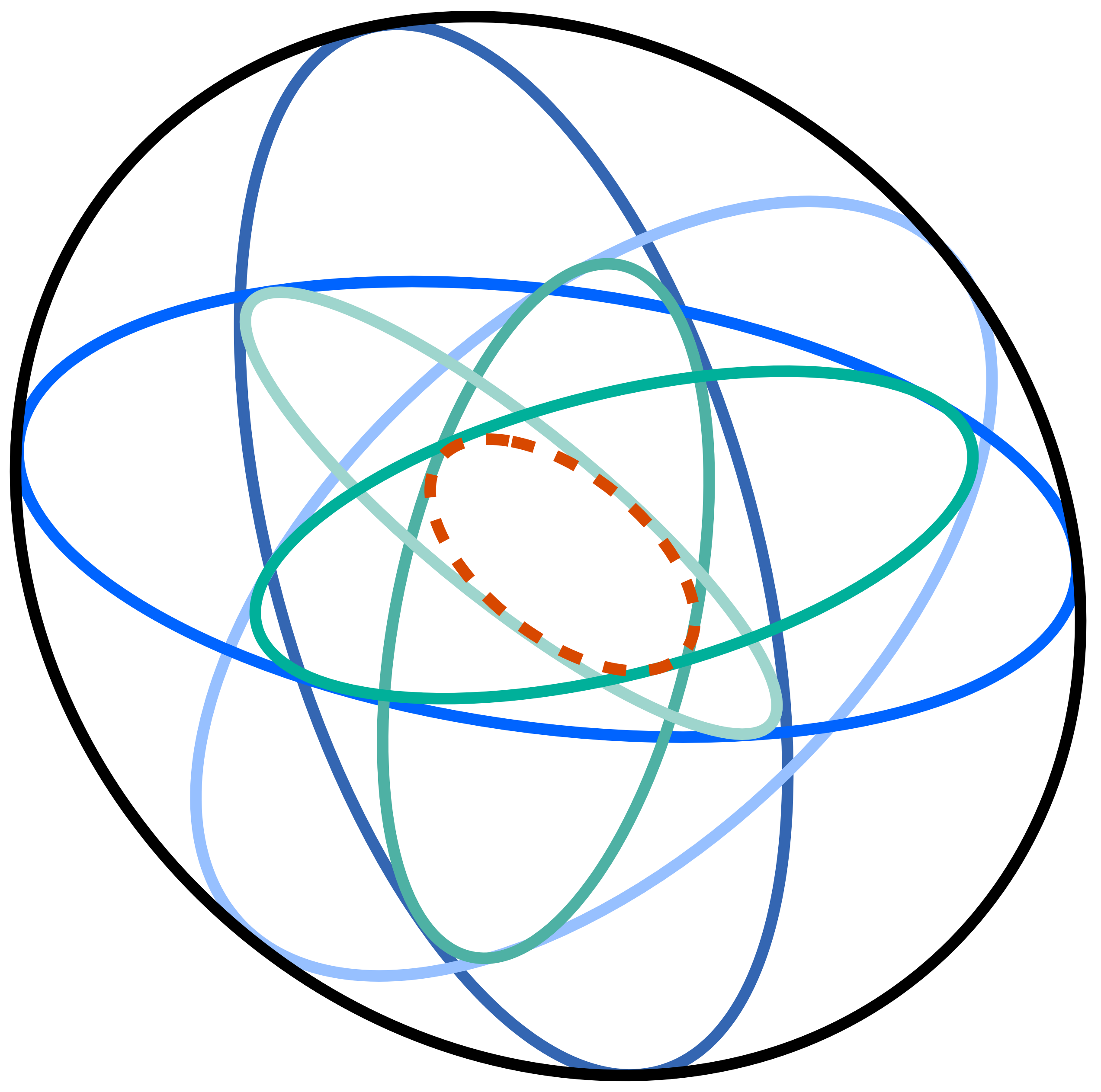

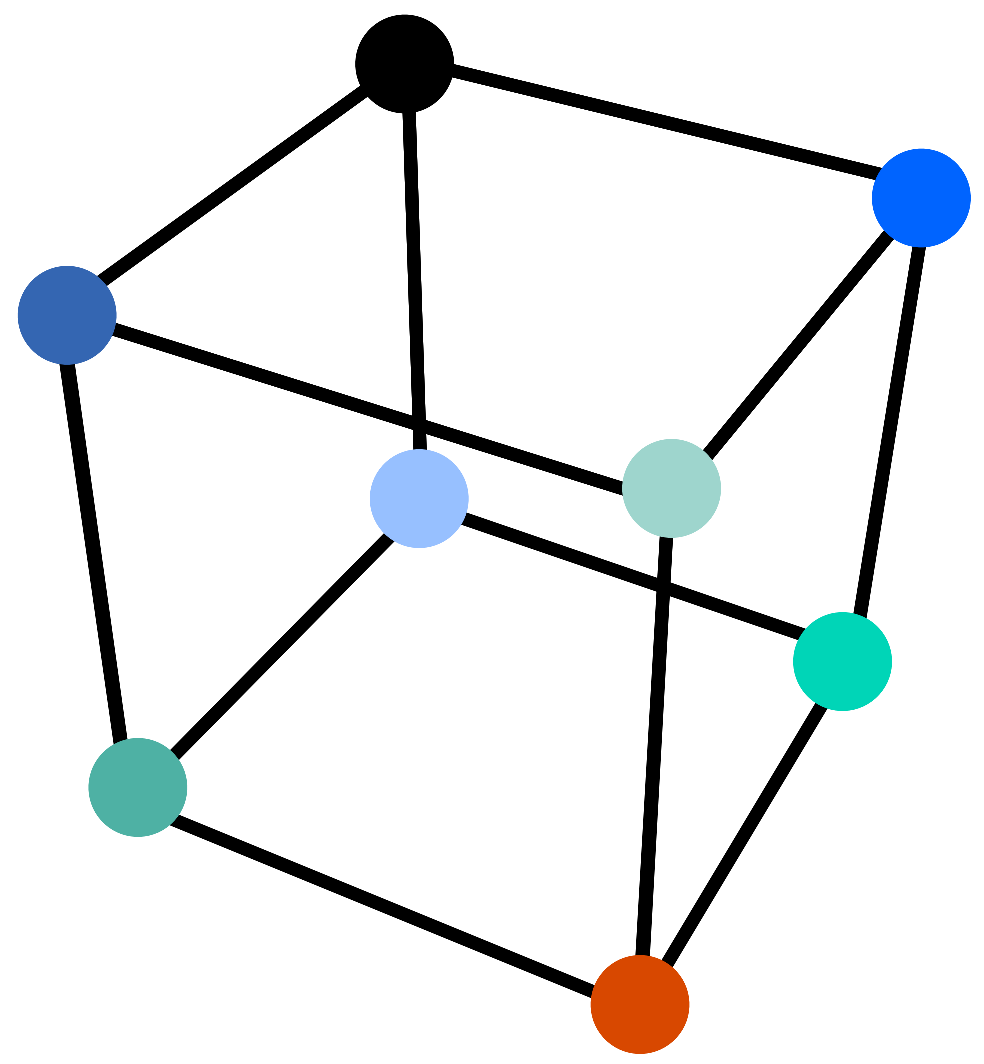

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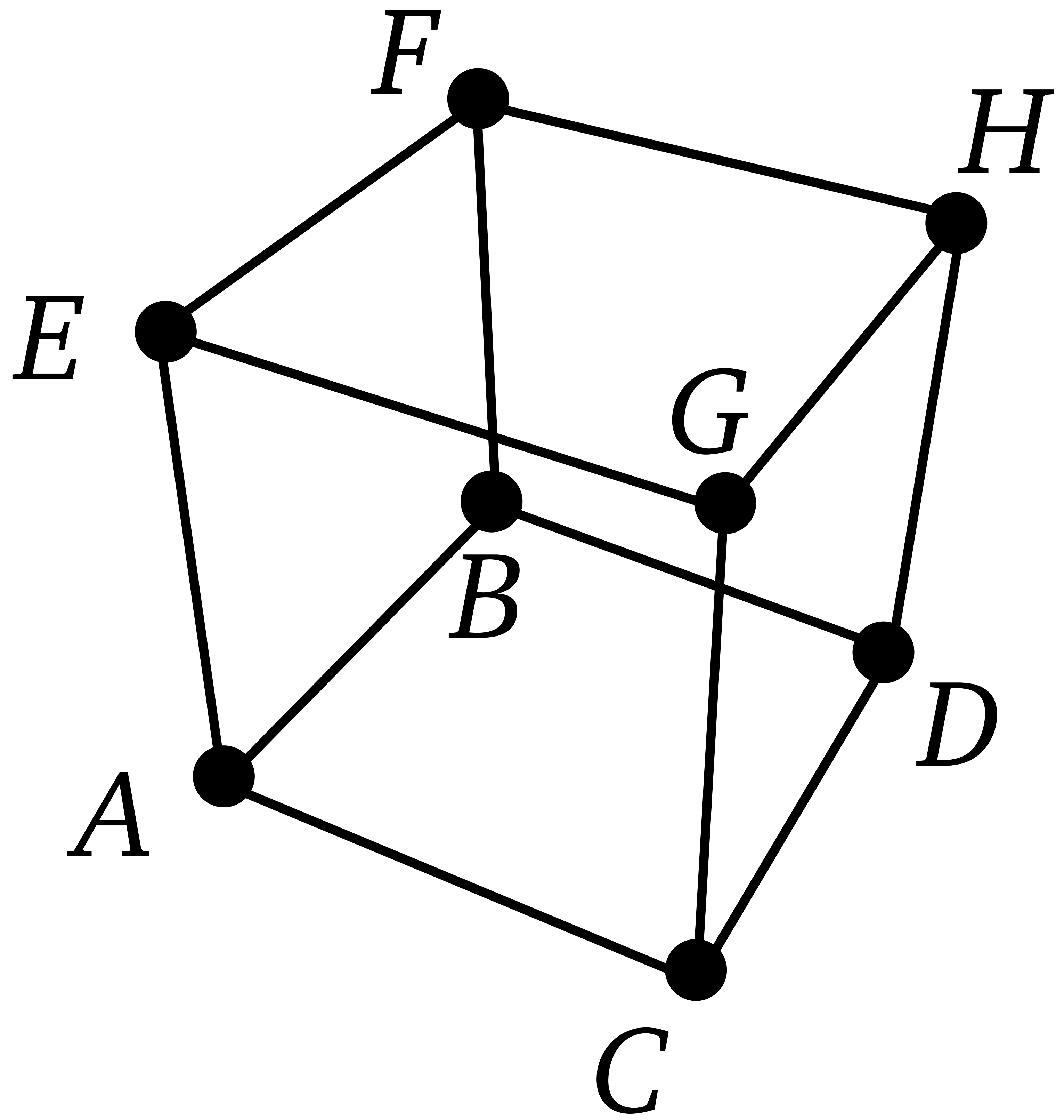

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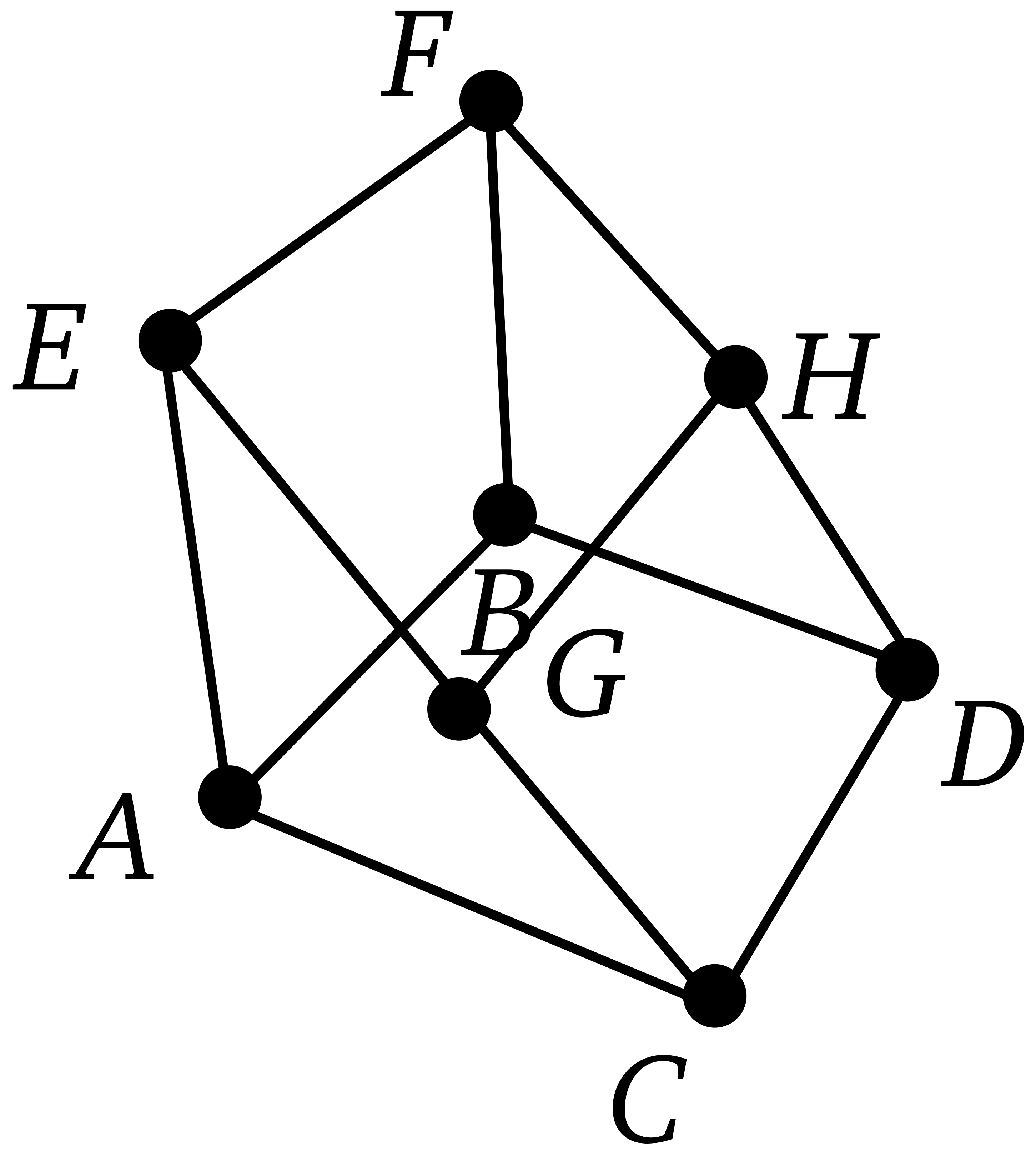

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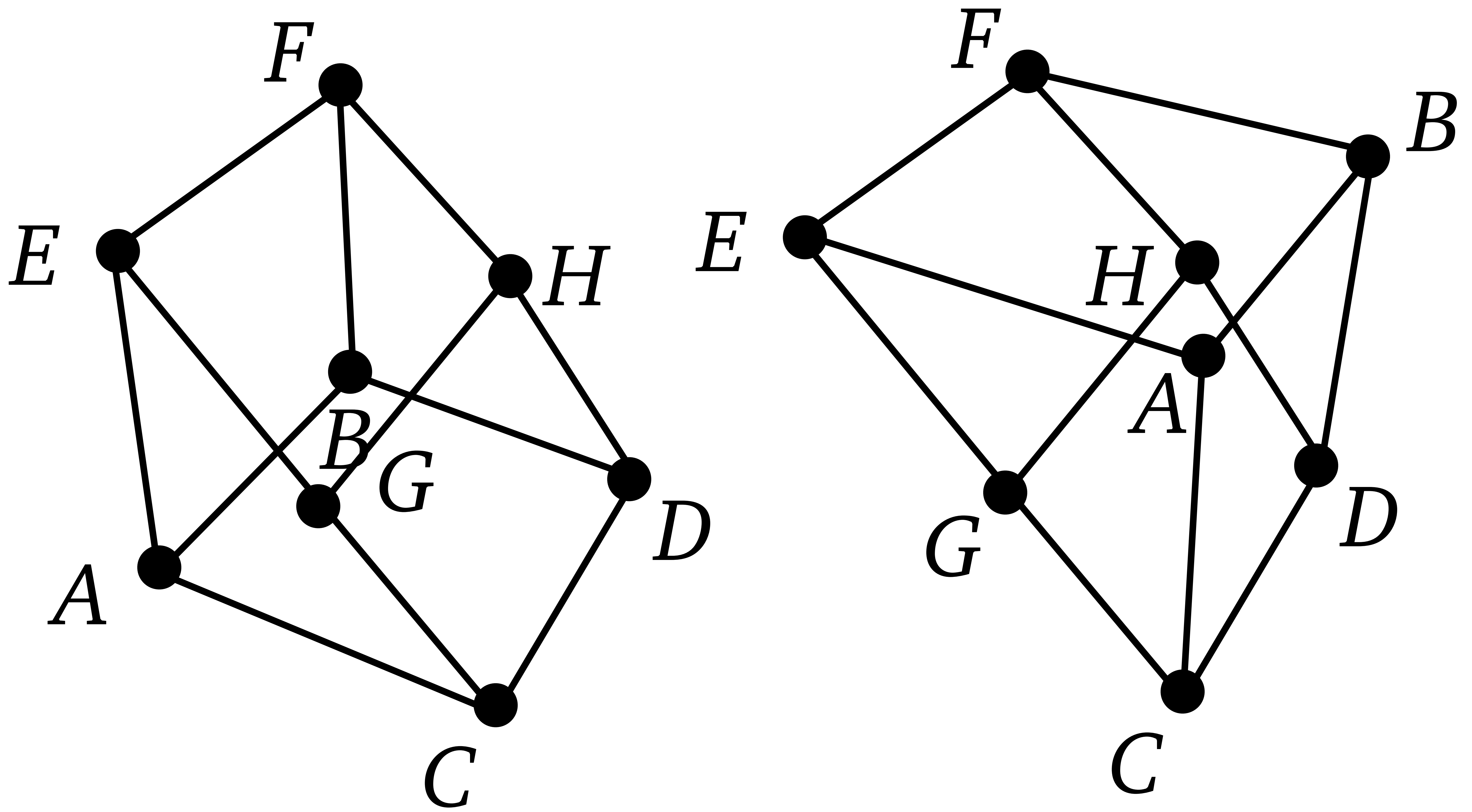

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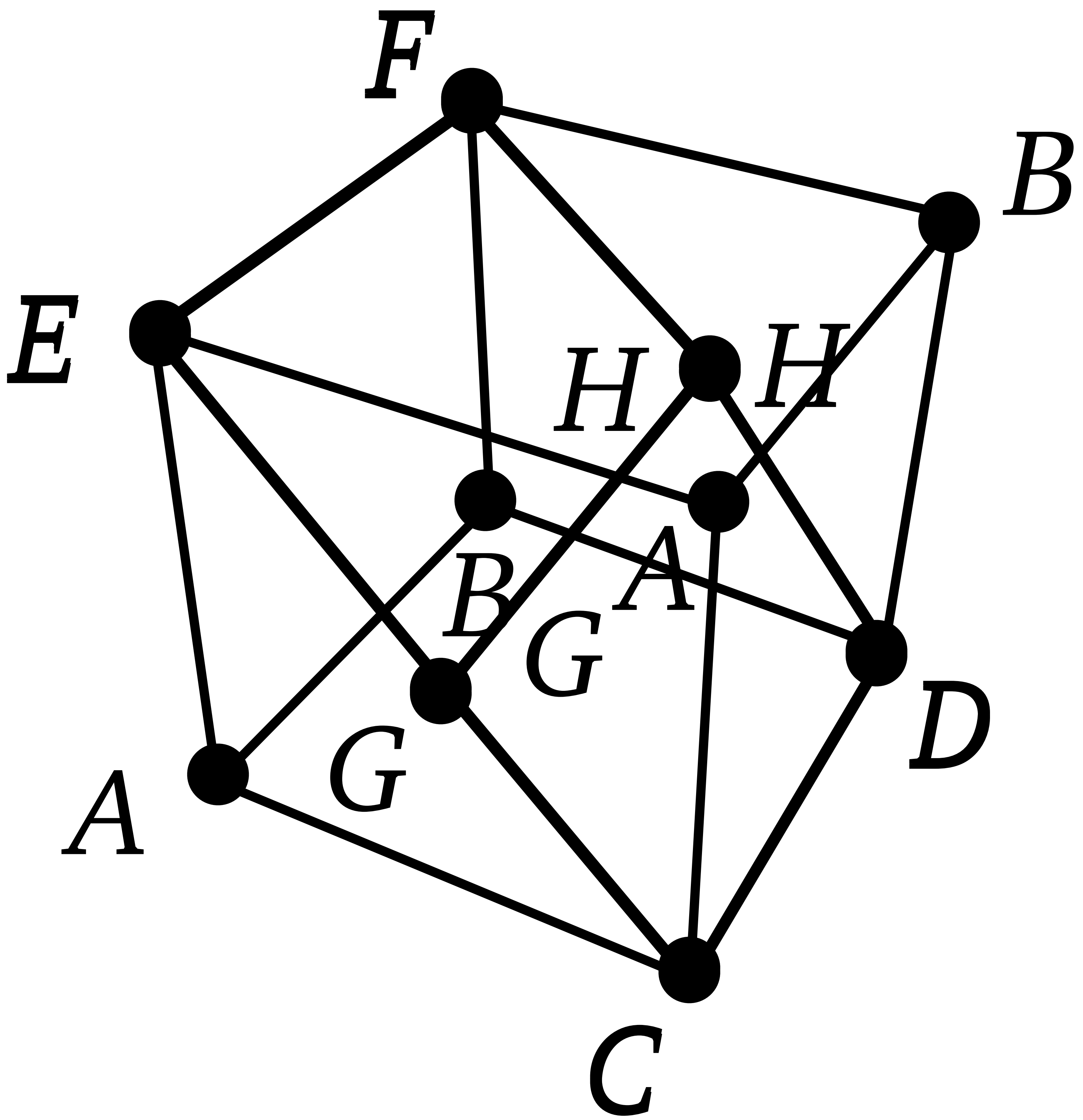

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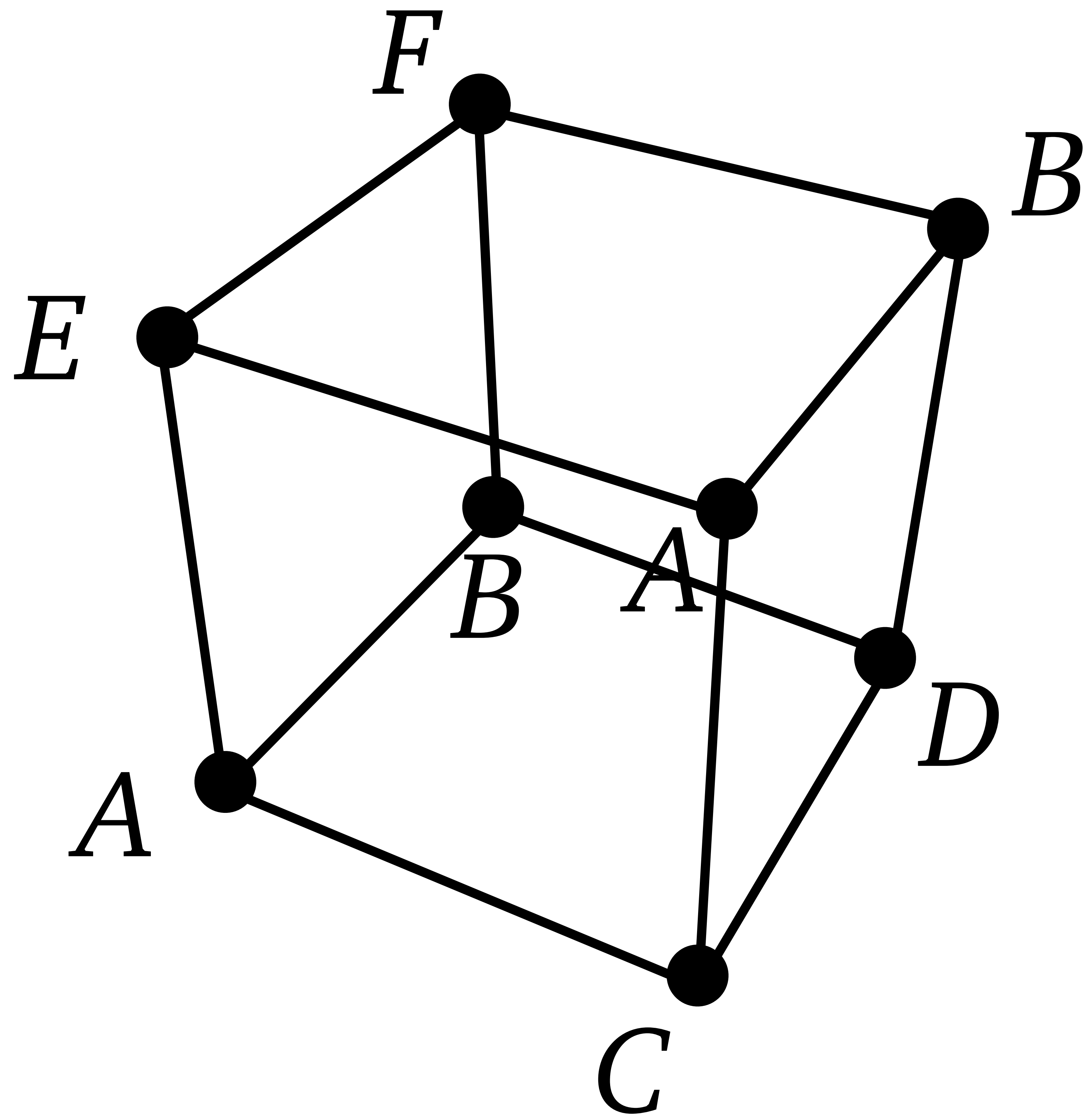

