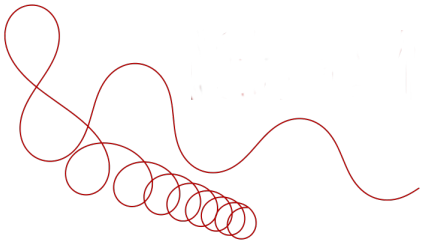
Titles & Abstracts

Adventures in integrable geometry.
Francis Burstall (University of Bath)



Hydrodynamic integrability and geometry.
David Calderbank (University of Bath)

Dispersionless integrable systems are, roughly speaking, the integrable PDE systems that describe special geometric structures on the domain manifold of the PDE. Prototypical examples include Einstein–Weyl geometry and quaternionic geometry. The method of hydrodynamic reductions is a systematic way to characterize such PDE systems as those admitting many solutions of a particular form. In many cases, this hydrodynamic integrability can be described in terms of special geometric structures on the codomain manifold of the PDE system. This talk will explain some geometry that underpins hydrodynamic integrability and a fairly general setting where this geometry reduces to the existence of special nets on the codomain of the PDE, including a generalization of conjugate nets.

A loop group approach to finding global solutions of the tt* equations.
Martin Guest (Waseda University)

The tt* equations are a special case of the equations for harmonic maps into the noncompact symmetric space Gl(n,R)/O(n). They provide interesting examples of harmonic maps related to SUSY field theory and the Hitchin/Higgs equations, and also a test case for the loop group Iwasawa factorization in the "noncompact case". In joint work with Alexander Its and Chang-Shou Lin ("Isomonodromy aspects of the tt* equations of Cecotti and Vafa I,II,III") we have solved the equations in some situations using a collection of disparate methods. A purely loop group-theoretic - and potentially more general - approach to these results will be proposed.

symmetry breaking.
Udo Hertrich-Jeromin (TU Wien)

Symmetry breaking is a notion widely employed in physics, in a variety of contexts. Being an inherently geometric notion, we shall investigate how (spontaneous) symmetry breaking appears in geometry.

Polynomial conserved quantities for symmetric R-spaces.
Callum Kemp (University of Bath)

Isothermic submanifolds are characterised by a family of flat connections $d + t\eta$ where η is a closed Lie algebra-valued 1-form. In fact we can define this family on many different bundles through representations of the Lie algebra. I aim to demonstrate which bundle is the best choice to tackle the question of polynomial conserved quantities.

On closed finite gap curves.
Martin Kilian (University College Cork)

I will outline the proof that closed finite gap curves are dense in the space of closed curves in the spaceforms. This is joint work with Sebastian Klein.

Isothermic Positons.
Katrin Leschke (University of Leicester)

Isothermic surfaces are surfaces which allow, away from umbilic points, a conformal curvature line parametrisation. This allows to introduce a complex spectral parameter and an isothermic surface is given by a family of flat connections with complex parameter. The simple factor dressing gives new isothermic surfaces by gauging the family with an appropriate dressing matrix with simple poles. Then Bianchi permutability allows to construct algebraically a common simple factor dressing from two given simple factor dressings with distinct poles. In this talk we discuss a Sym-type way to obtain the common simple factor dressing by differentiating with respect to the parameter in the case when the two simple factor dressings have the same poles.

This is joint work with J. Cho and Y. Ogata.

Harmonic surfaces in the Cayley plane.
Rui Pacheco (Universidade da Beira Interior)

In this talk we consider the twistor theory of nilconformal harmonic maps from a Riemann surface into the Cayley plane $\mathbb{O}P^2 \cong F_4/Spin(9)$. By exhibiting this symmetric space as a submanifold of the Grassmannian of 10-dimensional subspaces of the fundamental representation of F_4, techniques and constructions similar to those used in earlier works on twistor constructions of nilconformal harmonic maps into classical Grassmannians can also be applied in the Cayley plane case. The originality of our approach lies on the use of the classification of Nilpotent orbits in Lie algebras as described by D. Djoković. As a corollary of our results, we will show that, when the domain is a flat torus, finite uniton number and finite type cover all harmonic maps into $\mathbb{O}P^2$.

This is joint work with Nuno Correia and Martin Svensson partially supported by FCT, through the project UID/MAT/00212/2019.

