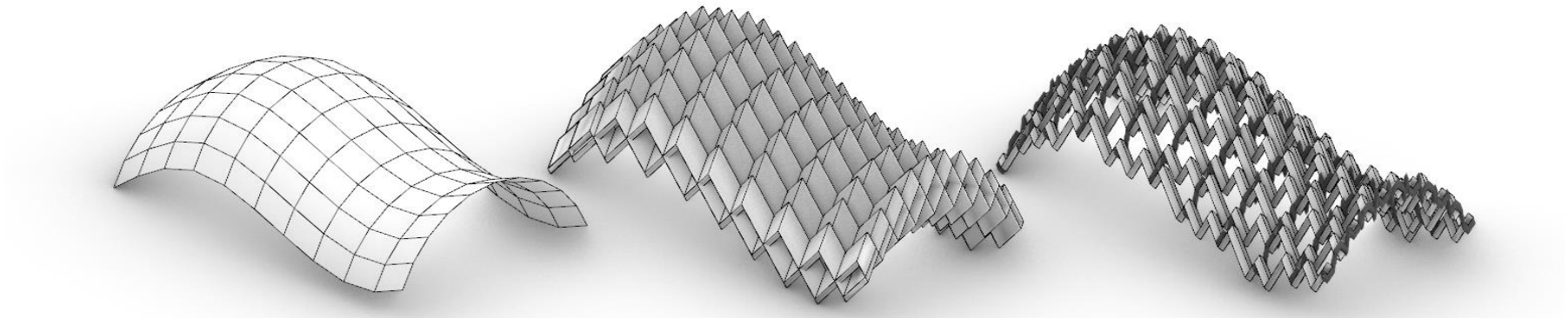
Rigid-foldable Origami: Complex Topologies

Kiumars Sharifmoghaddam

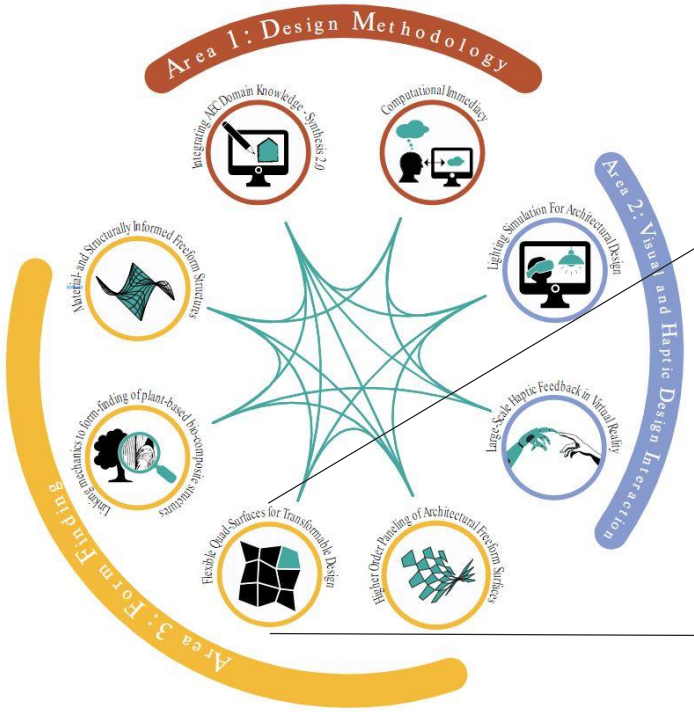
Workshop on Discrete Geometric Structures
September 2-6, 2024 TU Wien

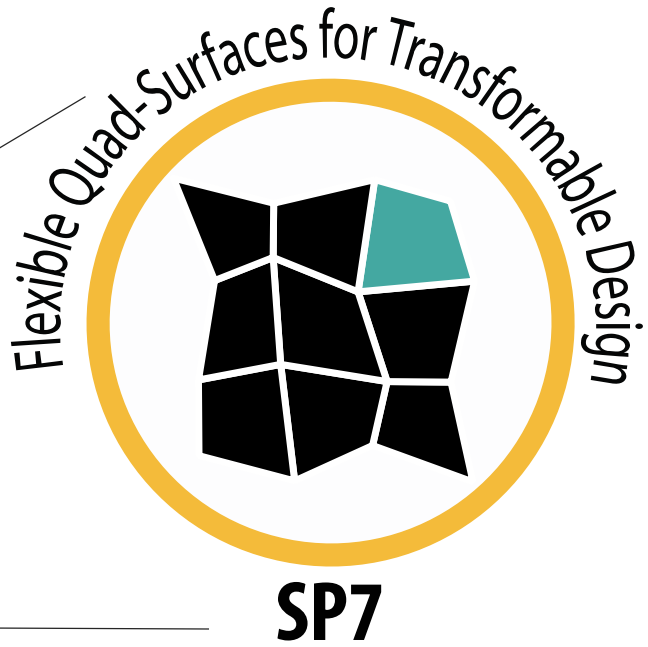


TU WIEN TECHNISCHE UNIVERSITÄT WIEN
GCD
DGD Discretization in Geometry and Dynamics SFB Transregio 109
FWF Der Wissenschaftsfonds.

Project Background

Advanced Computational Design





Ivan Izmestiev

Georg Nawratil

And +25 coauthors!

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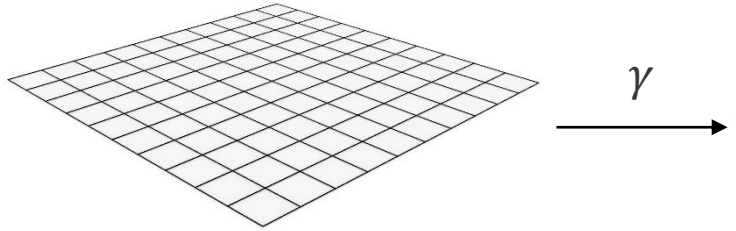
IACH LAB

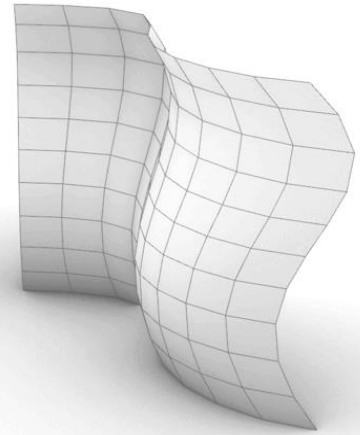
Introduction
Definitions and Settings

Flexible Planar Quad Surfaces (PQ-meshes)

- $\gamma: \mathbb{Z}^2 \to \mathbb{R}^3$ (Combinatorics of a regular grid)
- Conjugate nets (Planar quad faces)
- Isometric deformation (rigid quad faces + rotational joints)
- Topology of a disc



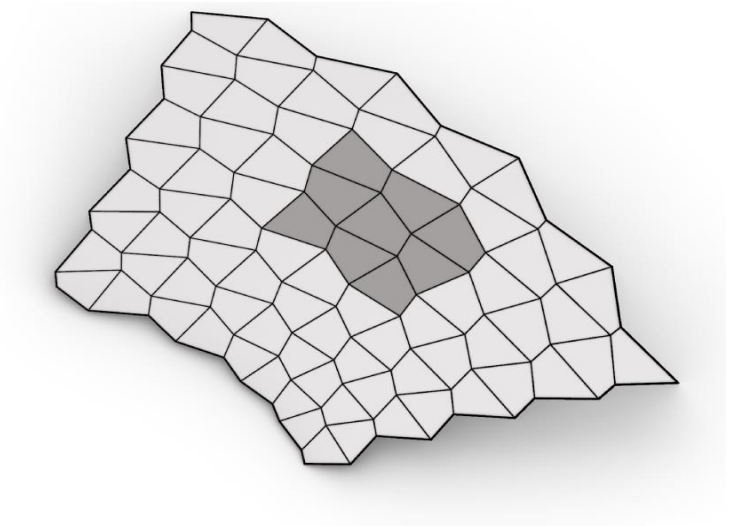
γ



Installation of *Poetic Structure: Art + Engineering + Architecture*.
Skidmore, Owings & Merrill LLP (SOM), 2018

Introduction
Flexible Quad-Surfaces in Discrete Setting

- Such a PQ-mesh is flexible, if and only if every 3×3 patch is flexible (rigidly foldable), [by Schief et al.]
- With **one degree-of-freedom** movement,
- **Not** necessarily **developable** and/or **flat-foldable**,
- Orthodiagonal (**T-hedra**) and Isogonal (**V-hedra**) are the two main classes. [Classification by Izmestiev]

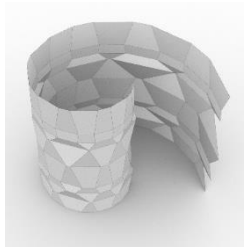


Contributions

T-hedra

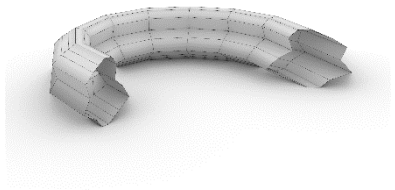
- Construction and motion simulation

Using Flexible Trapezoidal Quad-Surfaces for Transformable Design (IASS2021) K. Sharifmoghaddam, G. Nawratil, A. Rasoulzadeh, J . Tervooren



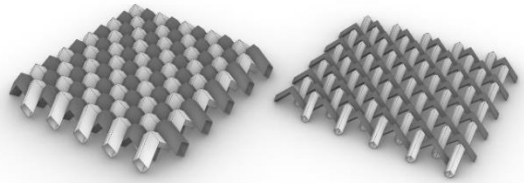
- T-hedral tubes

Generalizing Rigid-foldable Tubular Structures of T-hedral Type (Mechanics Research Communications 2023) K. Sharifmoghaddam, R. Maleczek, G. Nawratil



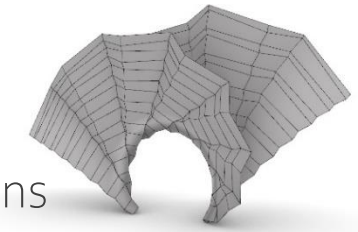
- Woven T-hedra

Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces (8OSME 2024) K. Sharifmoghaddam, K. Mundilova, G.Nawratil, T. Tachi



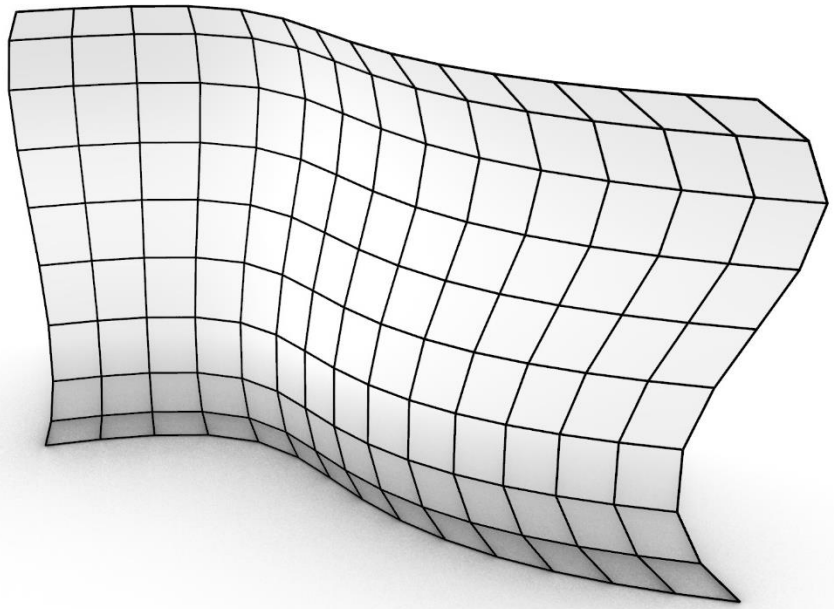
(Anti-)V-hedra

- Construction and motion simulation
- Non-convex quads
- Hybrid and novel variations

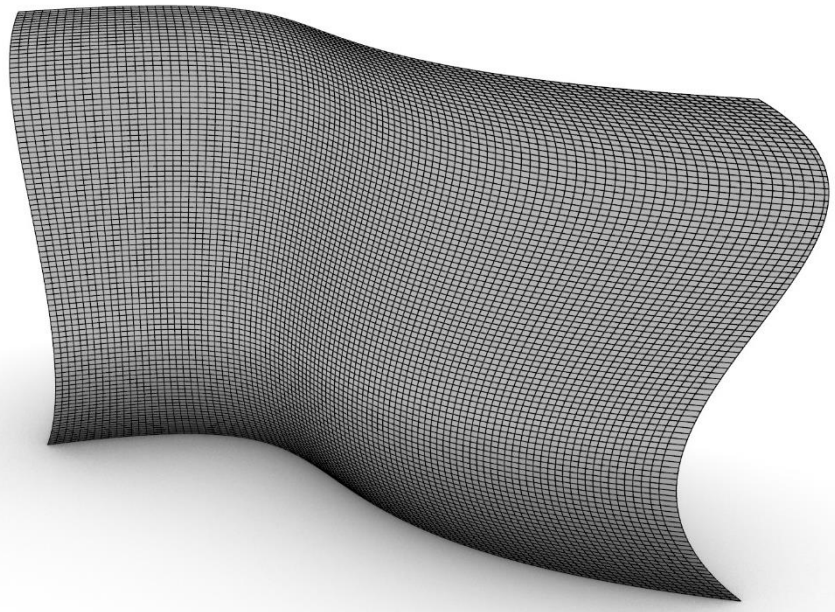


Interactive Design of Discrete Voss Nets and Simulation of their Rigid Foldings (CGTA 2024) M. Kilian et al.

Introduction
Trapezoidal Quad-Surfaces



T-hedron

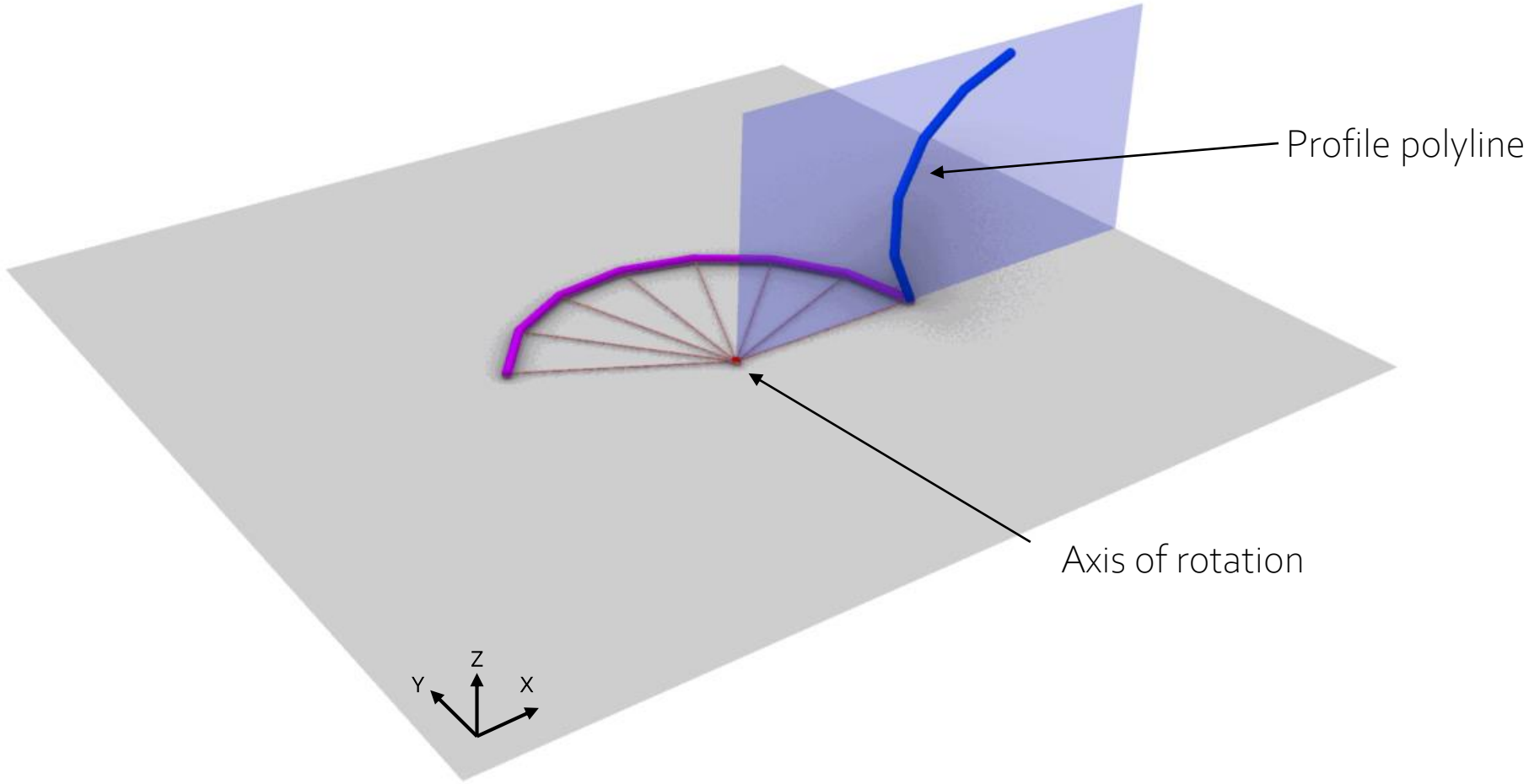


T-surface

Isometric Deformations of Discrete and Smooth T-surfaces (Journal of Computational Geometry) I. Izmestiev, A. Rasoulzadeh, J. Tervooren