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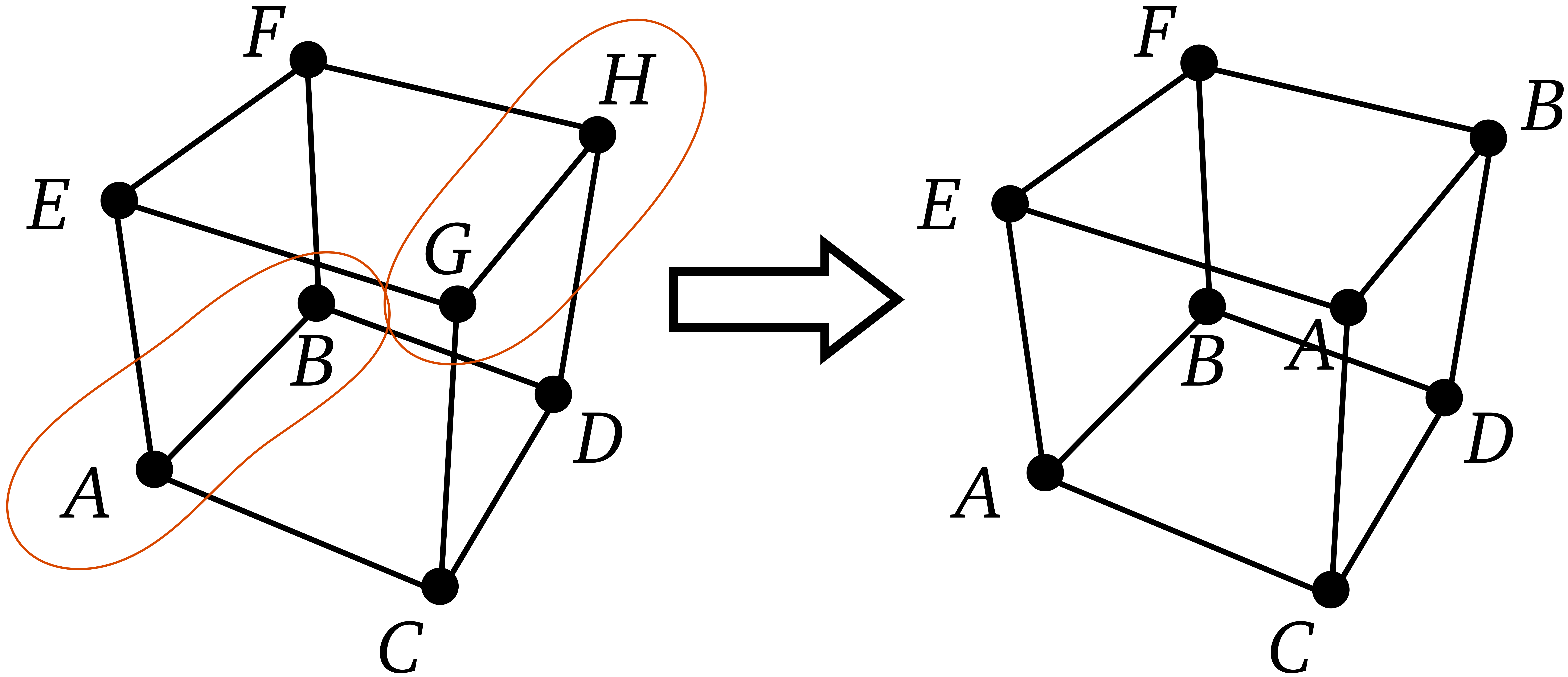

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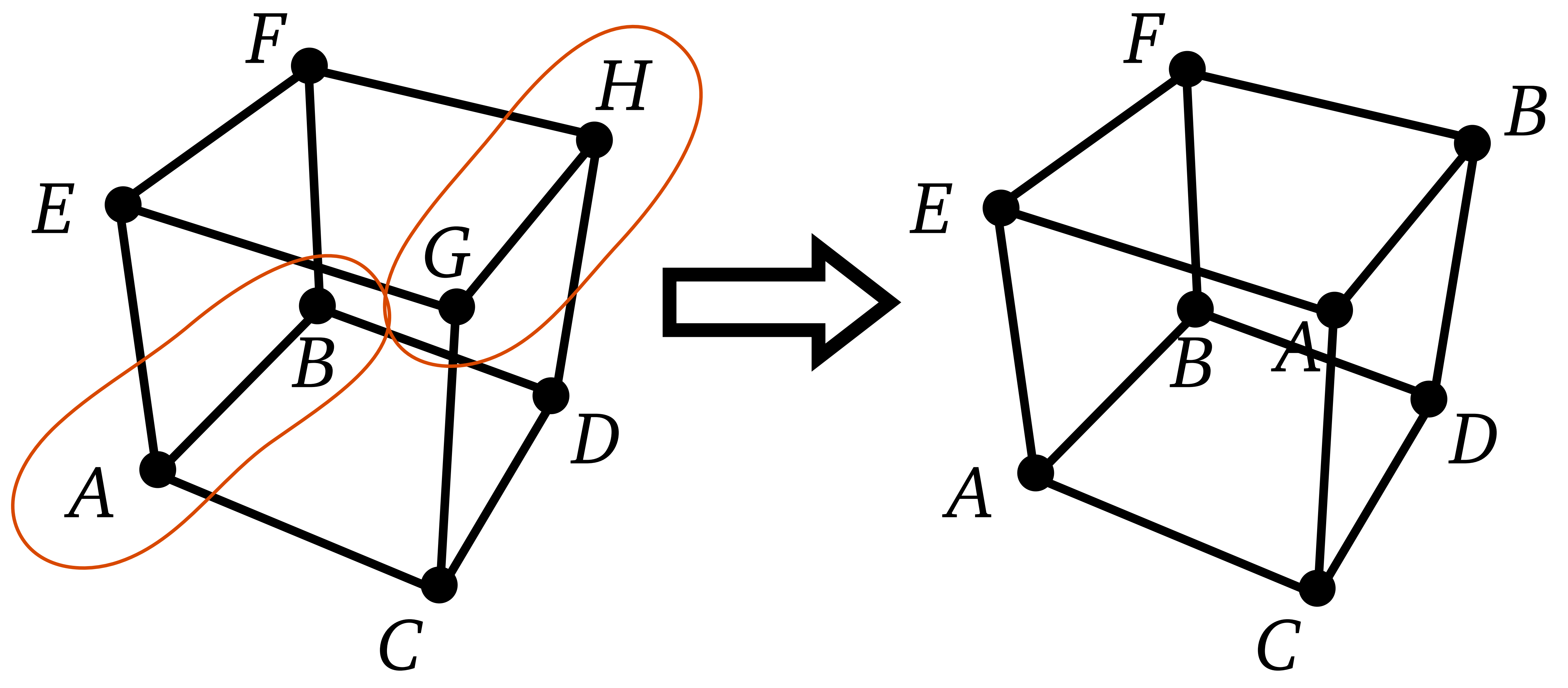

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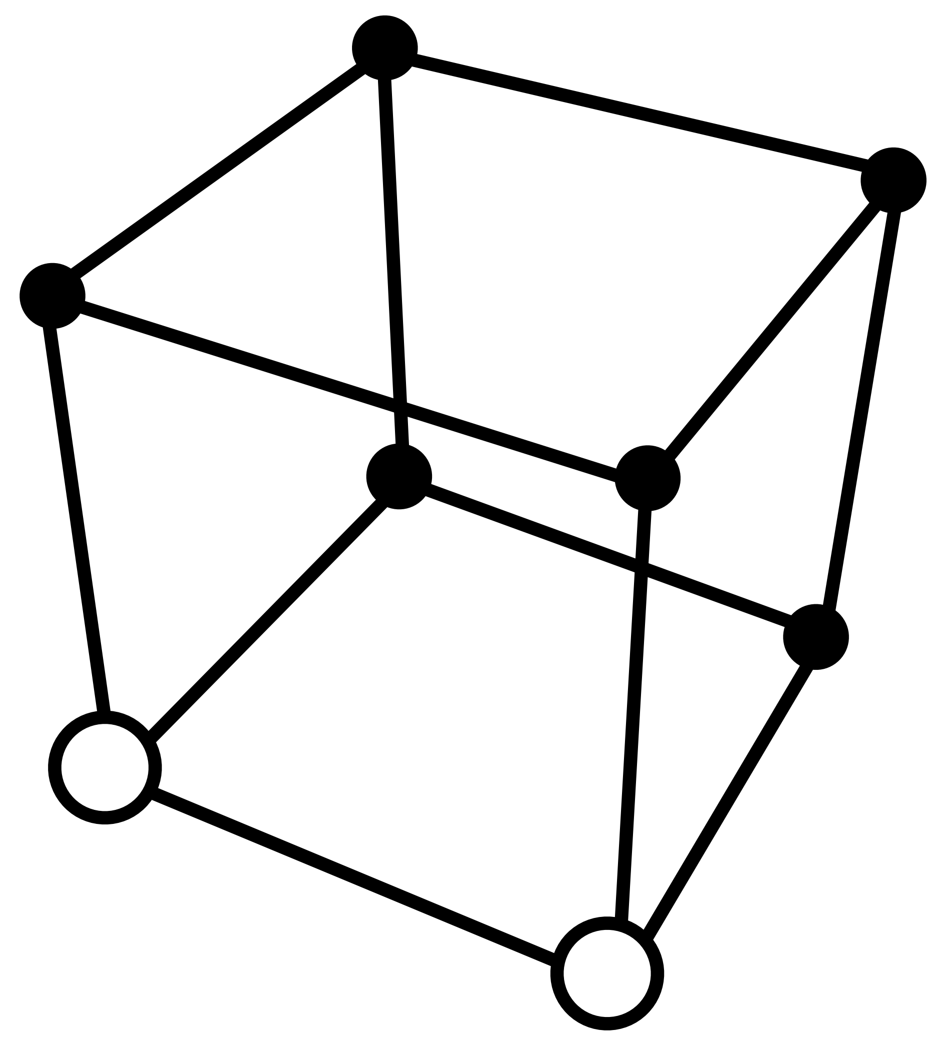

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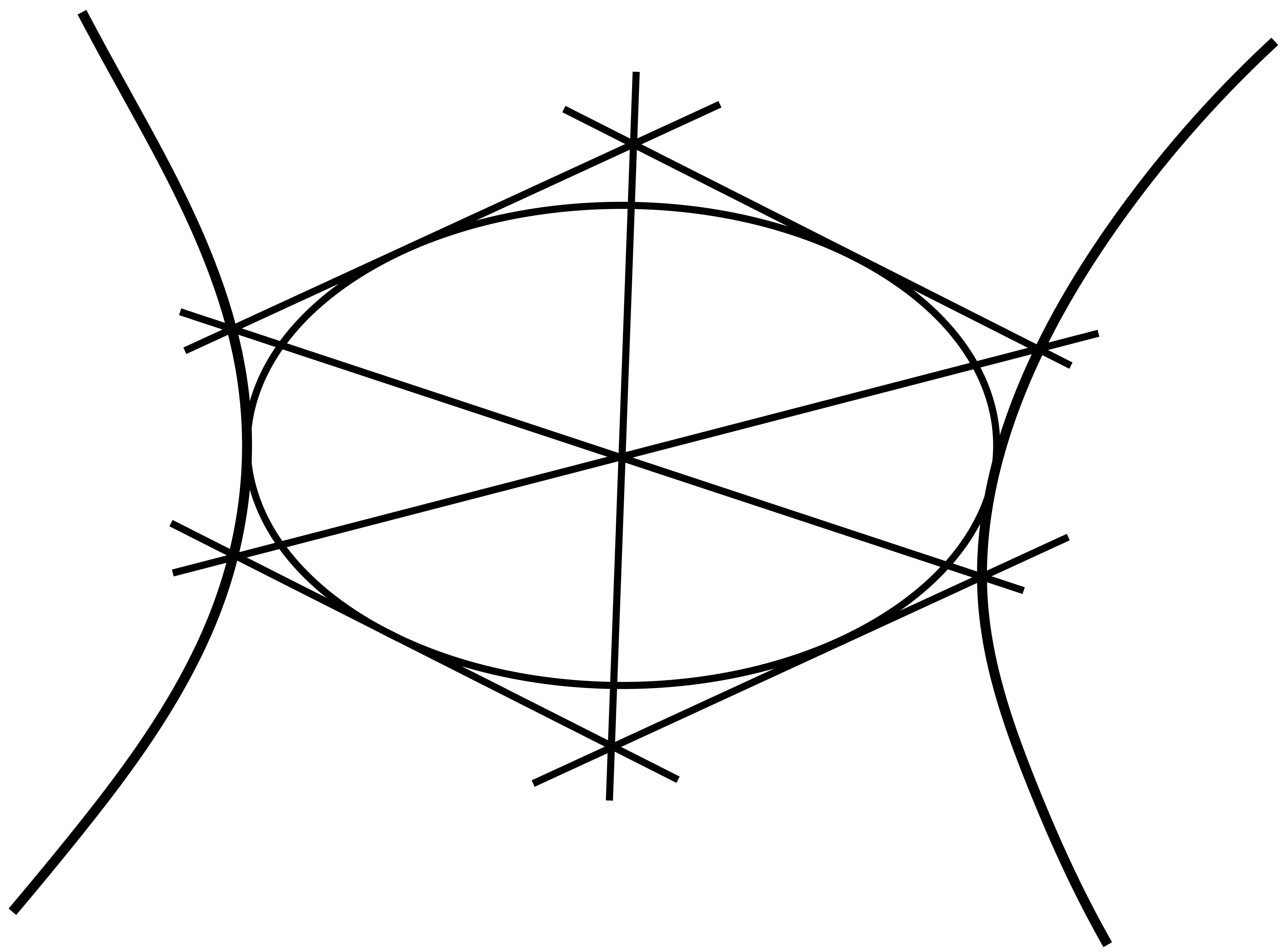

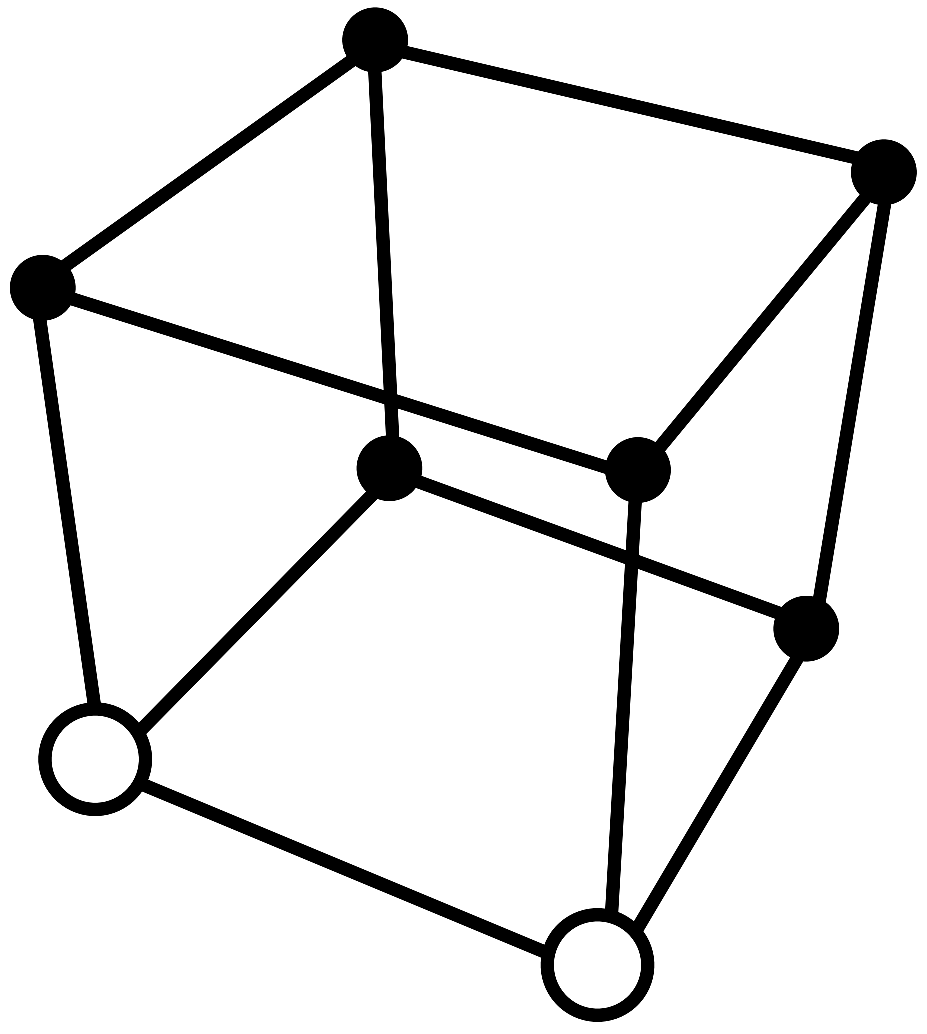

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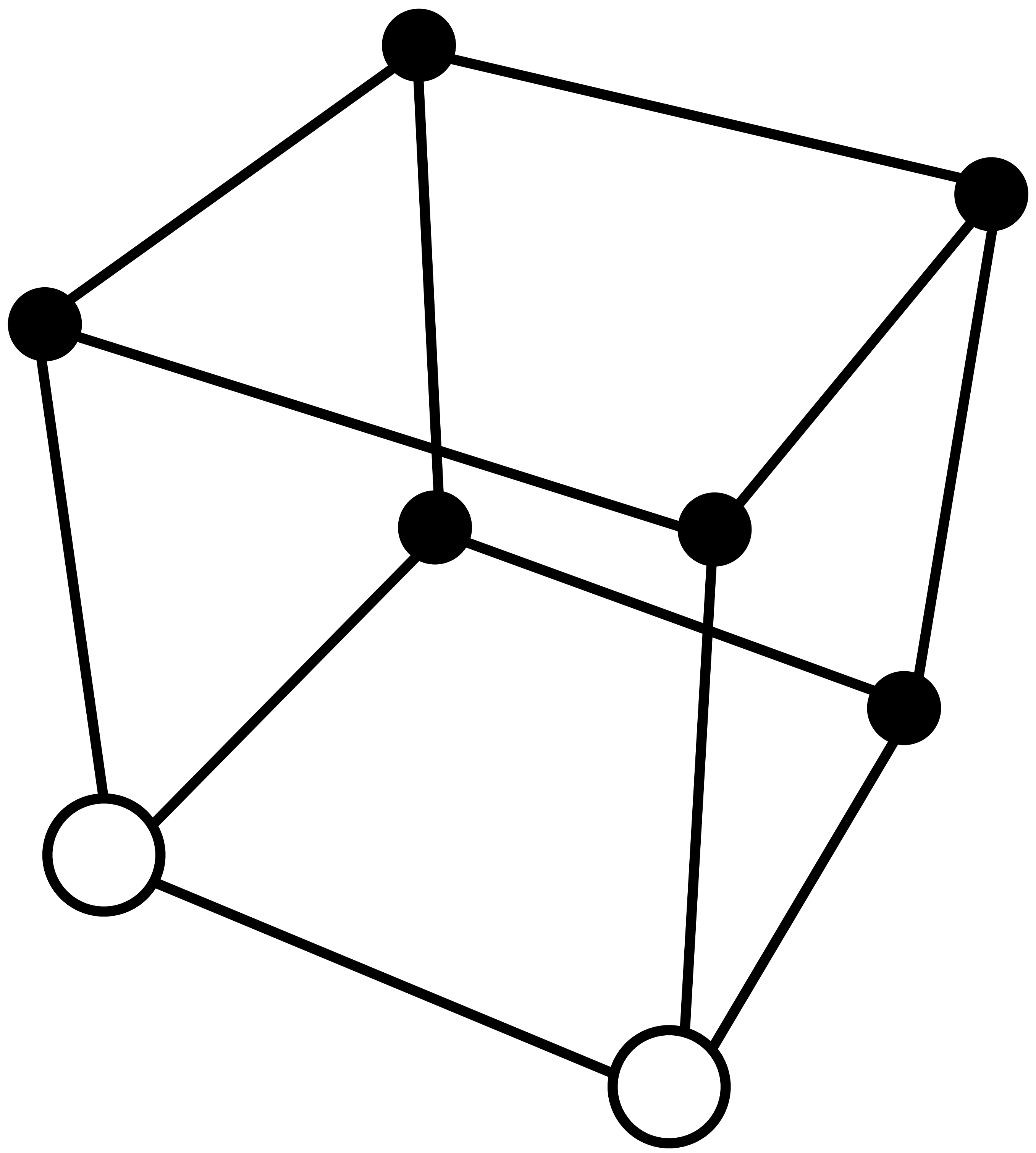

