Generalizing the Control Number for 6-dof UCU Hexapods with classic or eccentric U-joints

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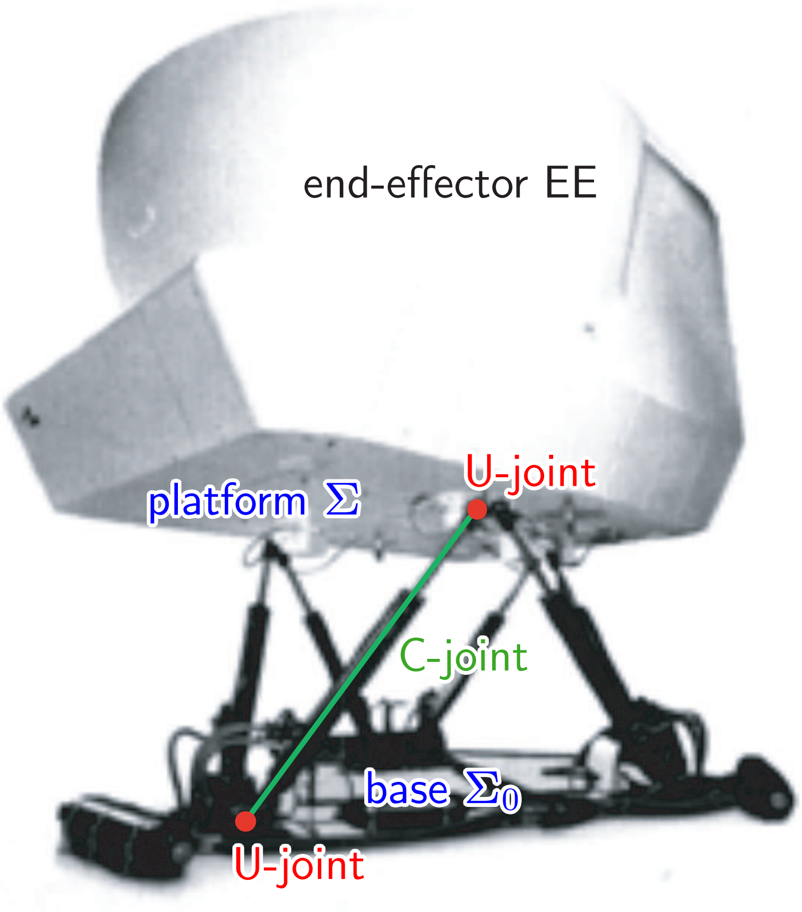
Overview

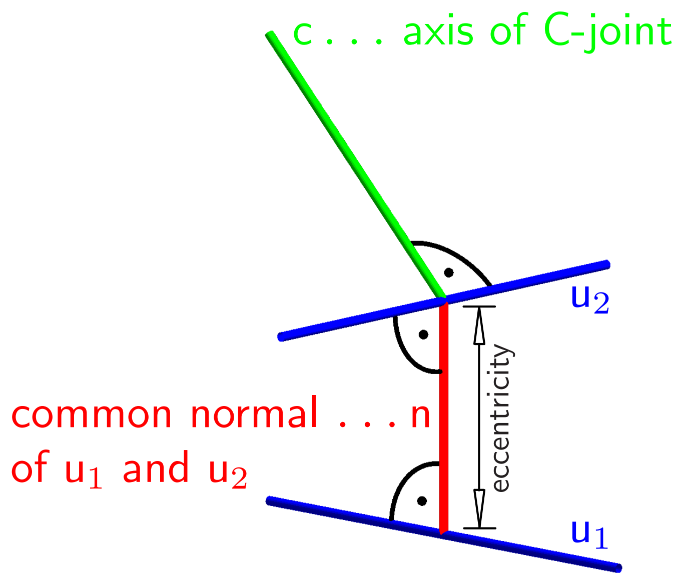


end-effector EE
U-joint
platform Σ
C-joint
base Σ_0
U-joint

1. Introduction

U-joint: It can be seen as a serial $2R$-chain with orthogonal axes u_1 and u_2. If these axes intersect each other, we have the *classic* U-joint, otherwise we get a so-called *eccentric* one.

Advantages of eccentric U-joints (cf. [3,4]):
- larger pivot range $\Longrightarrow$ extension of workspace,
- stiffer design $\Longrightarrow$ improvement of accuracy,
- cheaper production.



U-joints and C-joints are connected as follows:
- the lines u_2, c and n are copunctal,
- and c intersects u_2 orthogonally.

Remark: These assumptions keep the kinematic structure of the UCU-legs simple enough for practical application (cf. [4]). ◇

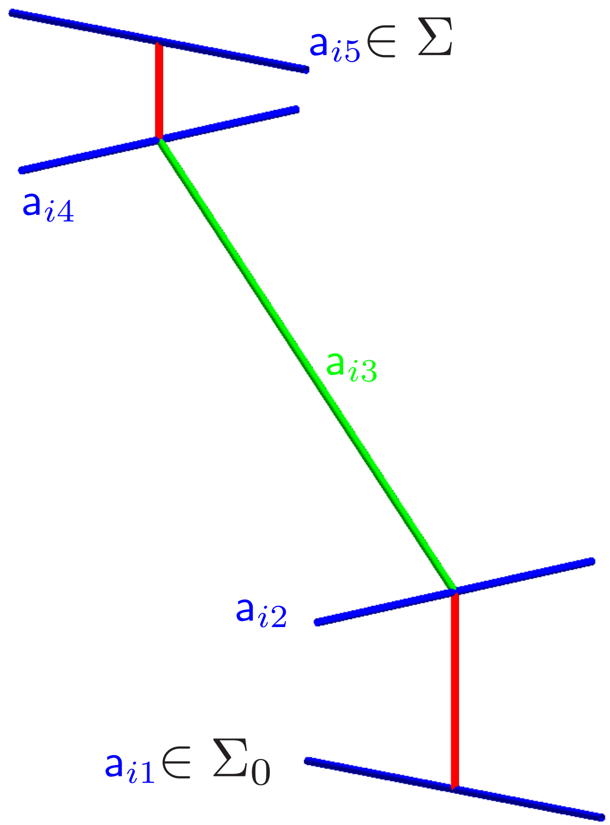
1. Introduction

For the UCU-legs, both types of U-joints are allowed, which are in all cases passive joints of the manipulator.