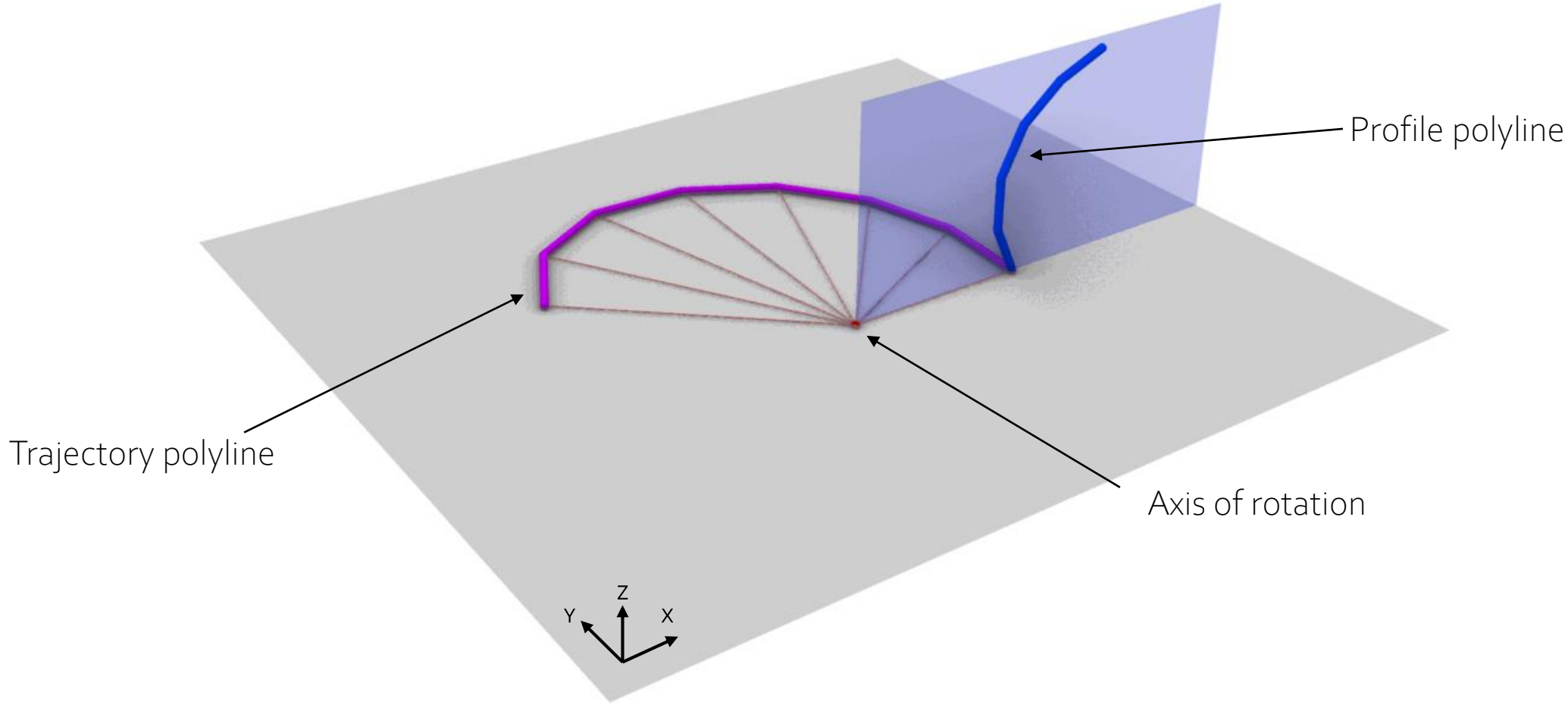
Construction of T-hedra

Surface of Revolution

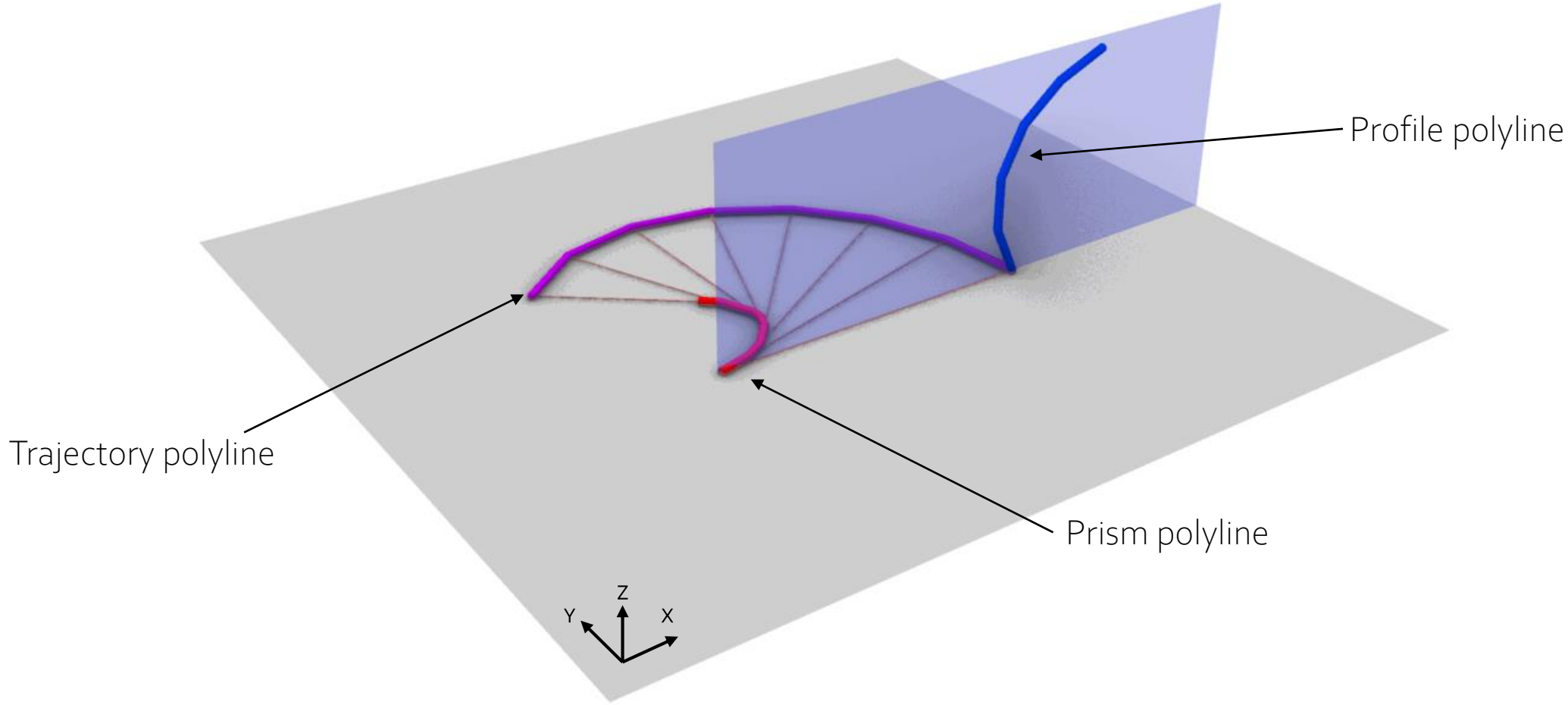


Construction of T-hedra

Stretch Rotation

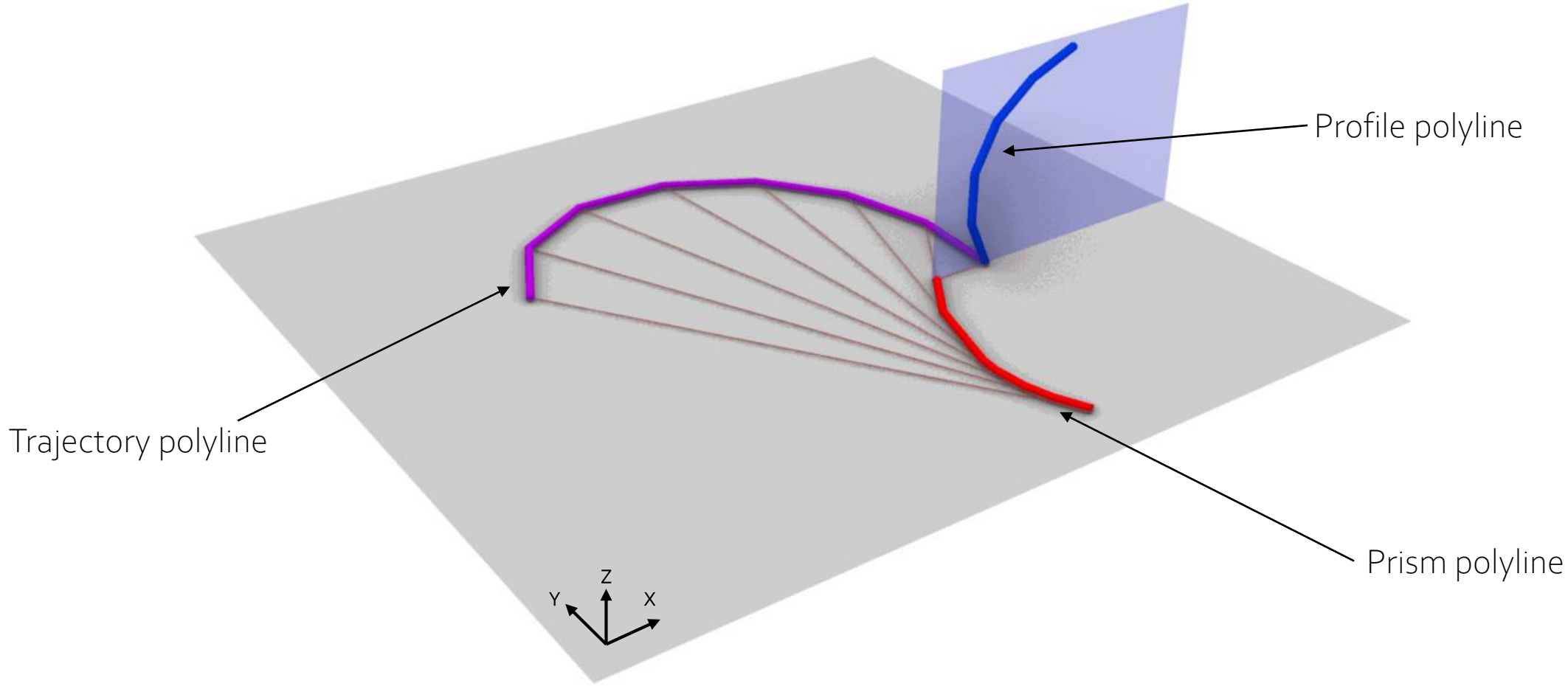


Construction of T-hedra
Moulding Surface



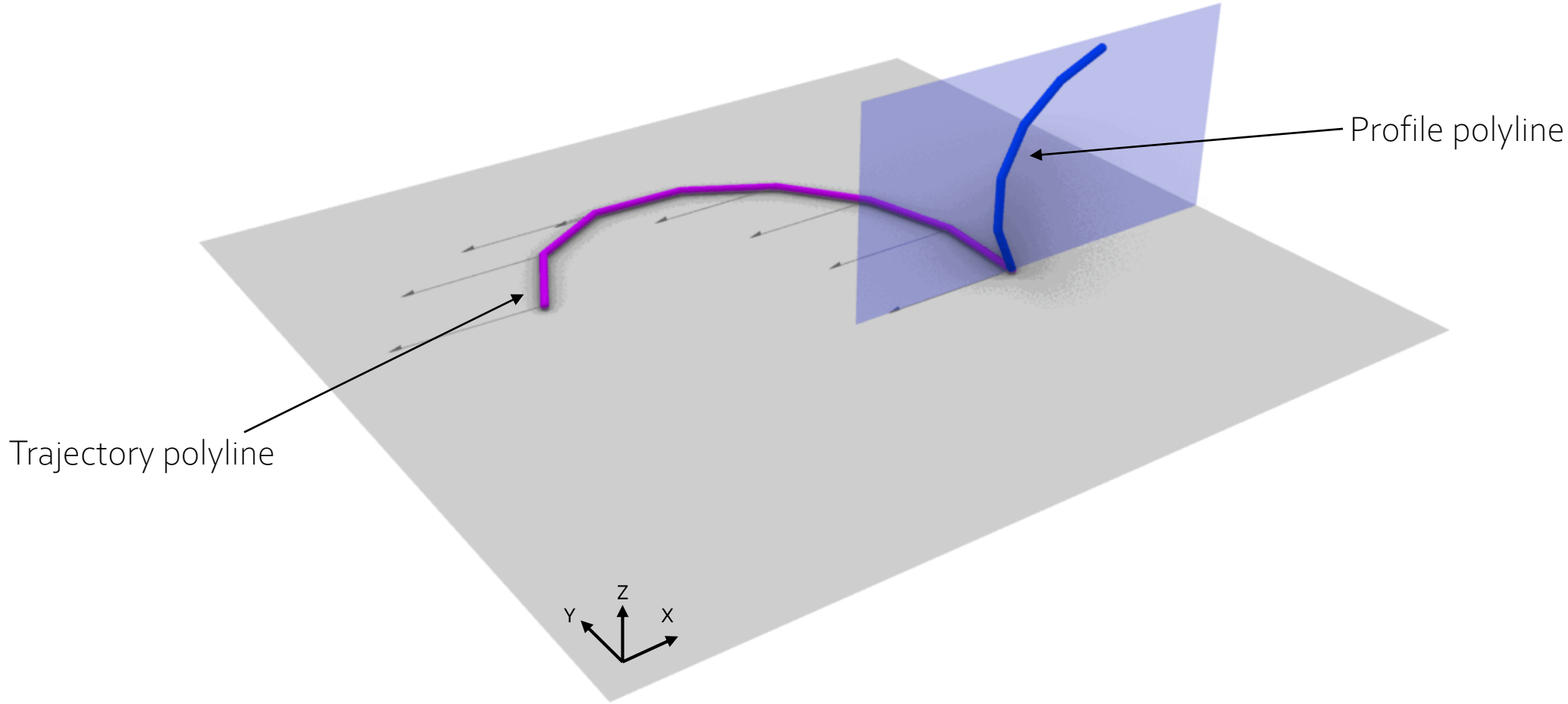
Construction of T-hedra

Profile Affine



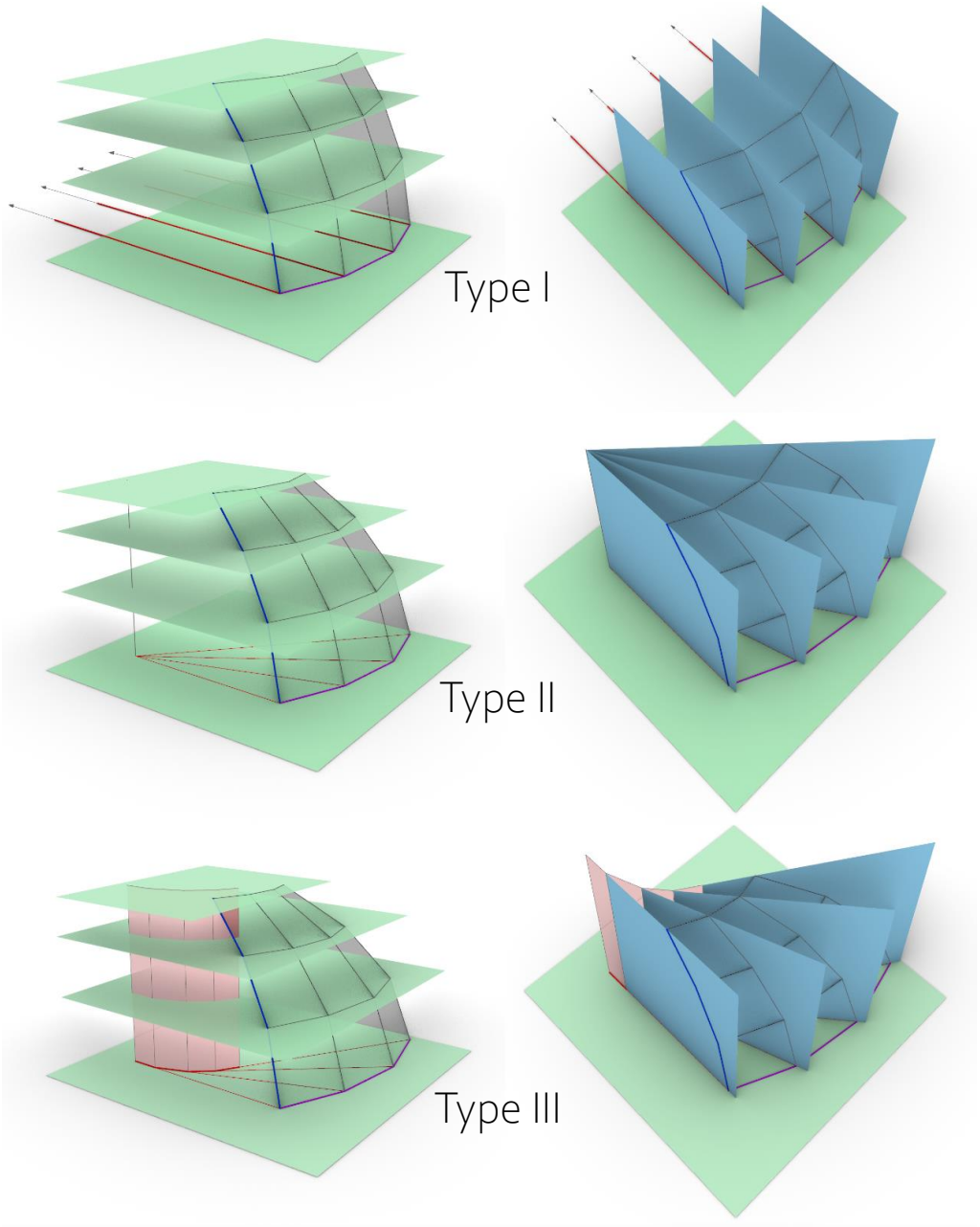
Construction of T-hedra

Translational Surface



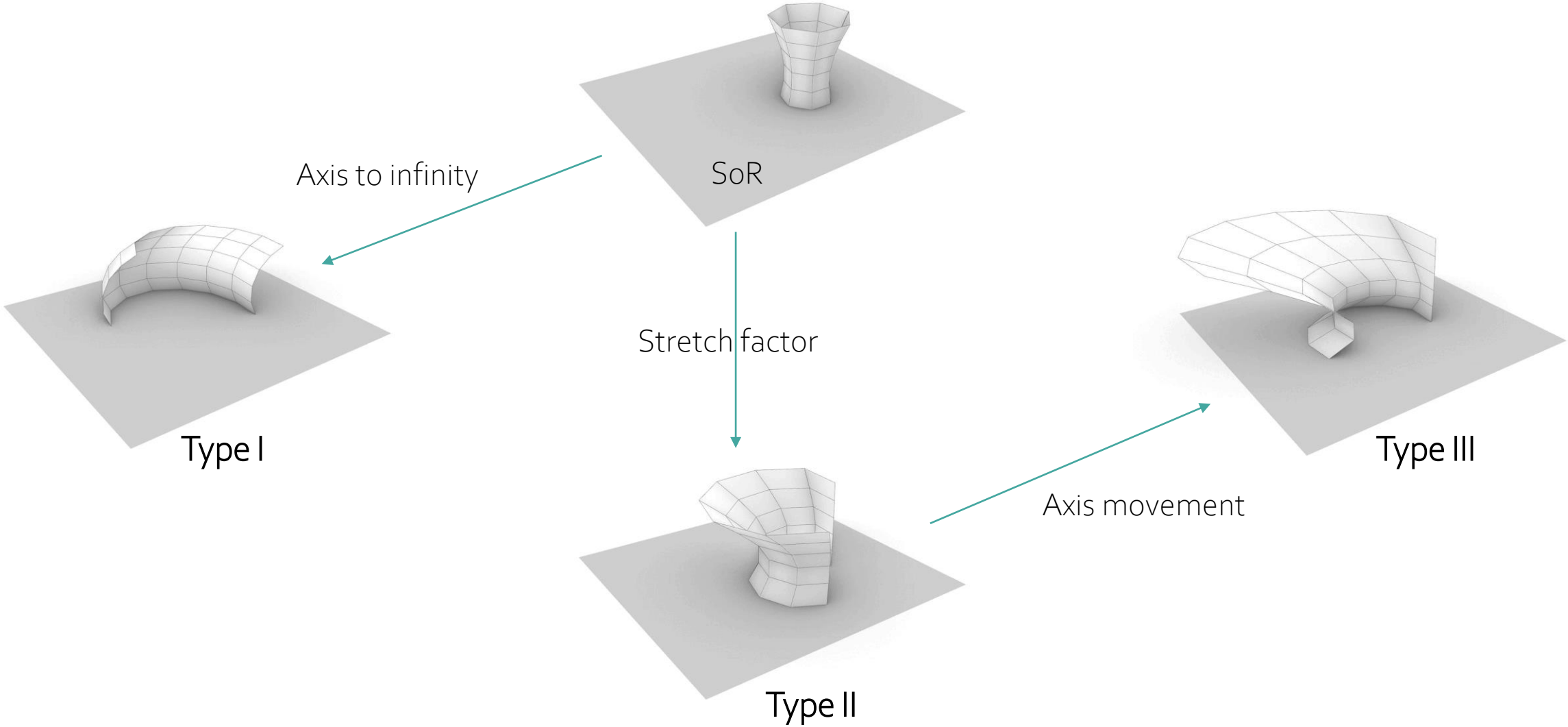
Properties of T-hedra

- Trapezoidal faces
- Planar coordinate polylines in any state of deformation
- Trajectory planes (green) are parallel
- Profile planes (blue) are ortogonal to the base plane
- Class-preserving Isometric deformation
- Can be developable and/or flat-foldable

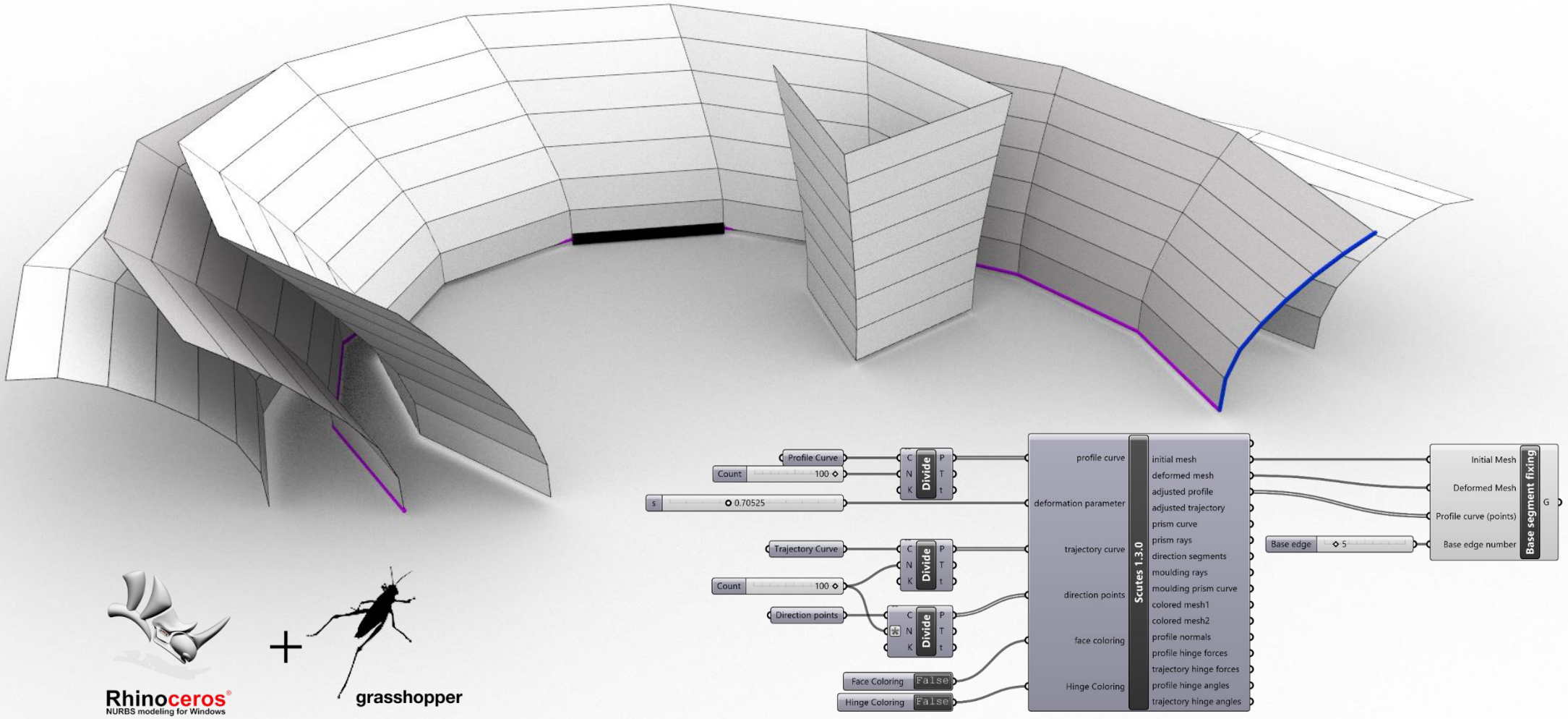


T-hedra

Isometric Deformations



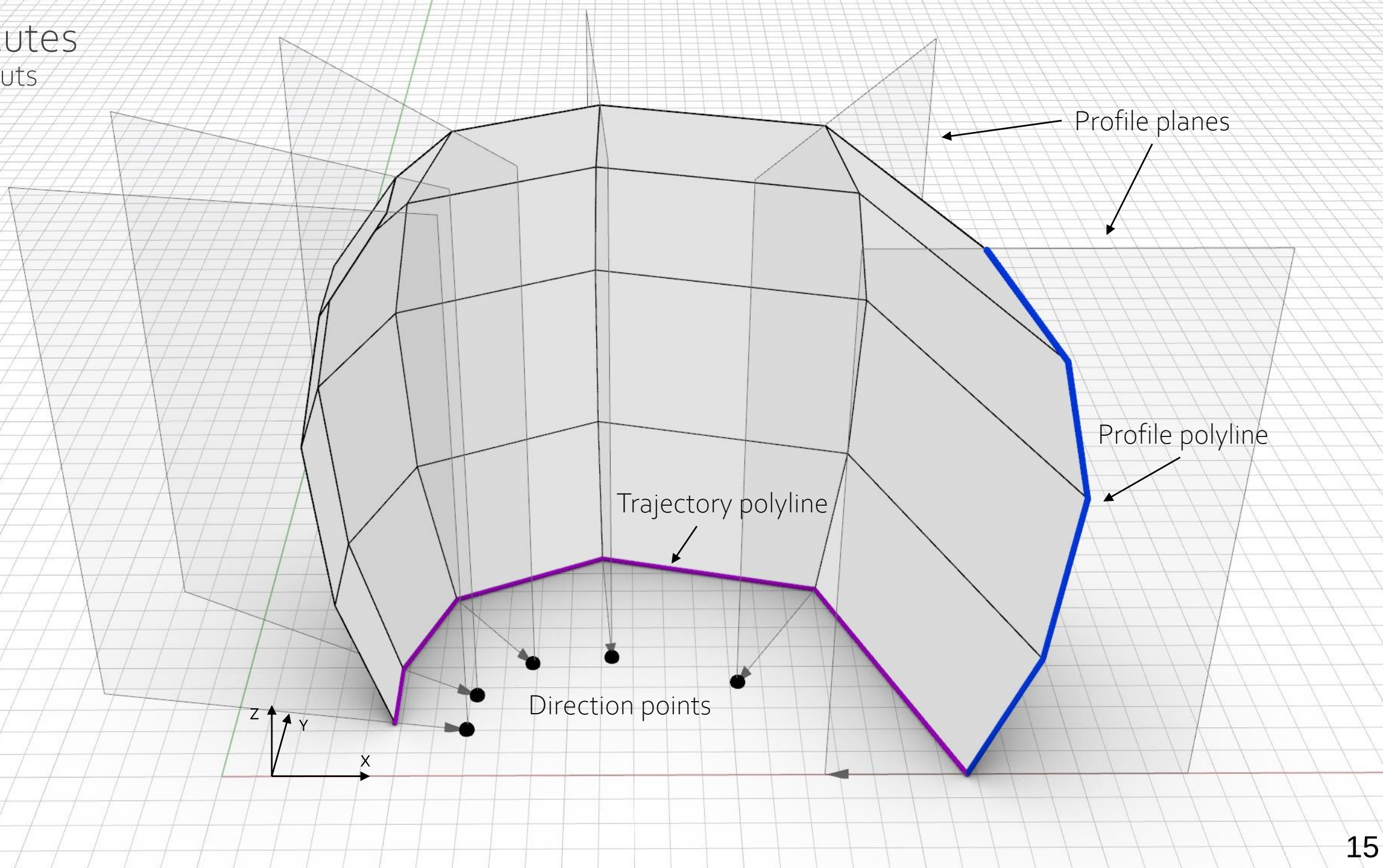
Scutes
Interactive Tool



Using Flexible Trapezoidal Quad-Surfaces for Transformable Design (IASS2021) K. Sharifmoghaddam, G. Nawratil, A. Rasoulzadeh, J . Tervooren