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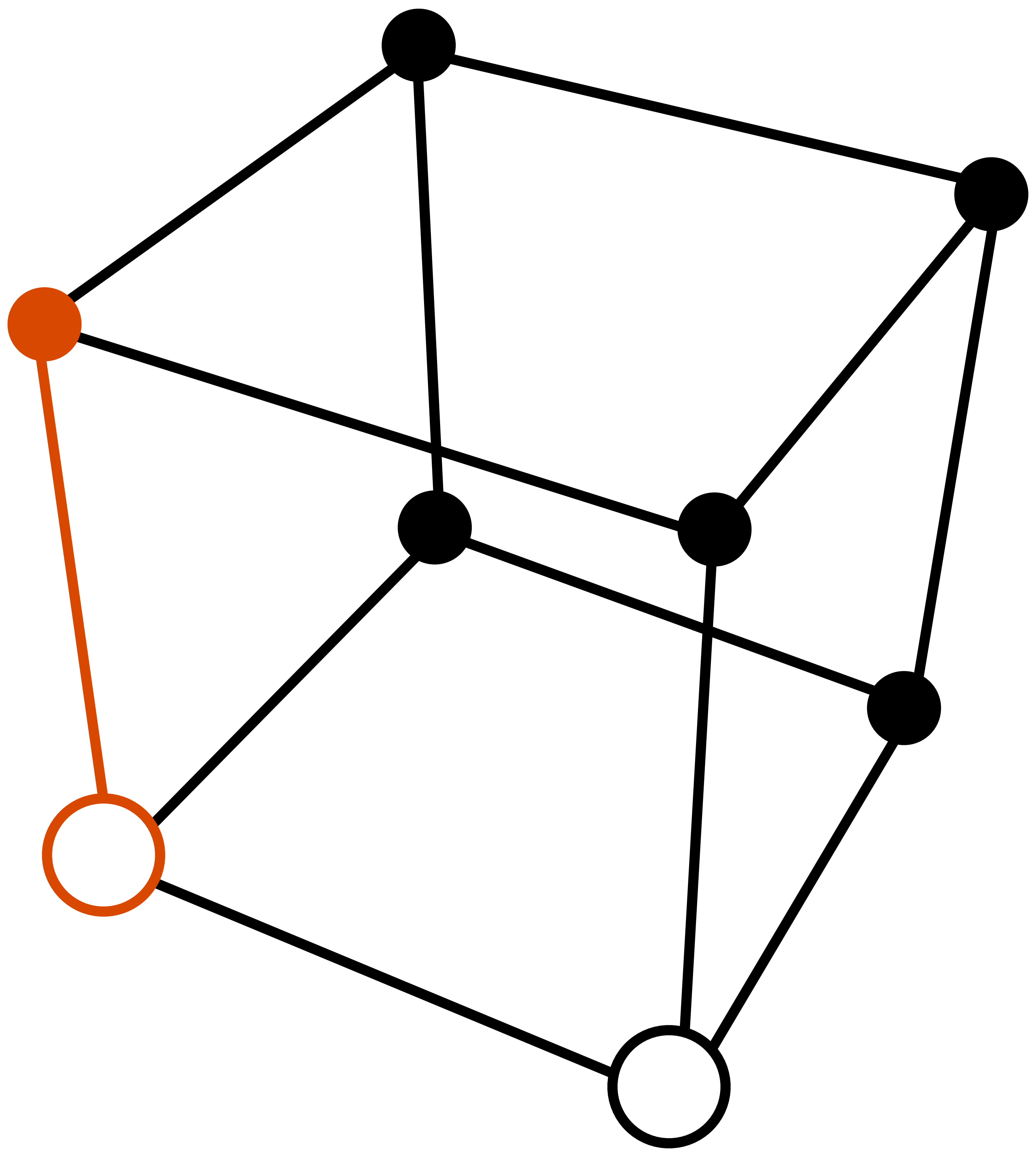

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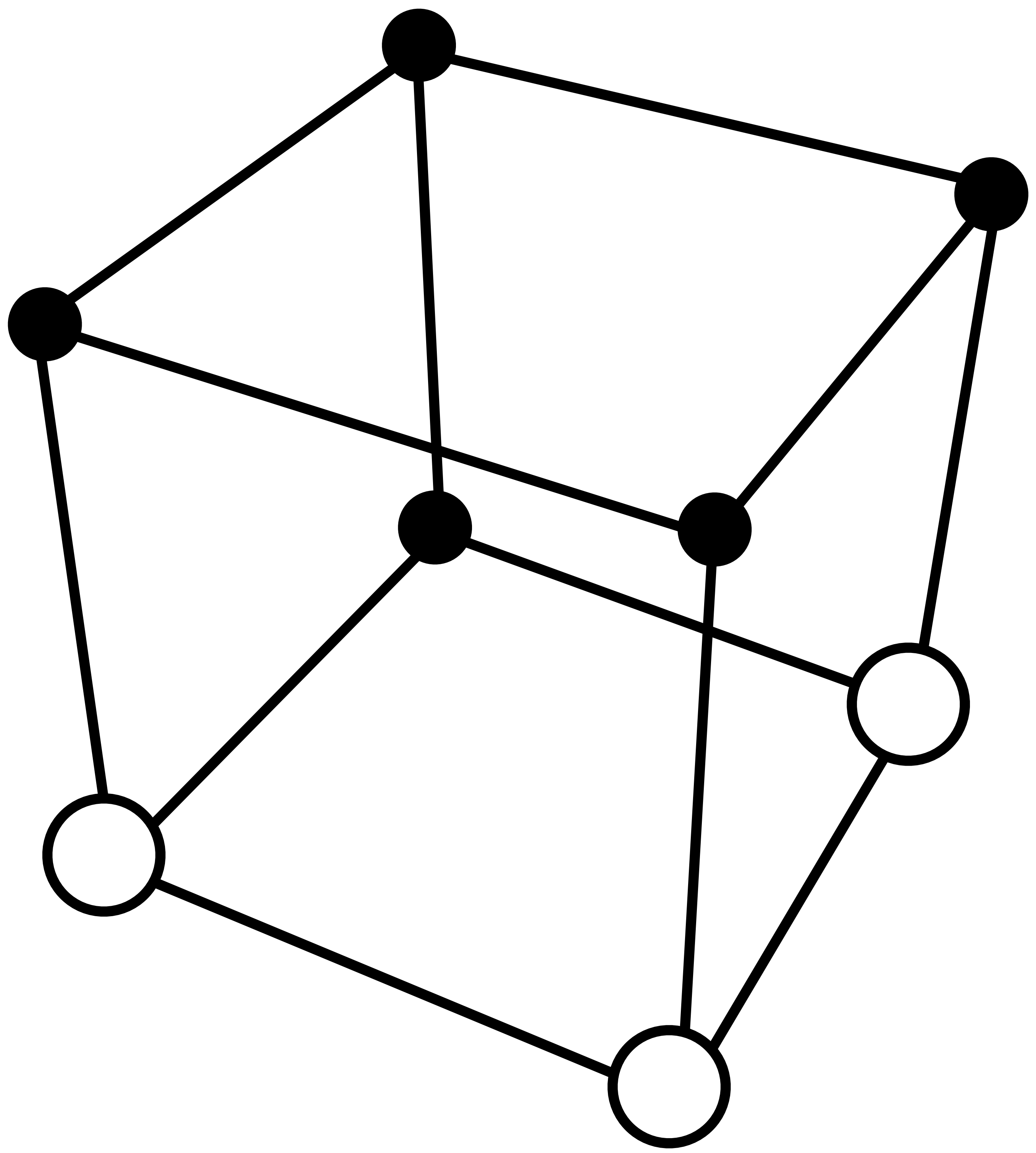

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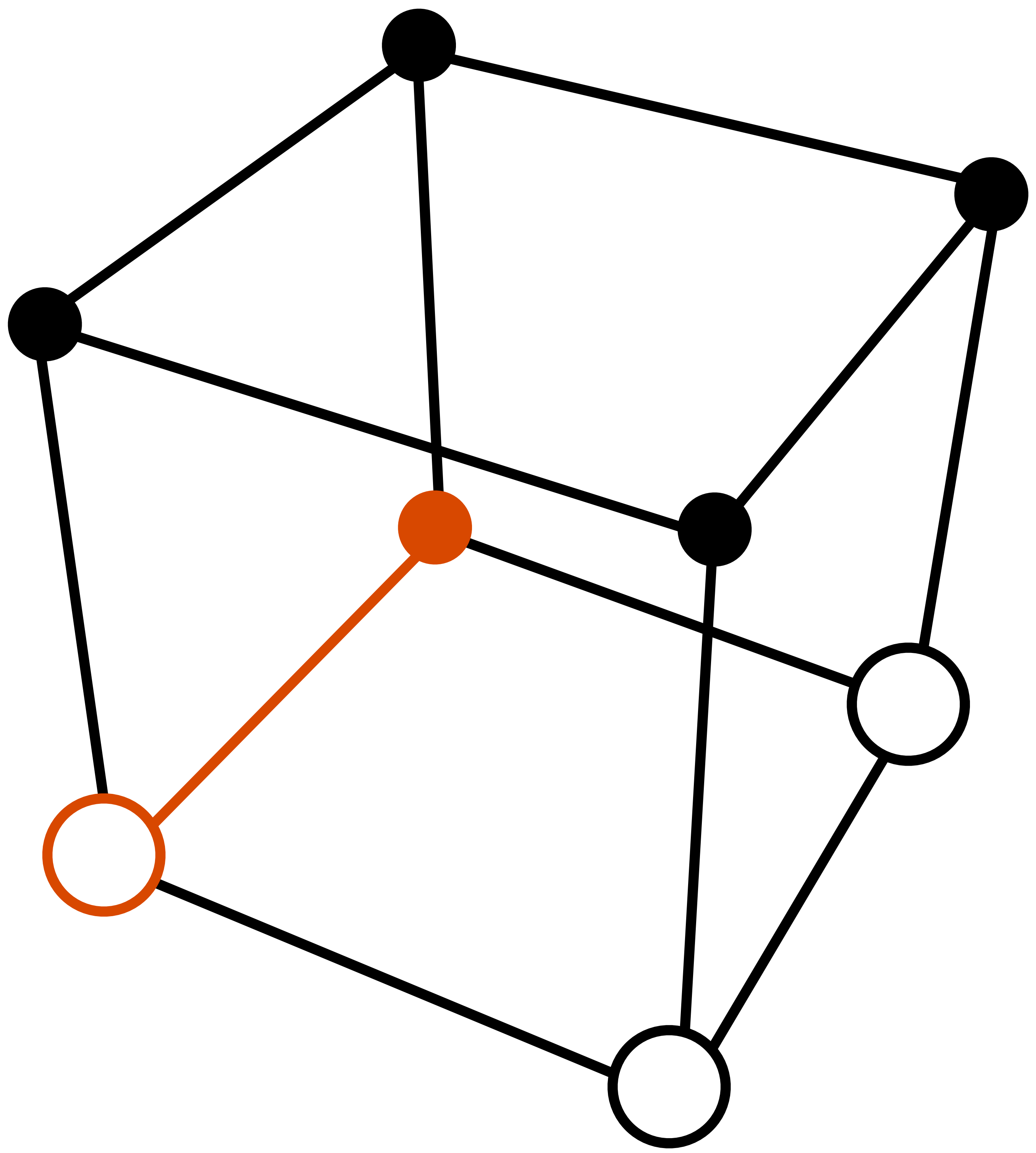

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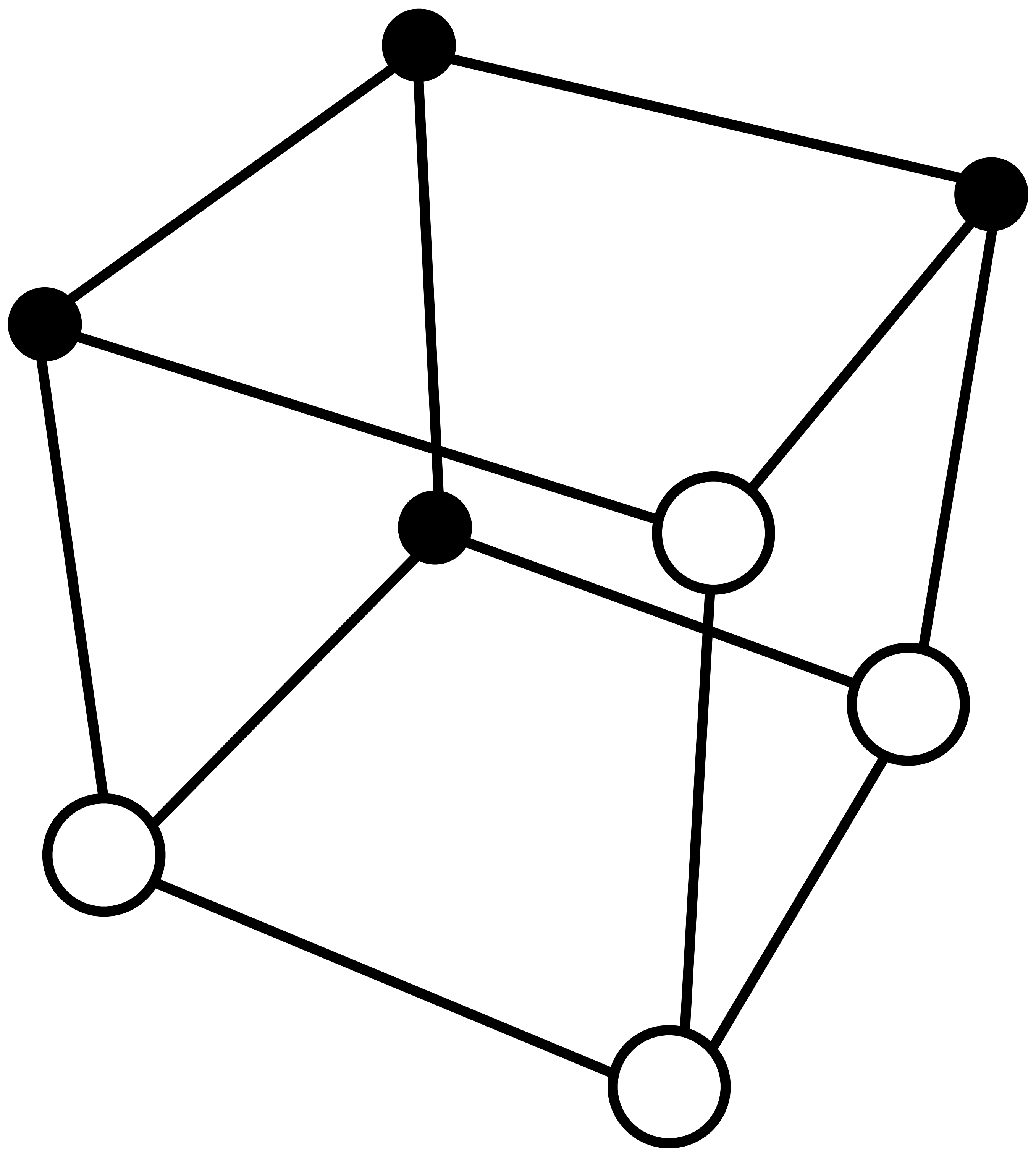

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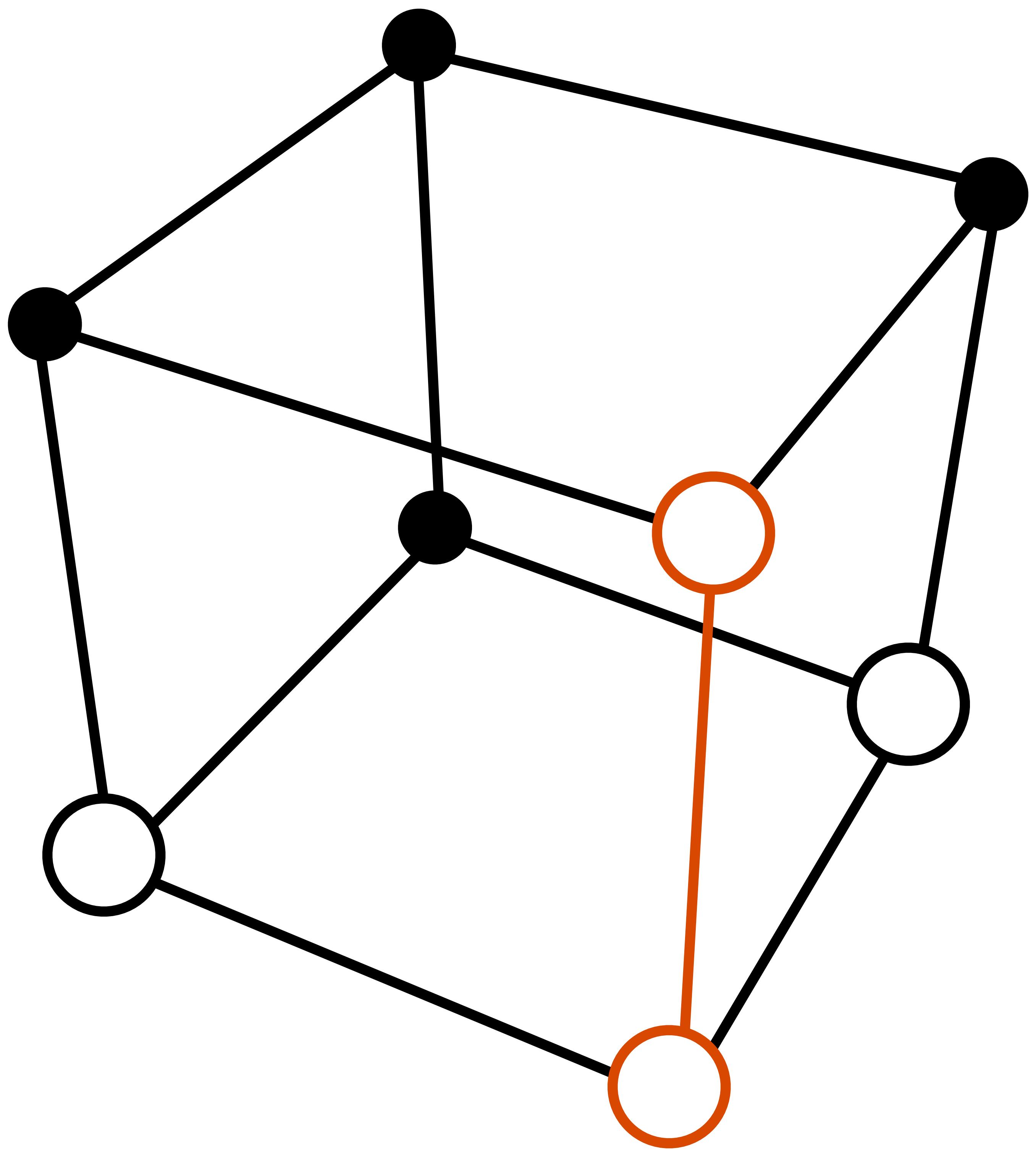

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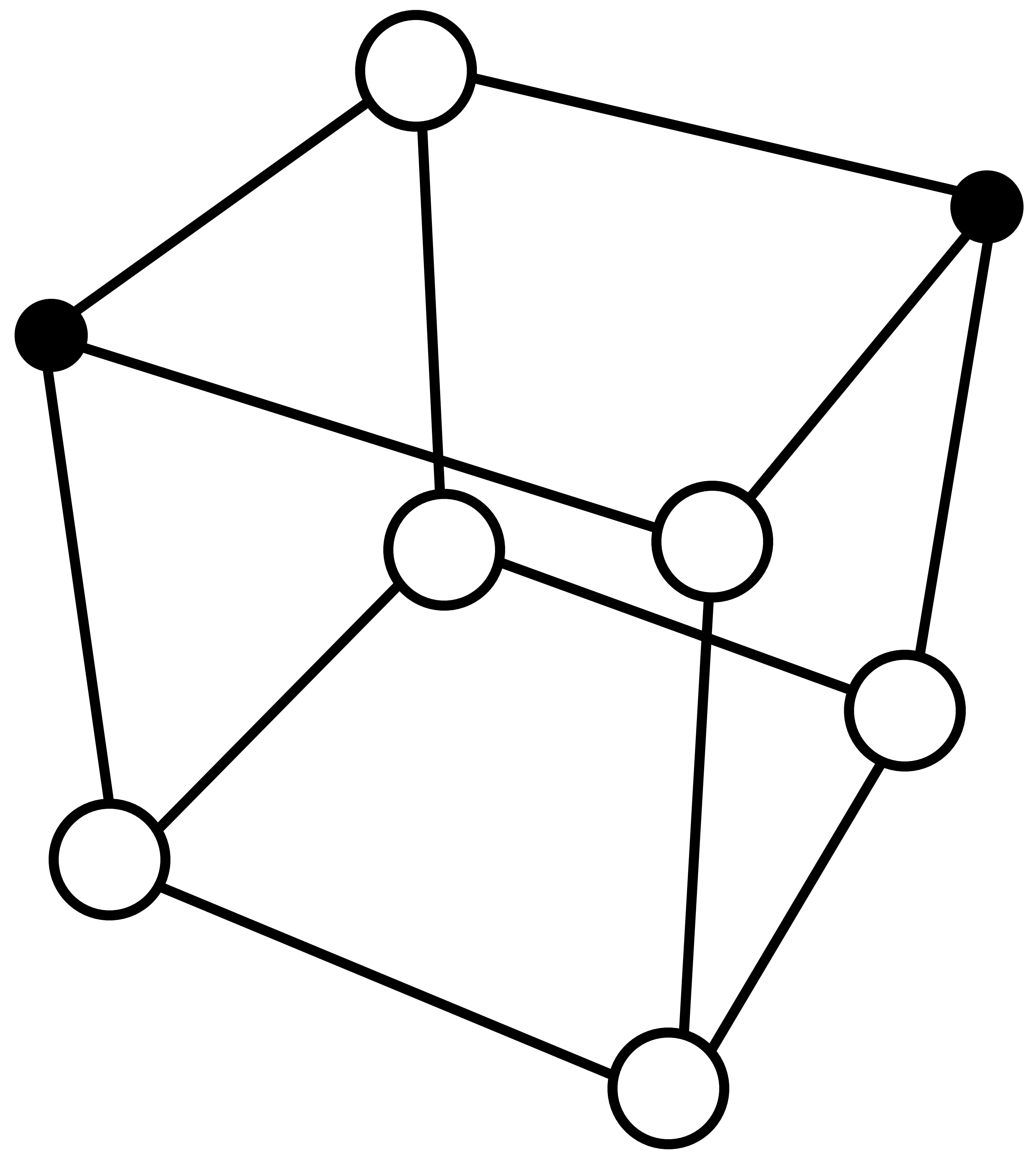

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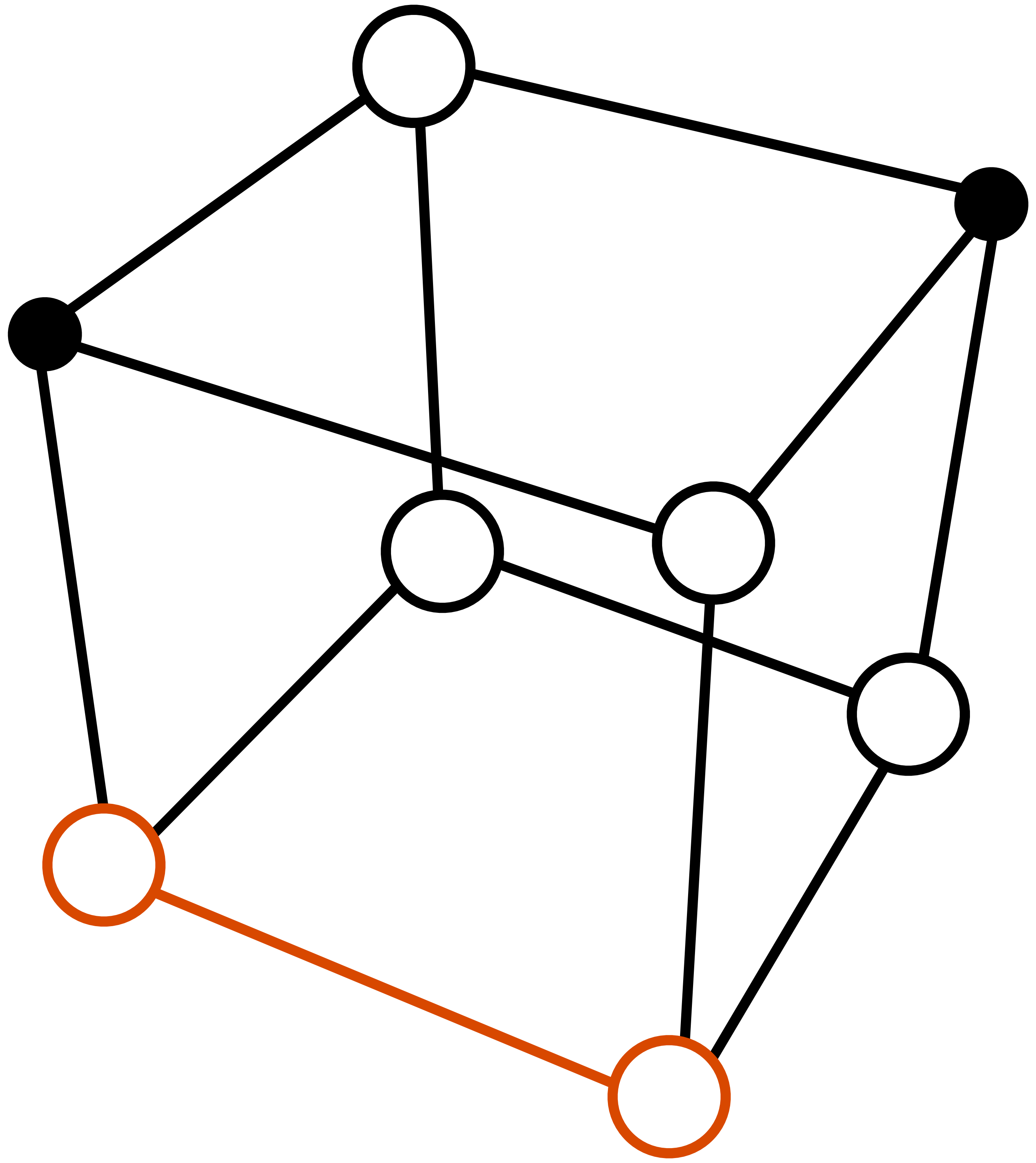