C-joint: As only the translation along c can be controlled actively, the C-joint can be replaced by a composition of an active P-joint along c and a passive R-joint with rotary axis c.

Therefore each leg connecting Σ_0 with Σ can be seen as a serial RRPRRR-chain, where the P-joint is active and the five R-joints are passive.

Notation:
We denote the j^{th} rotation axis of the i^{th} leg by a_{ij} for $i = 1, \ldots, 6$ and $j = 1, \ldots, 5$.



1. Dual vector calculus

The rotation axis a_{ij} is given by

$$\underline{\mathbf{a}}_{ij} = \mathbf{a}_{ij} + \varepsilon\widehat{\mathbf{a}}_{ij},$$

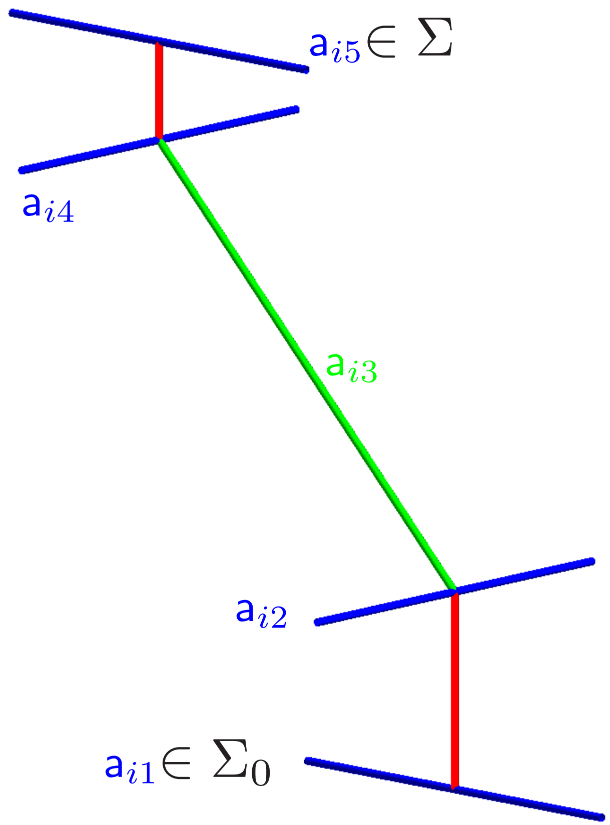
where $\mathbf{a}_{ij}$ is the unit vector along a_{ij}. $\widehat{\mathbf{a}}_{ij}$ is the so-called moment vector, which is given by $\mathbf{x}_{ij} \times \mathbf{a}_{ij}$, where $\mathbf{x}_{ij}$ is the coordinate vector of an arbitrary point $\mathsf{X}_{ij} \in \mathsf{a}_{ij}$.

ε is the dual unit, which has the property $\varepsilon^2 = 0$.

The screw for the prismatic joint of the i^{th} leg is given by

$$\underline{\mathbf{t}}_i = \mathbf{o} + \varepsilon\widehat{\mathbf{t}}_i,$$

where $\mathbf{o}$ denotes the zero vector and $\widehat{\mathbf{t}}_i = \mathbf{a}_{i3}$ the unit vector in direction of the translation.



2. Jacobian Matrix of the i-th leg

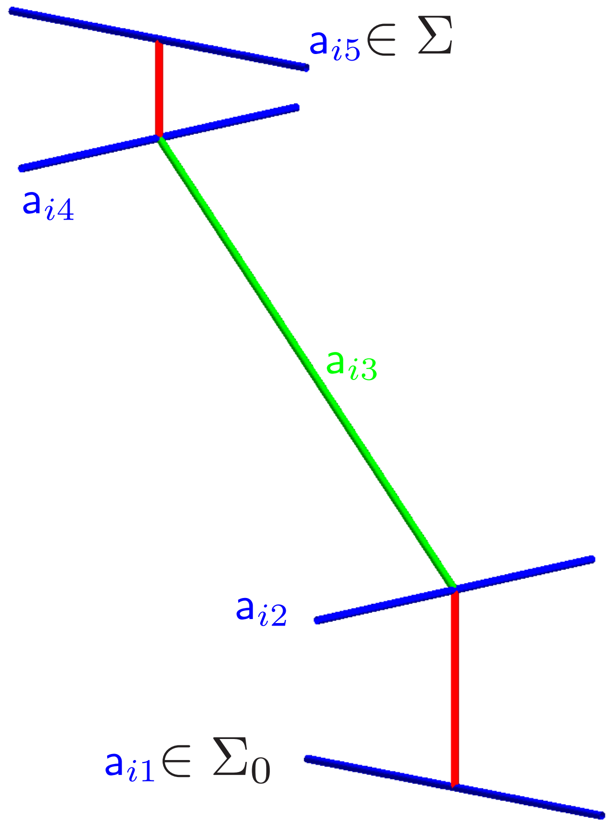
The Jacobian matrix $\mathbf{J}_i$ of the i^{th} leg (= serial RRPRRR-robot) can be written as (cf. [7]):

$$\mathbf{J}_i = \begin{pmatrix} \mathbf{a}_{i1} & \mathbf{a}_{i2} & \mathbf{a}_{i3} & \mathbf{a}_{i4} & \mathbf{a}_{i5} & \mathbf{o} \\ \widehat{\mathbf{a}}_{i1} & \widehat{\mathbf{a}}_{i2} & \widehat{\mathbf{a}}_{i3} & \widehat{\mathbf{a}}_{i4} & \widehat{\mathbf{a}}_{i5} & \widehat{\mathbf{t}}_i \end{pmatrix}.$$