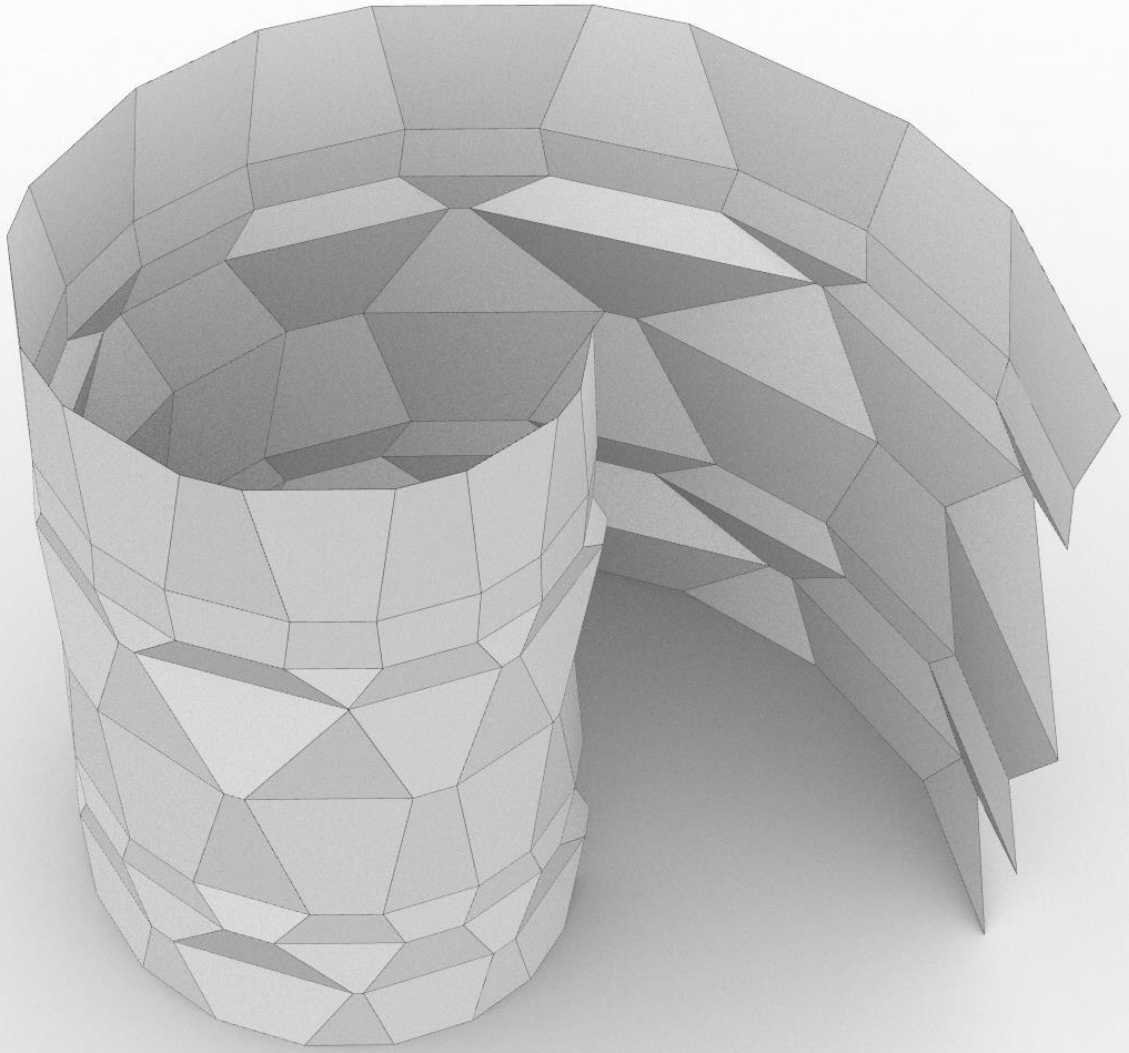
Scutes

Inputs

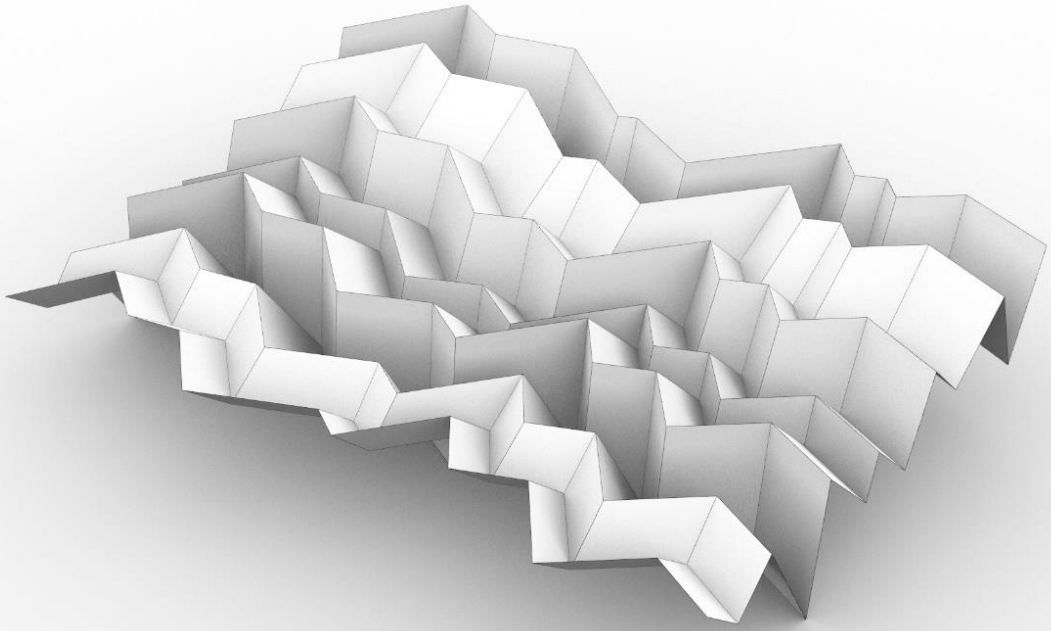


Scutes

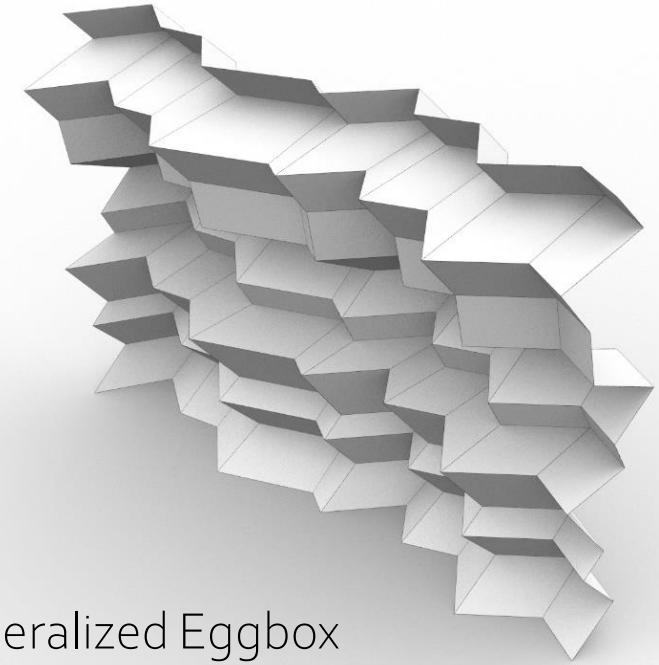
Developable and Flat-foldable T-hedra



Generalized Chickenwire



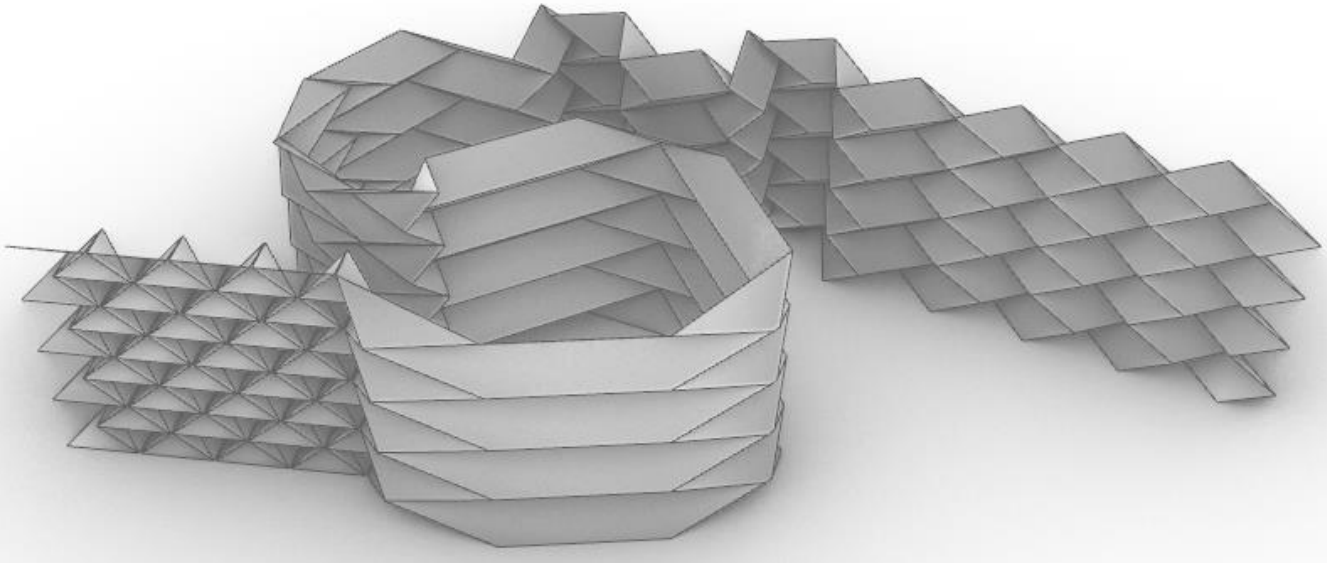
Generalized Miura-ori

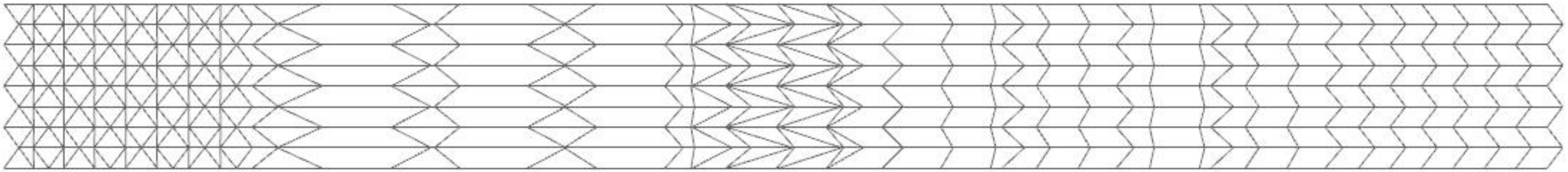


Generalized Eggbox

Trajectory Developable T-hedra

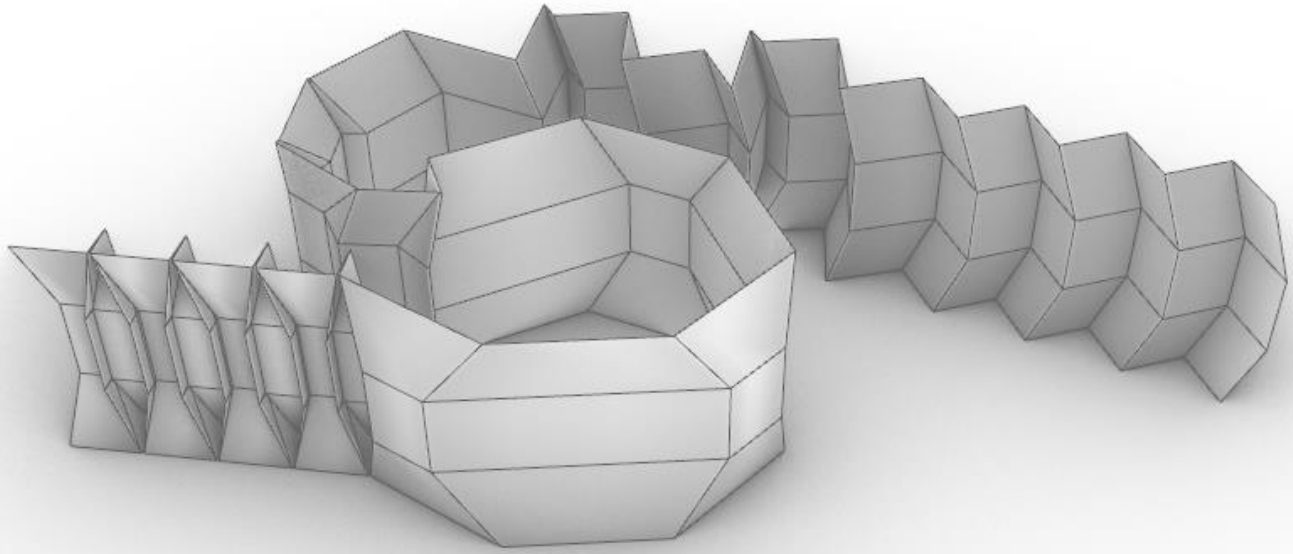
Hybrid pattern

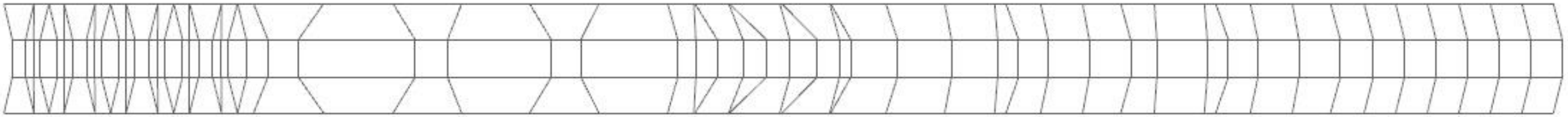




Trajectory Developable T-hedra

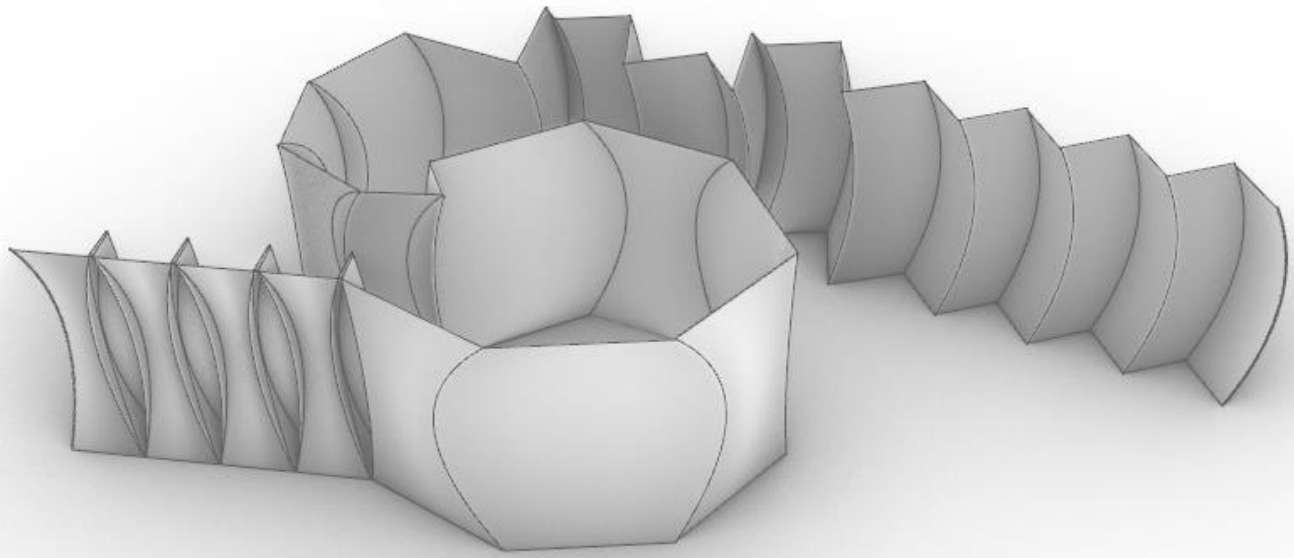
Hybrid pattern

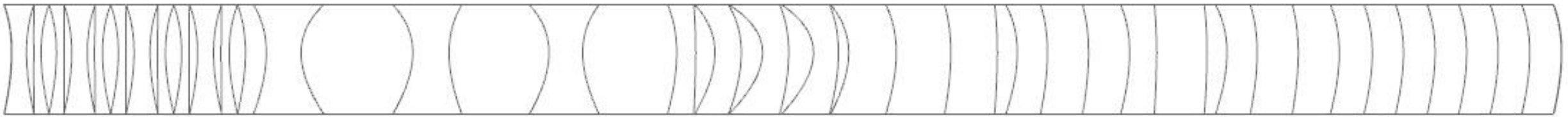




Trajectory Developable semi-discrete T-hedra

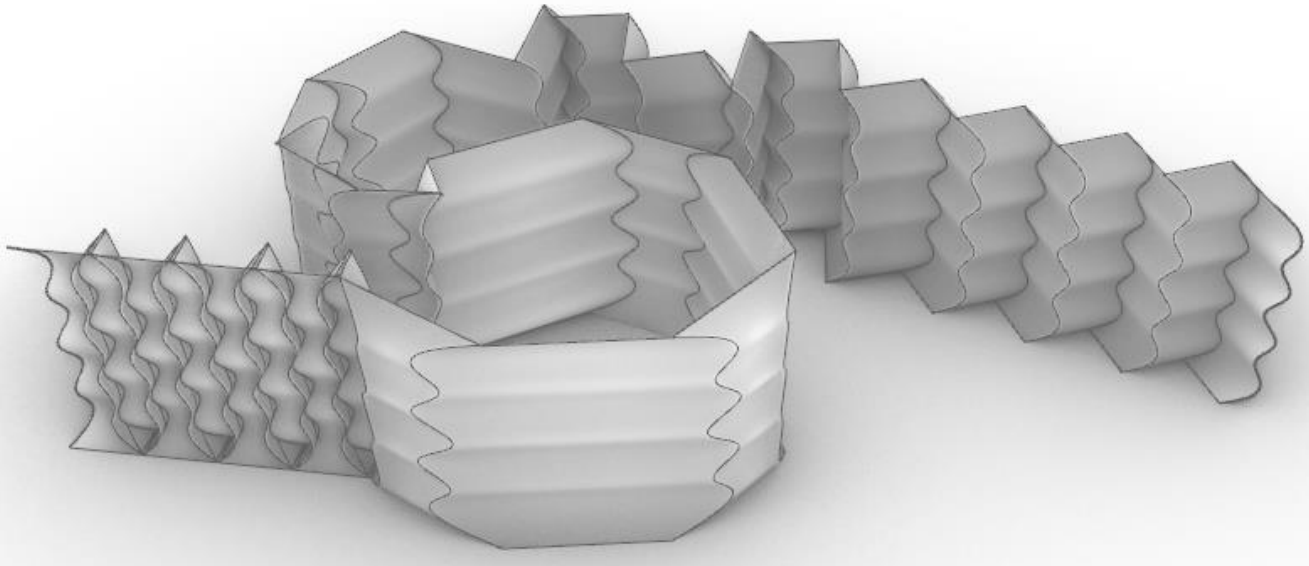
Hybrid pattern

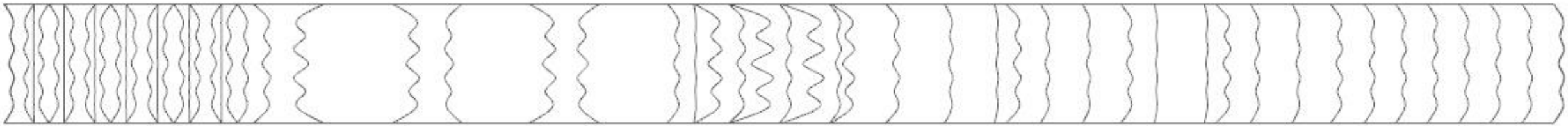




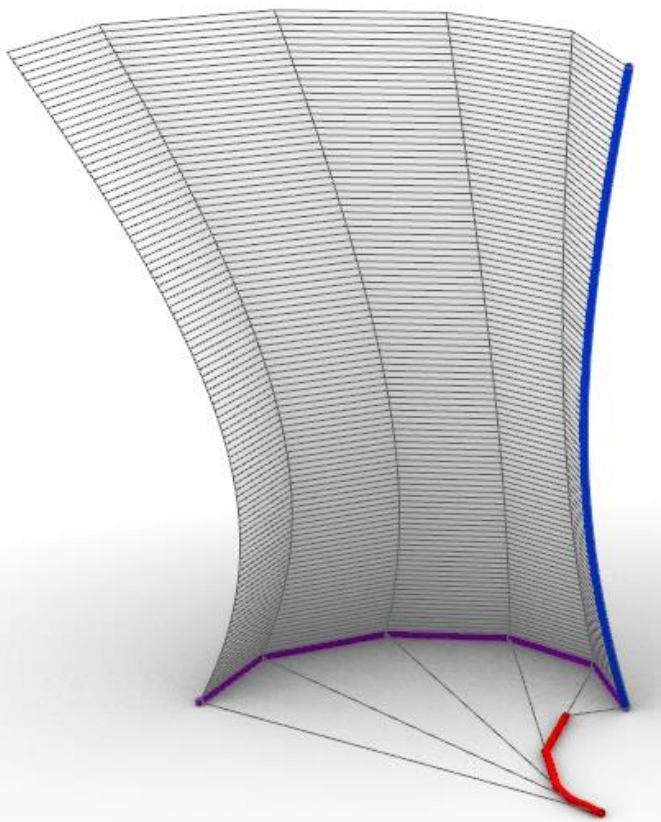
Trajectory Developable semi-discrete T-hedra