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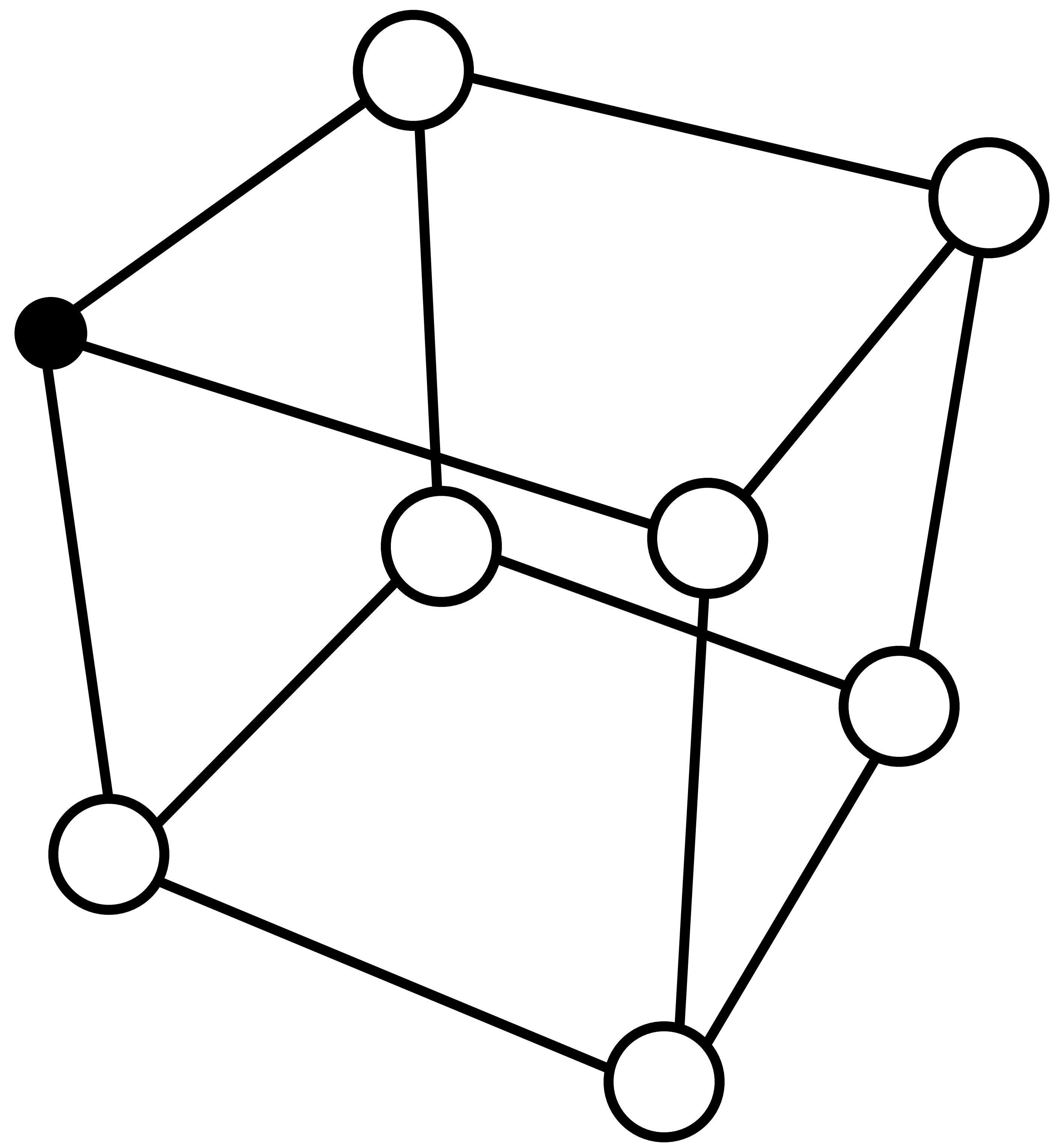

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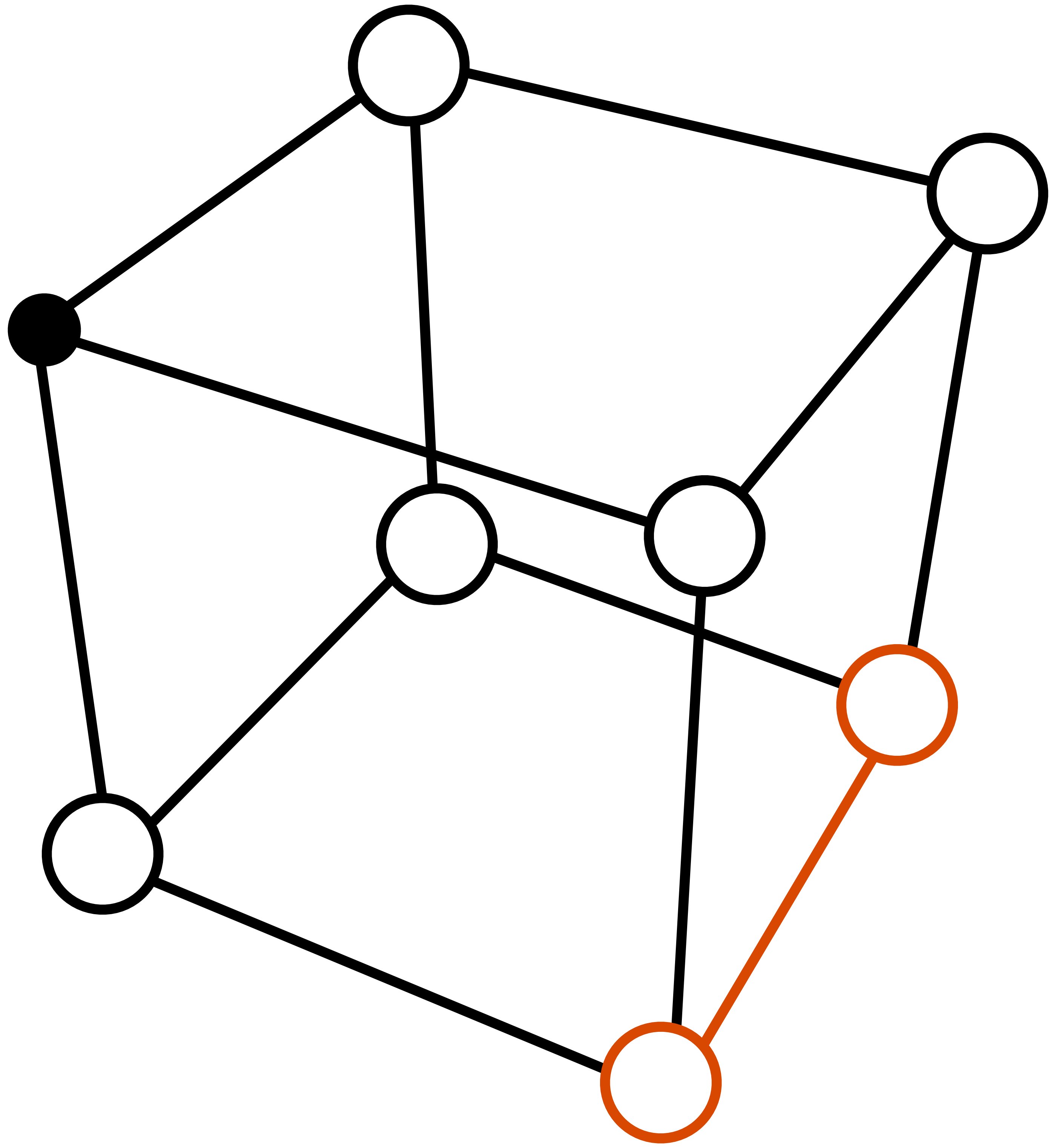

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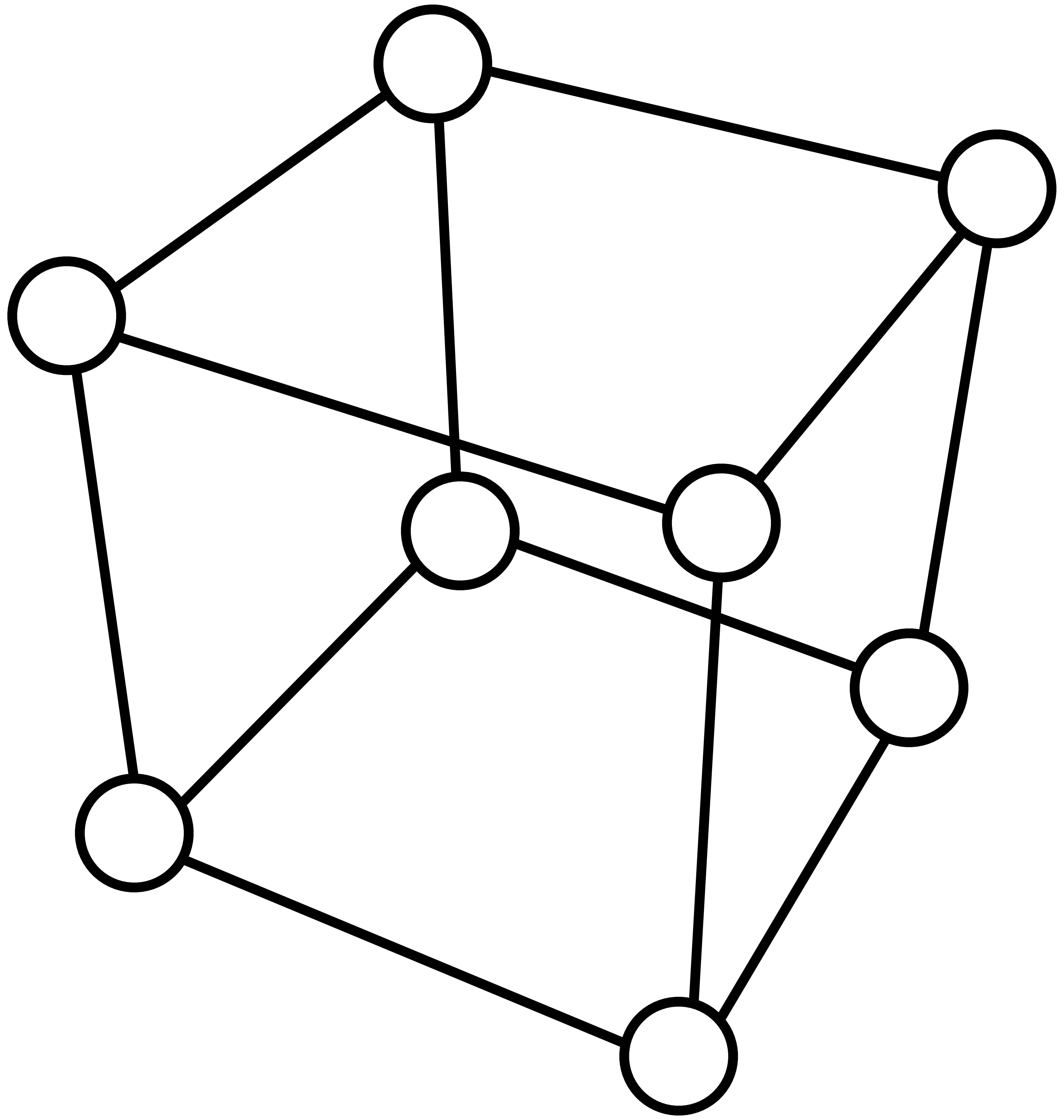

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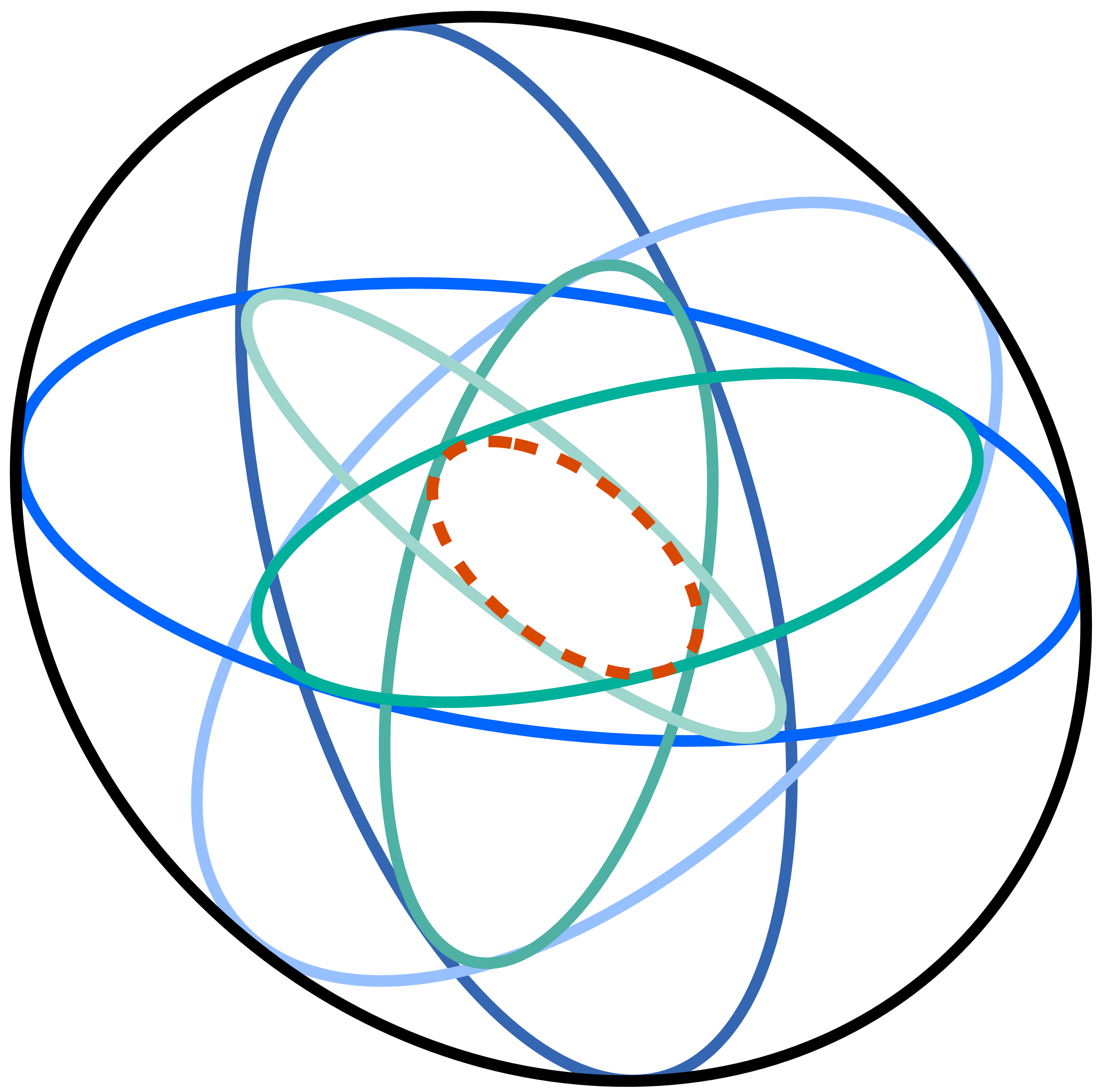