Therefore the instantaneous screw $\underline{\mathbf{q}} = \mathbf{q} + \varepsilon\widehat{\mathbf{q}}$ of Σ with respect to Σ_0 can be computed as

$$\begin{pmatrix} \mathbf{q} \\ \widehat{\mathbf{q}} \end{pmatrix} = \mathbf{J}_i \begin{pmatrix} \omega_{i1} \\ \vdots \\ \omega_{i5} \\ \tau_i \end{pmatrix},$$

ω_{ij} ... angular velocity of the j^{th} R-joint,
τ_i ... translatory velocity of the P-joint.



2. Jacobian Matrix of the EE

If we assume that $rk(\mathbf{J}_i) = 6$ for $i = 1, \ldots, 6$, we can compute

$$\mathbf{J}_i^{-1} \begin{pmatrix} \mathbf{q} \\ \widehat{\mathbf{q}} \end{pmatrix} = \begin{pmatrix} \omega_{i1} \\ \vdots \\ \omega_{i5} \\ \tau_i \end{pmatrix}.$$

By denoting the sixth row of $\mathbf{J}_i^{-1}$ by $(\widehat{\mathbf{j}}_i, \mathbf{j}_i)$, the 6×6 Jacobian matrix $\mathbf{J}$ of the platform can be written as

$$\mathbf{J} = \begin{pmatrix} \widehat{\mathbf{j}}_1 & \mathbf{j}_1 \\ \vdots & \vdots \\ \widehat{\mathbf{j}}_6 & \mathbf{j}_6 \end{pmatrix}.$$

$\mathbf{J}$ transforms the instantaneous screw of the platform into the translatory velocity of the active joints, i.e.

$$\mathbf{J} \begin{pmatrix} \mathbf{q} \\ \widehat{\mathbf{q}} \end{pmatrix} = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_6 \end{pmatrix}.$$

Remark: The instantaneous screw $\underline{\mathbf{j}}_i := \mathbf{j}_i^T + \varepsilon \widehat{\mathbf{j}}_i^T$ equals an instantaneous rotation around the carrier line of the i^{th} P-joint. Therefore $(\mathbf{j}_i, \widehat{\mathbf{j}}_i)$ are the spear coordinates of the axis a_{i3}. ◇

2. $rk(\mathbf{J}_i) < 6\ldots$ leg singularity

Geometrically, this means that the five rotary axes and the axis of the translation (ideal line) belong to a so-called *linear line complex* (cf. Section 3 of [17]). In this case, there exists a non-trivial linear-combination of the zero screw $\underline{\mathbf{o}}$, i.e.

$$\omega_{i1}\underline{\mathbf{a}}_{i1} + \omega_{i2}\underline{\mathbf{a}}_{i2} + \omega_{i3}\underline{\mathbf{a}}_{i3} + \omega_{i4}\underline{\mathbf{a}}_{i4} + \omega_{i5}\underline{\mathbf{a}}_{i5} + \tau_i\underline{\mathbf{t}}_i = \underline{\mathbf{o}}.$$

a) $\tau_i \neq 0$: The translatory velocity of the i^{th} active joint cannot be transmitted onto the EE, as the velocity ratio $(\tau_1 : \ldots : \tau_i : \ldots : \tau_6) = (0 : \ldots : 1 : \ldots : 0)$ causes an instantaneous standstill of Σ, i.e. $\underline{\mathbf{q}} = \underline{\mathbf{o}}$.

b) $\tau_i = 0$: Now there is an infinitesimal redundant mobility of the leg itself (but not of Σ). In the worst case, this can result in a self-motion of the leg.

Remark: In a leg singularity, the leg loses $6 - rk(\mathbf{J}_i)$ dofs. If an infinitesimal screw belonging to the lost dofs is applied to Σ, this can yield a breaking of the leg. $\diamond$

2. $rk(\mathbf{J}) < 6 \ldots$ EE singularity

The hexapod is in an EE singularity, if and only if, the carrier lines of the P-joints belong to a linear line complex. In this case, there exists at least a screw $\underline{\mathbf{q}} \neq \underline{\mathbf{o}}$ with

$$\mathbf{J}\begin{pmatrix}\mathbf{q}\\ \widehat{\mathbf{q}}\end{pmatrix} = \begin{pmatrix}\mathbf{o}\\ \mathbf{o}\end{pmatrix}.$$

Therefore Σ is infinitesimal movable while all active joints are locked. In the worst case, an EE singularity can result in a self-motion of Σ.