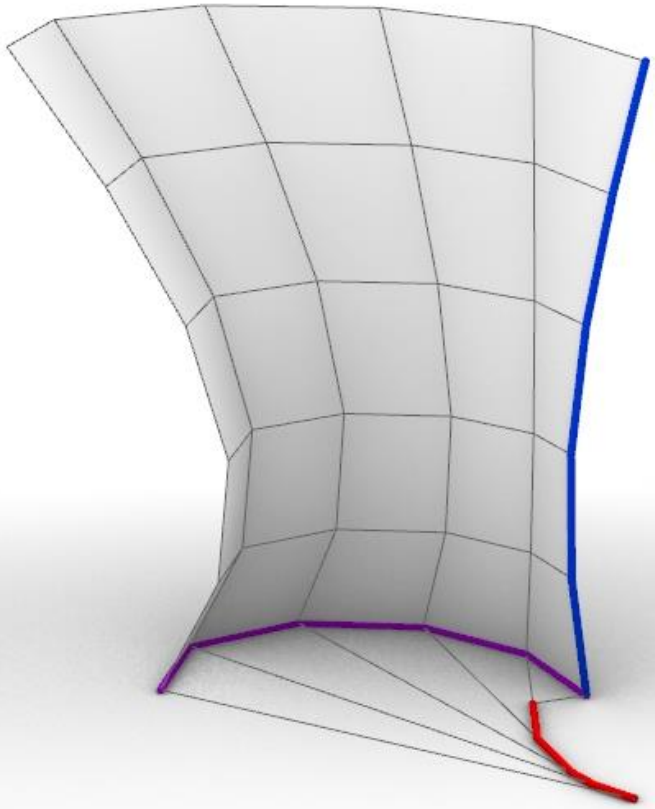
Hybrid pattern

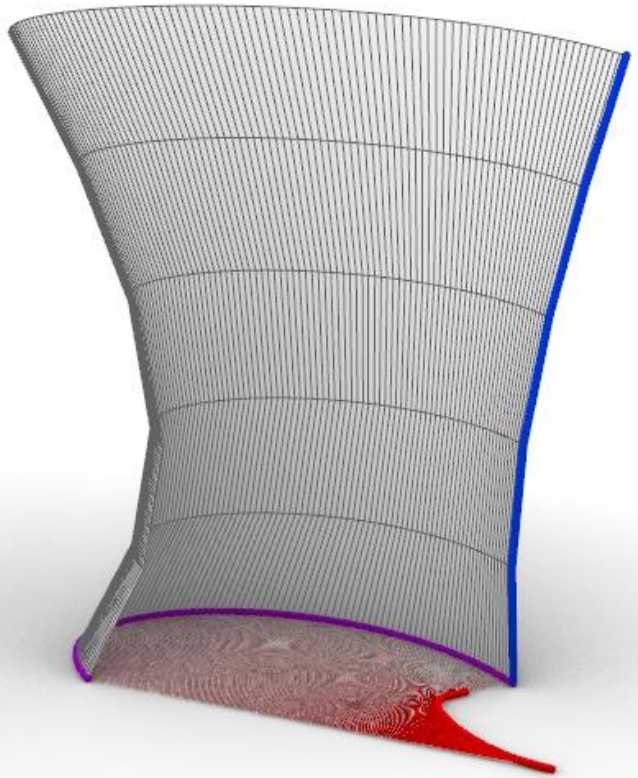




Discrete and Semi-discrete
Construction

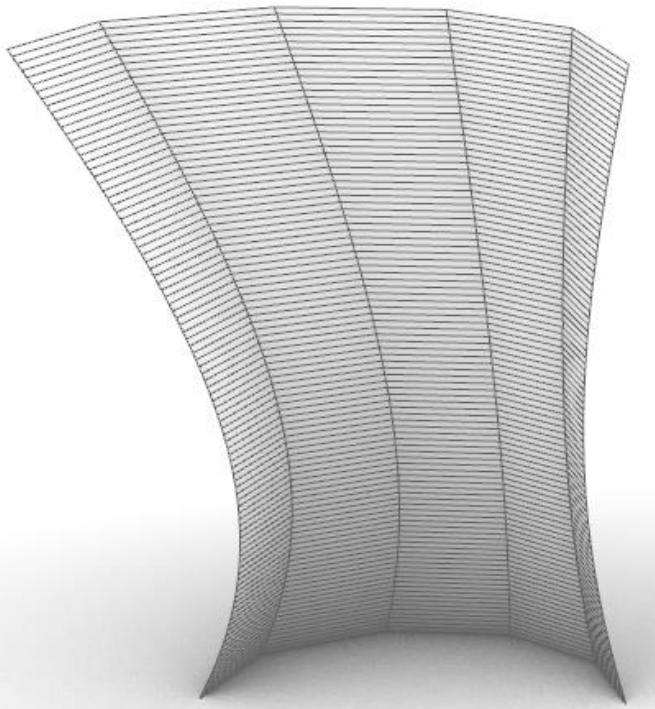


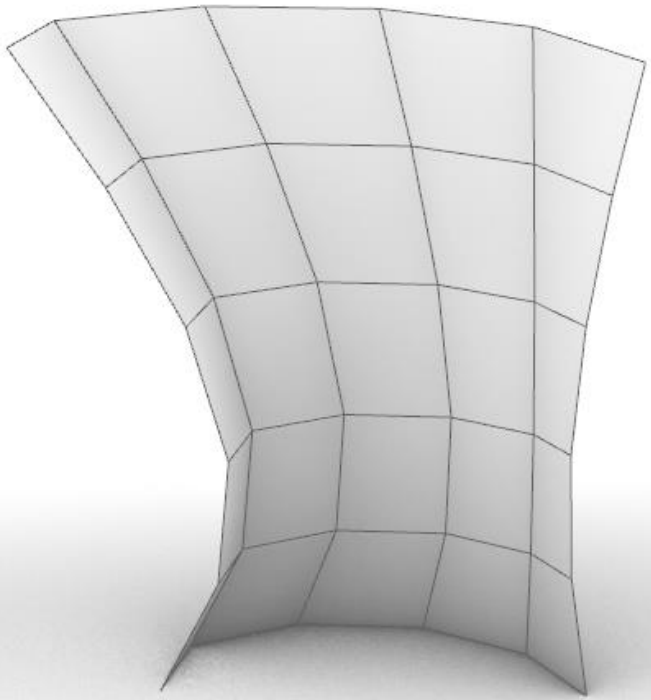


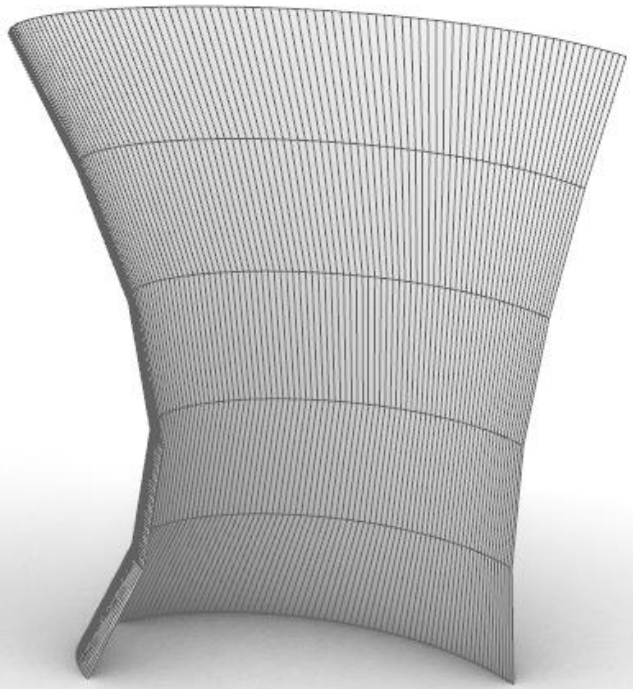


Discrete and Semi-discrete

Developability - Piecewise

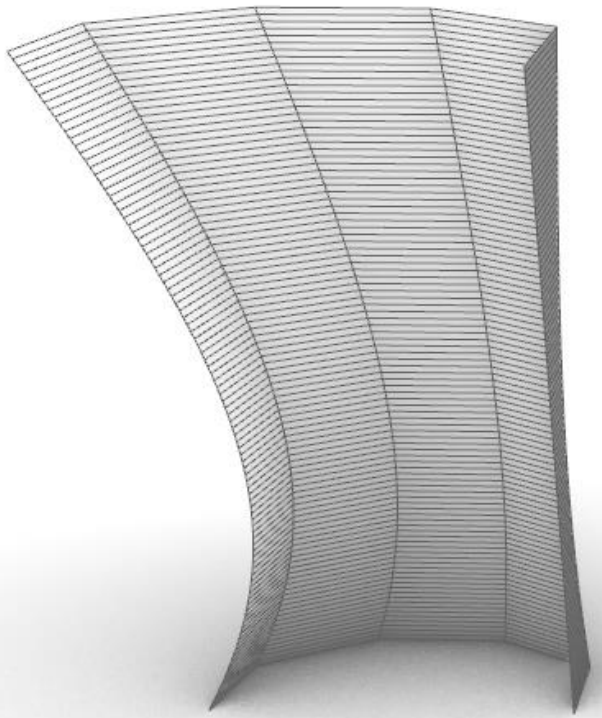


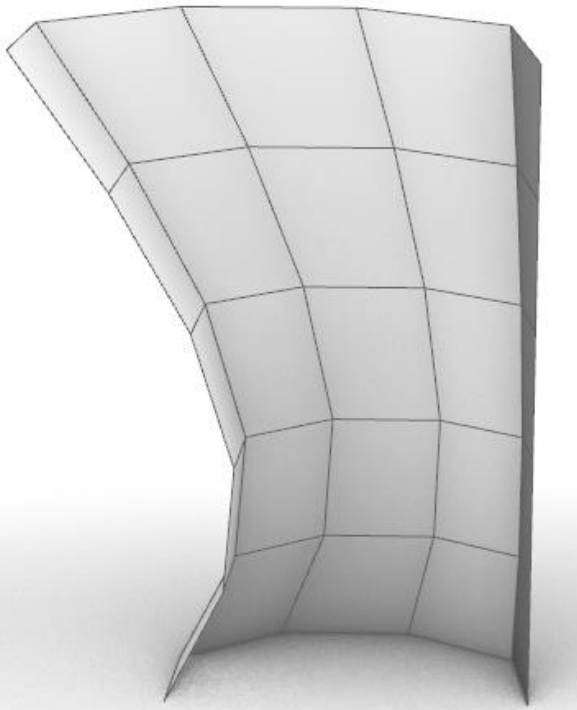


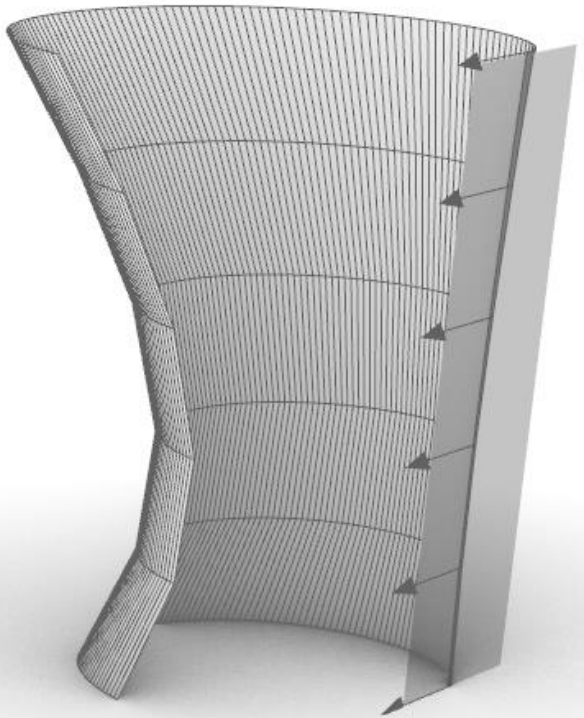


Discrete and Semi-discrete

Flexion Limits - Vertical

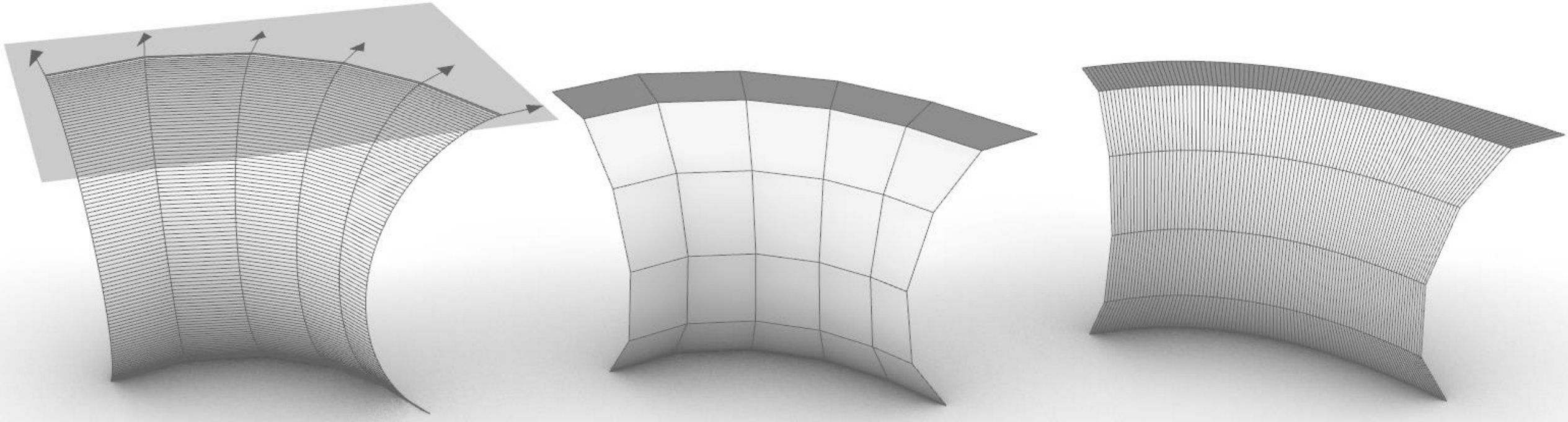






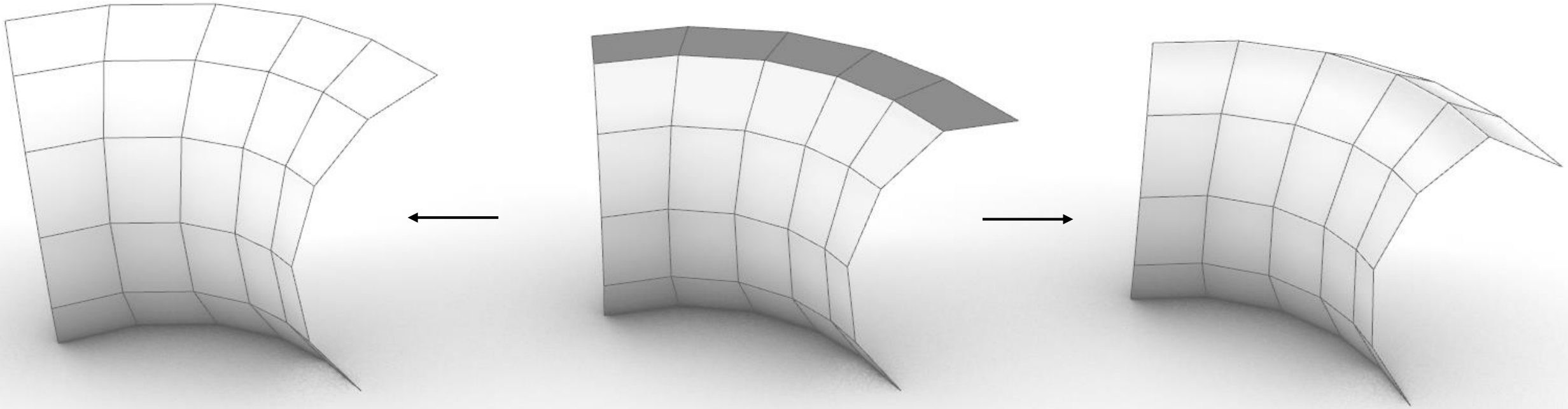
Discrete and Semi-discrete

Flexion Limits - Horizontal

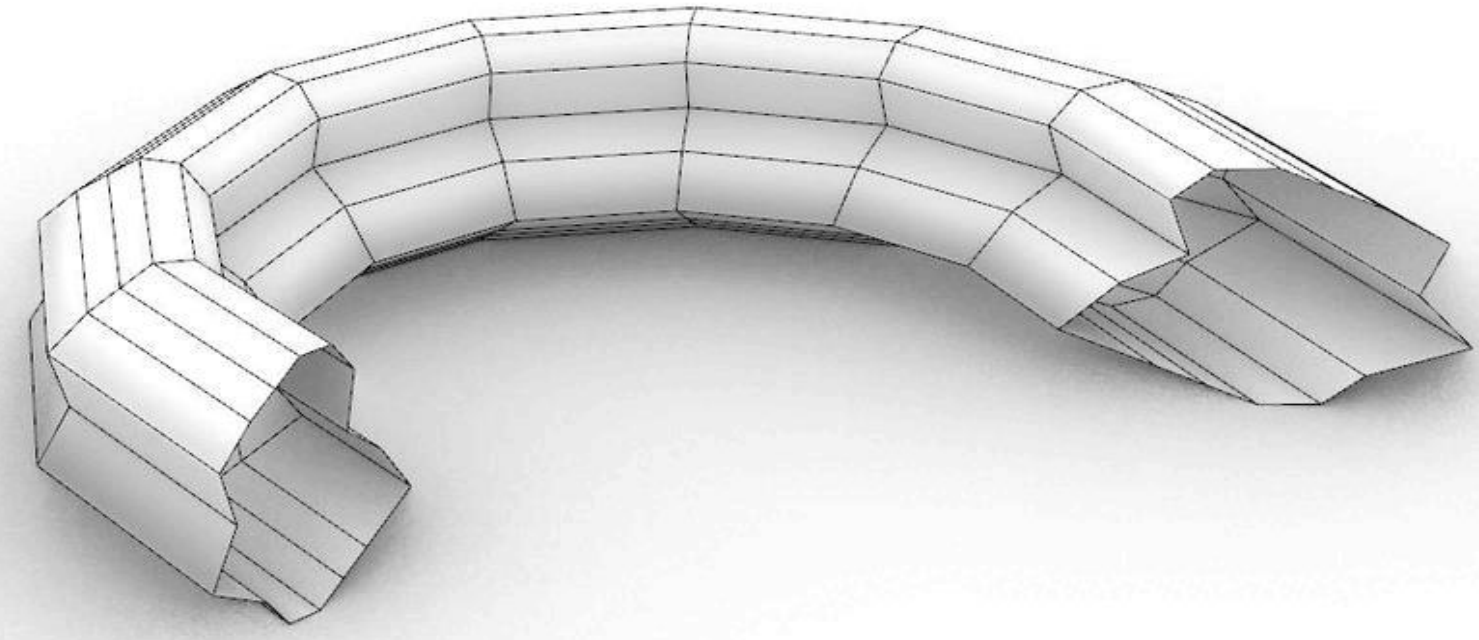


Discrete and Semi-discrete

Flexion Limits - Bifurcation

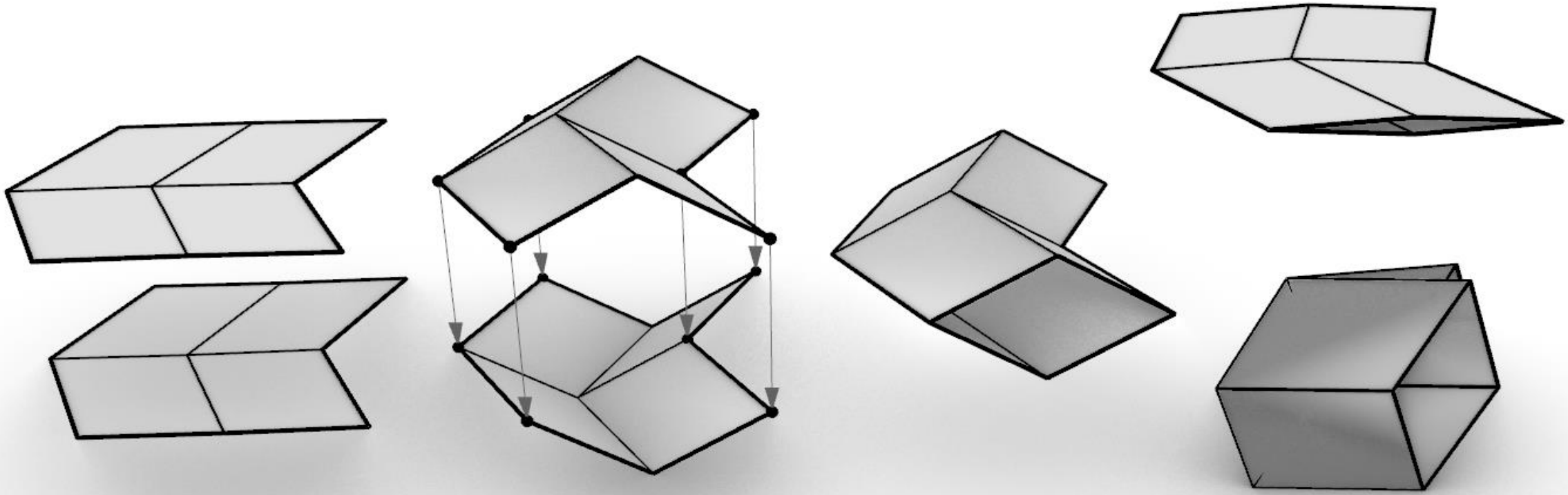


T-hedral Tubes

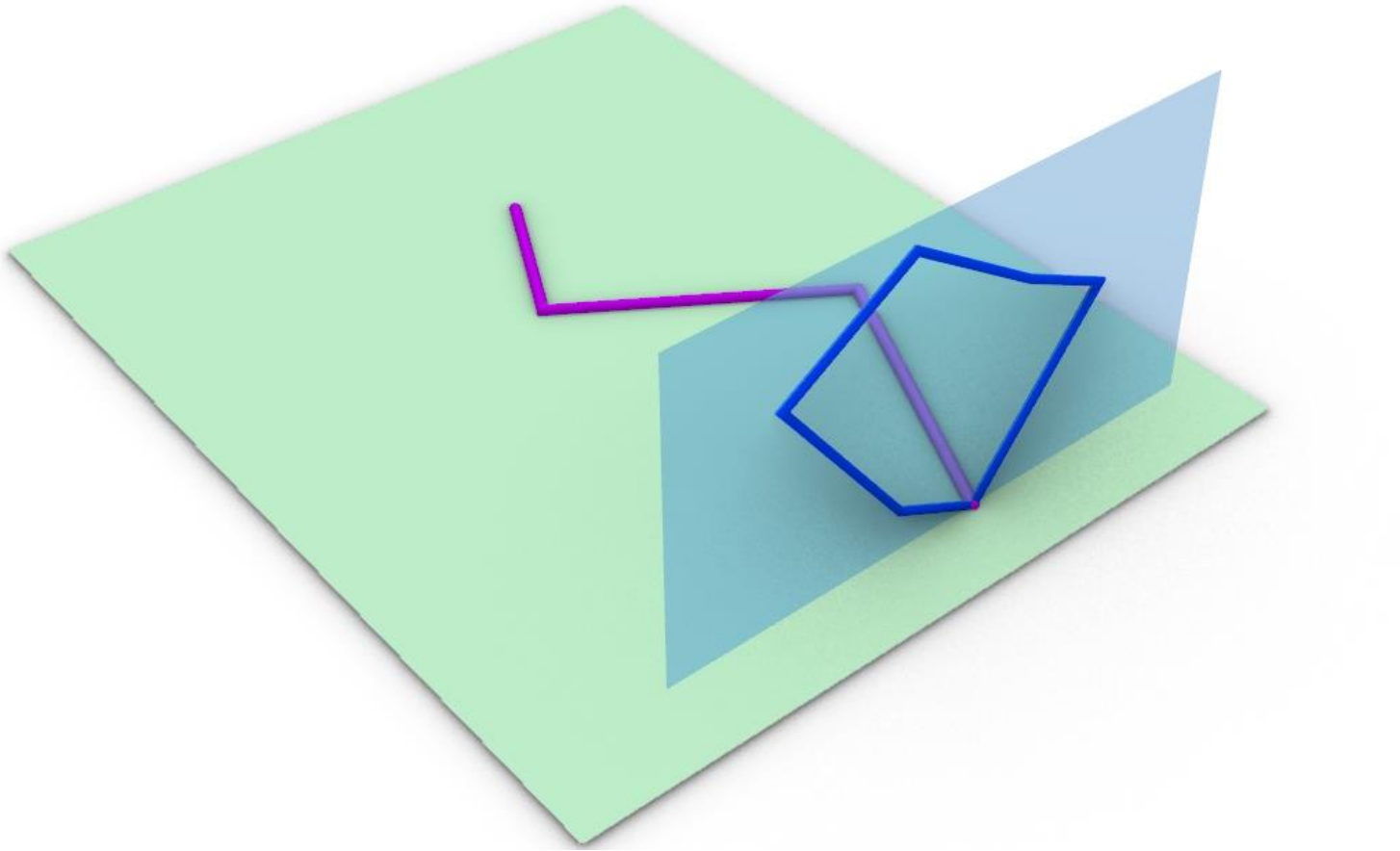


T-hedral Tubes

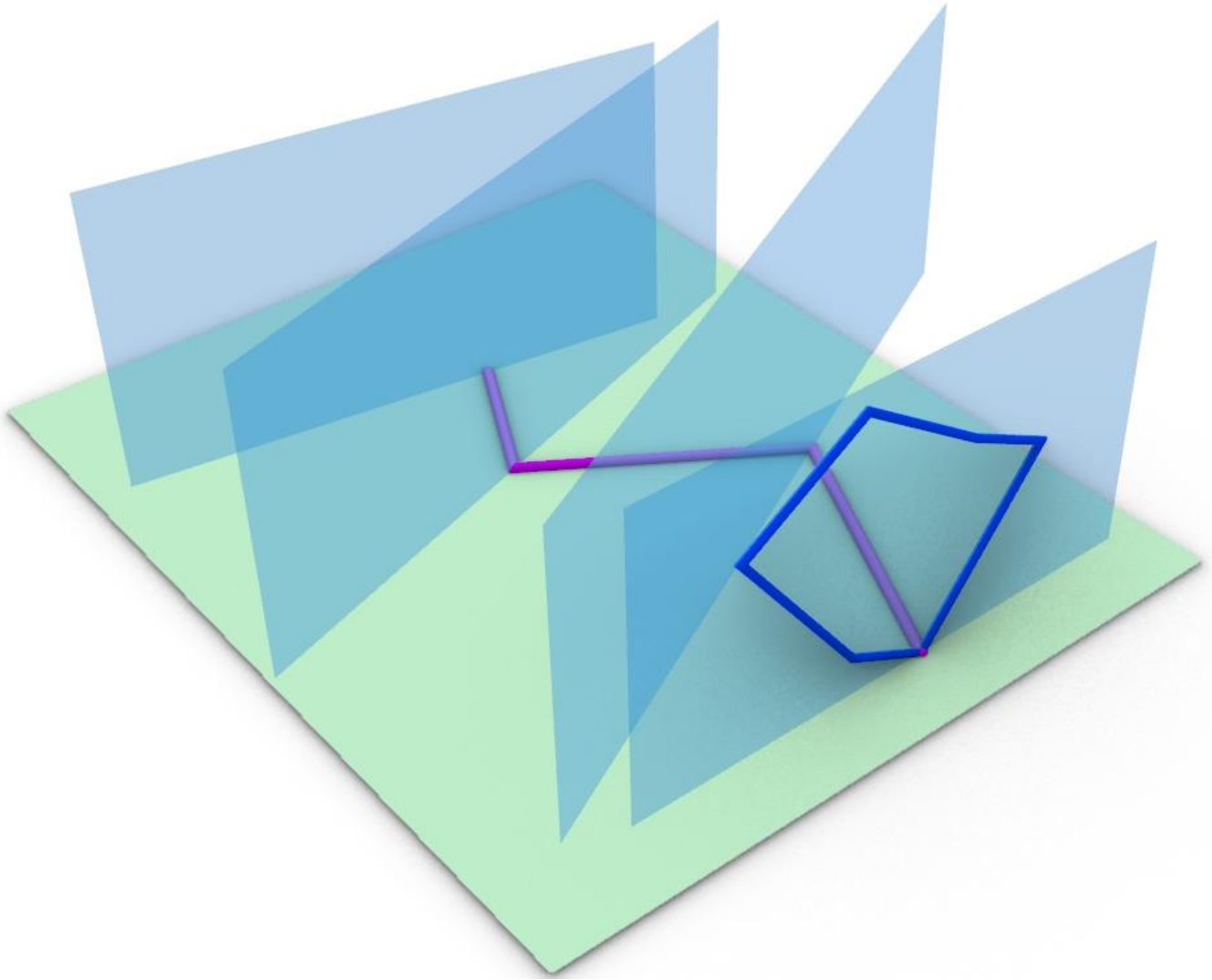
Unit cell



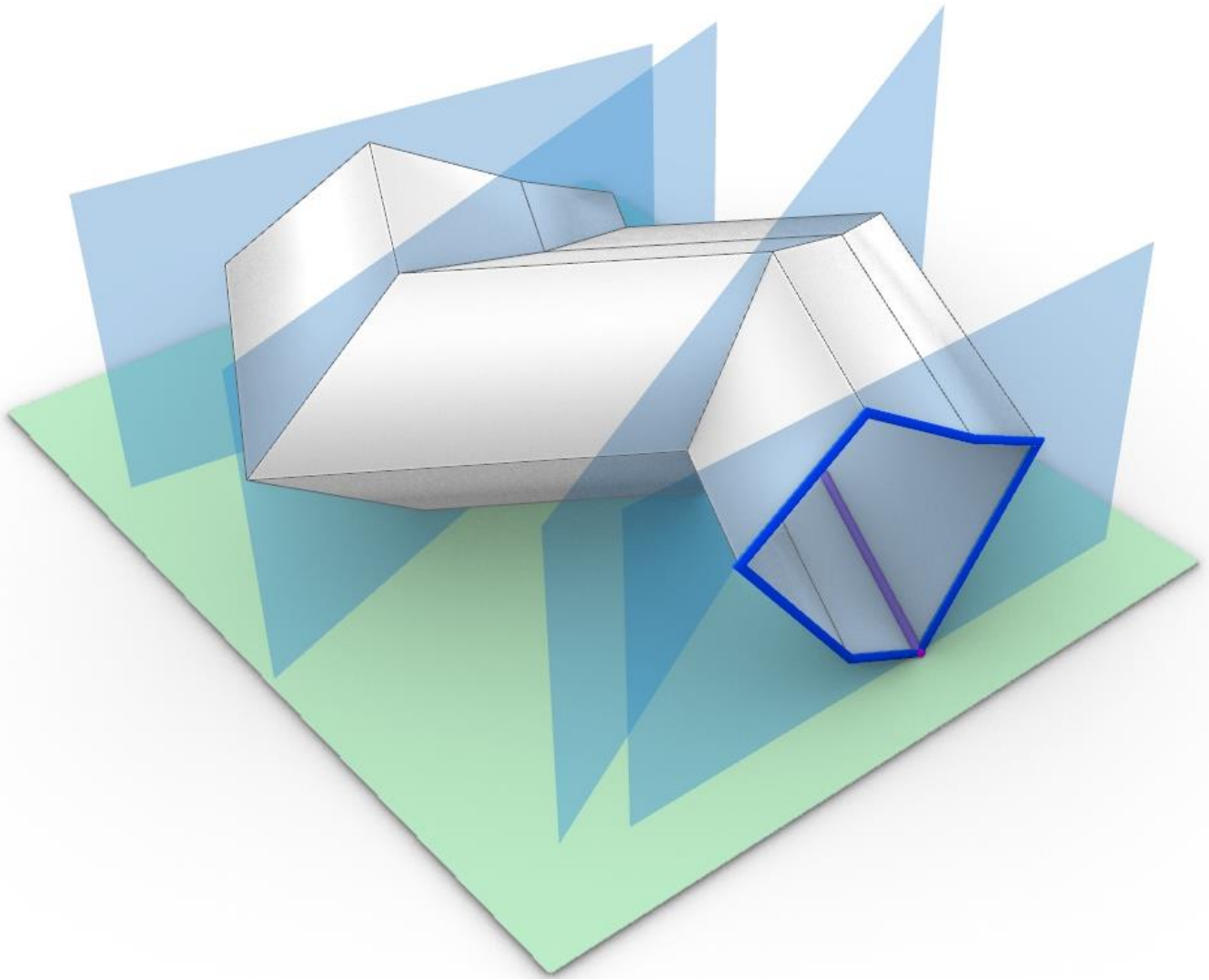
T-hedral Tubes
Construction



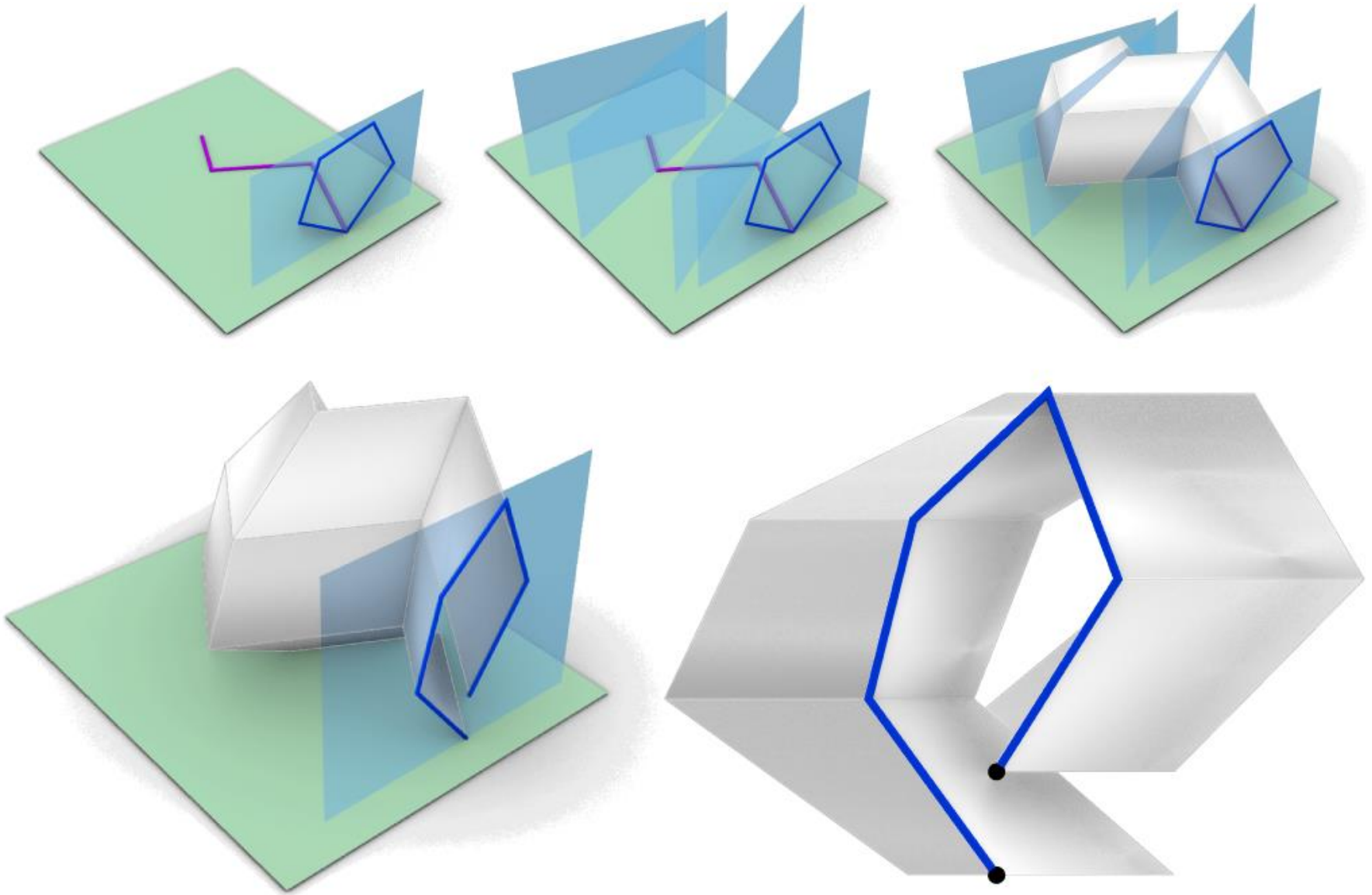
T-hedral Tubes
Construction



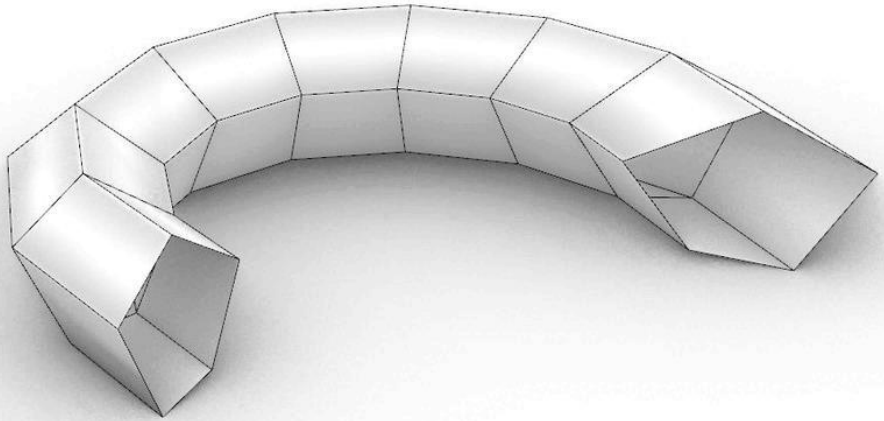
T-hedral Tubes
Construction



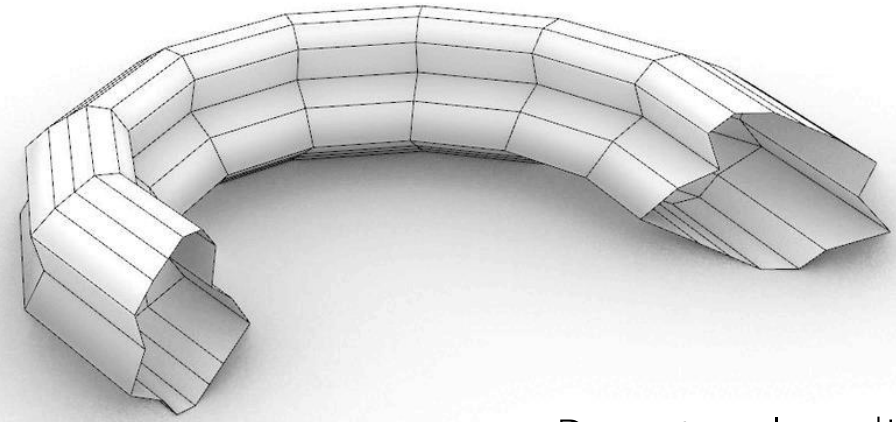
T-hedral Tubes
Invalid Profile



T-hedral Tubes
Invalid Profile



Opens up!

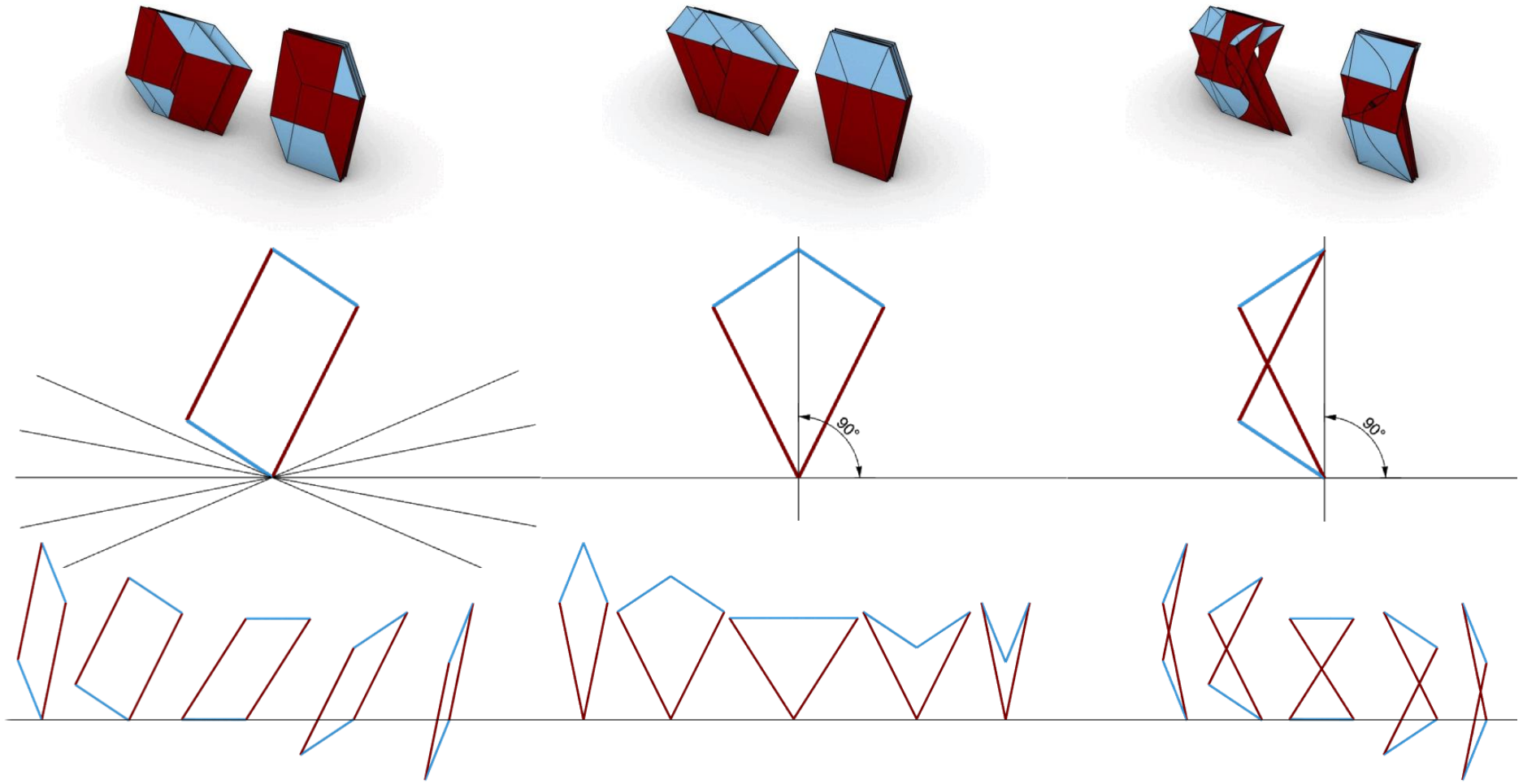


Remains closed!