Finding isometric immersions of surfaces via variational calculus.
Franz Pedit (UMass Amherst)

We will discuss a variational problem with parameters whose minimizers are isometric immersions of a given Riemannian surface in 3-space. One of the parameters controls the Willmore energy, which can be used as a regularizer preventing excessive creasing/crumpling of the resulting isometric image in 3-space of the abstract surface. The exposition will be augmented by numerous images of isometrically immersed surfaces produced by a numerical algorithm based on our theory.

Spherical curvature lines.
Mason Pember (TU Wien)

In this talk we will investigate surfaces that have one family of spherical curvature lines. Following a result of Blaschke, we see how these surfaces can be obtained via a Lie sphere geometric notion of parallel transport. We shall then consider when such surfaces are additionally Omega-surfaces.

This is joint work with Joseph Cho and Gudrun Szewieczek.

Minimal surfaces under constrained Willmore transformation.
Áurea Quintino (Universidade de Lisboa)

The class of constrained Willmore (CW) surfaces in space-forms constitutes a Möbius invariant class of surfaces with strong links to the theory of integrable systems, with a spectral deformation [Burstall–Pedit–Pinkall], defined by the action of a loop of flat metric connections, and Bäcklund transformations [Burstall–Q.], defined by a dressing action by simple factors.
Constant mean curvature (CMC) surfaces in 3-dimensional space-forms are [Richter] examples of constrained Willmore surfaces, characterized by the existence of some polynomial conserved quantity [Q.–Santos]. Both CW spectral deformation and CW Bäcklund transformation preserve [Q.–Santos] the existence of such a conserved quantity, defining, in particular, transformations within the class of CMC surfaces in 3-spaces, with, furthermore [Q.–Santos], preservation of both the space-form and the mean curvature, in the latter case.
A classical result by Thomsen characterizes, on the other hand, isothermic Willmore surfaces in 3-space as minimal surfaces in some 3-dimensional space-form. CW transformation preserves [Burstall–Pedit–Pinkall, Burstall–Q.] the class of Willmore surfaces, as well as the isothermic condition, in the particular case of spectral deformation [Burstall–Pedit–Pinkall]. We define, in this way, a CW spectral deformation and CW Bäcklund transformations of minimal surfaces in 3-dimensional space-forms into new ones, with preservation of the space-form in the latter case.

Global behavior of smooth surfaces with Weierstrass-type representations.
Wayne Rossman (Kobe University)

An important subclass of Omega surfaces is those with Weierstrass-type representations, which facilitate the study of global behavior – including topological types, end asymptotics, singularities, and type change of metrics. One can hope this would provide hints about global behavior on other Omega surfaces as well. In this vein, I wish to give an overview of long-term joint work with M. Umehara, K. Yamada, and others.

When isothermic surfaces feel special.
Susana Duarte Santos (Universidade de Lisboa)

Among isothermic surfaces, one can find the so-called special isothermic surfaces, discovered by Darboux in connection with deformations of quadrics. We show that these surfaces have a simple explanation in terms of the integrable systems approach to iso-

thermic surfaces. As fruits of this analysis, the classical theory of special isothermic surfaces is generalized to arbitrary codimension, and a hierarchy of natural classes of isothermic surfaces emerges, filtered by a non-negative integer d. These "special" isothermic surfaces (of type d) are characterized by the existence of a polynomial conserved quantity (of degree d) of the associated 1-parameter family of flat connections, and the classical theory fits into that hierarchy by considering $d = 2$ in codimension 1. An overview of the theory developed is presented. This "special" and inspiring journey was done with Fran Burstall, paving the way to new subsequent research works.

Harmonic maps of finite and infinite uniton number.
John Wood (University of Leeds)

We describe our joint work with M.J. Ferreira and B.A. Simões which builds on the work of F.E. Burstall and M.A. Guest to find explicit formulae for all harmonic maps of finite uniton number into the orthogonal group ; we see how these formulae are related to the free Weierstrass representation for minimal surfaces. Then we describe joint work with A. Aleman and R. Pacheco which gives new criteria for the finiteness of the uniton number of a harmonic map : one of these criteria generalizes work of Burstall and F. Pedit.