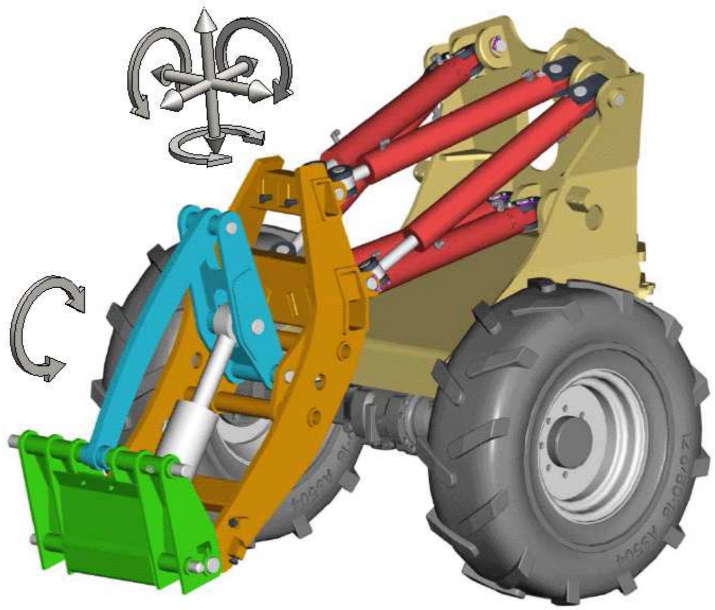
Remark: This singularity study also shows, that the hexapods under consideration only have line-based singularities, even though the last three joints of each leg are not equivalent with a S-joint, if an eccentric U-joint is used at Σ. Therefore these are more general parallel manipulators with line-based singularities than those characterized in Section 4 of [2]. $\diamond$

3. Performance Index: Motivation

In future applications, the hexapod's motion is controlled directly by skilled workers and not by highly-qualified academics.

Therefore there is an interest in an index, which gives the operator a feedback about the closeness of a given non-singular hexapod-configuration to the next singular one.

As there does not exist a distance metric in the pure mathematical sense, if rotational and translatory dofs are involved, we are looking for a performance index PI, which assigns to each configuration $\mathcal{C}$ a scalar $PI(\mathcal{C}) \in \mathbb{R}$.



Wheel loaders are coupled by hexapods with different EEs (cf. research project MOBIMA)

3. Desirable properties of a PI

1. $PI(\mathcal{C}) \geq 0$ for all $\mathcal{C}$ of the configuration space,
2. $PI(\mathcal{C}) = 0$ if and only if $\mathcal{C}$ is singular,
3. $PI(\mathcal{C})$ is invariant under Euclidean motions of the reference frame,
4. $PI(\mathcal{C})$ is invariant under similarities,
5. $PI(\mathcal{C})$ has a geometric/kinematic meaning,
6. $PI(\mathcal{C})$ is computable in real-time.

The PI has to evaluate the closeness to different types of singularities simultaneously, as separated computations of the closeness to EE and leg singularities go at the expense of the computation time, and one is confronted with the problem of combining the obtained values to a single meaningful closeness index.

But exactly this clear geometric/kinematic meaning is of importance for identifying a critical value, which indicates that a given configuration is too close to a singularity for guaranteeing a save performance of the hexapod.

3. Review

As the set of singular poses is solely determined by the hexapods geometry, a PI, which makes demands to evaluate the closeness to the next singularity, should only depend on geometric/kinematic properties of the inspected non-singular pose.

Therefore such a PI must not depend on the EE. As a consequence, all known condition number indices [8,14,15,21] as well as the local singularity transmission index [11] are out of question.

Moreover the requested index must not depend on non-kinematic parameters as mass or stiffness, which exclude also the indices presented in [1,6,18].

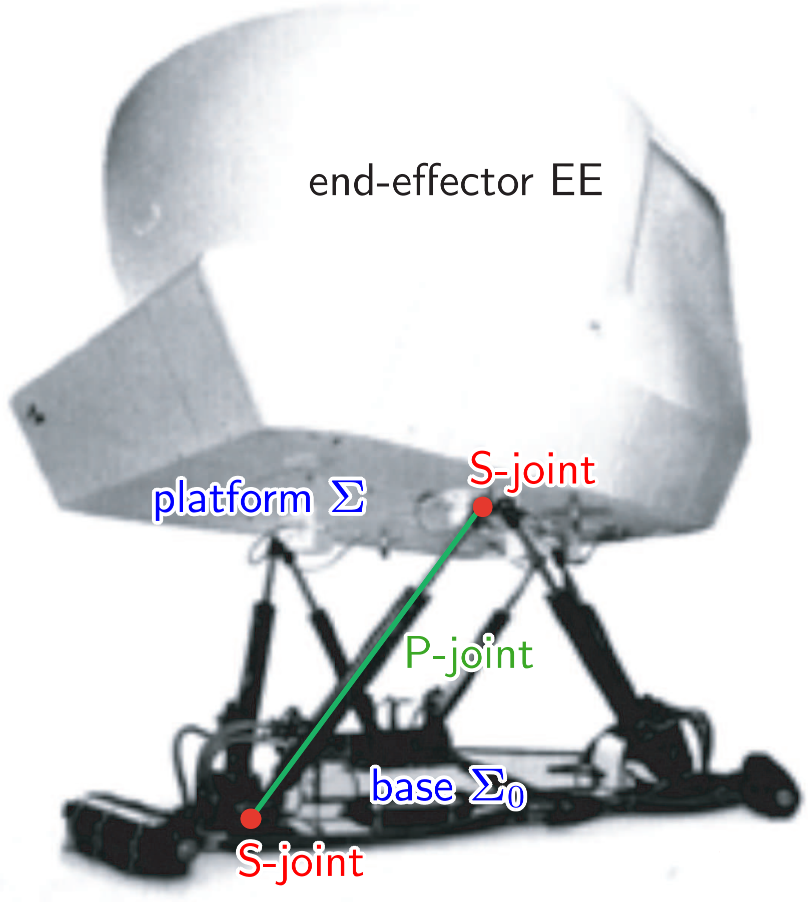
Also the manipulability [29] and the best fitting linear line complex [16,19], which are EE independent PIs, cannot master our demands (cf. presented paper).