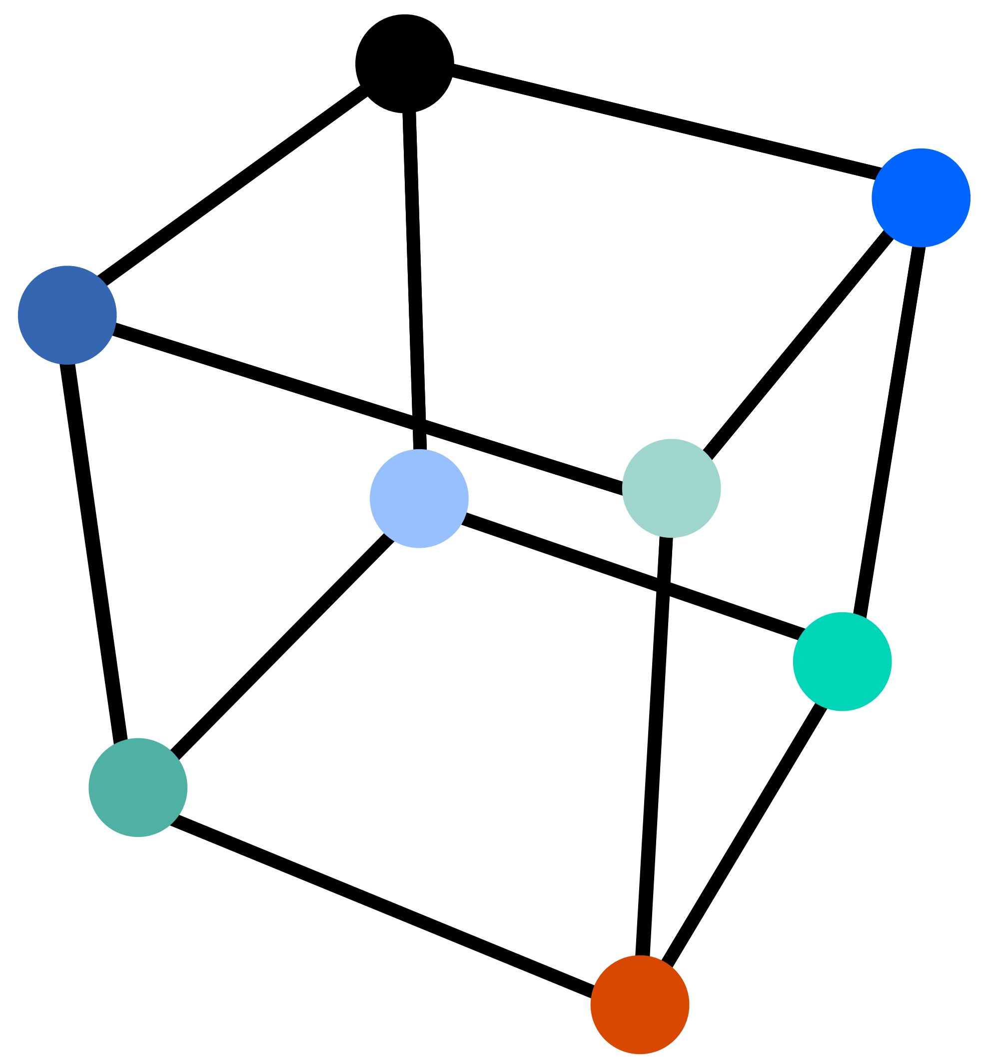

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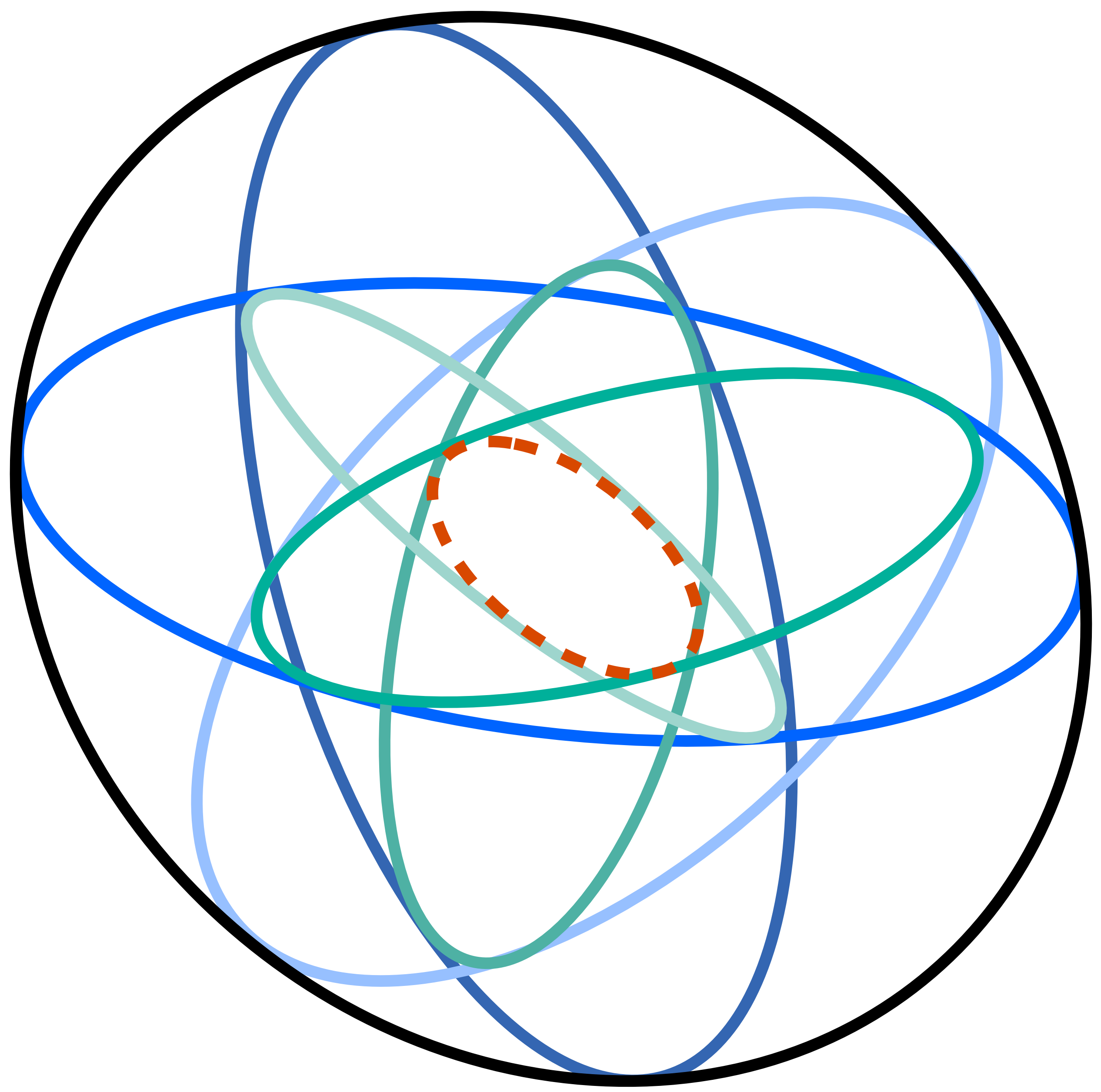
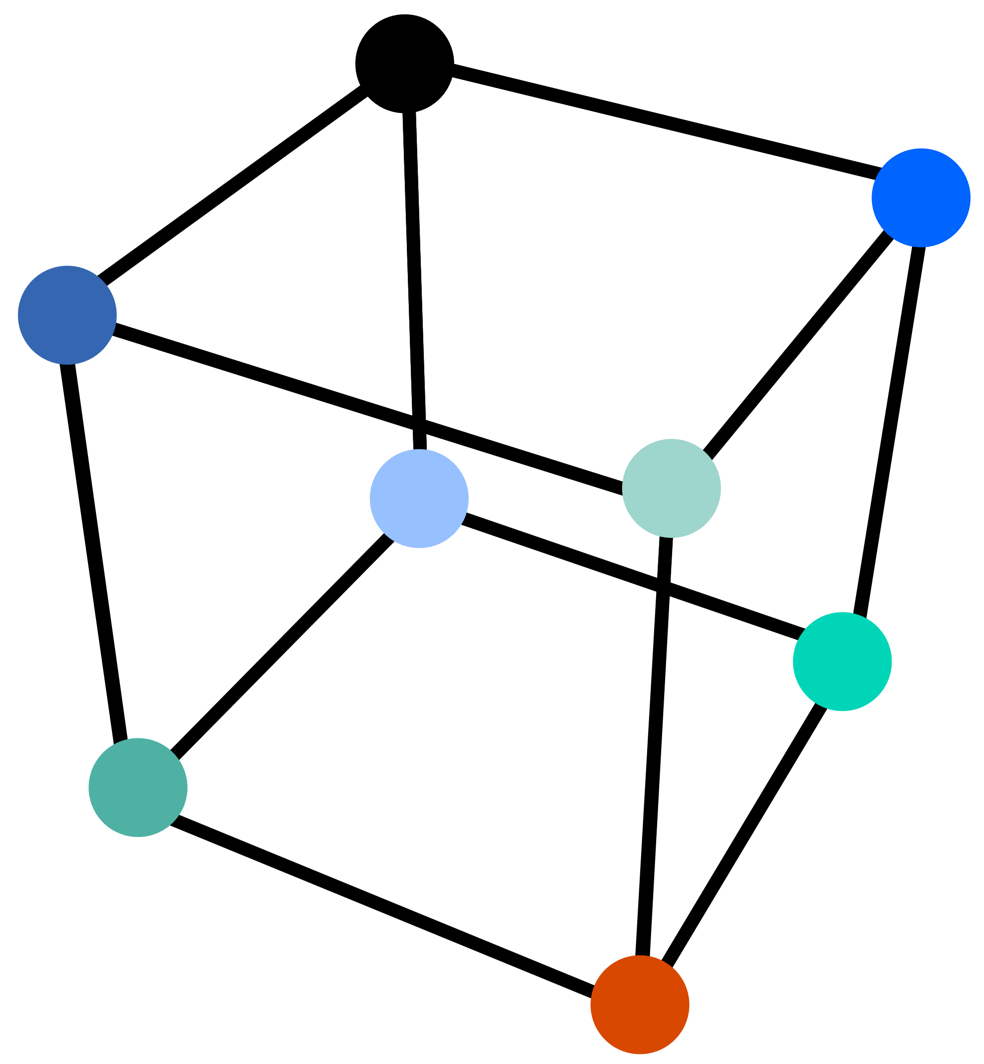
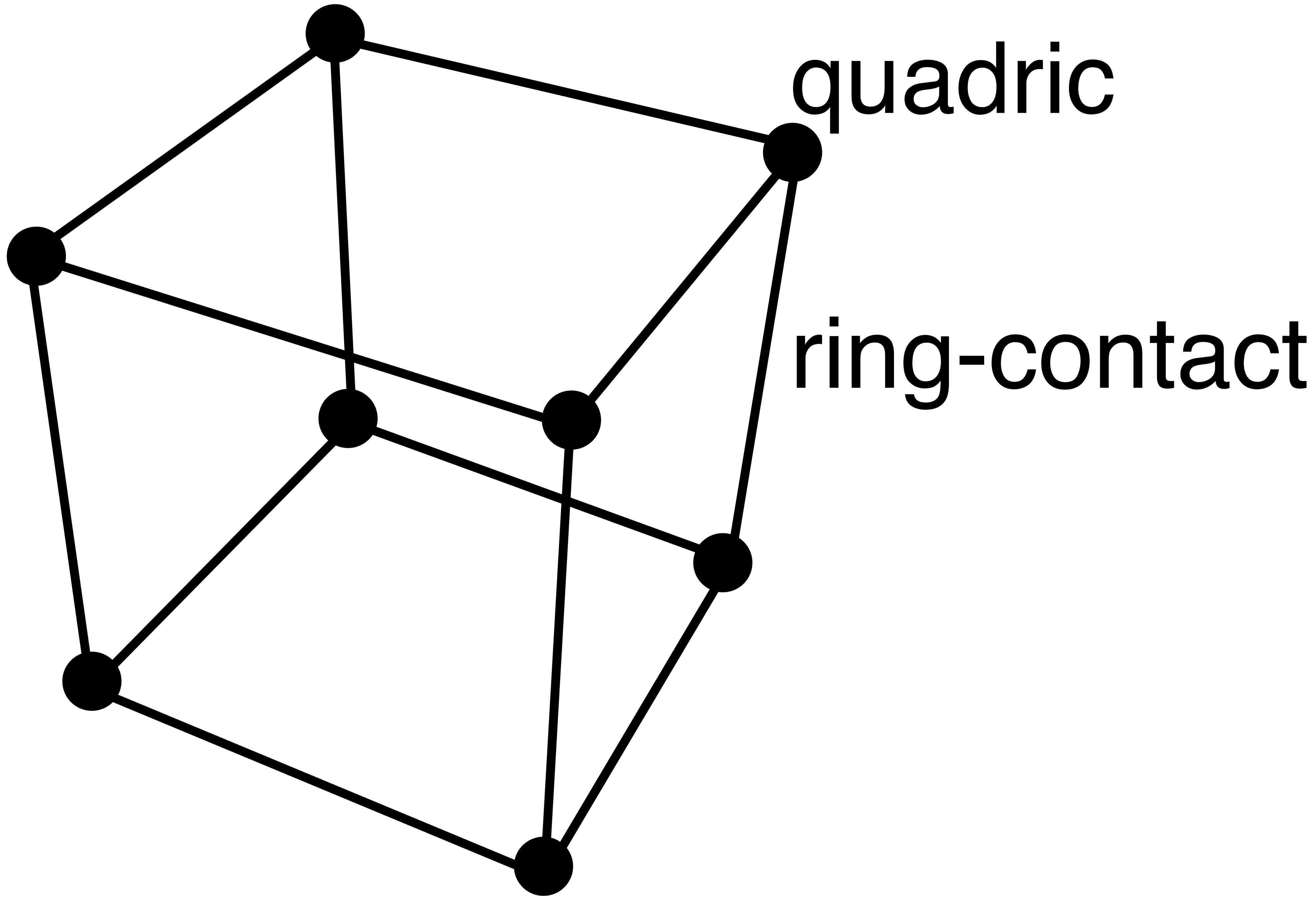
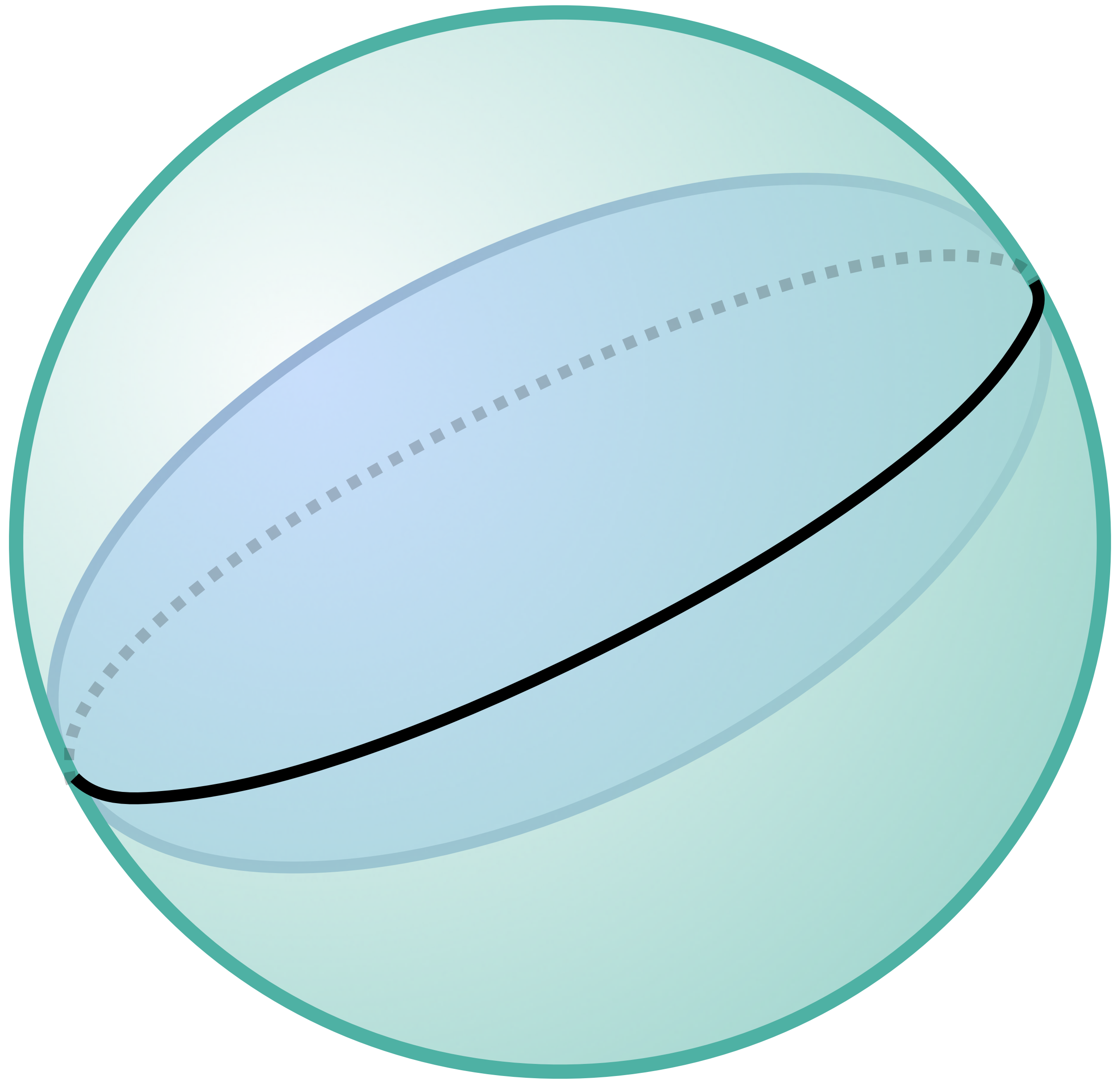




Penrose's 3D Approach

- The 8-conic theorem is a slice/view of an **8-quadric theorem**.

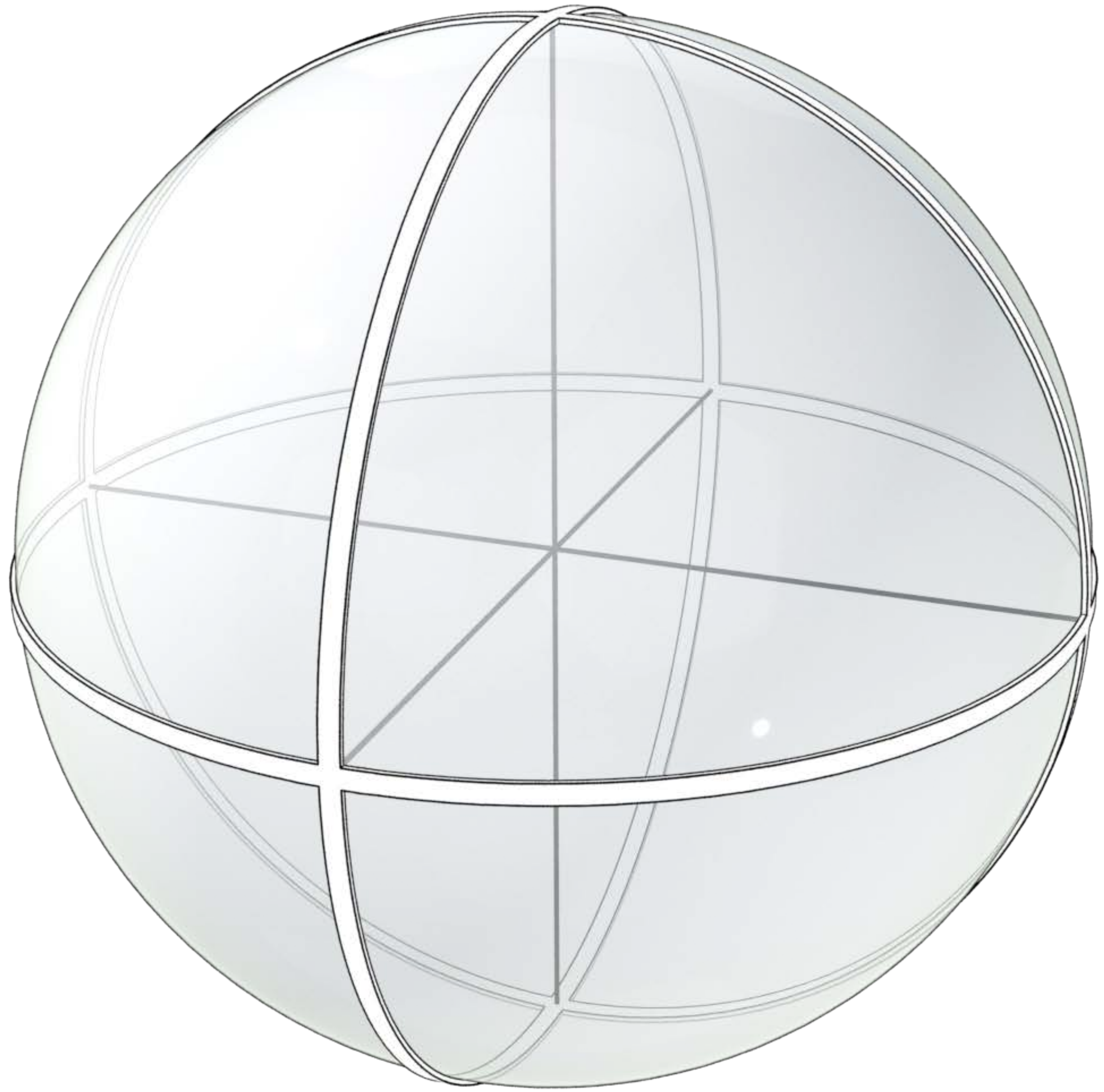


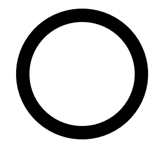


two quadrics in ring contact

- **Theorem** *If 7 of the vertices are given, the 8th one uniquely exists.*

Eight-quadric theorem





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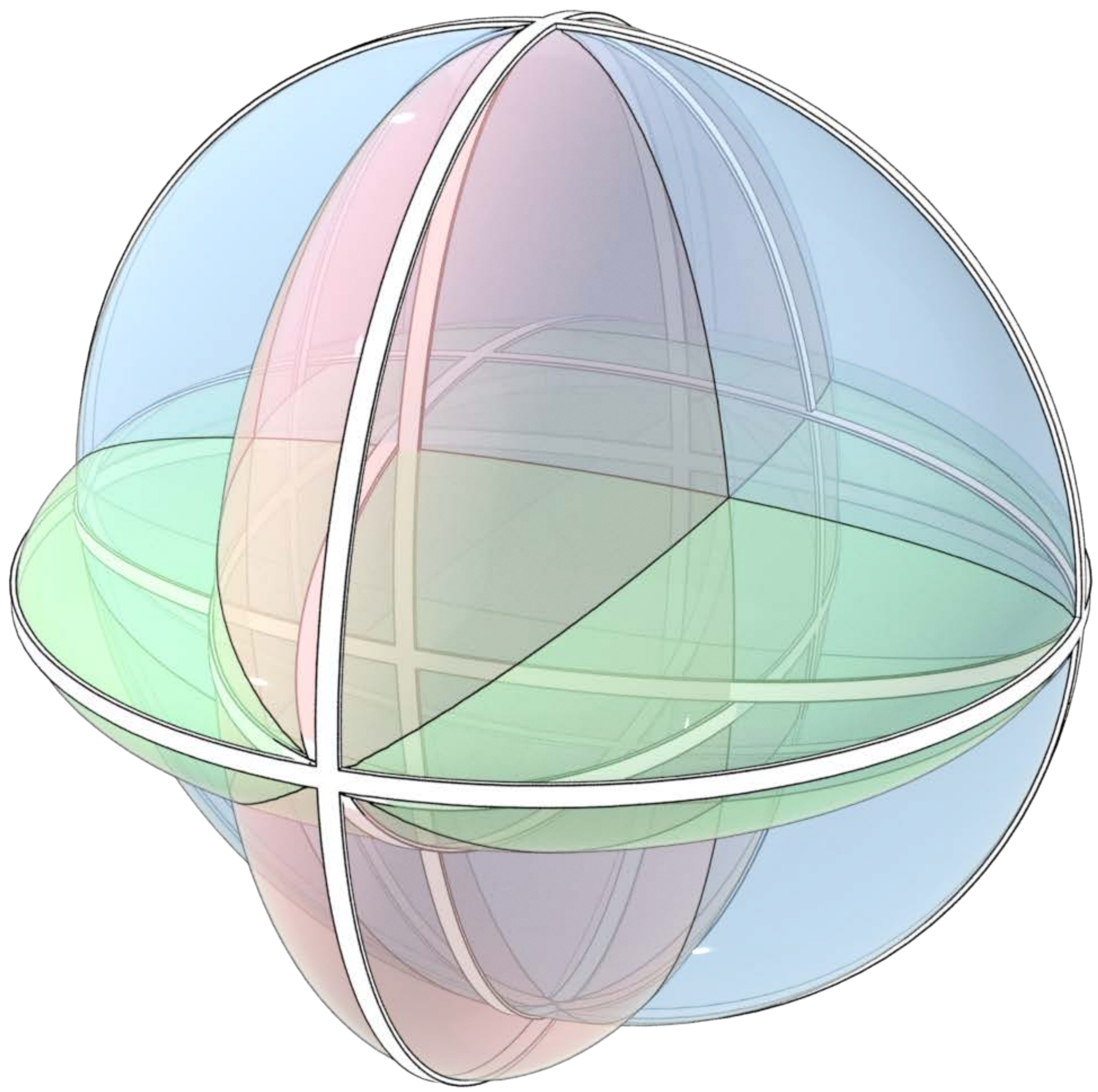

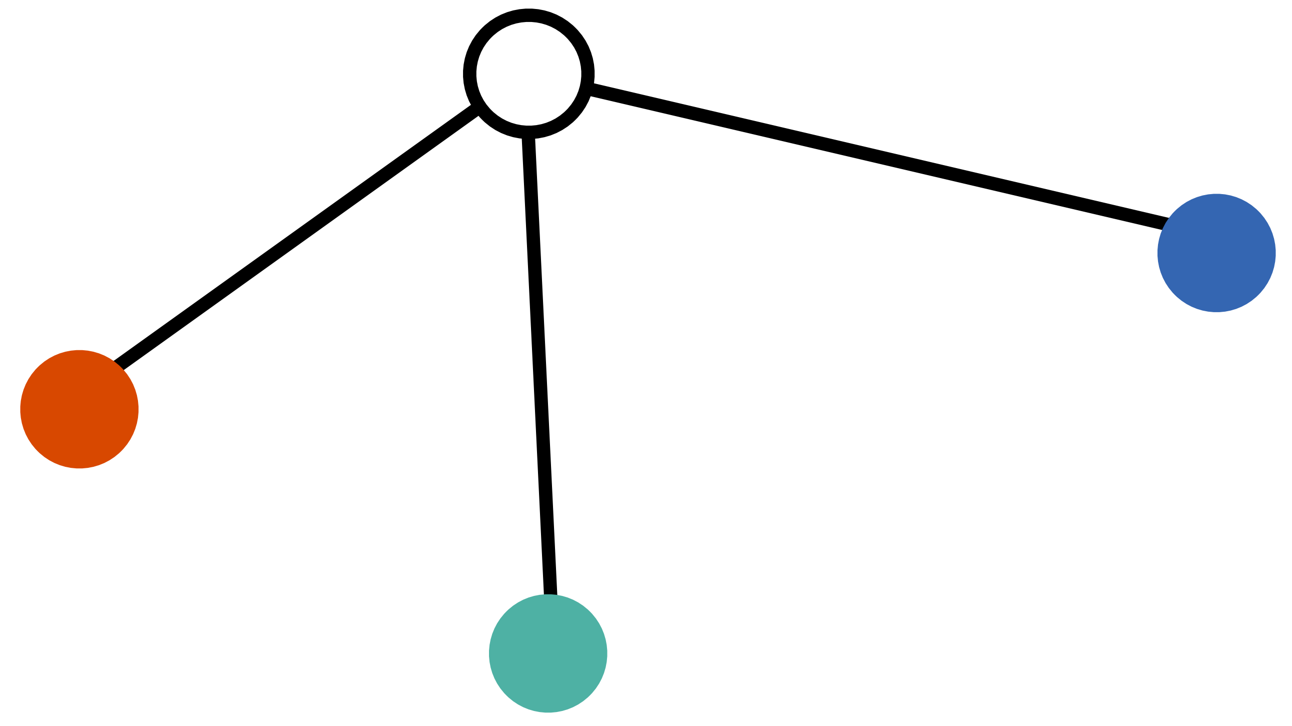

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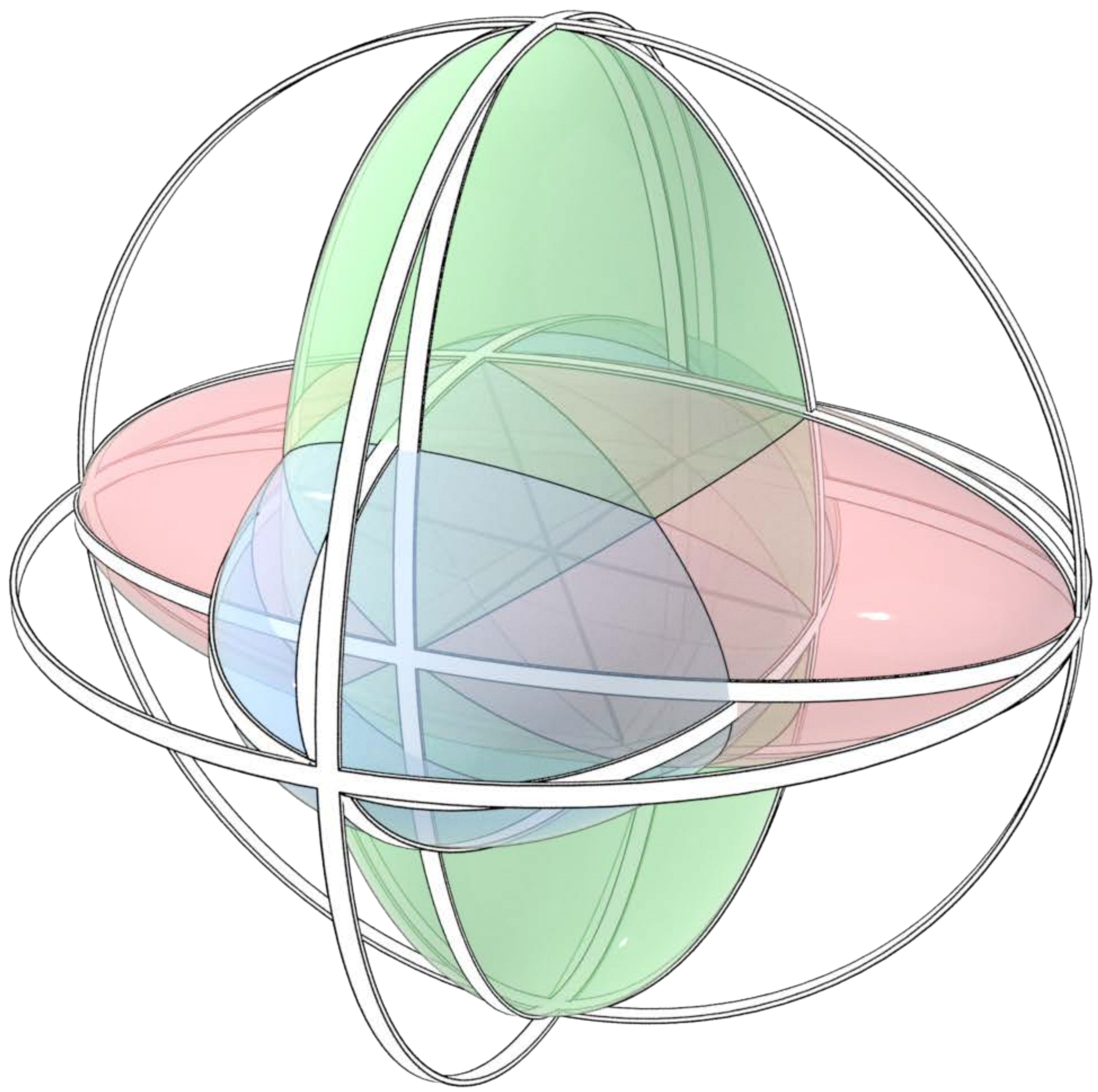