- A property of the profile (cross-section) not the trajectory

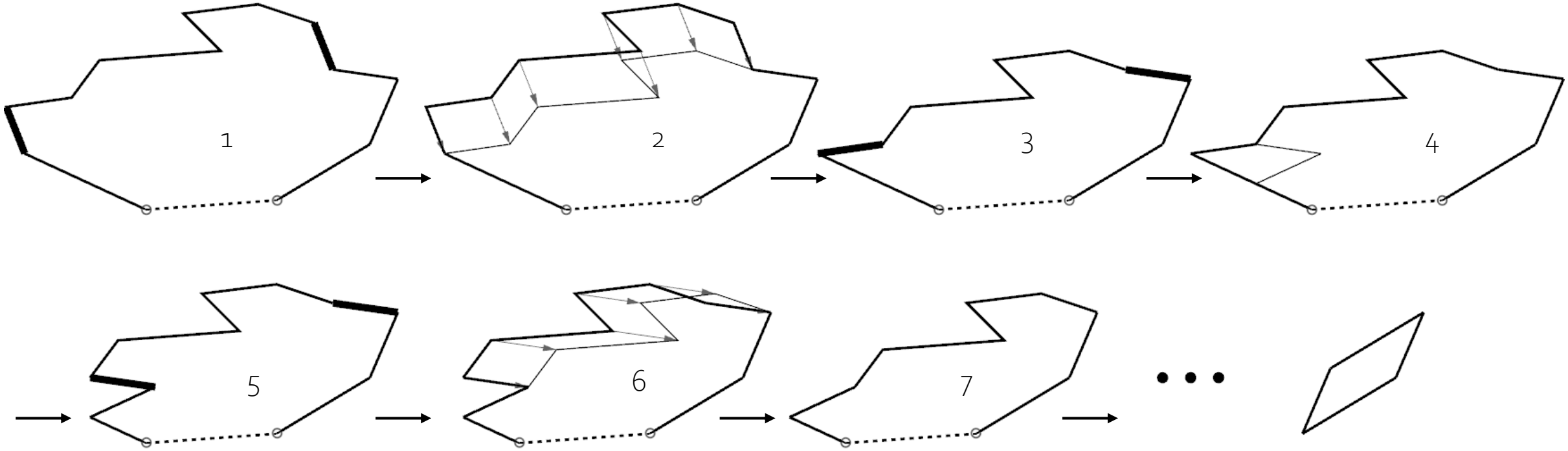
T-hedral Tubes

All Different Quad Cross-sections



T-hedral Tubes

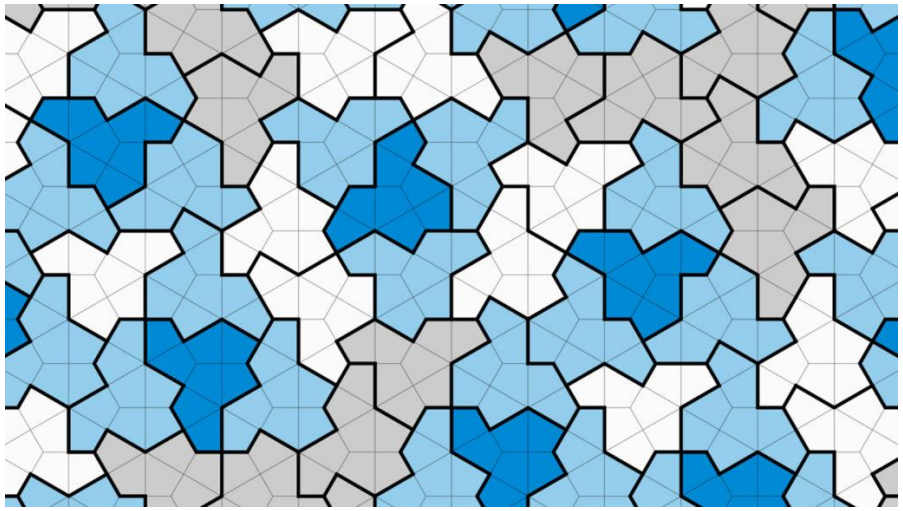
Profile Polyline Closure [sufficient] Condition



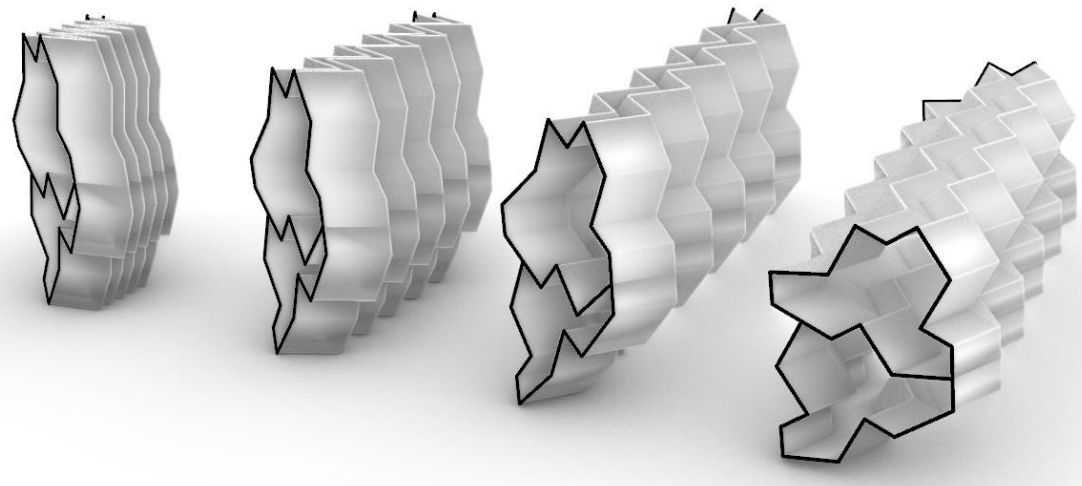
- The segments are in pairs with the same length and the same (non-zero) absolute value of the slope
- Can be constructed by adding or subtracting parallelograms and deltoids with vertical symmetry line

T-hedral Tubular Structures

Aperiodic tiling - Einstein tile

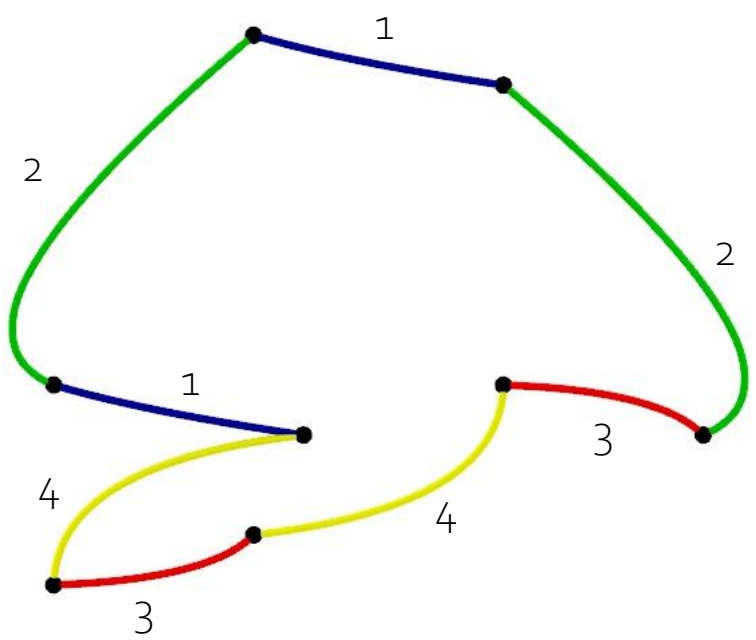


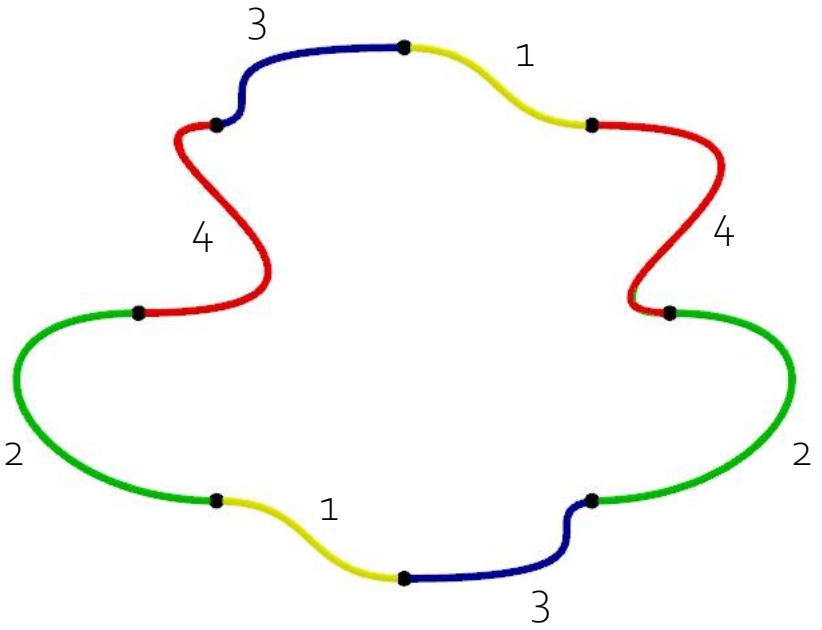
https://static.scientificamerican.com/sciam



T-hedral Tubes

Profile Poly-curve Closure Condition



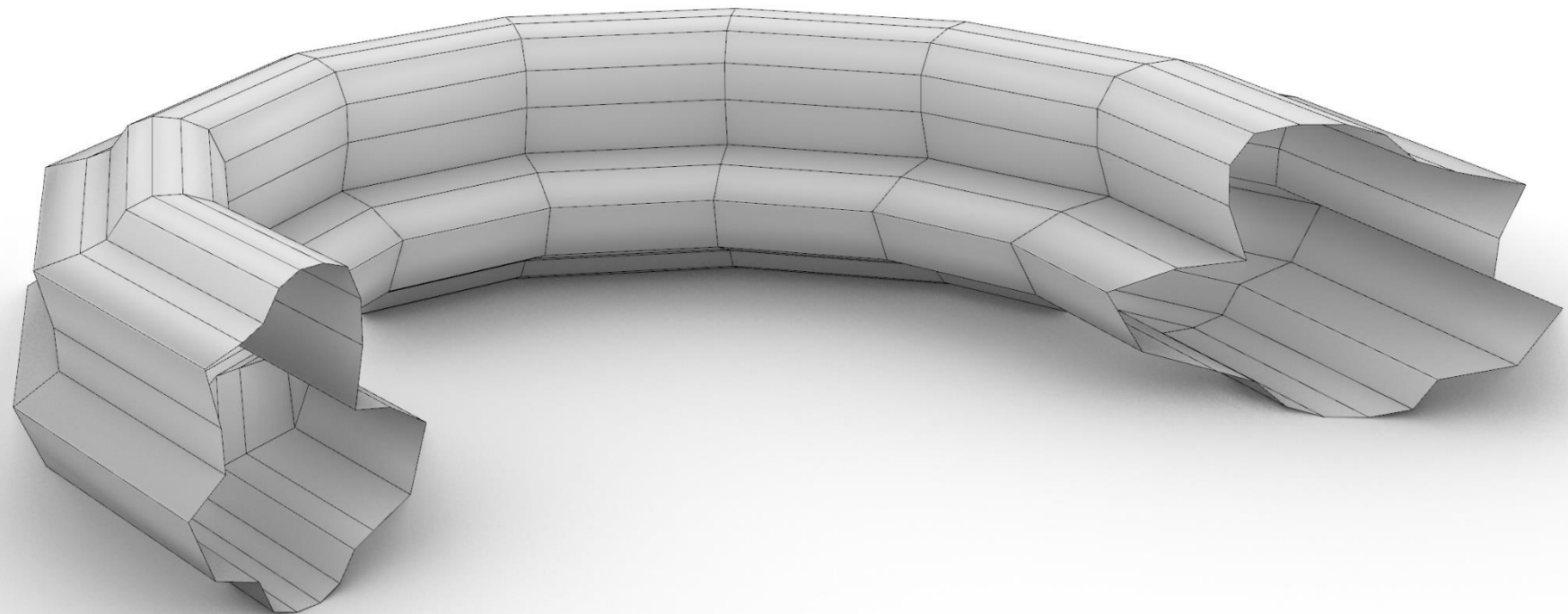


Curved segments must be in pairs with either of these relationships:

1. Identical
2. Vertically mirrored
3. Horizontally mirrored
4. 180 degrees rotated

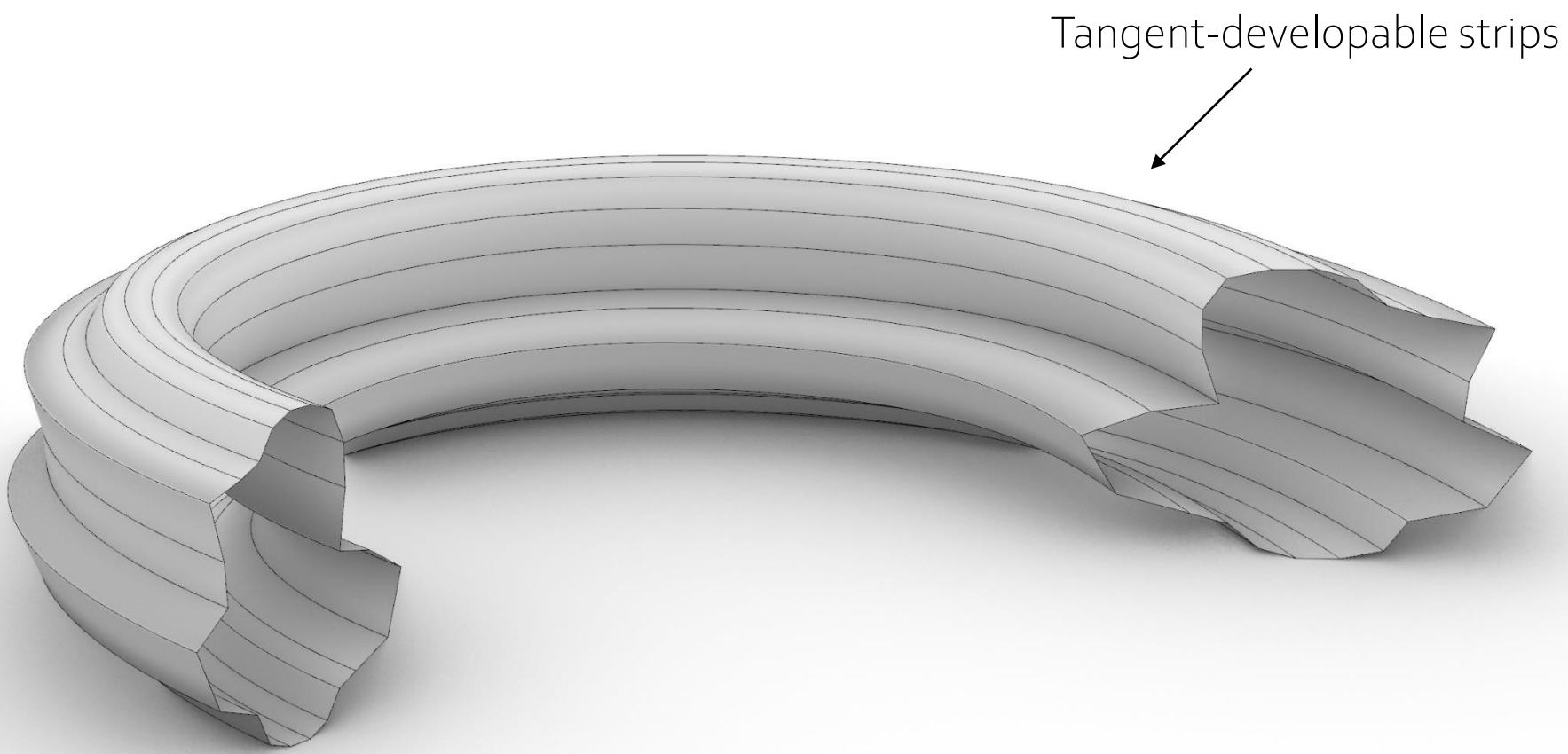
T-hedral Tubes

Discrete

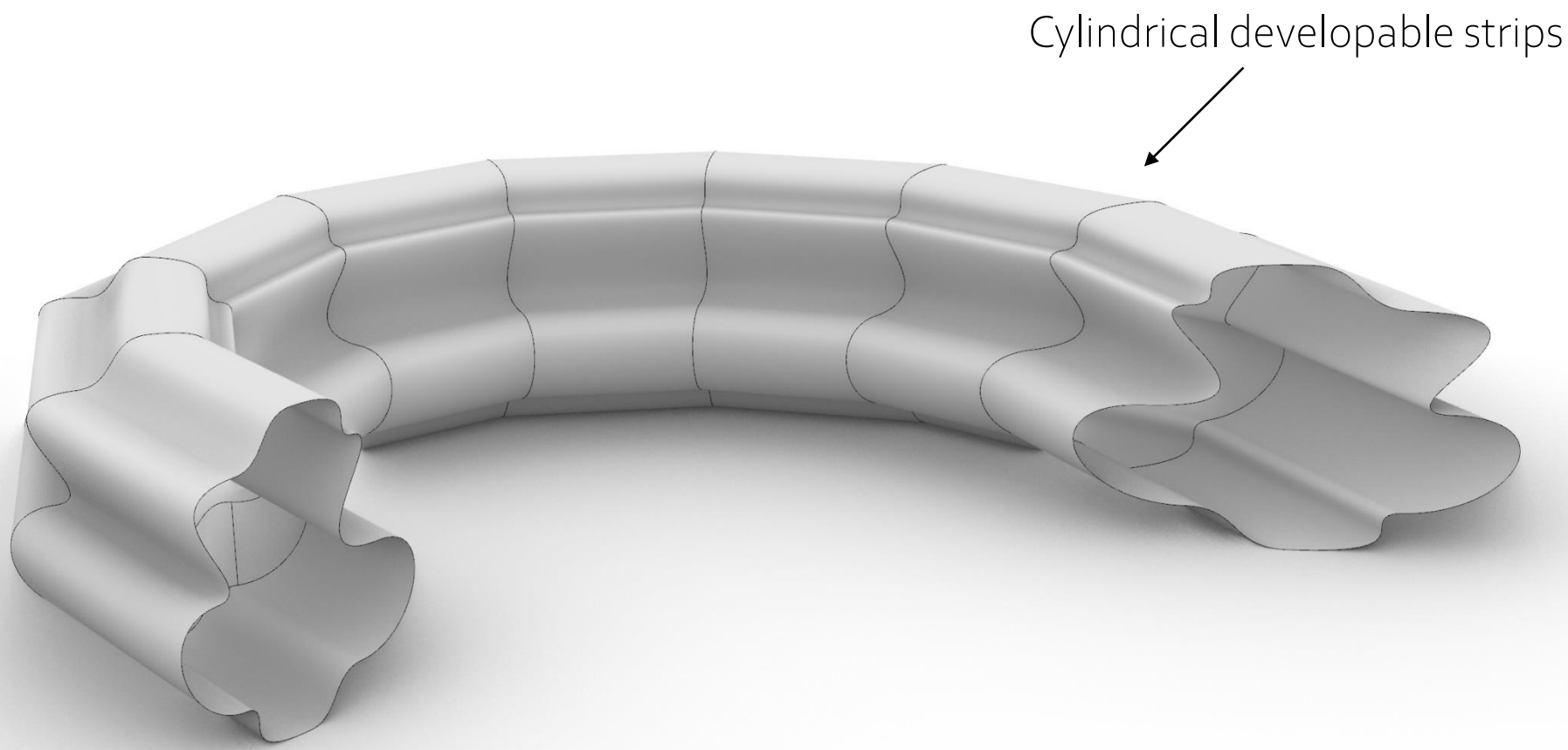


T-hedral Tubes

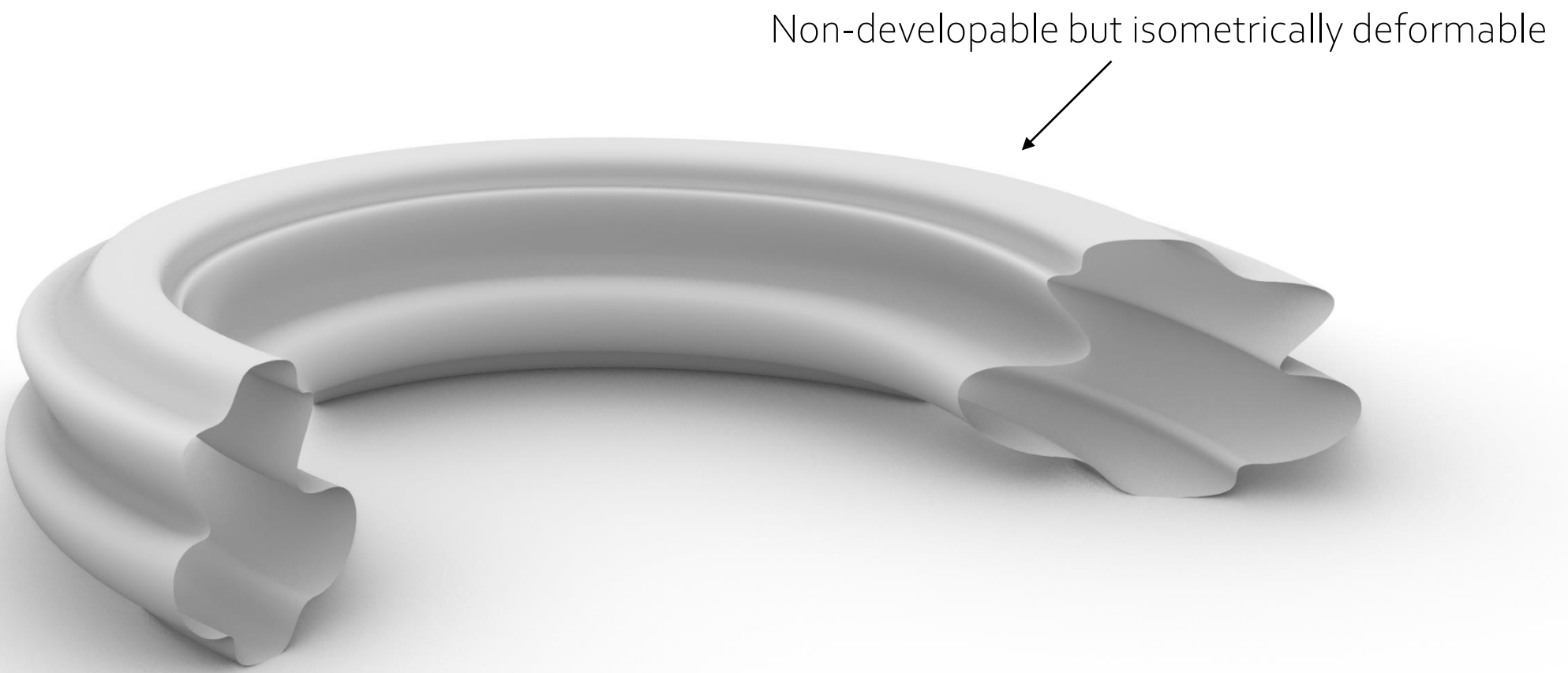
Horizontal Semi-discrete



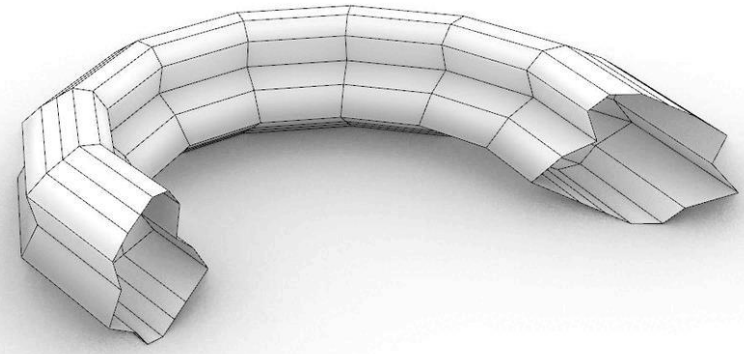
T-hedral Tubes
Vertical Semi-discrete

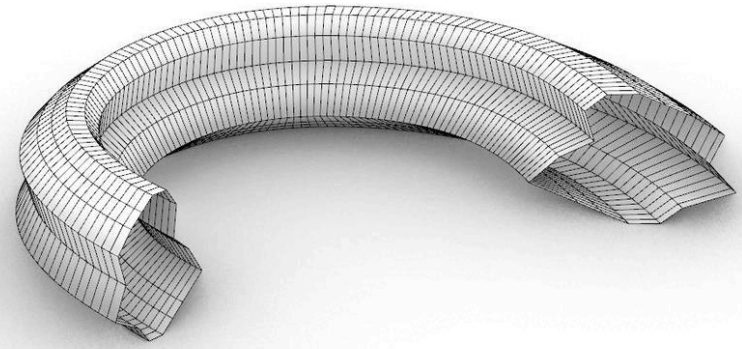


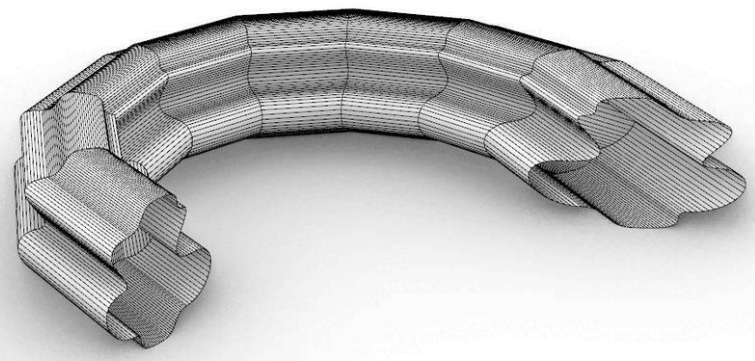
T-hedral Tubes
Smooth



T-hedral Tubes
Isometric Deformations

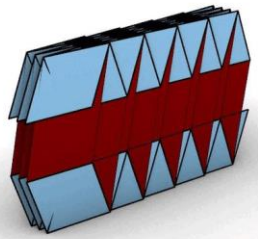


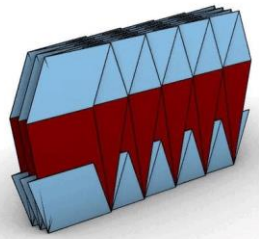


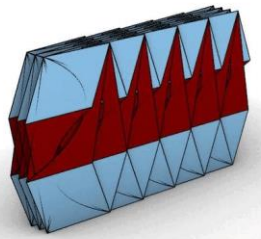


T-hedral Tubular Structures

Sandwiches

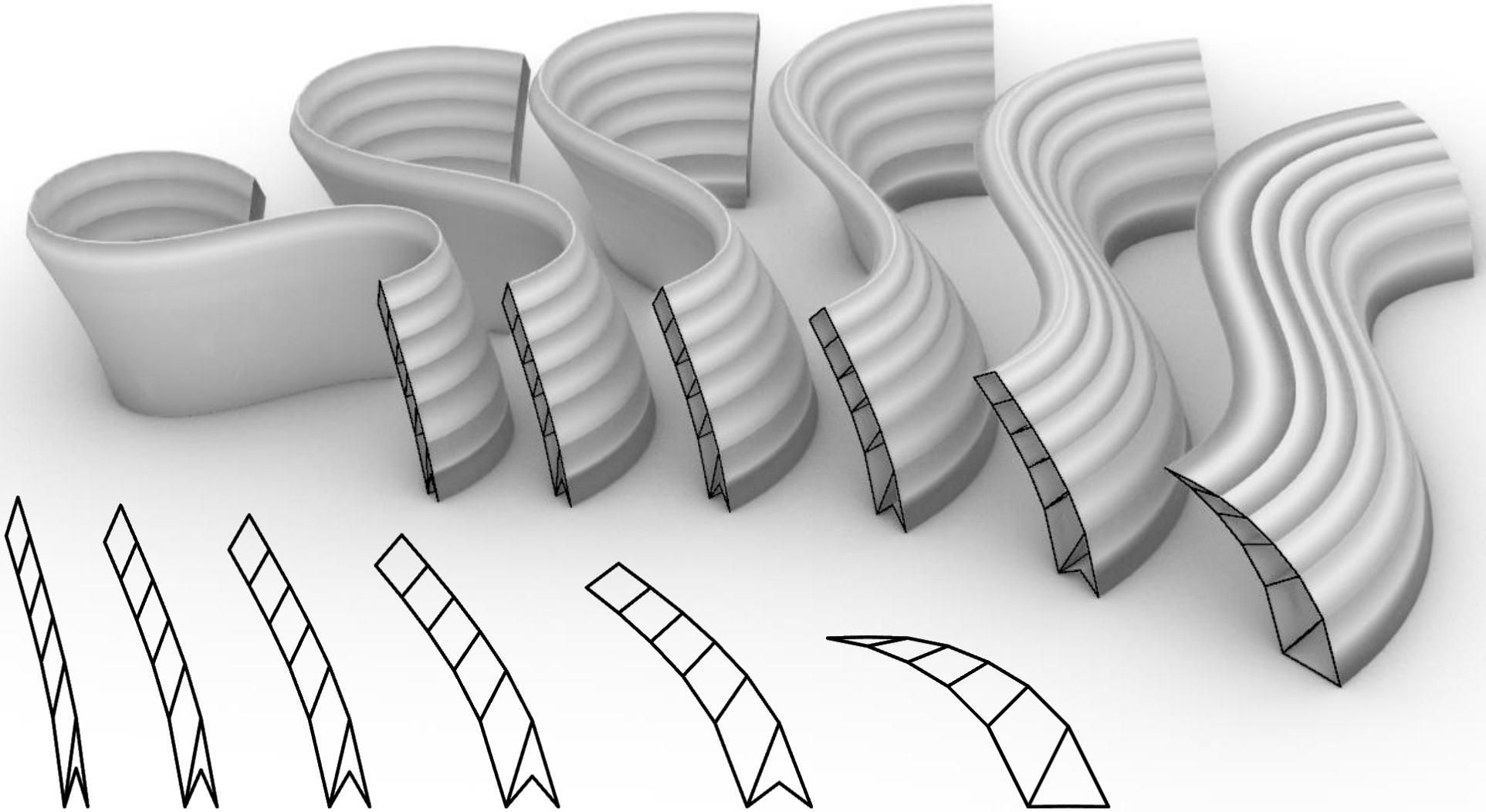






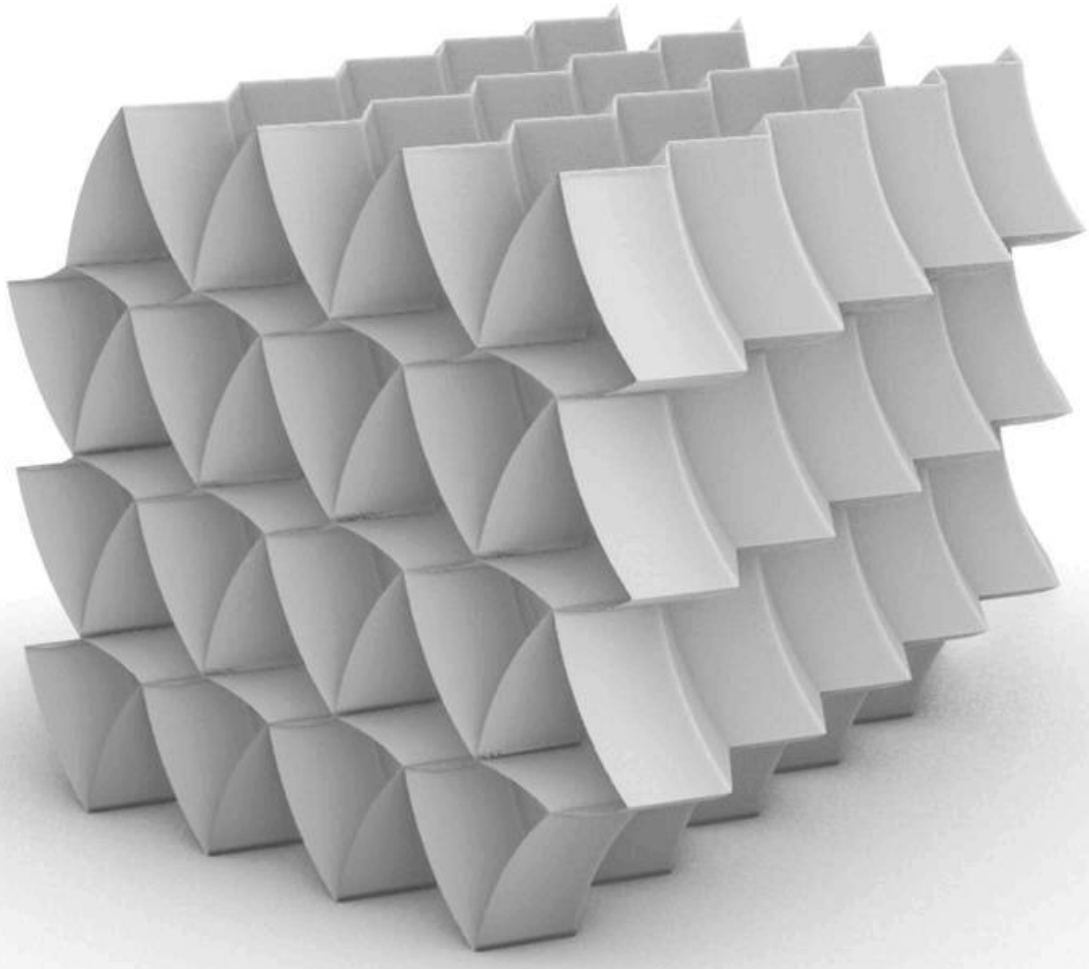
T-hedral Tubular Structures

Tubular surface (semi-discrete)



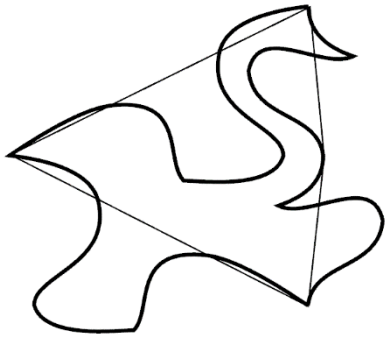
T-hedral Tubular Structures

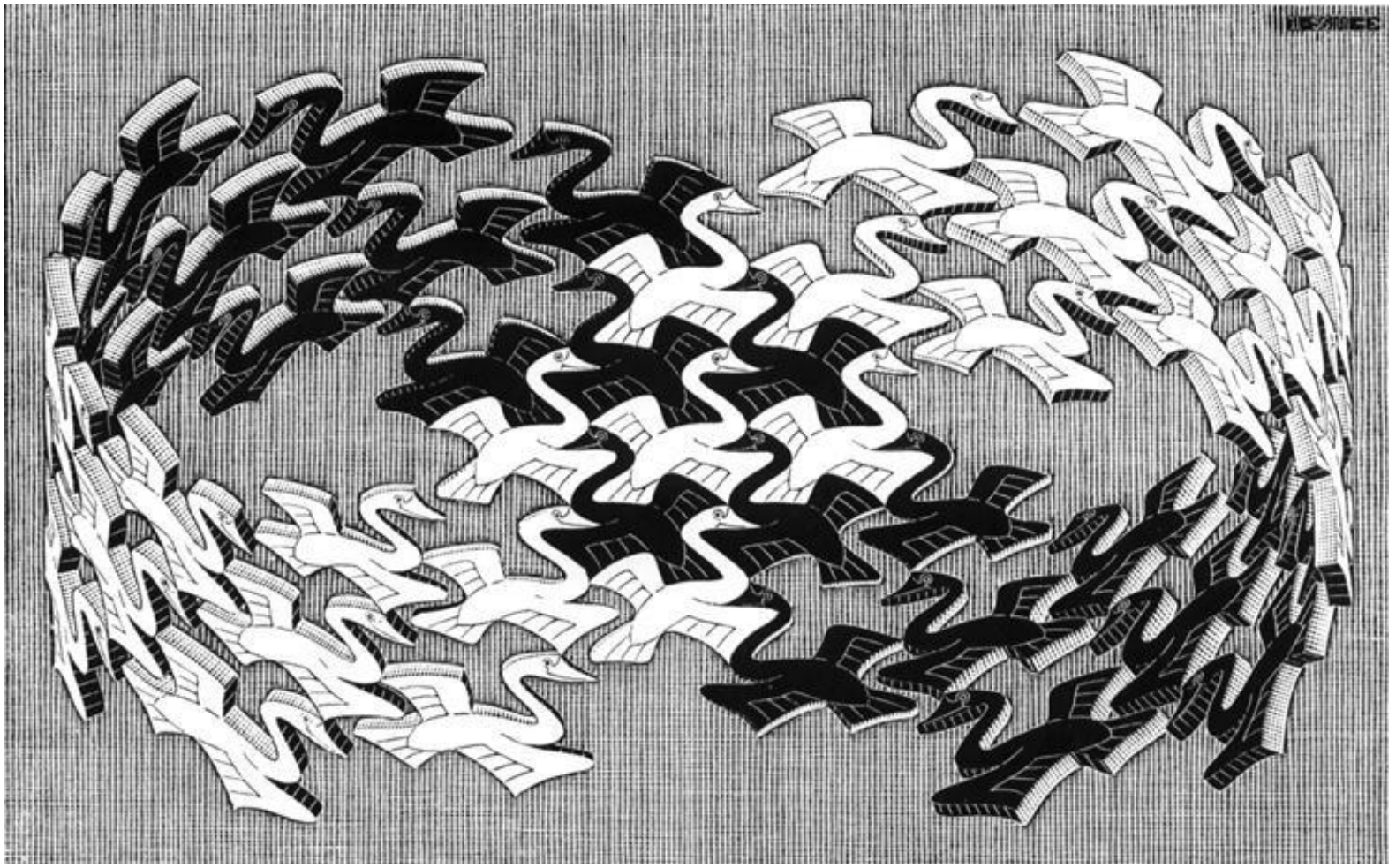
Metamaterials (semi-discrete)



T-hedral Tubular Structures

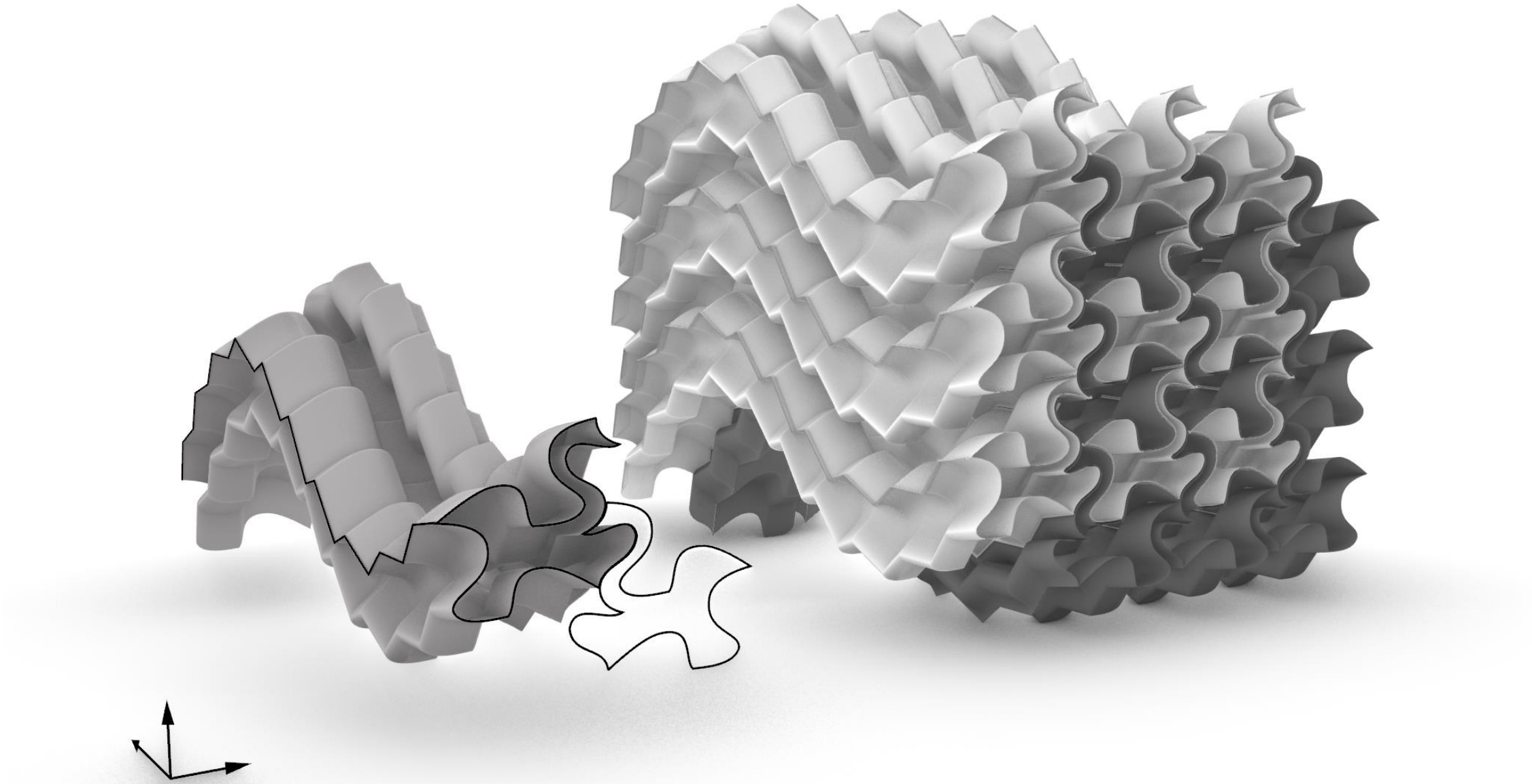
M. C. Escher's Birds





Swans, 1956, wood engraving, M.C.Escher

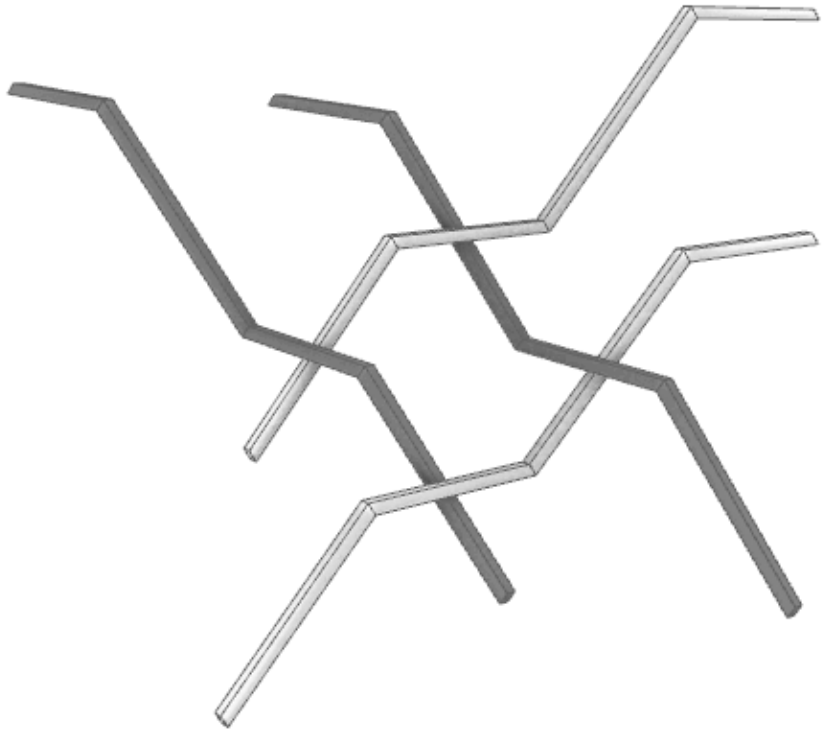
When Escher meets Miura!

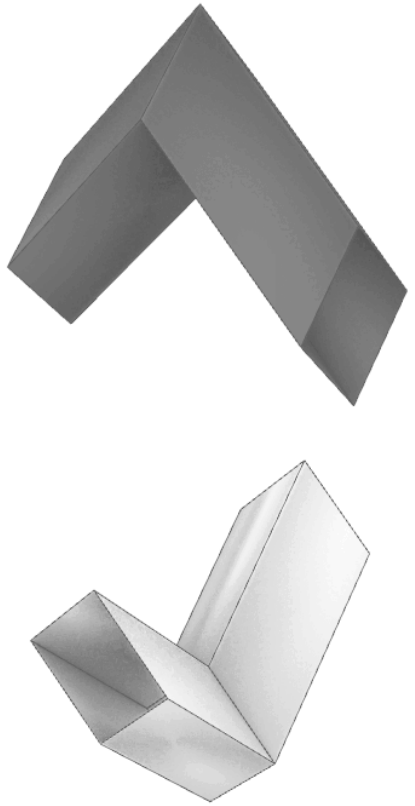


Generalizing Rigid-foldable Tubular Structures of T-hedral Type (Mechanics Research Communications) K. Sharifmoghaddam, R. Maleczek, G. Nawratil

Woven Rigid-foldable Tubes

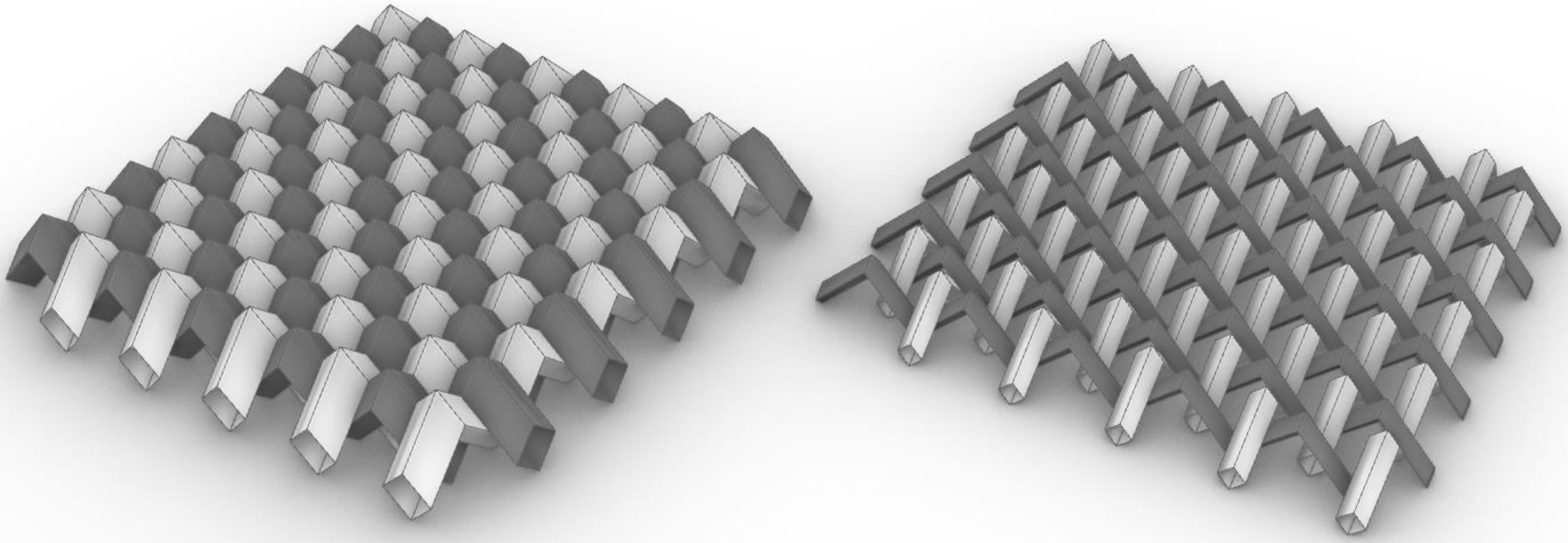
Can we really weave them?