3. Control Number CTN for SGPs

The CTN fulfills all six demands, but until now it is only defined for evaluating the closeness to EE singularities of Stewart Gough platforms (SGP).

In an EE singularity of a SGP, there exists an infinitesimal motion of Σ while all P-joints are locked.

In practice, configurations must be avoided, where minor (or even zero) variations of the leg lengths have uncontrollable large effects on the instantaneous displacement of Σ.



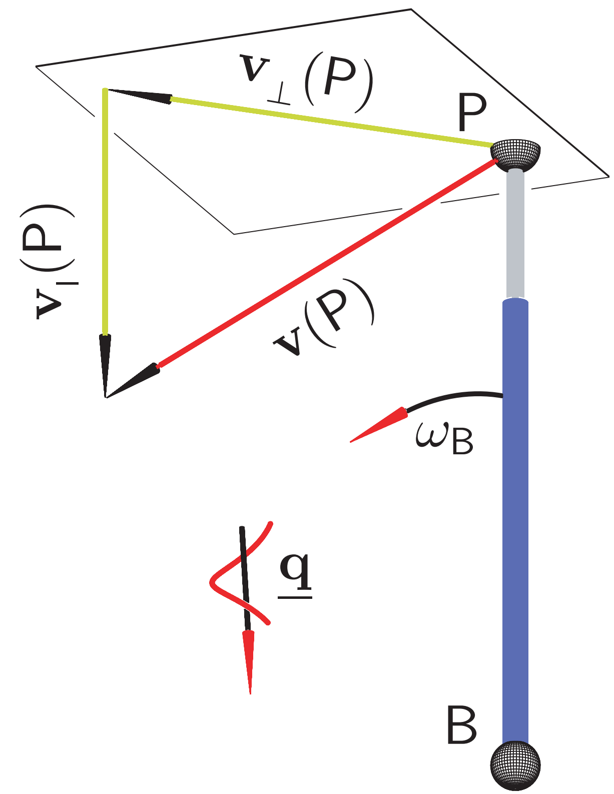
3. Control Number CTN for SGPs

The question is, which measurable parameter of SGPs indicates the circumstance of uncontrollability in a natural way and has a geometric/kinematic meaning.

The answer to this question are the angular velocities of the S-joints (see figure).

We computed the maximum λ_{max} and the minimum λ_{min} of the sum of the squared angular velocities of the passive joints under the normalizing condition that the sum of the squared translatory velocities of the active joints equals 1.

$$CTN := \sqrt{\frac{\lambda_{min}}{\lambda_{max}}} \in [0,1].$$



Remark: For details please see [13,14,15]. ◇

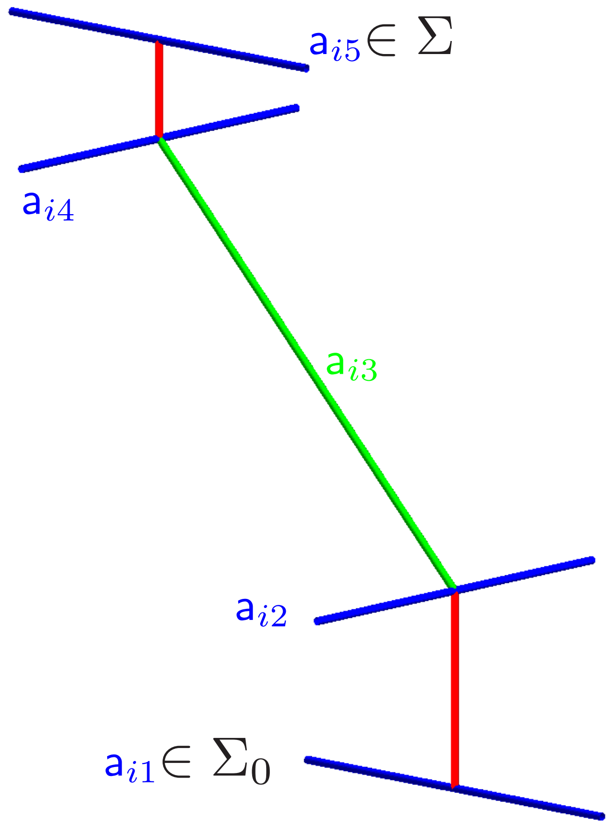
4. Basic Theorem for the Generalized CTN

Theorem 1. A leg singularity of type (a) with $rk(\mathbf{J}_i) = 5$ cannot exist.

Proof: As $\mathsf{a}_{i1}, \ldots, \mathsf{a}_{i5}$ intersect (or are even identical with) the carrier line of the i^{th} P-joint (= line a_{i3}), the linear line complex $\mathcal{L}$ spanned by $\mathsf{a}_{i1}, \ldots, \mathsf{a}_{i5}$ is singular.

If $\underline{\mathbf{a}}_{i1}, \ldots, \underline{\mathbf{a}}_{i5}$ are linearly independent ($\Rightarrow rk(\mathbf{J}_i) = 5$) $\mathcal{L}$ is uniquely determined.

Therefore a leg singularity of type (a) with $rk(\mathbf{J}_i) = 5$ exists, if and only if the axis t of $\underline{\mathbf{t}}_i$ intersects a_{i3}. But this can never happen, as t is the ideal line of the plane orthogonal to a_{i3}. $\square$



4. Consequences of the Basic Theorem

As a consequence of Theorem 1 a leg singularity of type (a) can only occur if $rk(\mathbf{J}_i) < 5$ holds, but this implies the existence of a leg singularity of type (b).