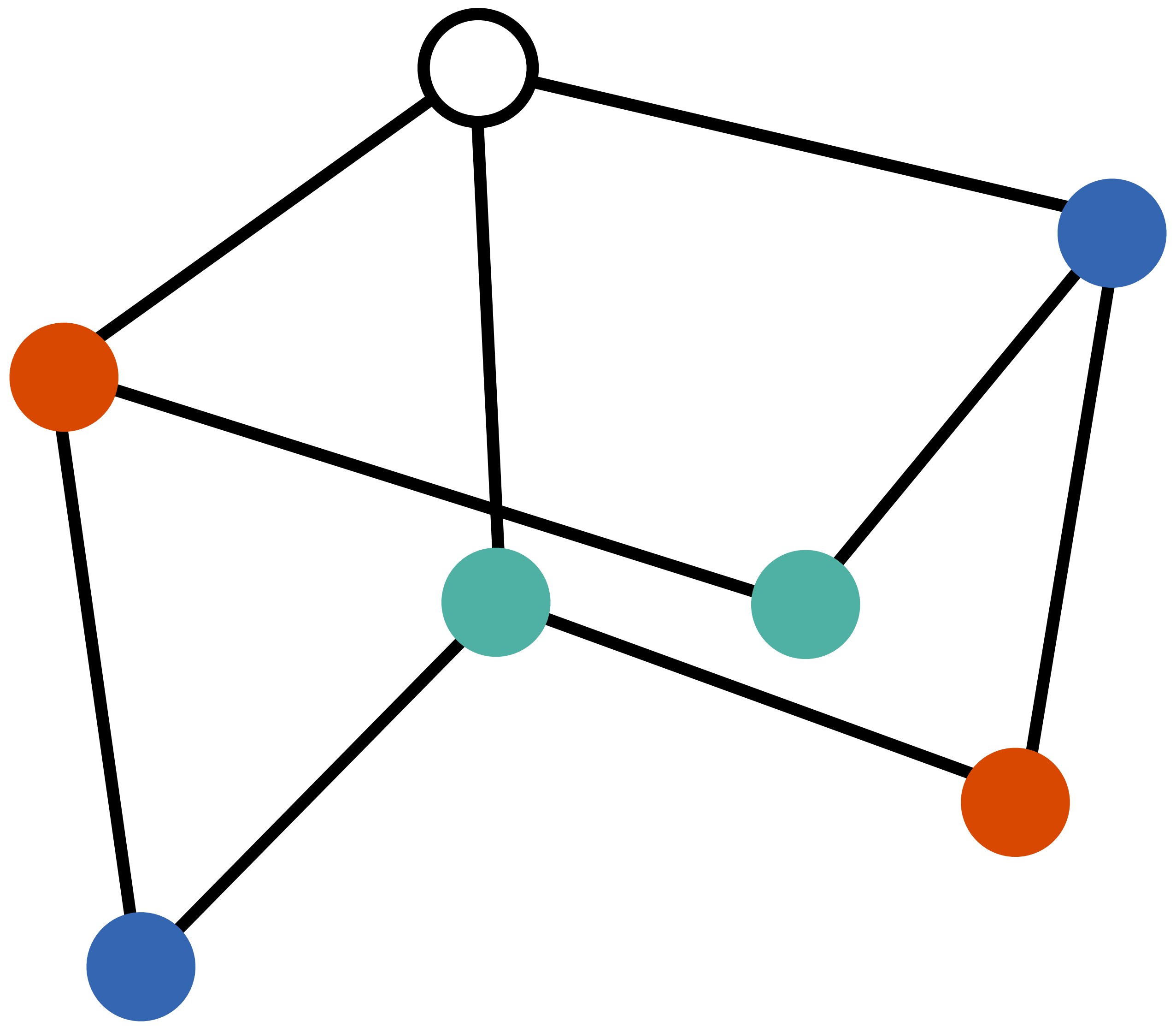

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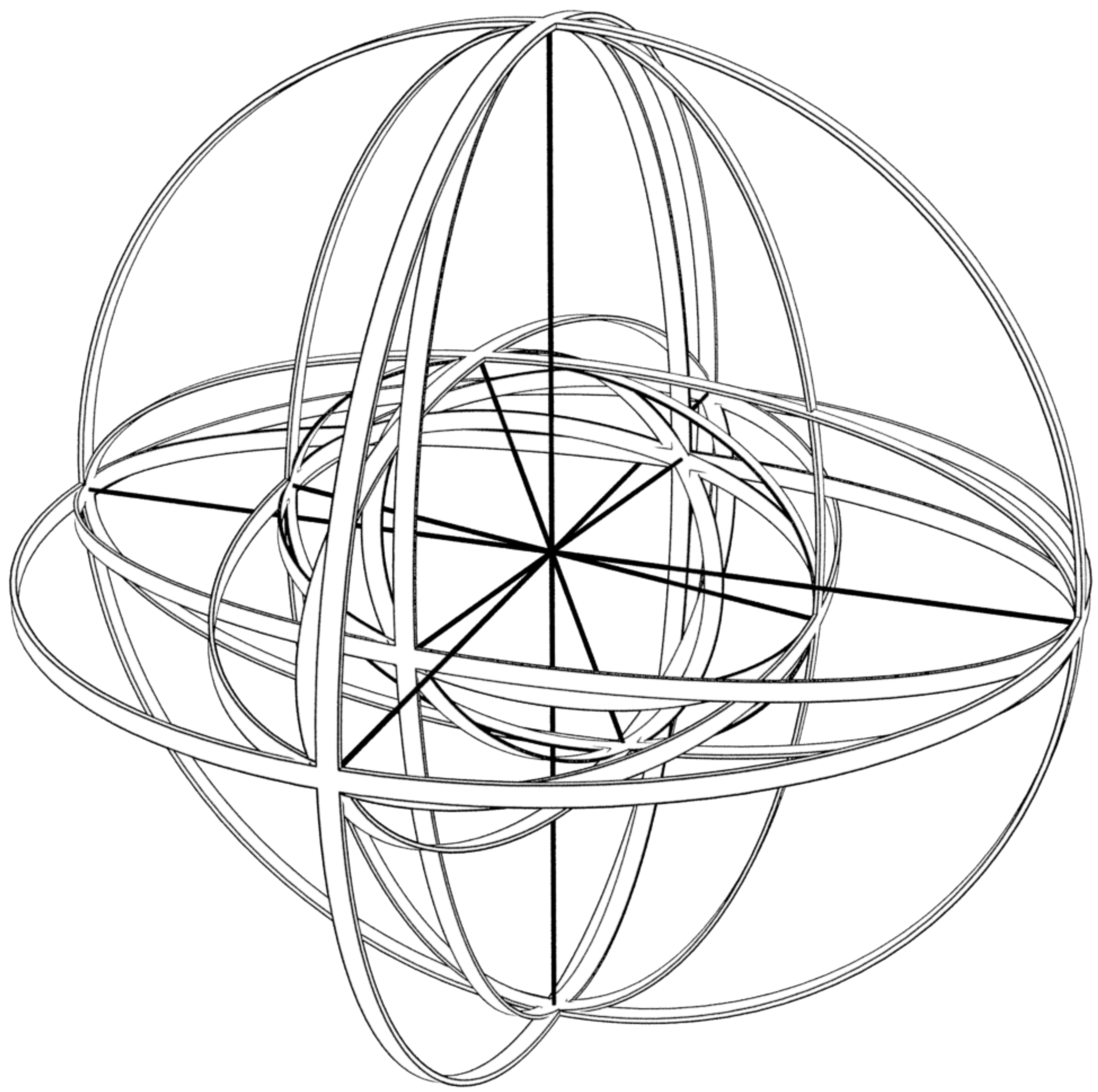

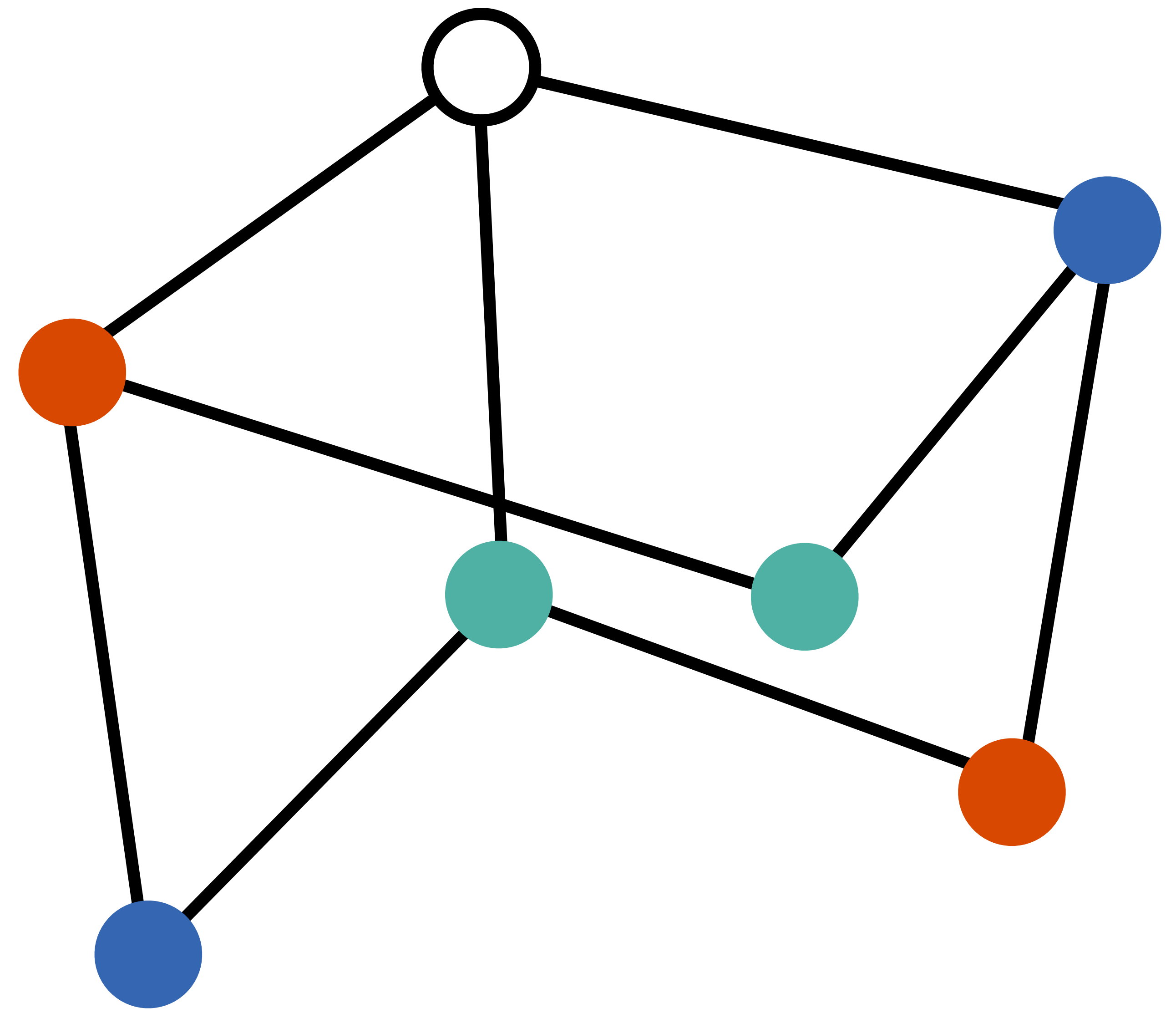

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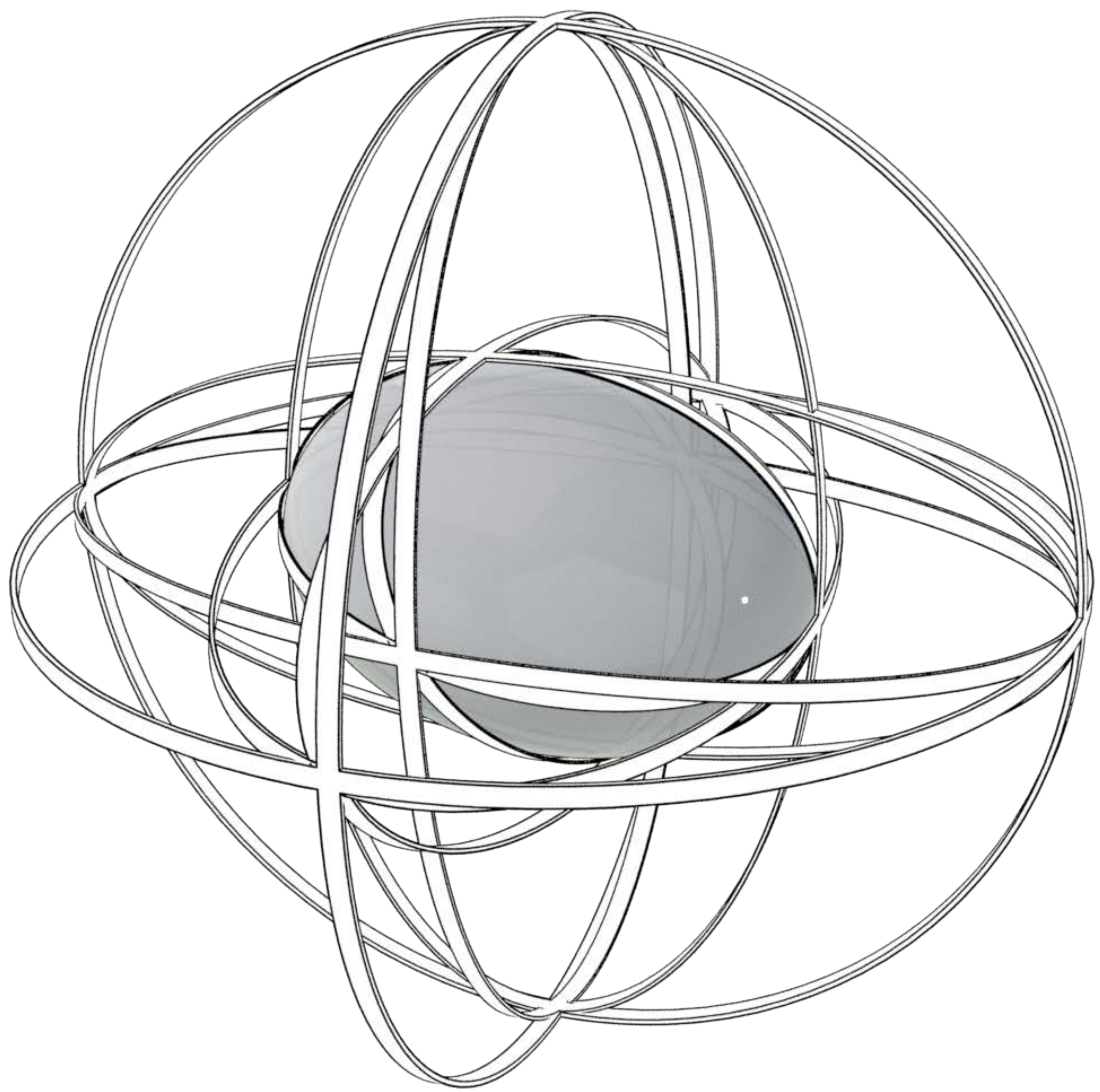