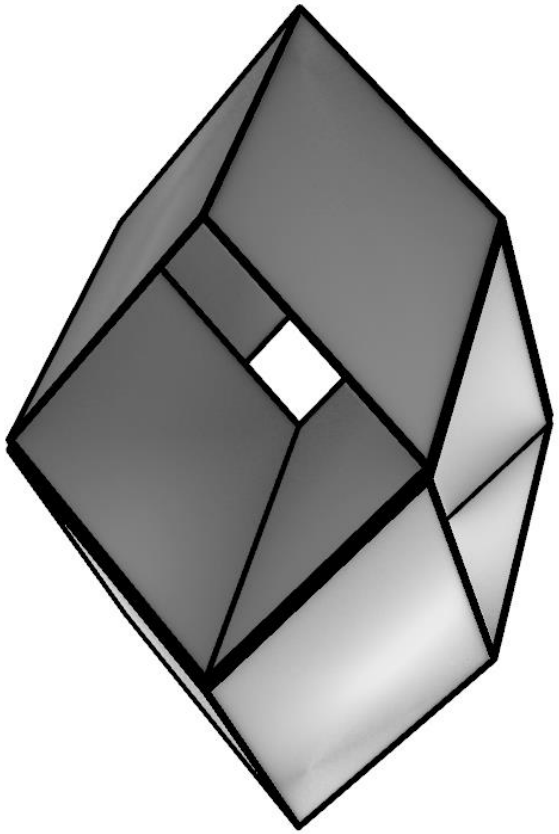
Woven Rigid-foldable Tubes

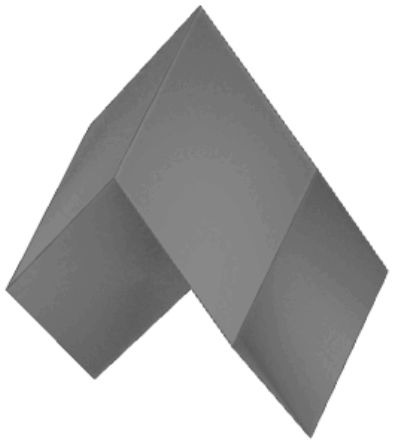
Dense and Sparse

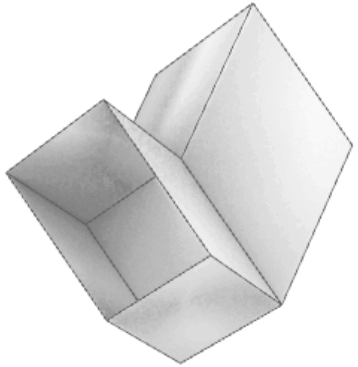


Woven Rigid-foldable Tubes

Flip-flop

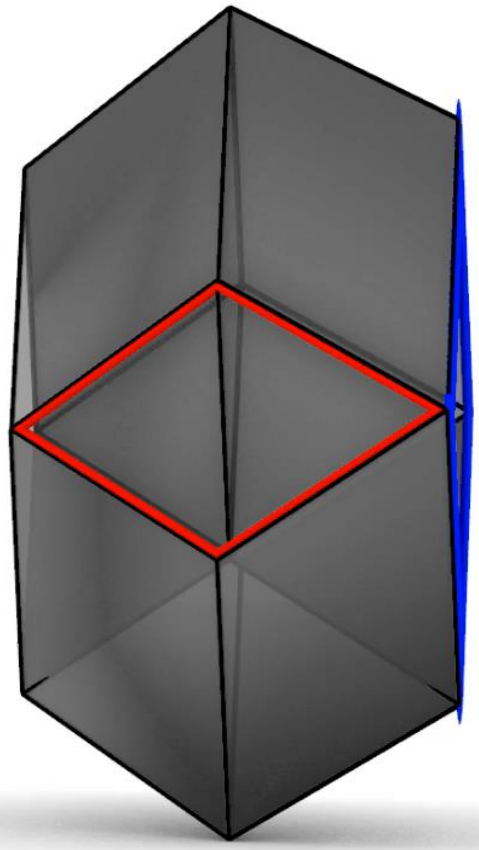




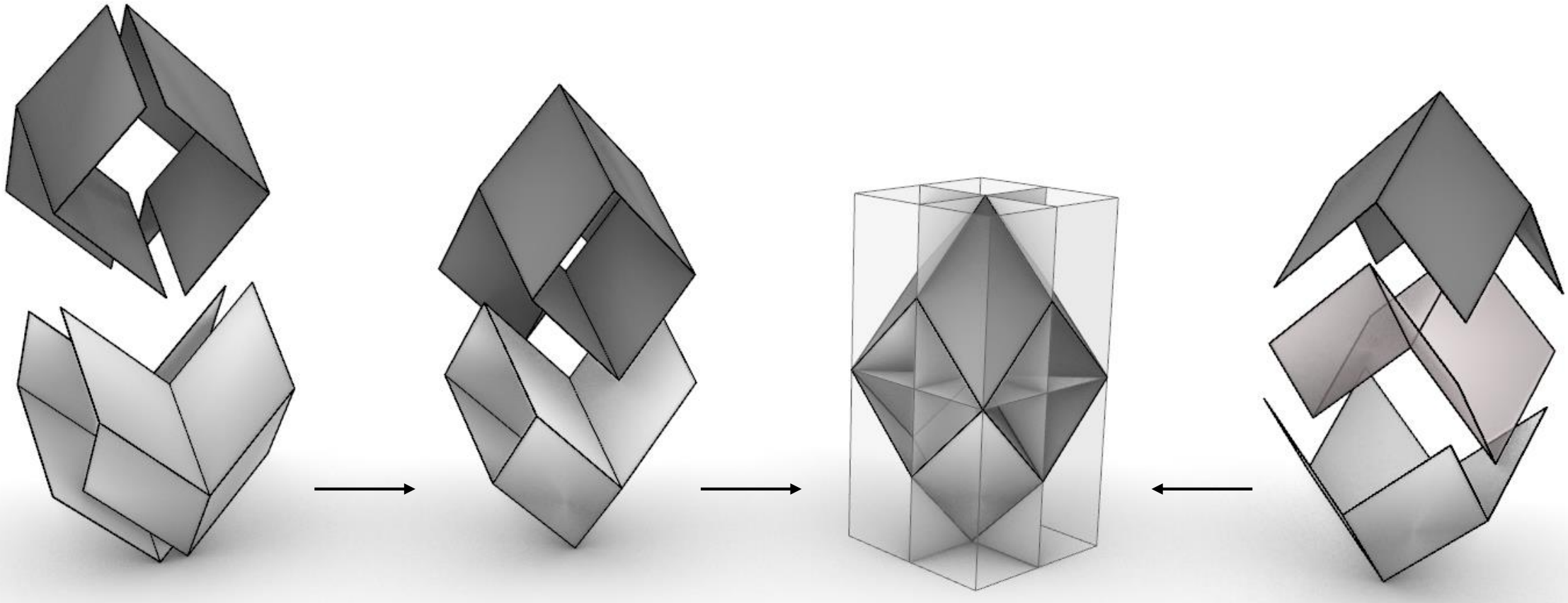


Flip-flop

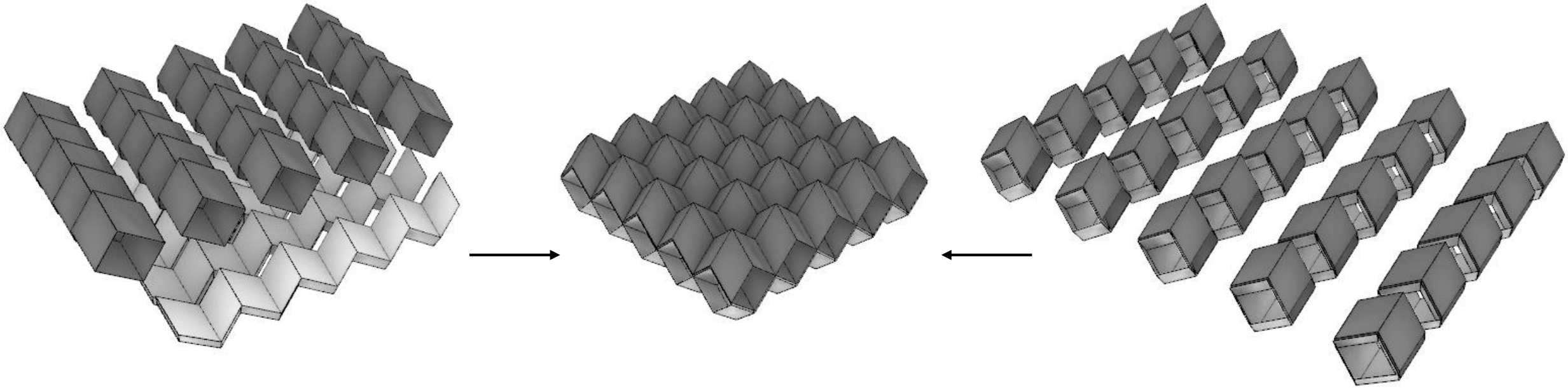
Bidirectional flat-foldability



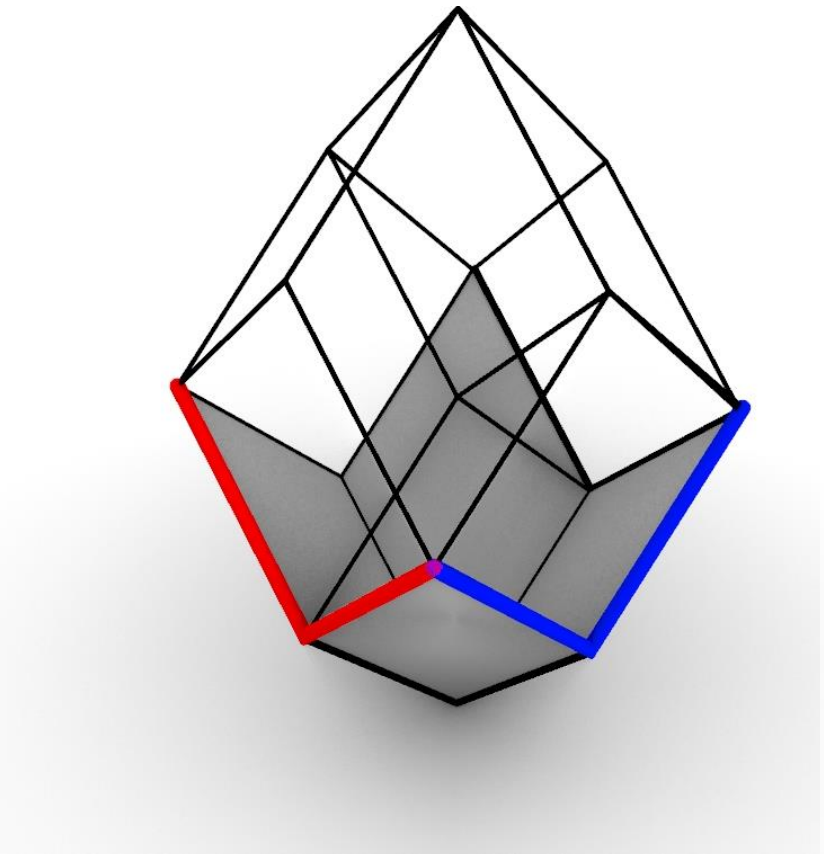
Flip-flop
Decomposition

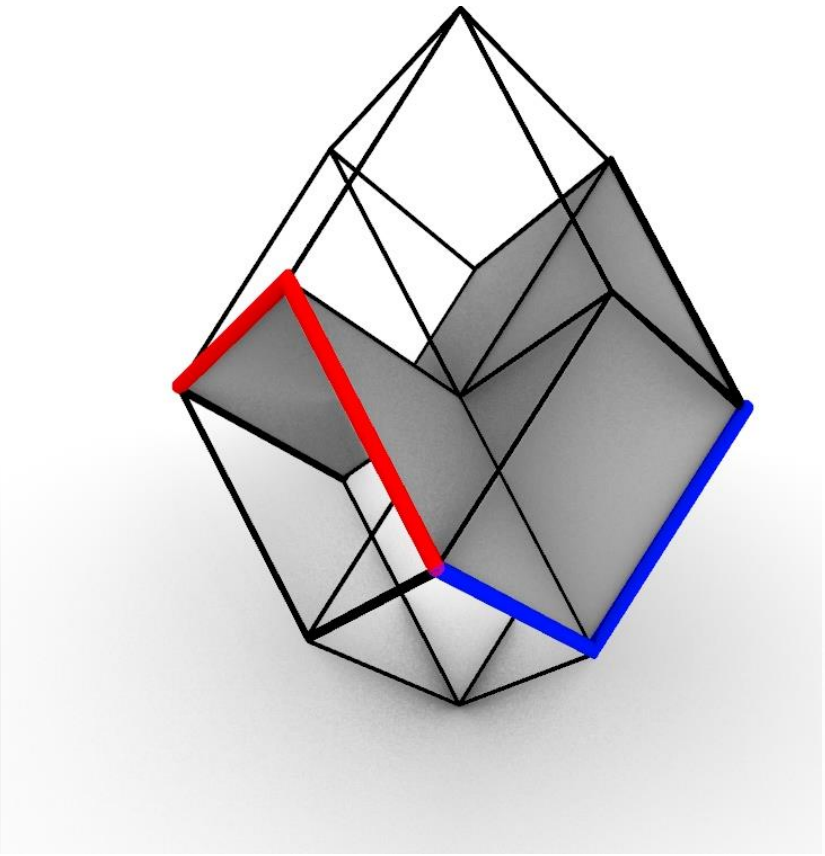


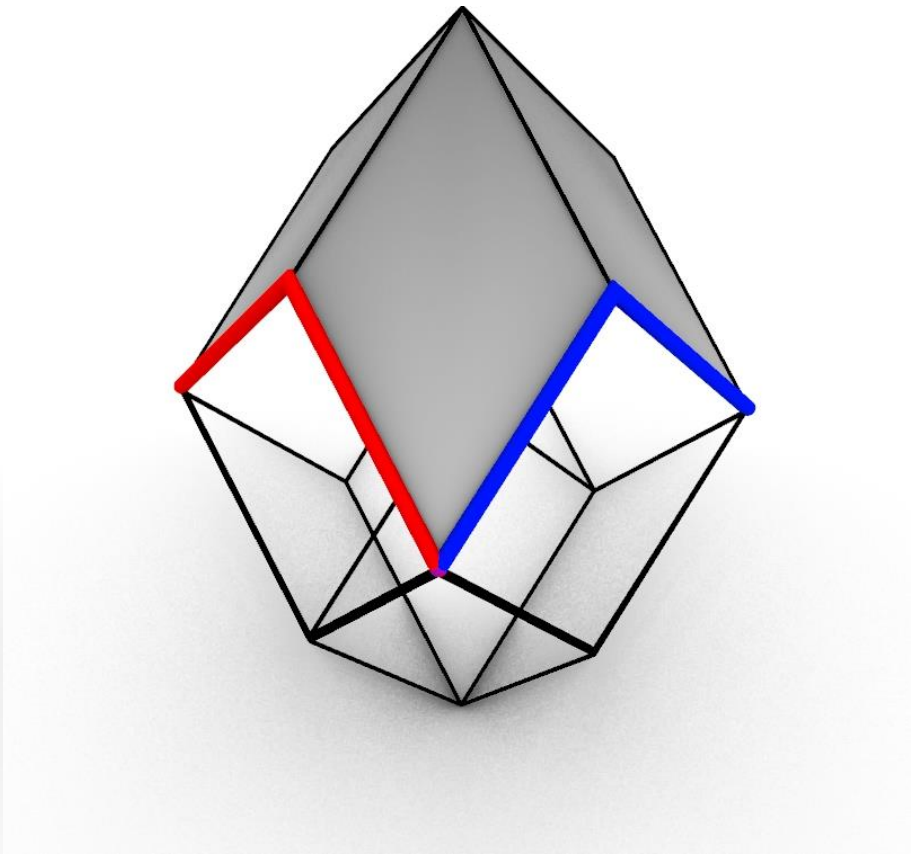
Interleaving tubes
Decomposition



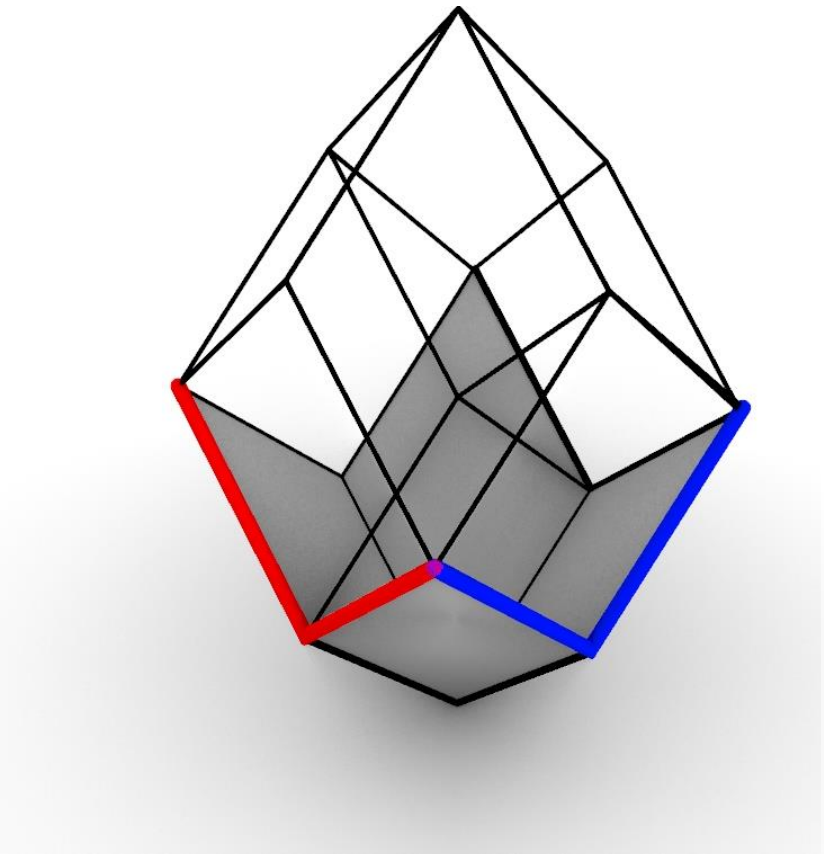
Flip-flop
Decomposition

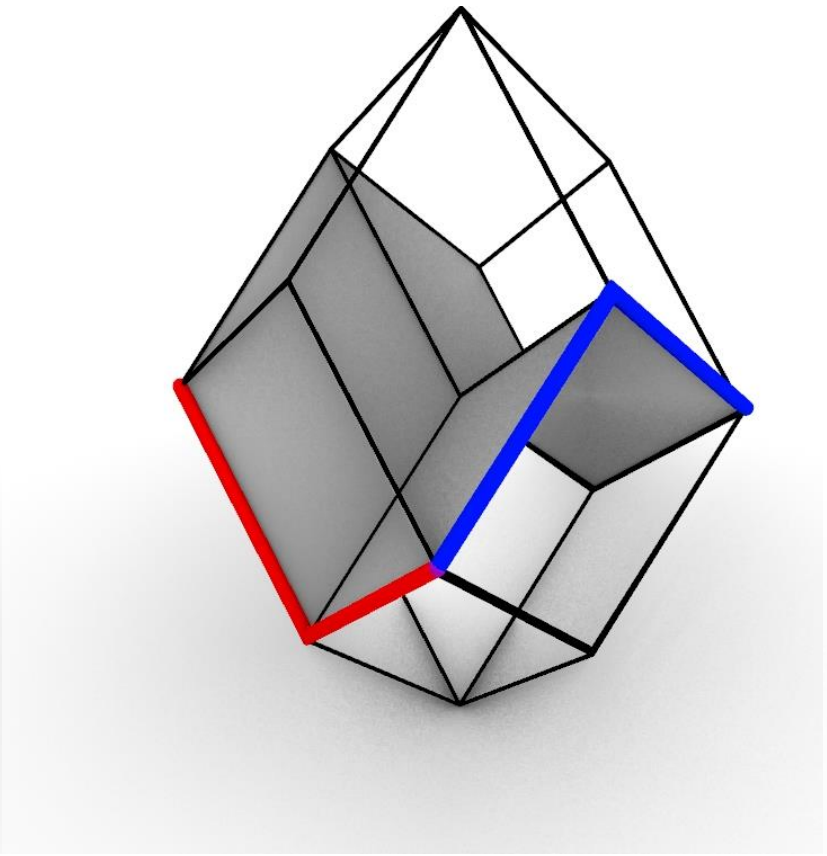


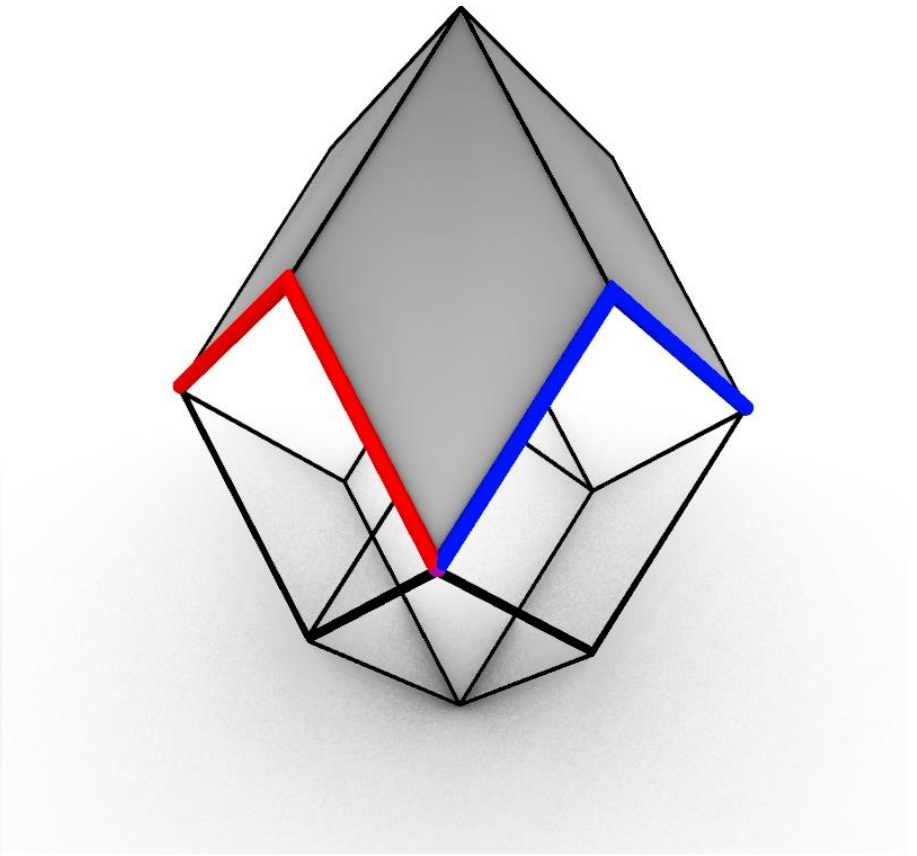




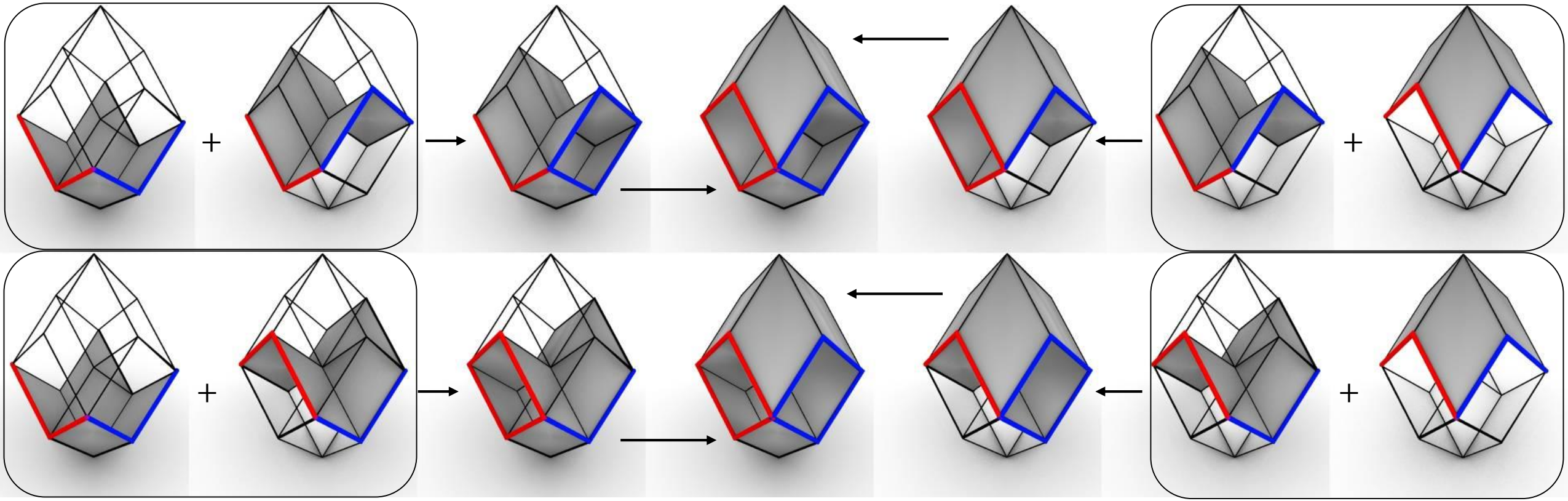
Flip-flop
Decomposition





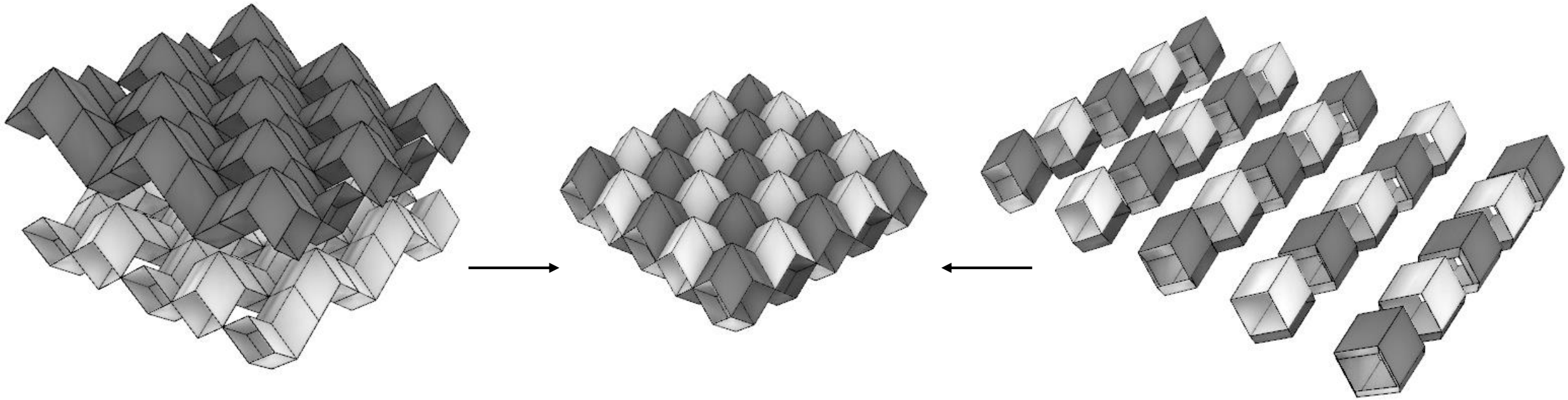


Flip-flop
Two Types



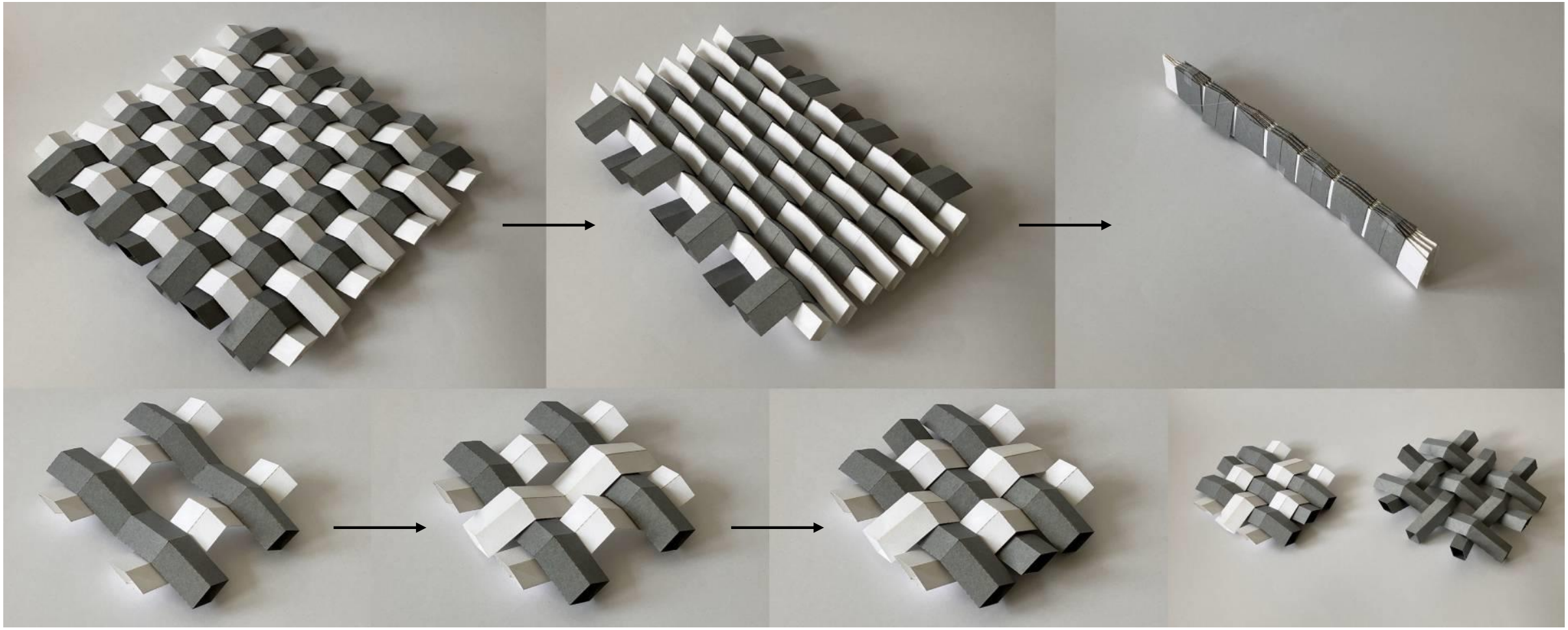
Woven Rigid-foldable Tubes

Decomposition



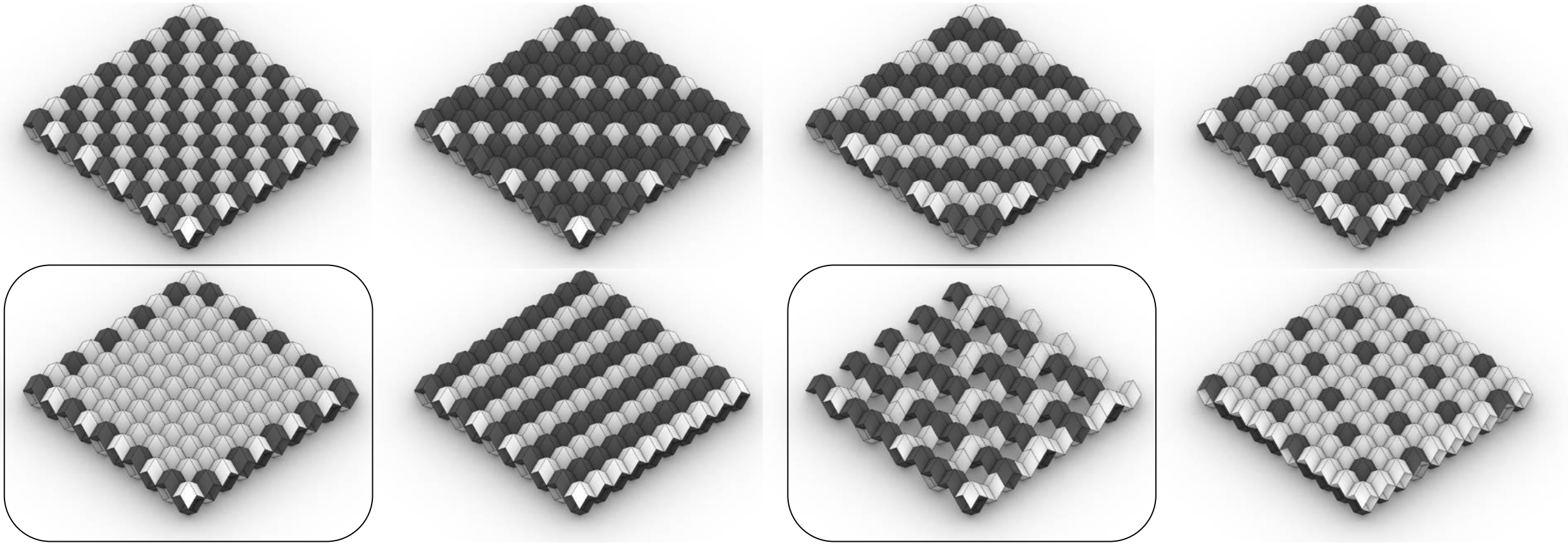
Woven Rigid-foldable Tubes

Fabrication



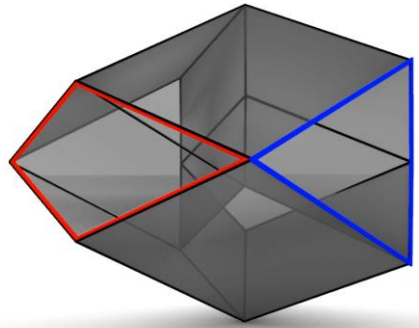
Woven Rigid-foldable Tubes

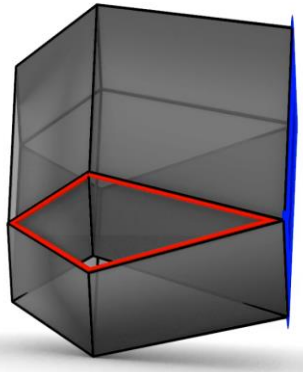
Woven tubes

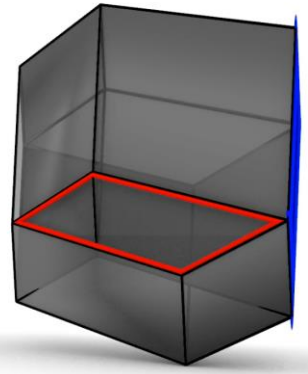