$\Longrightarrow$ The PI only has to indicate EE singularities and leg singularities of type (b).

The common characteristic property of these two singularities is that there exists an infinitesimal mobility while all active joints are fixed.

Therefore the so-called Generalized Control Number $GCTN$ can be used as index:

We computed the maximum λ_+ and the minimum λ_- of the sum of the squared angular velocities of the passive joints under the normalizing condition that the sum of the squared translatory velocities of the active joints equals 1.

4. Computation and Definition of GCTN

λ_+ and λ_- is the largest resp. smallest general eigenvalue of $\mathbf{Z}$ with respect to $\mathbf{N}$, with

$$\text{objective function } \zeta\text{: } \sum_{i=1}^{6}\sum_{j=1}^{5}\omega_{ij}^2 \qquad \Longleftrightarrow \qquad \zeta(\underline{\mathbf{q}}):\ (\mathbf{q}^T, \widehat{\mathbf{q}}^T)\mathbf{Z}\begin{pmatrix}\mathbf{q}\\ \widehat{\mathbf{q}}\end{pmatrix},$$

and

$$\text{normalizing condition } \nu\text{: } \sum_{i=1}^{6}\tau_i^2 = 1 \qquad \Longleftrightarrow \qquad \nu(\underline{\mathbf{q}}):\ (\mathbf{q}^T, \widehat{\mathbf{q}}^T)\mathbf{N}\begin{pmatrix}\mathbf{q}\\ \widehat{\mathbf{q}}\end{pmatrix} = 1.$$

$$GCTN := \sqrt{\frac{\lambda_-}{\lambda_+}} \in [0,1].$$

4. Main Theorem on $GCTN$

Theorem 2. The $GCTN$ fulfills all six stated requirements.

Proof: Due to the definition of the index, all demands, with exception of the second one, are trivially fulfilled. Therefore we only comment on demand 2:

The value of λ_+ equals ∞, if and only if, the manipulator is in an EE singularity or leg singularity of type (b), as only in these configurations an instantaneous self-mobility exists while all active actuators are locked.

Hence it remains to check the case $\lambda_- = 0$: In this case all passive joints have an instantaneous standstill. As a consequence an instantaneous change of the EE's orientation is not possible and therefore only a pure translation can be performed at this moment. A pure translation can only be done if all six legs are parallel to each other, but this already implies $rk(\mathbf{J}) \leq 3$, as the six carrier lines of the P-joints belong to a bundle of lines (cf. page 142 of [17]). $\square$

4. Additional Information from GCTN

If a given configuration is indicated by a small $GCTN$-value to be close to a singularity, the corresponding eigenvector $\mathbf{e}_+$ of λ_+ can be used to detect whether the given configuration is close to either an EE or a leg singularity by computing

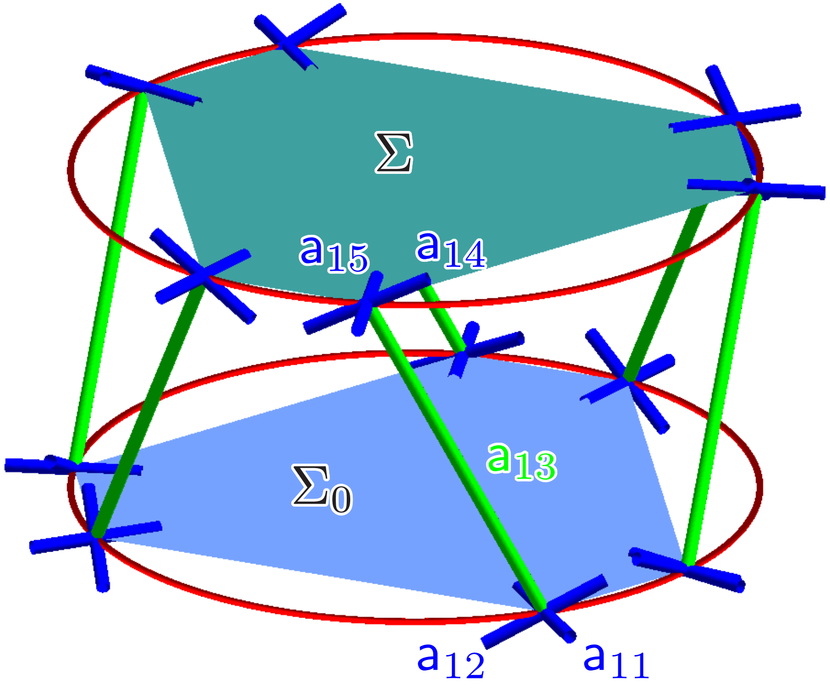
$$\mu_i(\underline{\mathbf{q}}_+) := \sum_{j=1}^{5} \omega_{ij}^2 \quad \text{with} \quad (\mathbf{q}_+, \widehat{\mathbf{q}}_+) := \mathbf{e}_+ \quad \text{for} \quad i = 1, \ldots, 6.$$