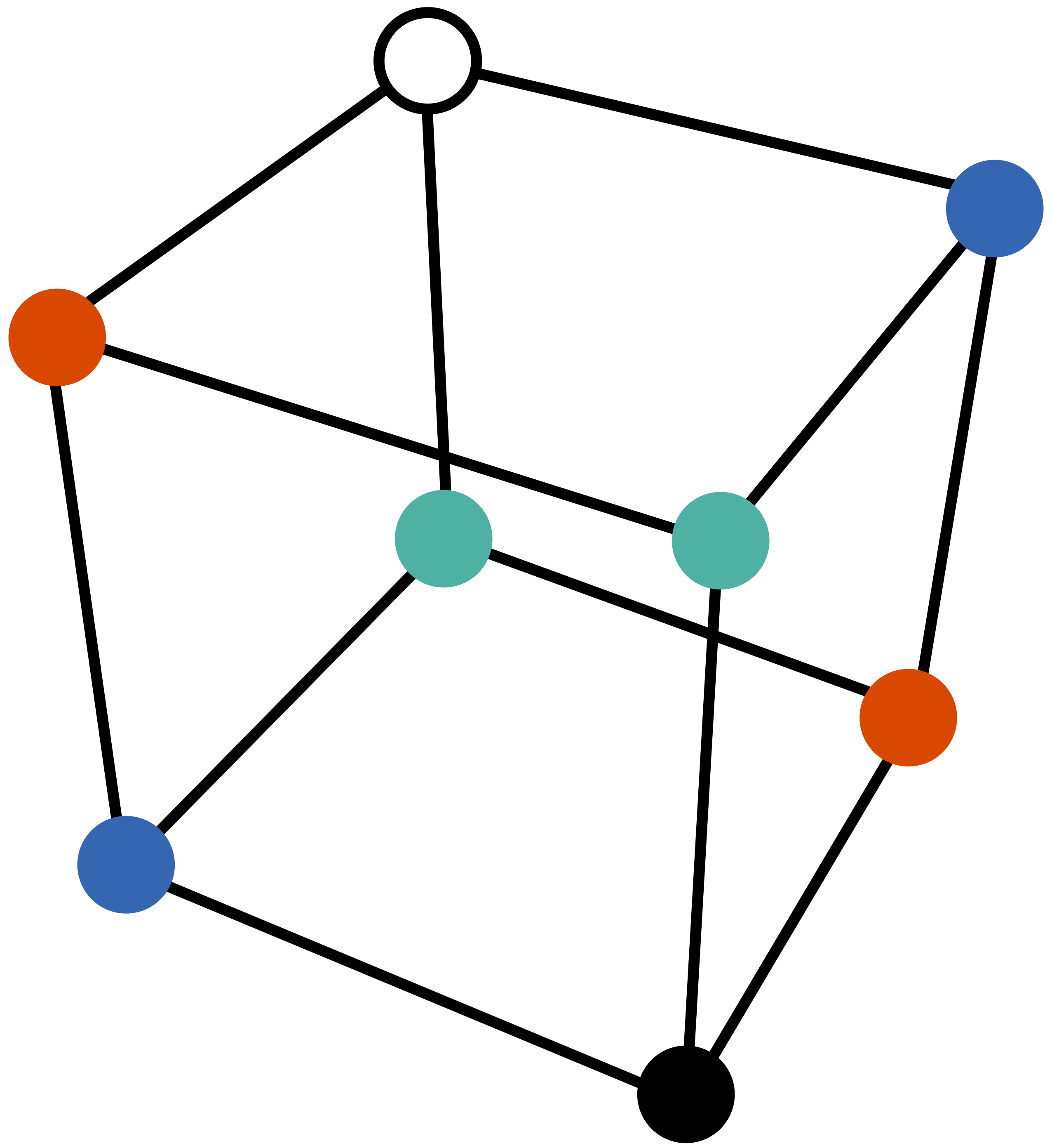

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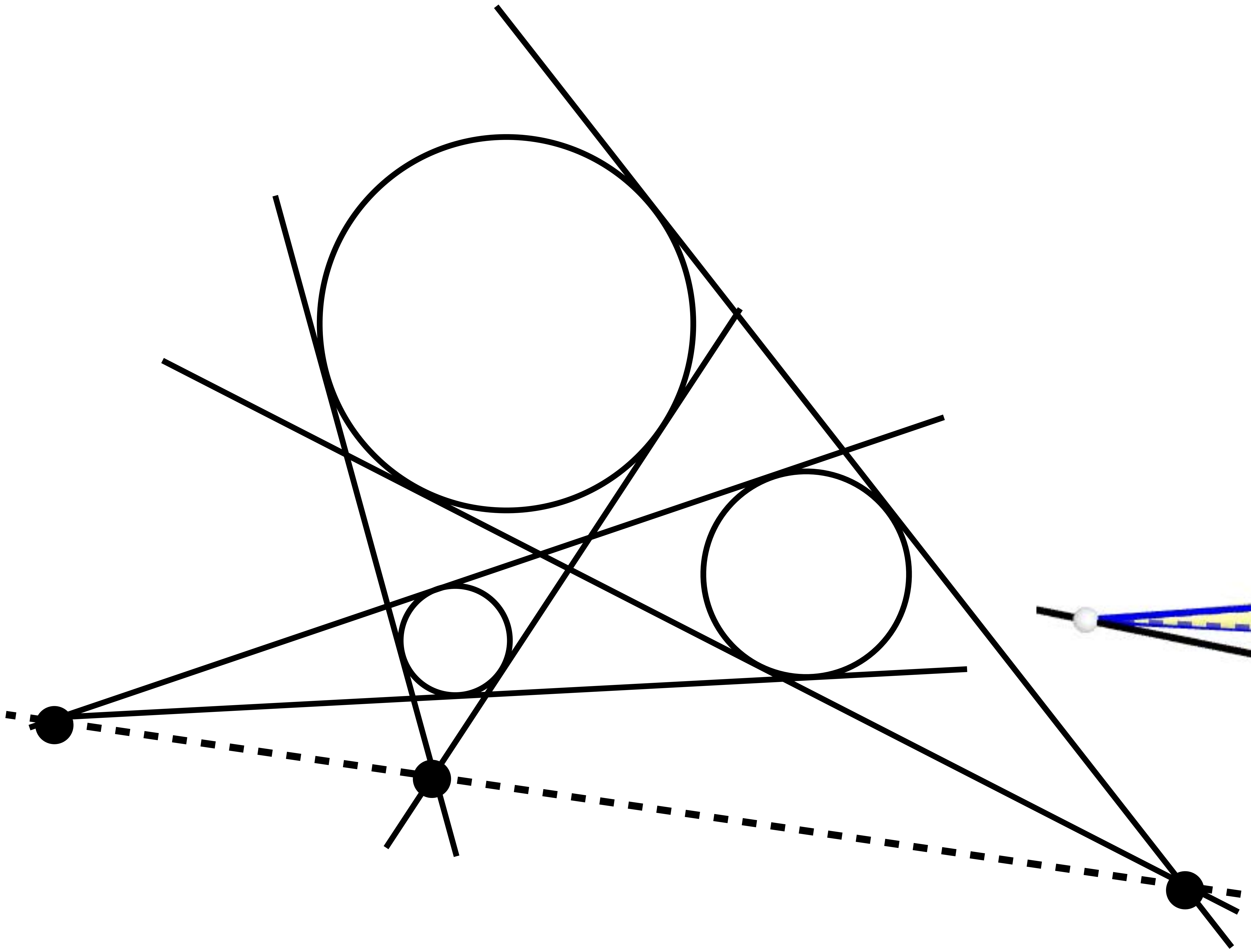
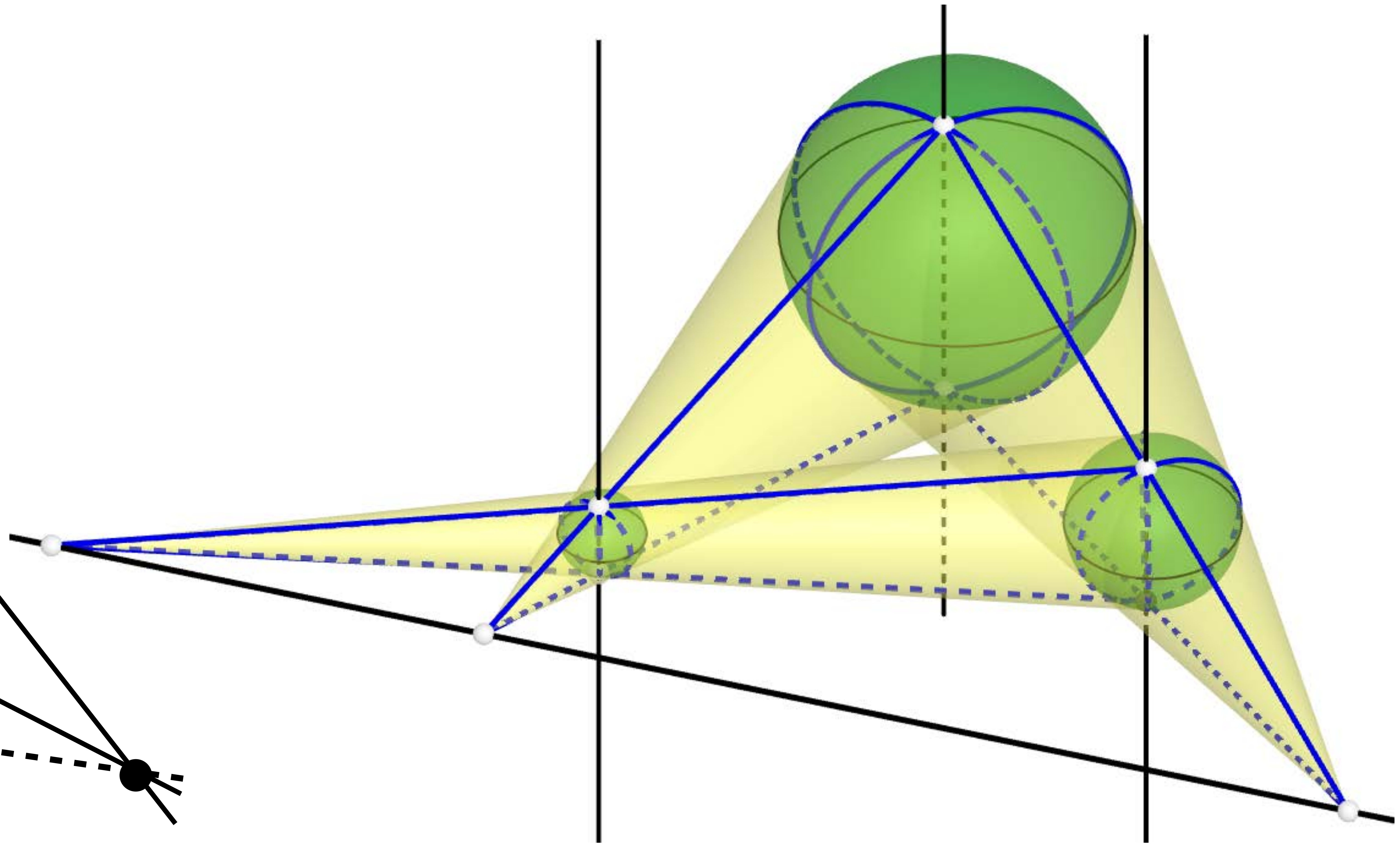
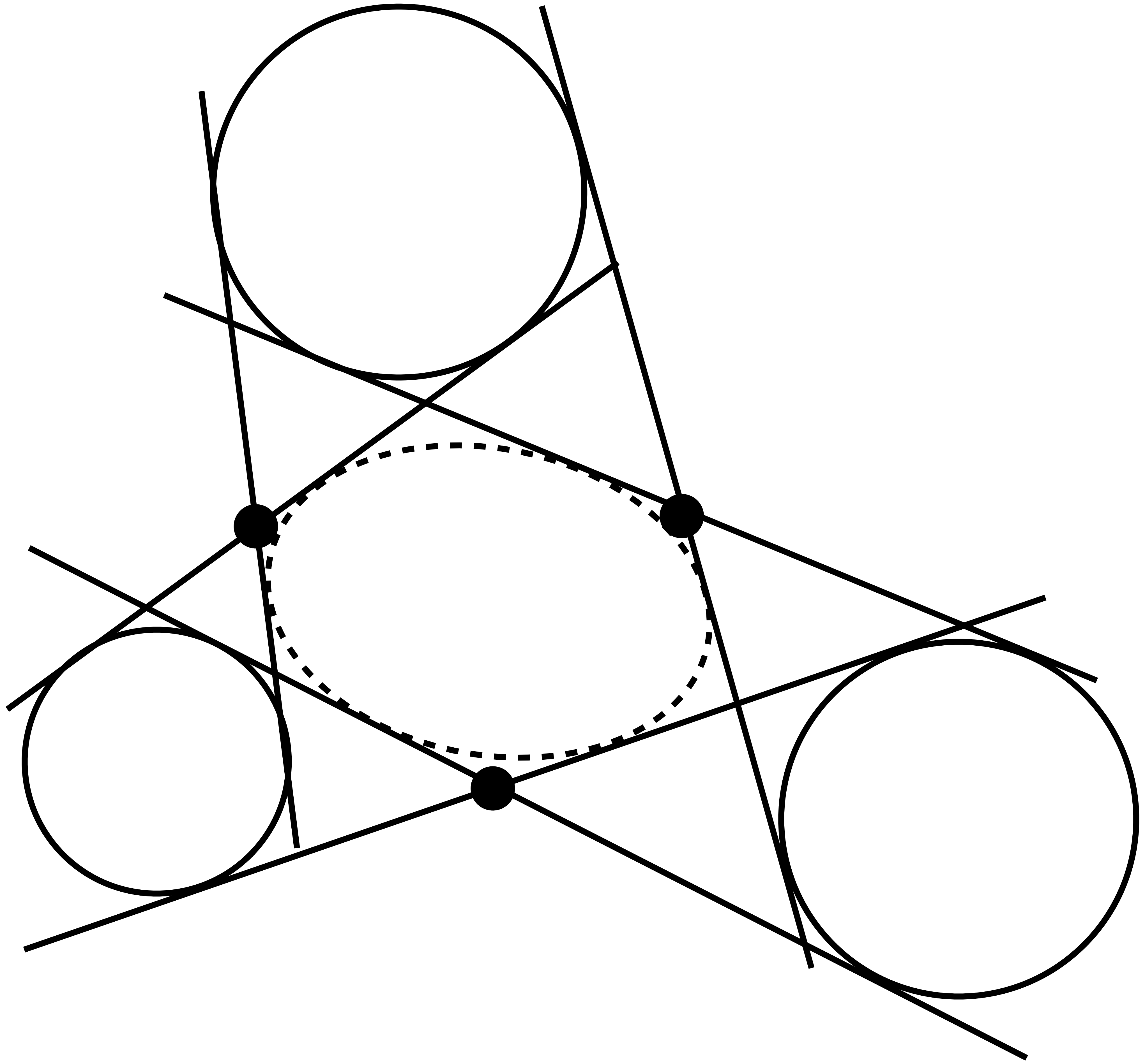
Special cases of 8-quadric configuration

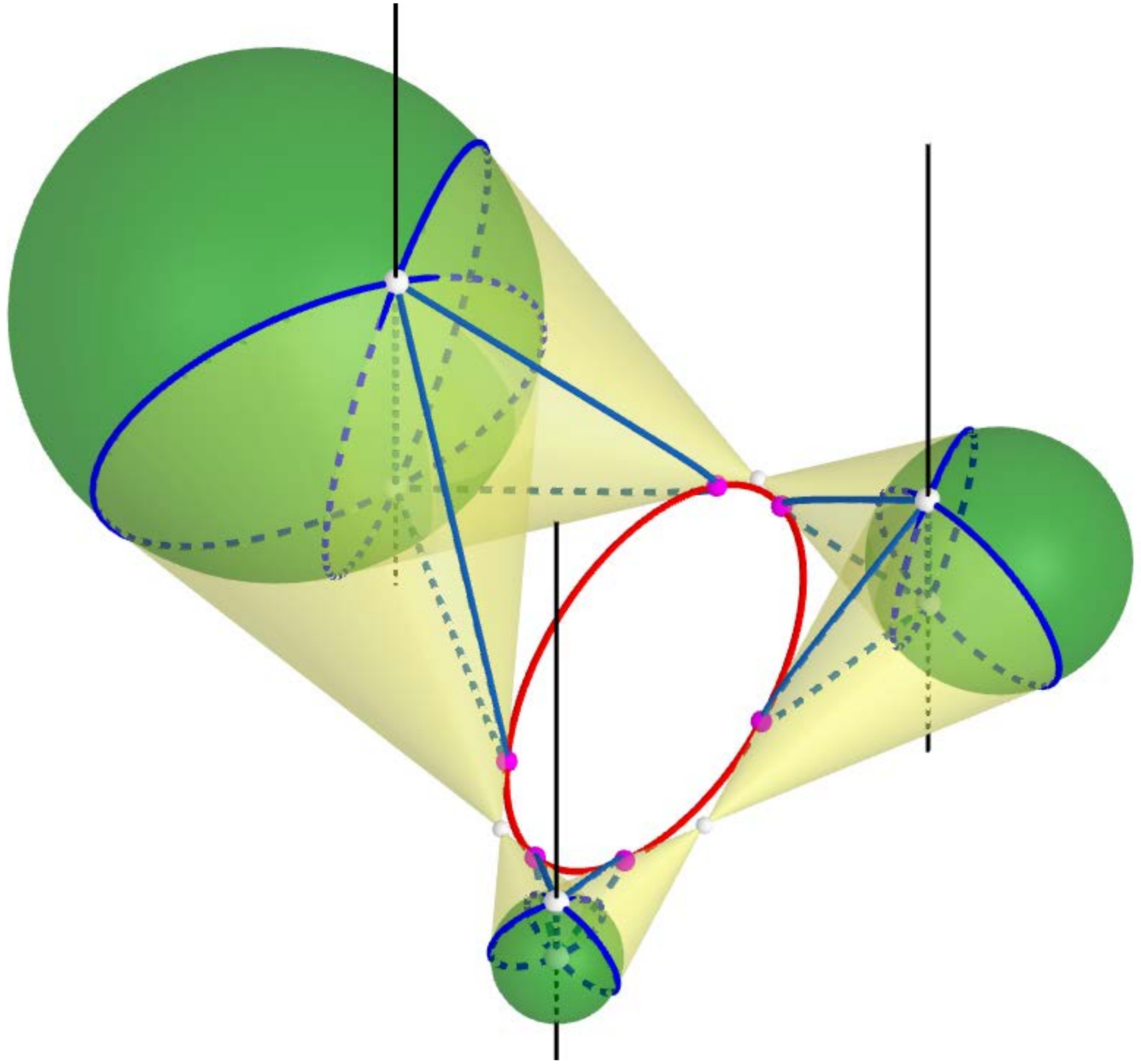
3D proof of the Monge theorem (Monge)



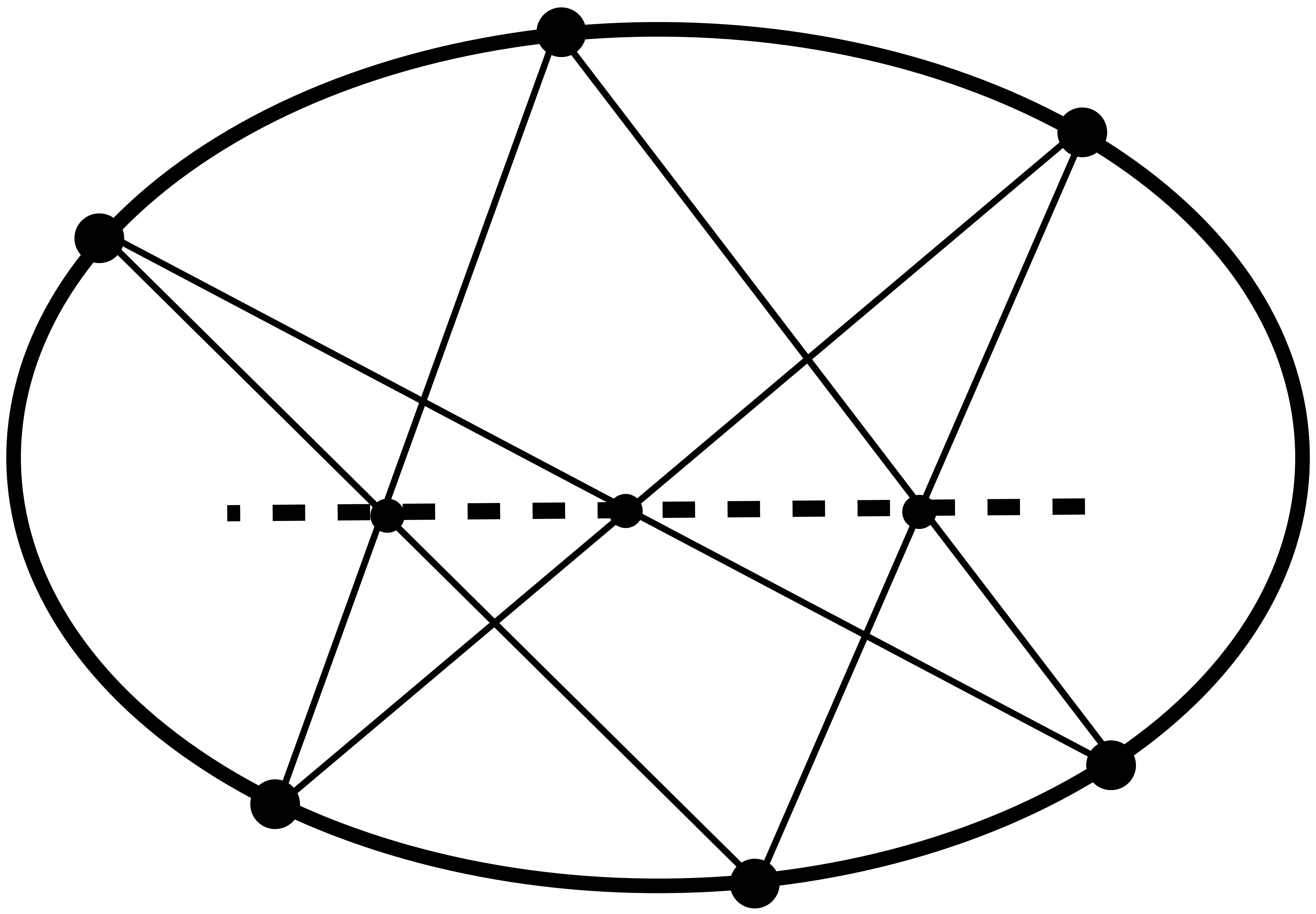


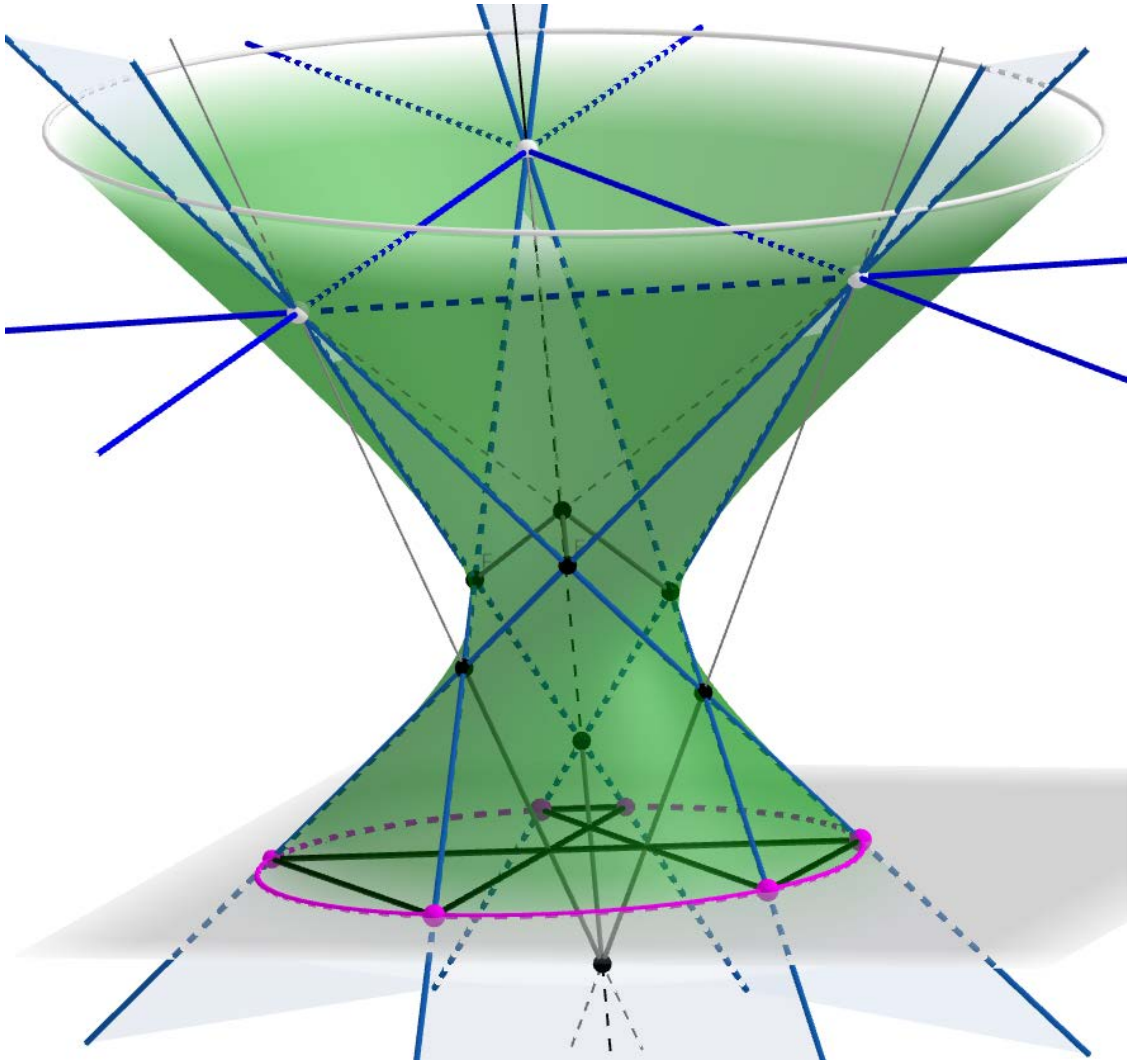
3D proof of the Monge-like theorem



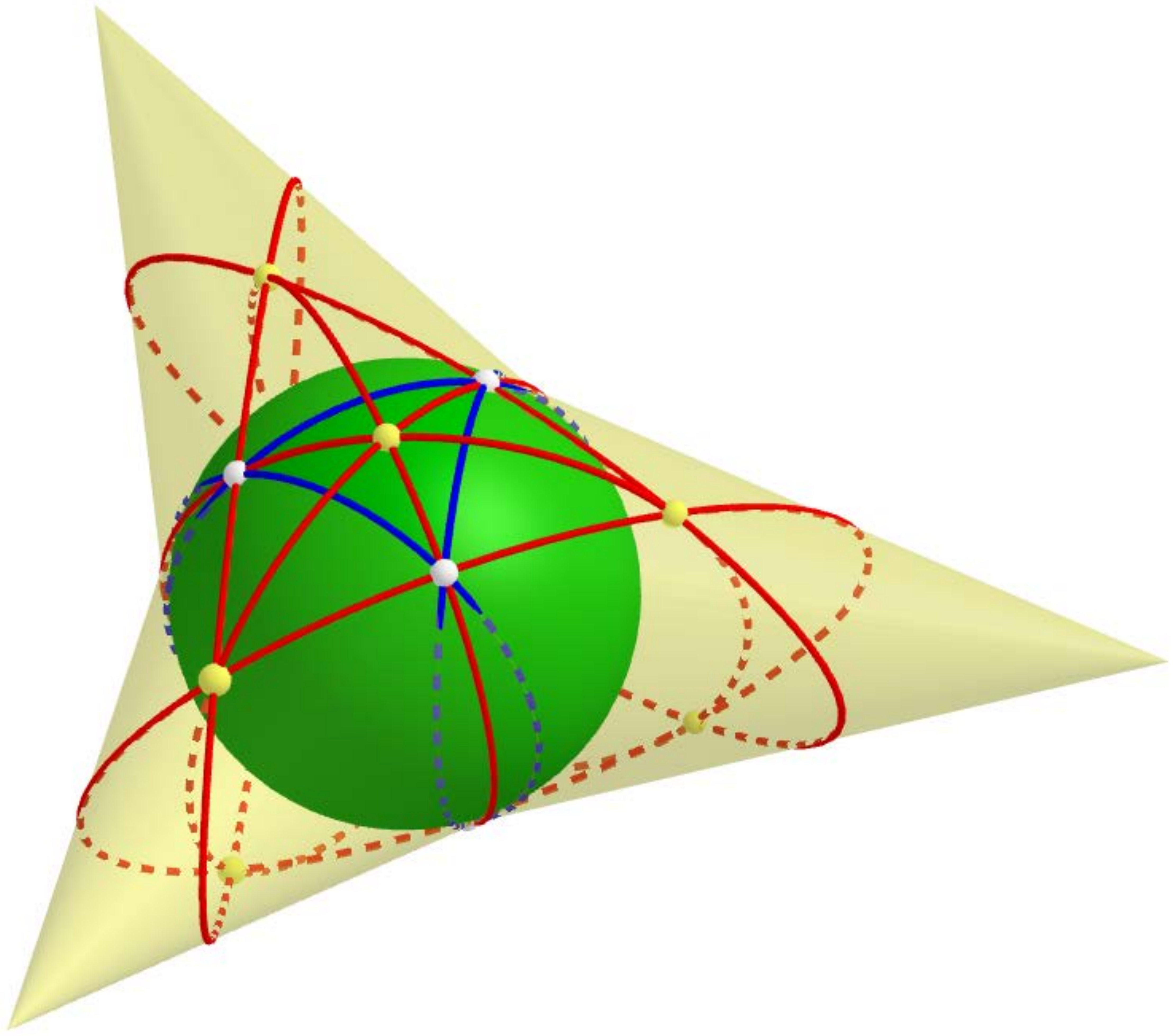


3D proof of the Pascal theorem (Dandelin 1826)