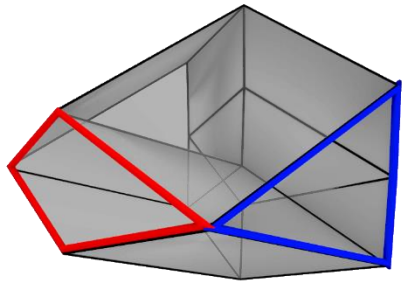
Woven Rigid-foldable Tubes

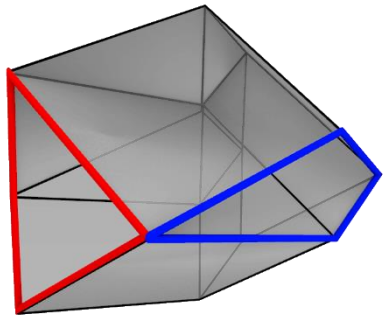
Generalization of flip-flop

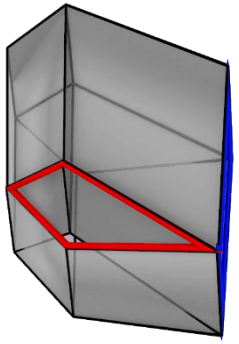


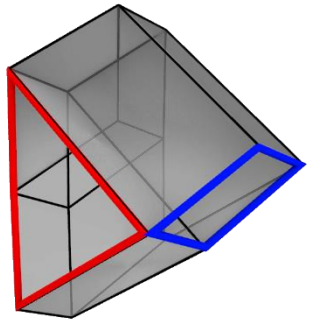


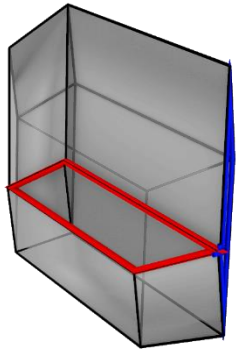


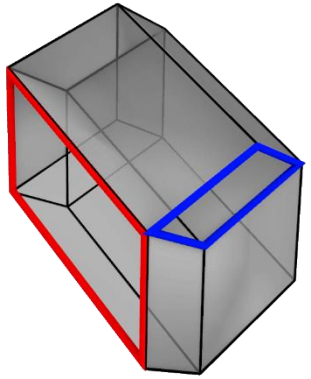




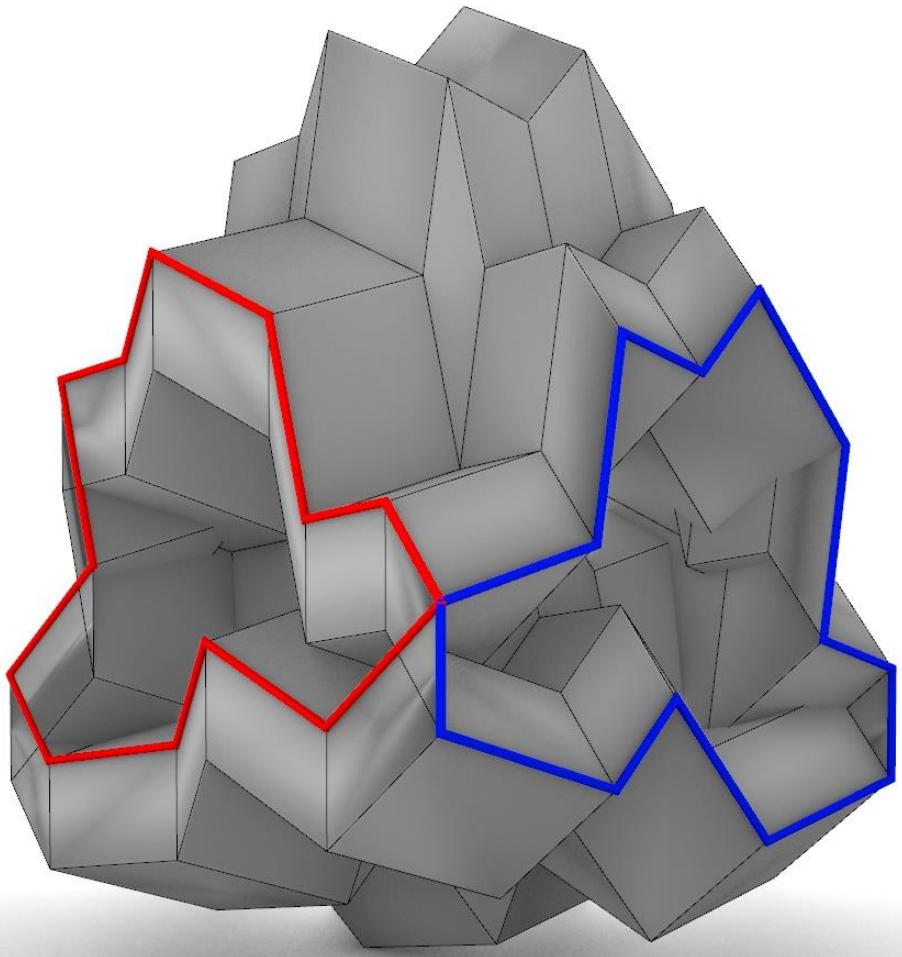


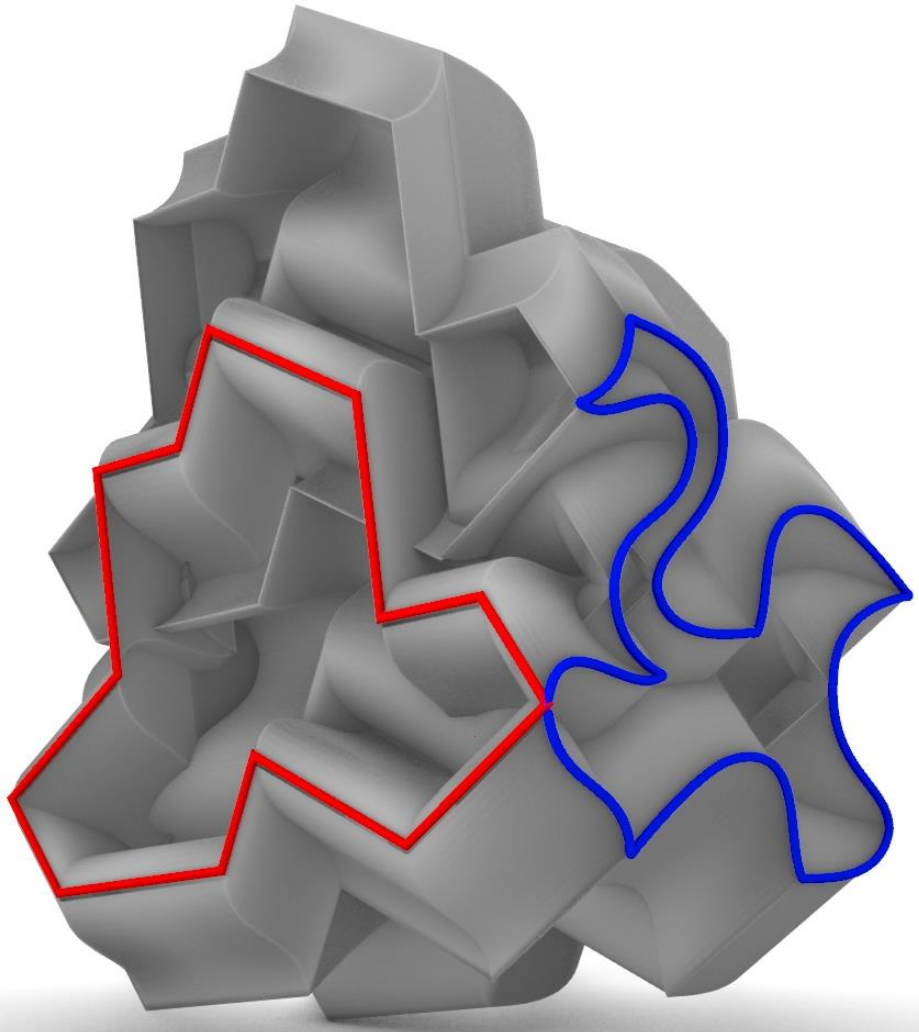






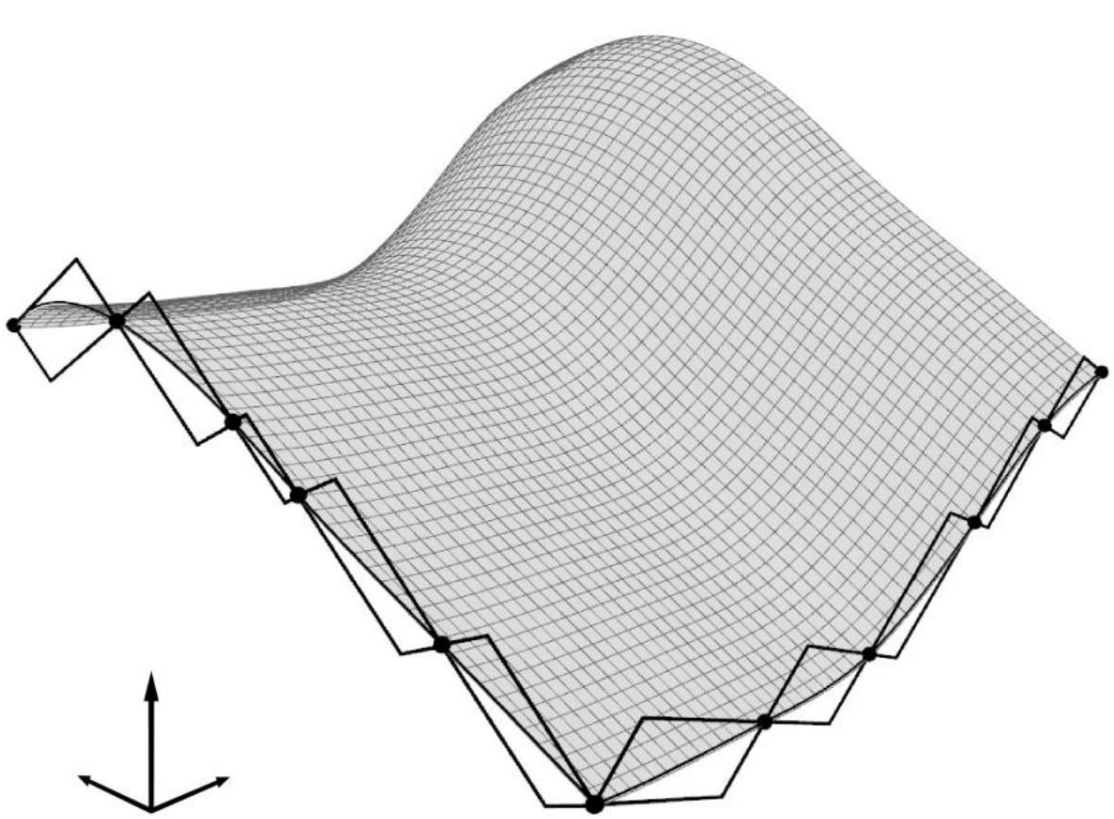
When Einstein meets Escher!

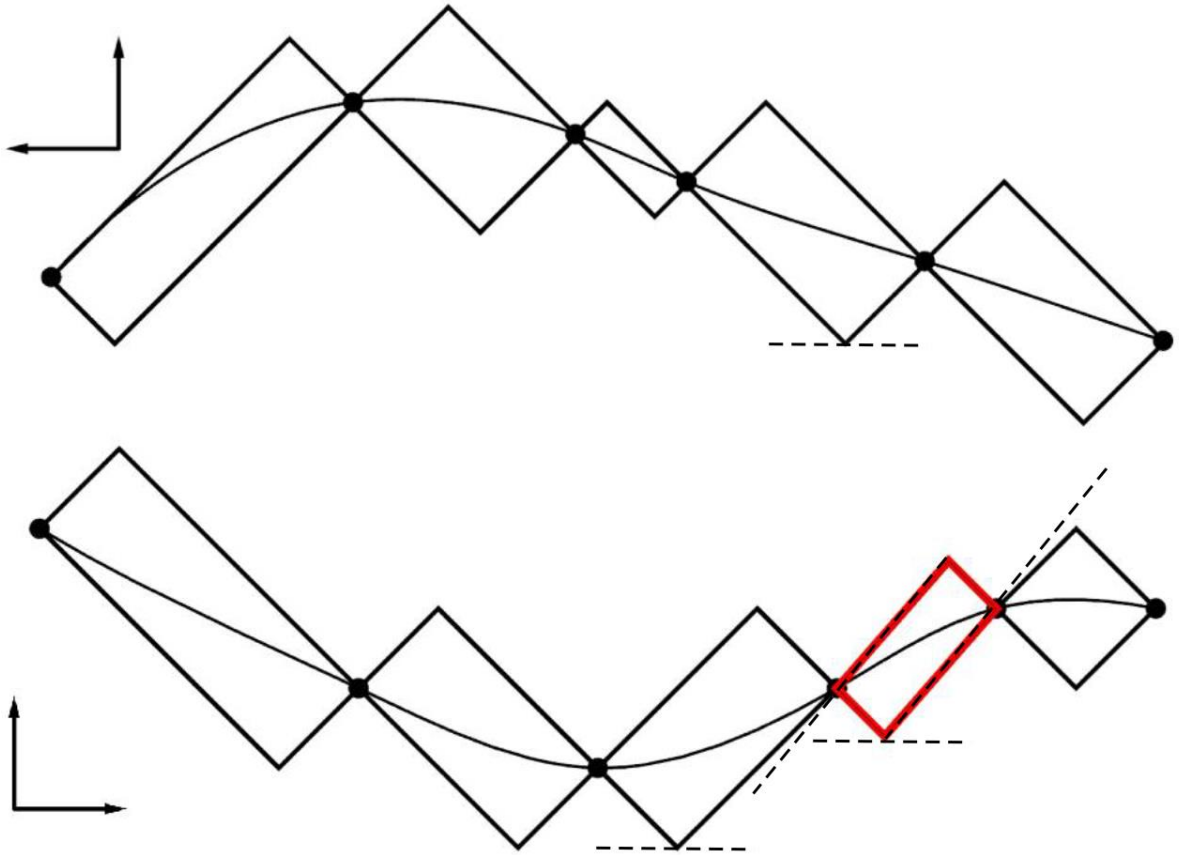




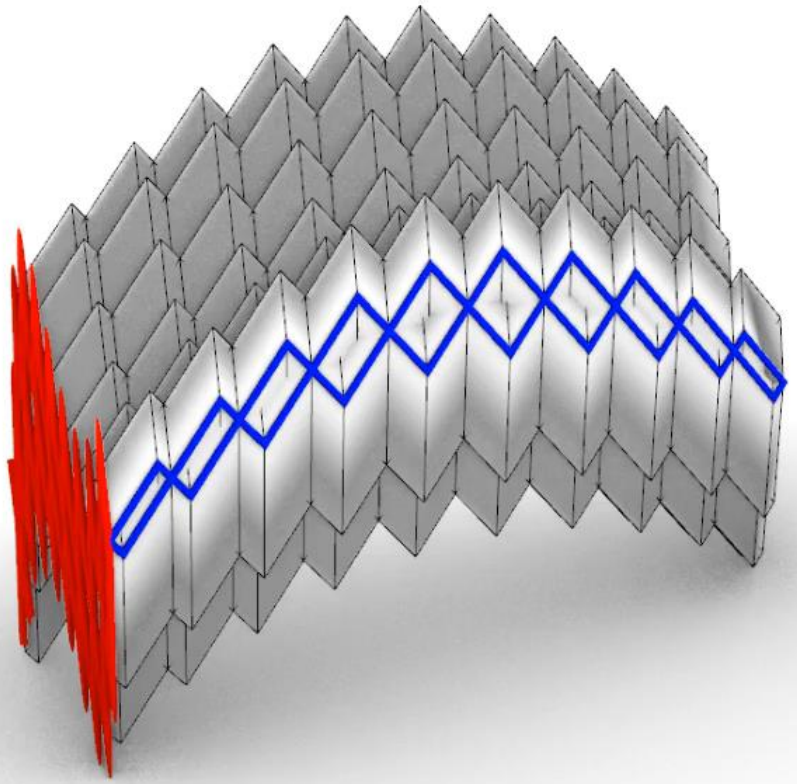
Woven Rigid-foldable Tubes

Woven tubes along a translational surface



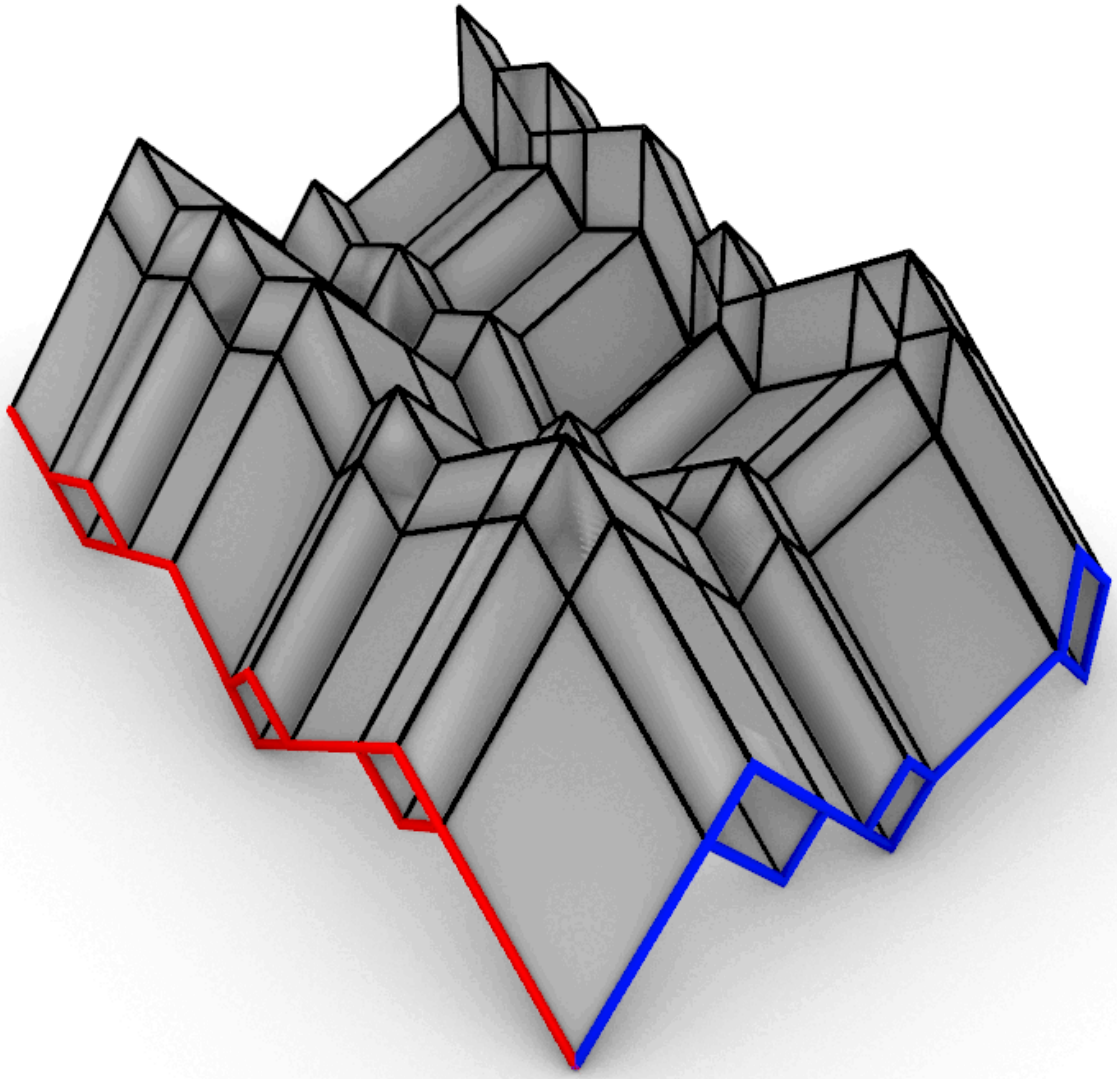


Woven Rigid-foldable Tubes



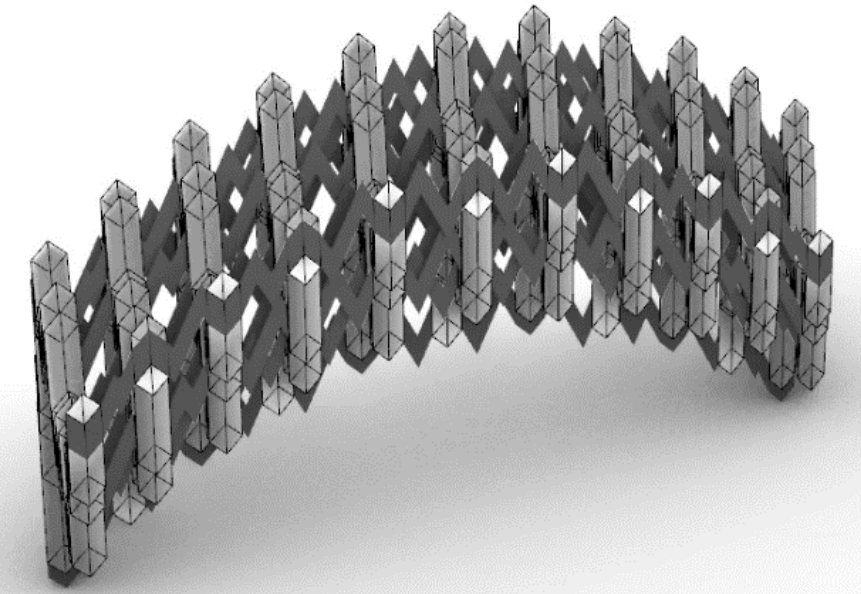
Woven Rigid-foldable Tubes

Woven tubes along a translational surface (sparse)



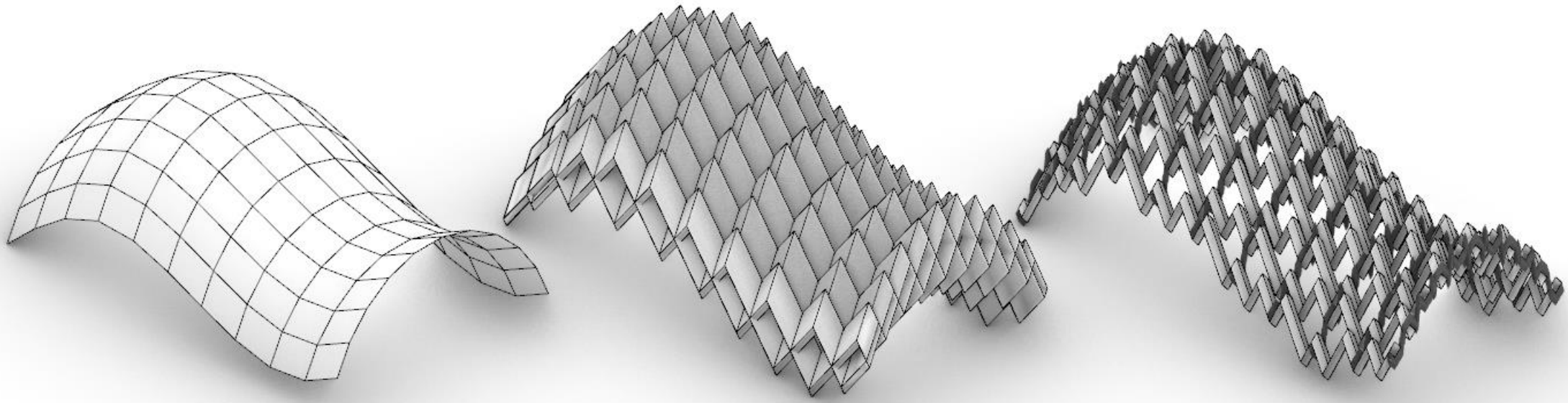
Woven Rigid-foldable Tubes

Woven tubes along a translational surface (sparse)

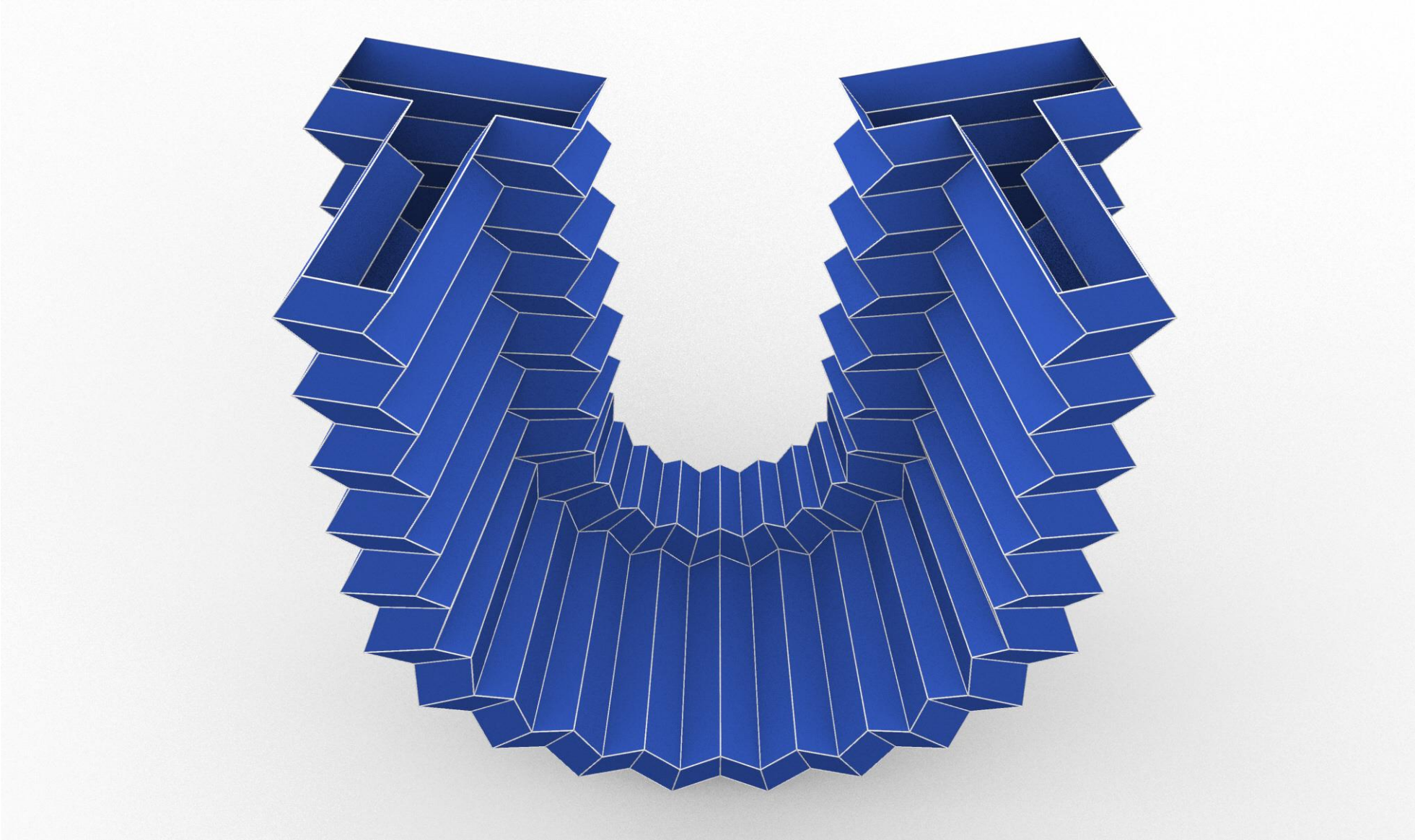


Implemented by Klara Mundilova

Woven Rigid-foldable Tubes



Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces (8OSME) K. Sharifmoghaddam, K. Mundilova, G.Nawratil, T. Tachi



Thank You!