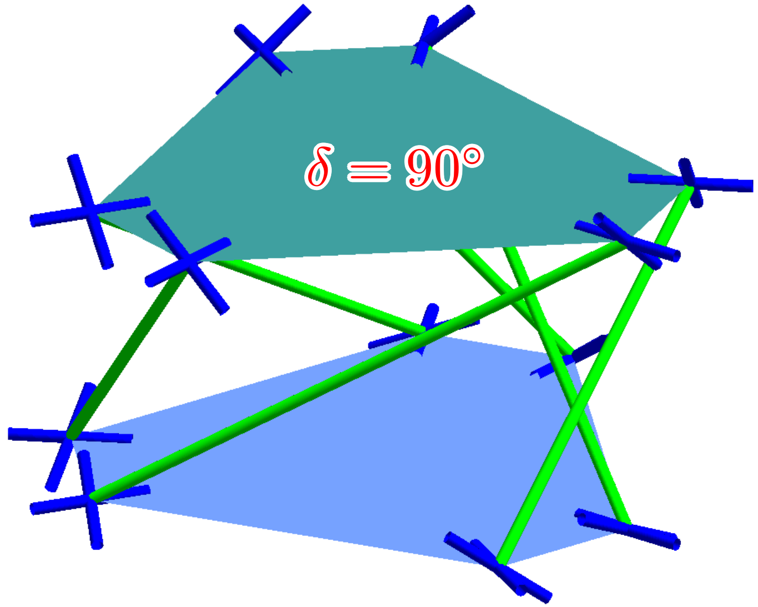
If $\mu_i(\underline{\mathbf{q}}_+)$ is close to λ_+, then the manipulator is in the neighborhood of a leg singularity of the i^{th} leg, as $\mu_1(\underline{\mathbf{q}}_+) + \ldots + \mu_6(\underline{\mathbf{q}}_+) = \lambda_+$ holds. Otherwise, we are in the neighborhood of an EE singularity.

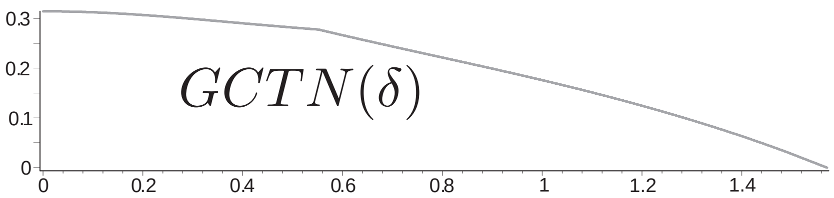
Remark: Similar to the CTN, the $GCTN$ can be used as well

- for manipulators with more than six legs (redundant hexapods),
- to optimize the kinematic design (isotropic central configuration). ◇