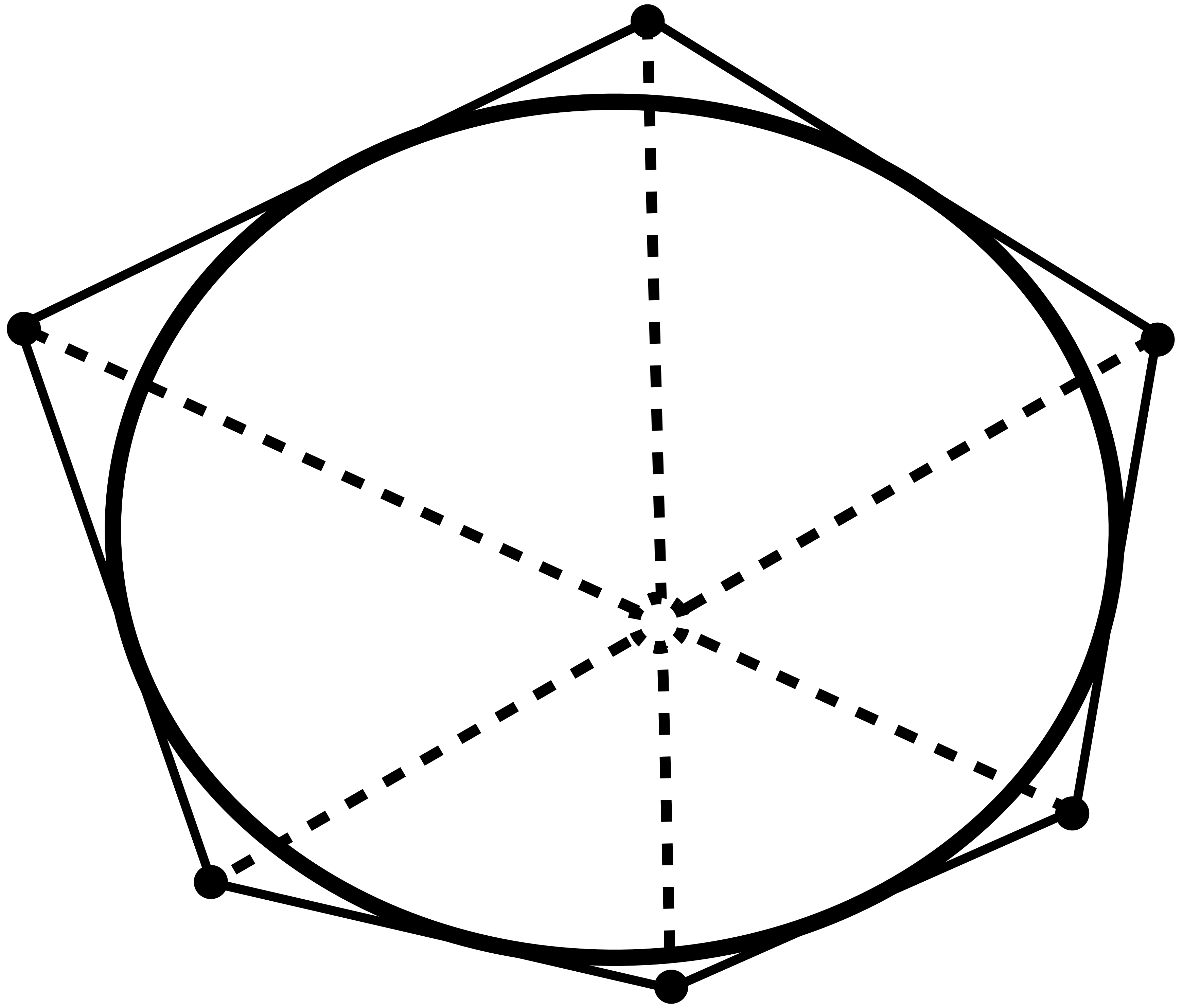


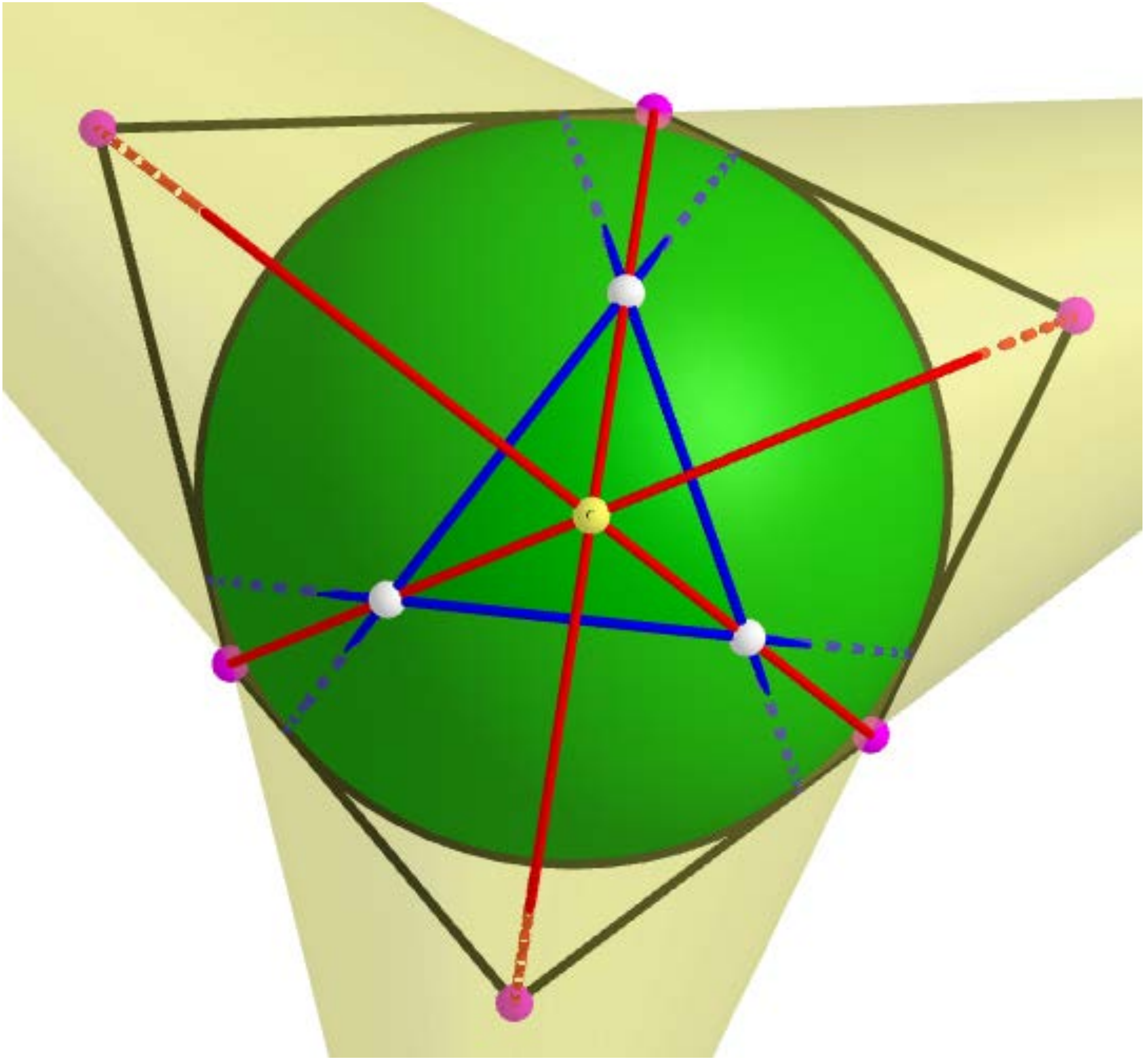


3 cones in ring contact a sphere

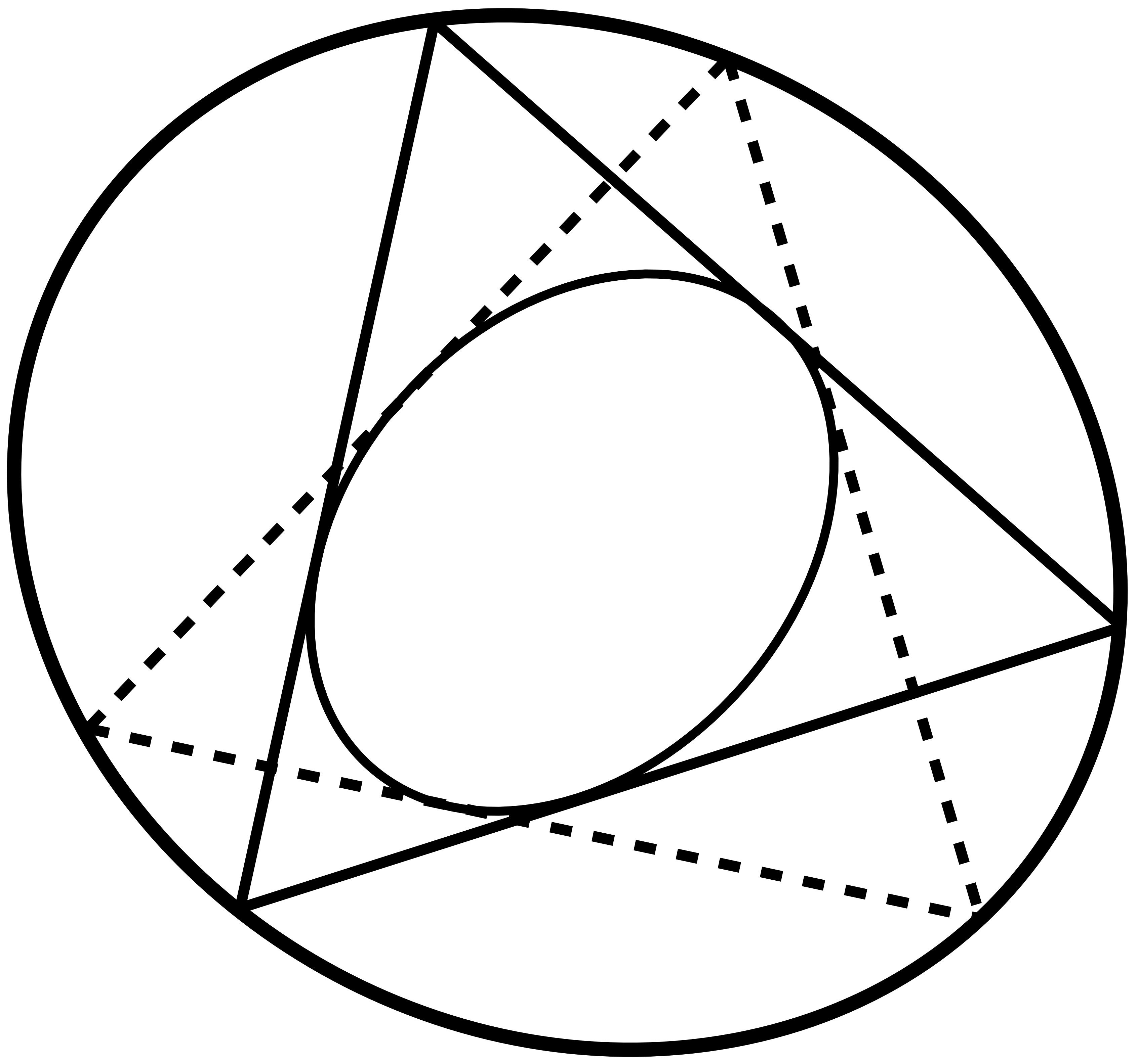


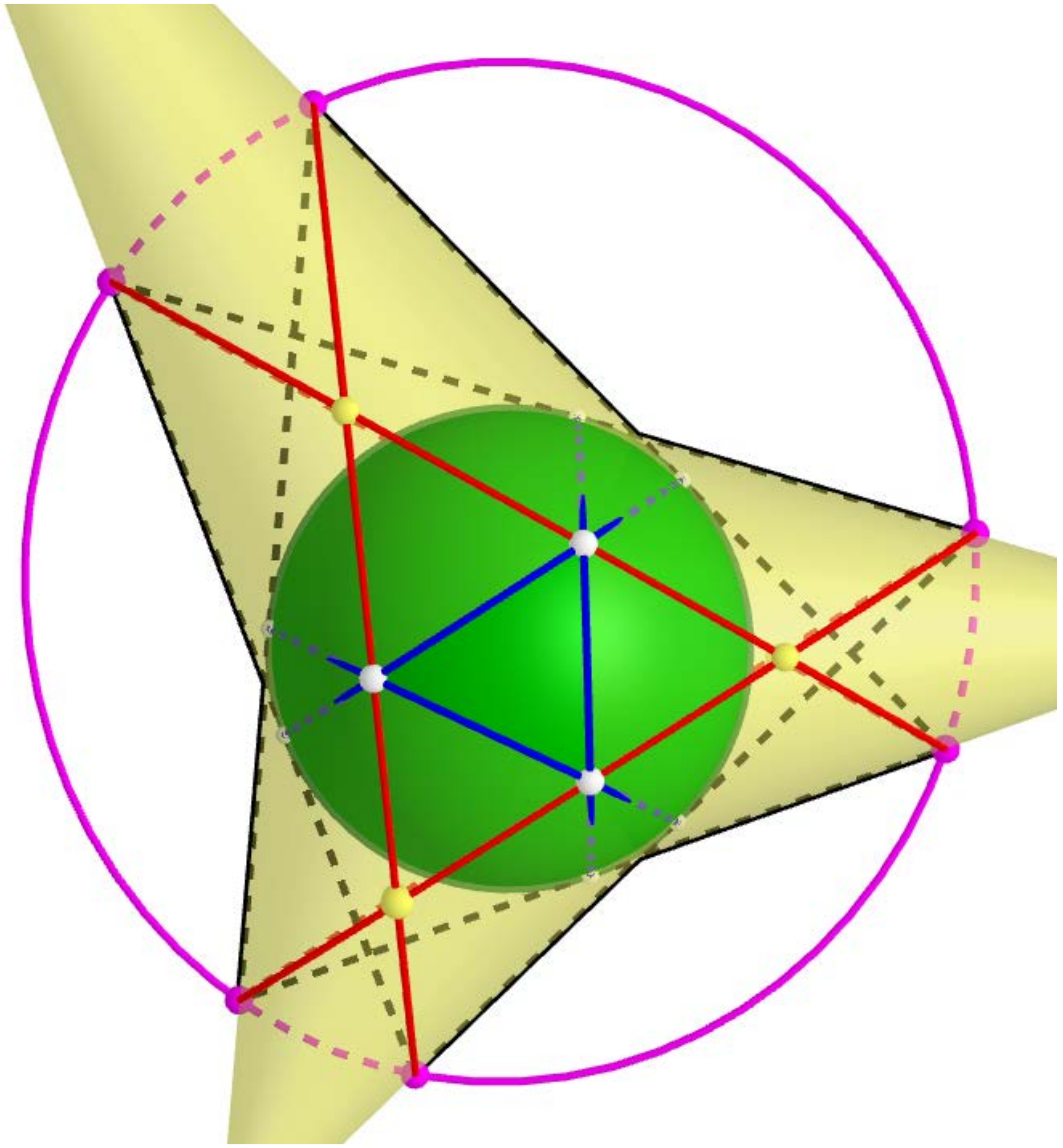
3D proof of the Brianchon theorem





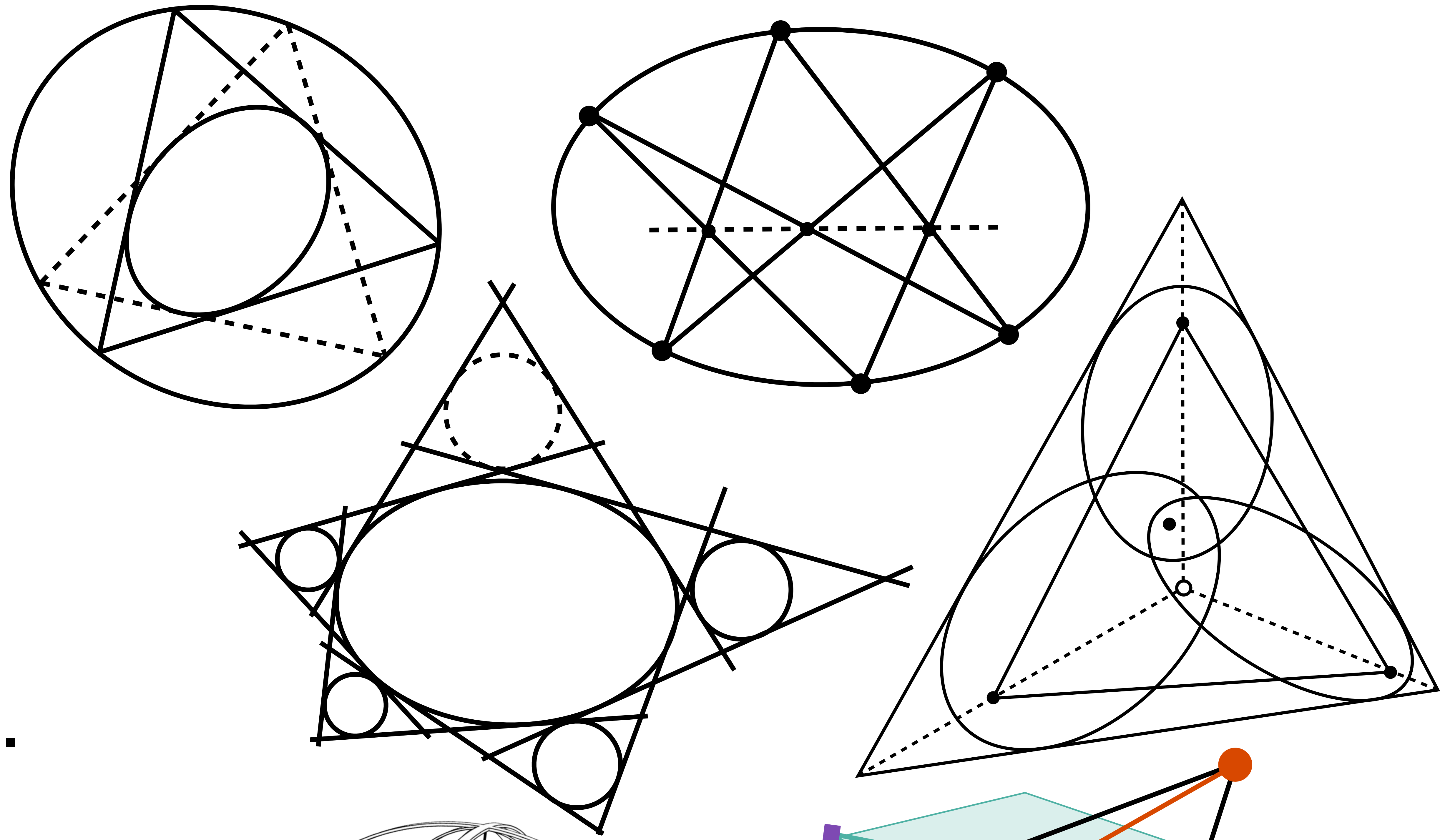
3D proof of the Poncelet porism

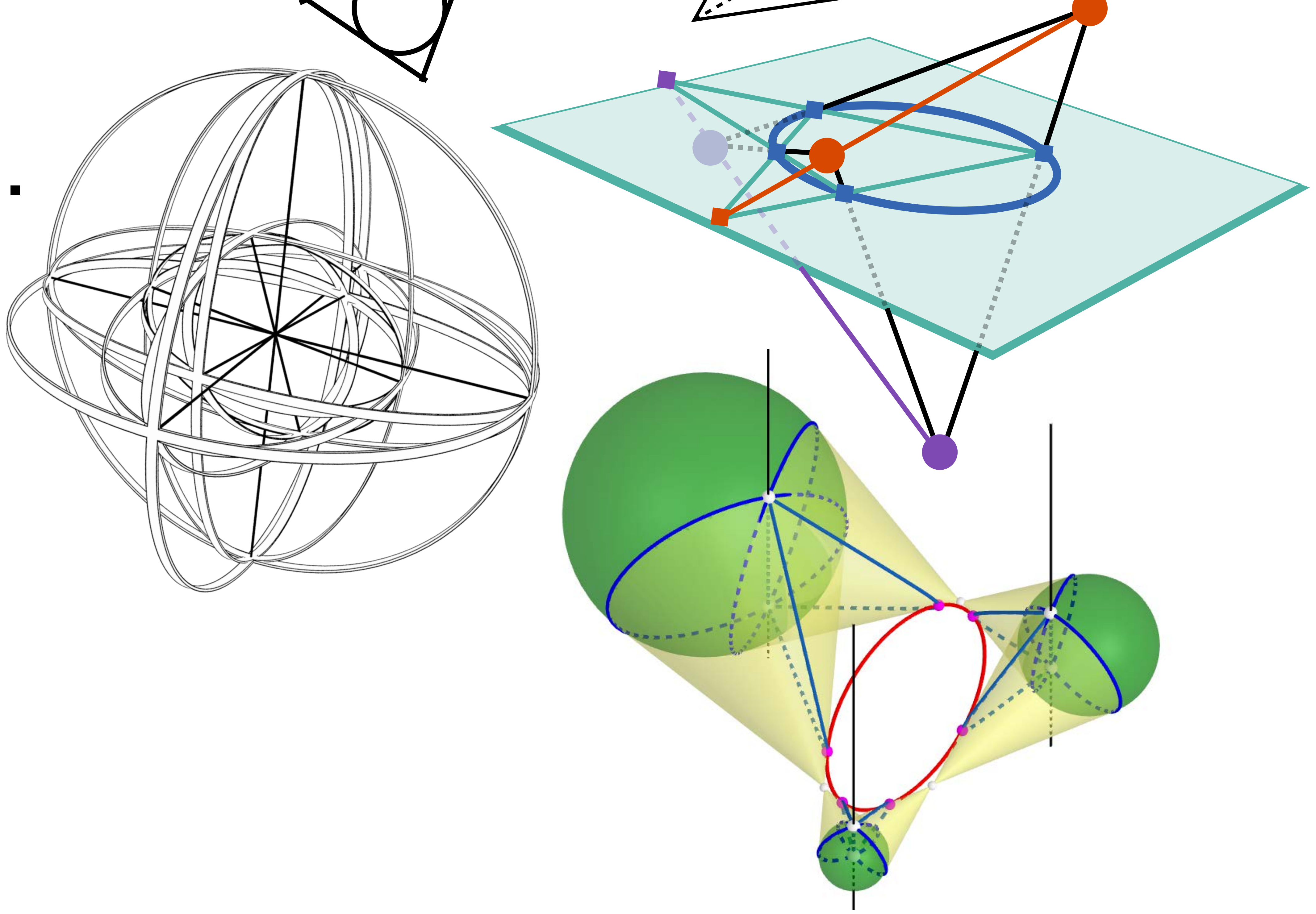




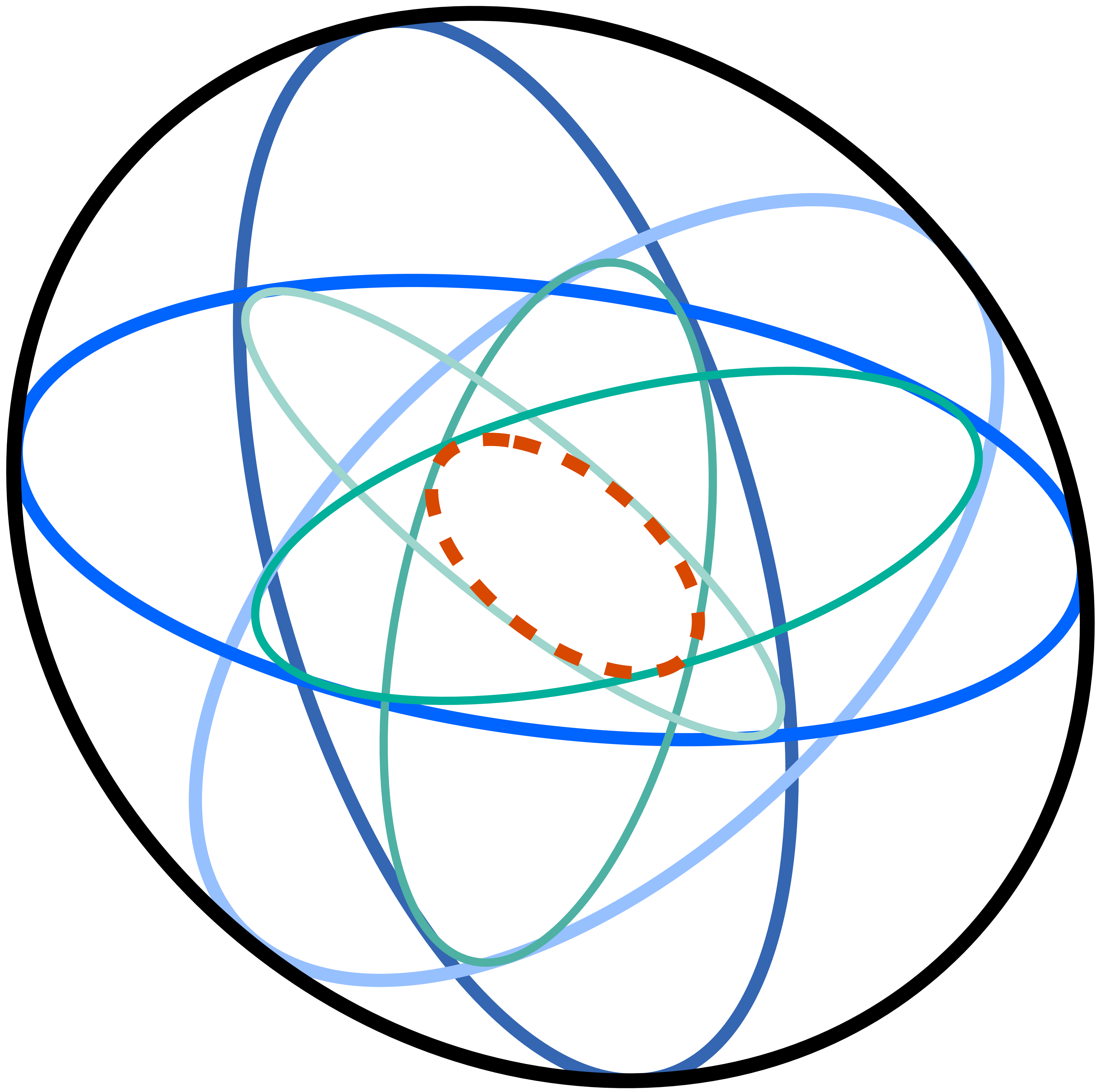
Summary

- 8-conic theorem unifies many projective geometry theorems.
- 8-conic theorem can be stacked to many more theorems.
- Beautiful structure in the 5D of conics.
- 3D proof by Penrose is intuitive.
- The 3D configuration unifies many 3D proofs of planar theorems.





Thank you



Collaborators

Russell Arnold

Charles Gunn

Thomas Neukirchner

Sir Roger Penrose

Publications

Just submitted to arXiv.

Stay tuned to my (Albert Chern's) publication page.