5. Example

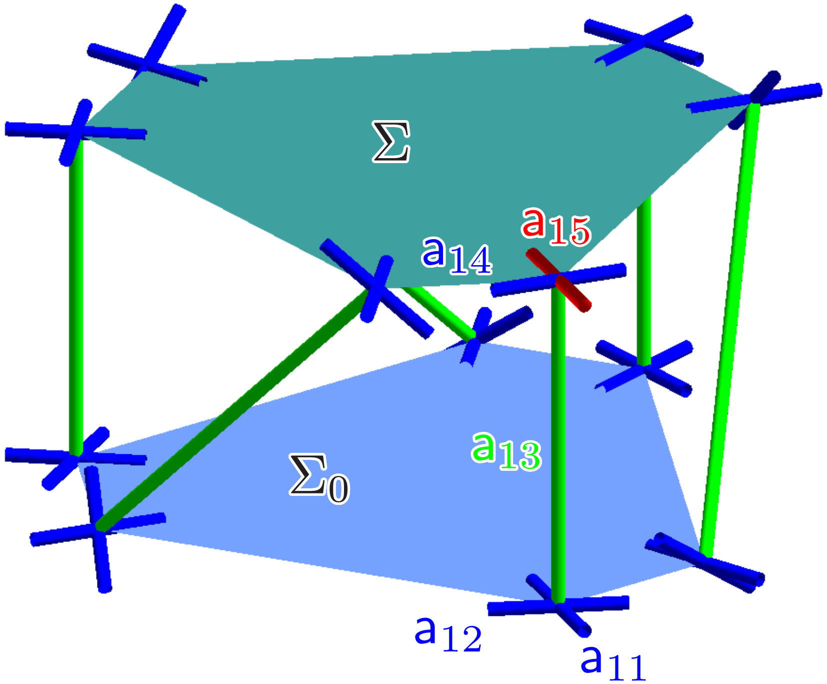


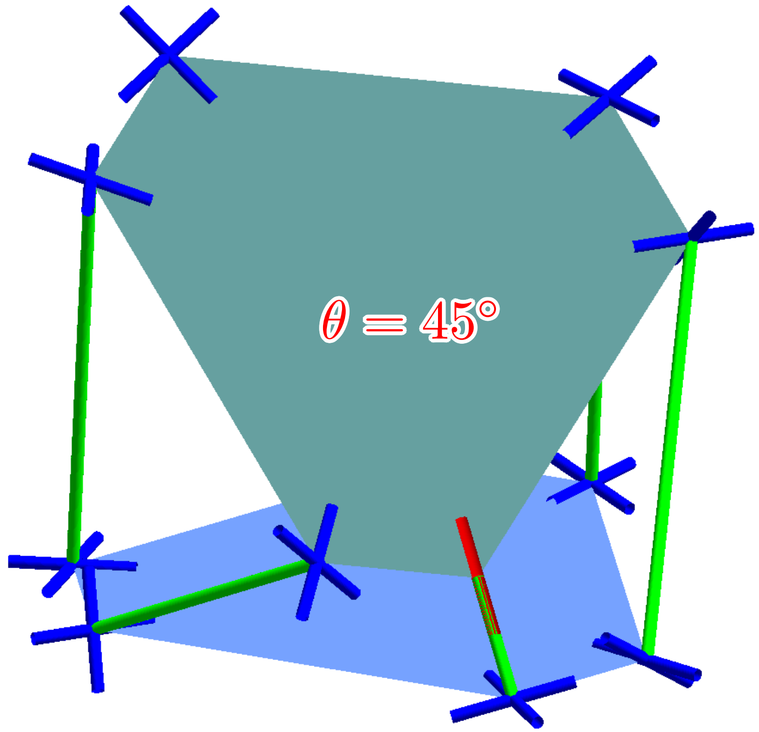


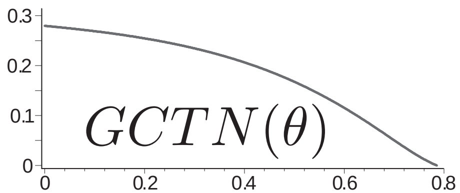


$\delta = 85°$: $GCTN \approx 0.034$, $\lambda_+ \approx 2268$ and $\mu_1 = \mu_3 = \mu_5 \approx 509$, $\mu_2 = \mu_4 = \mu_6 \approx 247$. According to the prognosticate behaviour, we are close to an EE singularity.

5. Example





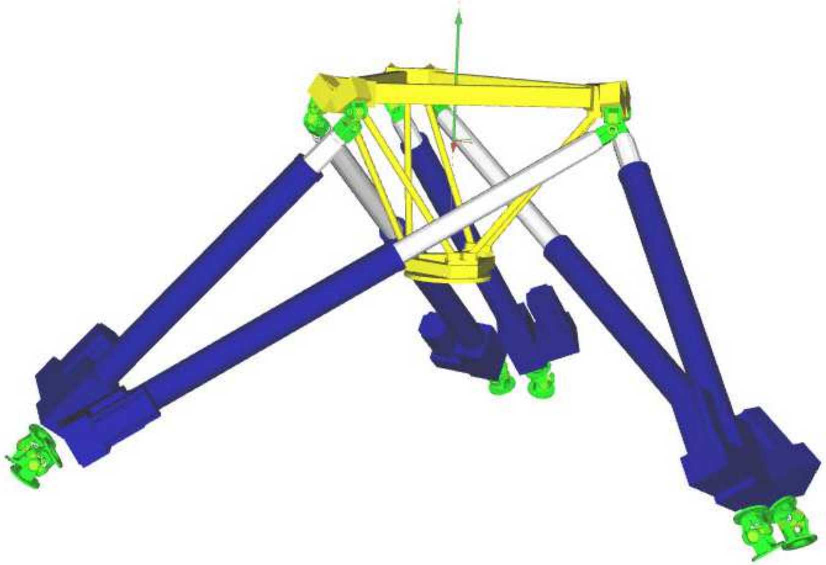


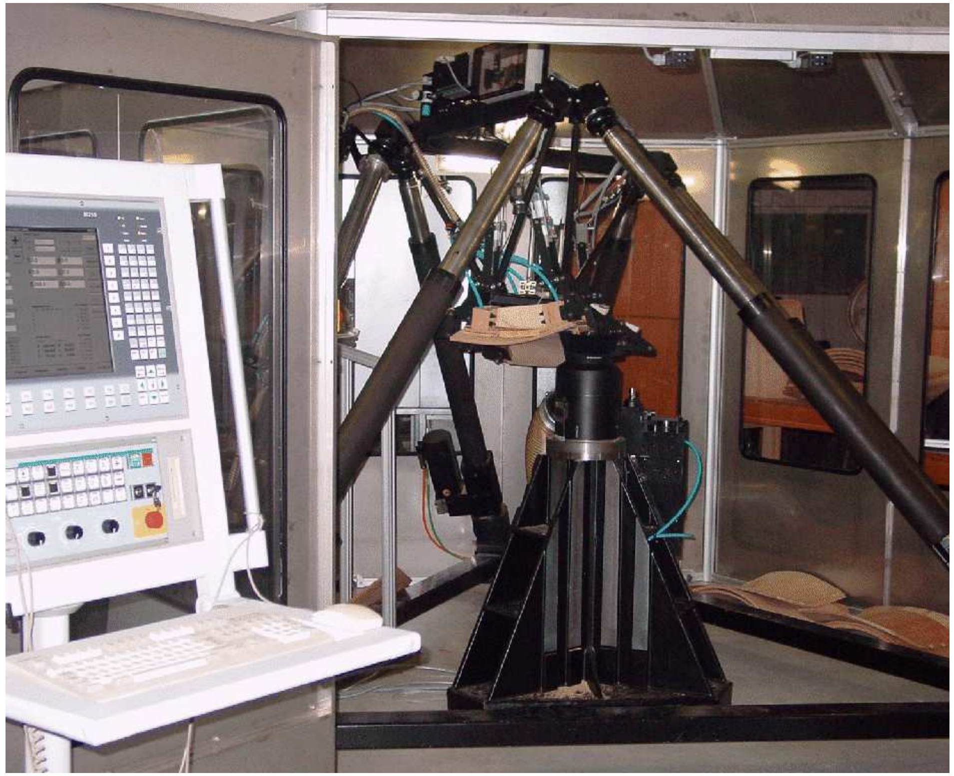
$\theta = 40°$: $GCTN \approx 0.052$, $\lambda_+ \approx 631$ and $\mu_1 \approx 601$, which shows that we are close to a leg singularity of the first leg.

6. Outlook

• Determination of isotropic designs, i.e. $GCTN = 1$ in central configuration.

• Bernd Kauschinger and his team (TU Dresden) try to identify a critical $GCTN$-value for FELIX-1.





6. References and Acknowledgments

All references refer to the list of publications given in the presented paper.

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