Kinematics of a Human Steering a Car

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***Abstract*— This paper presents an analysis of the kinematic manipulability of the human arm while steering a car. The human arm is modeled as 7-axis robot and a specialized measure of manipulability for the problem at hand is introduced. The analysis of different steering scenarios shows that optimal manipulability yields handling scenarios that are intuitive to a human operator. Furthermore the shoulder joint position is optimized to find the optimal seat position.**

I. INTRODUCTION

There are various approaches for steering a car, including different grips and angles at which the hand is in contact with the steering wheel. Since the human arm can be modeled as a 7-axis serial manipulator [1] its kinematics can be easily found using the Denavit-Hartenberg (DH) convention. The analysis of the manipulability of such a kinematic model shows that intuitive approaches of steering a car are similar to those found by optimizing a specialized measure of manipulability.

II. MATHEMATICAL MODELING

The process of steering was modeled using a serial manipulator with 7 rotational joints. The spherical joint at the shoulder was modeled by 3 intersecting rotational joints. Considering only the shoulder position the torso can be neglected. Further, two symmetric arms are assumed. Thus, the model contains 8 coordinate frames attached to the body, see Table I for the DH-parameters. Frame 1 is located in the shoulder and is assumed to be inertially fixed. The frames 1 to 7 and q_i, $i \in [1;7]$ are the joint angles of the arm, with $q_{1,2,3}$ for the shoulder, q_4 for the elbow and $q_{5,6,7}$ for the wrist. Frame 8 is used to describe the offset of the contact point on the steering wheel relative to the wrist. Furthermore, d_2 is half the distance between the shoulders, d_4 is the length of the upper arm, d_6 the length of the lower arm and r_8 is the distance from the wrist to the contact point of the hand with the steering wheel.

n	ϑ_n	d_n	r_n	α_n
1	0	0	0	$\pi/2$
2	$q_1 - \pi/2$	0.15m	0	$\pi/2$
3	$q_2 + \pi/2$	0	0	$\pi/2$
4	$q_3 + \pi/2$	0.31m	0	$\pi/2$
5	q_4	0	0	$-\pi/2$
6	$q_5 - \pi/2$	0.19m	0	$-\pi/2$
7	$q_6 - \pi/2$	0	0	$-\pi/2$
8	q_7	0	0.05m	0

TABLE I

DH PARAMETERS OF THE MATHEMATICAL MODEL.

The initial position of the seat relative to the steering wheel was chosen such that the shoulder is rotated $q_1 = 45°$ forwards from the vertical position and the elbow joint is $q_4 = 105°$. Additionally, the steering wheel was tilted by 20°. For the desired pose of the hand on the steering wheel different scenarios of steering are investigated. Each scenario is defined by desired positions and orientations of the contact point between the hand and the steering wheel during a steering maneuver. Figure 1 shows the links of the arm model and the trace of the end effector frame of a steering maneuver.

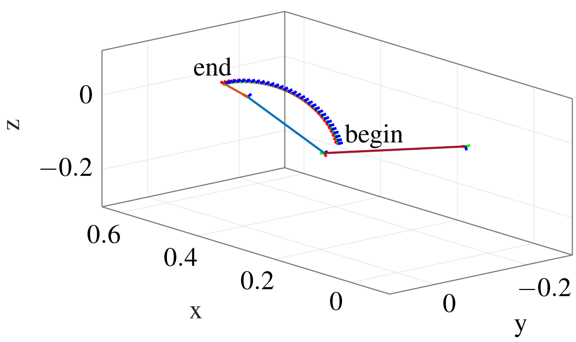

Fig. 1. Kinematic model for steering a car. Modeled arm with shoulder at (0,0,-0.15). end effector coordinate frames are traced along the steering wheel for a 90° turn.

III. OPTIMIZATION

For simulation, the position and orientation of the end effector frame are given as a reference pose. In order to solve the inverse kinematics problem an optimization was used. Compared to an analytical solution this allows for a relative weighting of the hand orientation and position and yields realistic movements of the hand. The joint angles of the model are then used to analyze the manipulability of the scenario. The test case is a rotation of the steering wheel of 90°.

For scenario 1 (s1) the right hand touches the steering wheel at the right side. The orientation is given by the back of the hand pointing radially outwards and the hand pointing perpendicular to the steering wheel. Scenario 2 (s2) is given by the driver grabbing the steering wheel at the top from below and pulling it downwards. Thus, the wrist points radially outwards and the back of the hand is orthogonal to the steering wheel center.

The end effector position p and the wrist orientation

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quaternion o are used in the cost function

$$J(q) = \sum_{k=1}^{N} (f(q_k) - p_{k,ref})^{\mathrm{T}} (f(q_k) - p_{k,ref}) + (o_{v,k} - o_{v,k,ref})^{\mathrm{T}} W_o (o_{v,k} - o_{v,k,ref}) + (q_{k+1} - q_k)^{\mathrm{T}} W_{\Delta q} (q_{k+1} - q_k) + q_k^{\mathrm{T}} W_q q_k \,, \tag{1}$$

with k being the discrete time index, $f(q_k)$ the position of the hand, and $o_{v,k}$ the vector part of its orientation quaternion. The constant diagonal weighting matrices W_o, $W_{\Delta q}$, and W_q determine the relative influence of the individual terms. This allows for the orientation to be less restrictive in the solution of the inverse kinematics problem, thus resulting in natural movements of the wrist.

IV. Manipulability

The resulting joint angles of the hand model are used to analyze the manipulability of the different steering scenarios. Given the joint angular velocities $\dot{q}$ the end effector linear velocities $\dot{p}$ can be calculated according to

$$\dot{p} = J\dot{q} \,, \tag{2}$$

with the Jacobian matrix $J \in \mathbb{R}^{3\times 7}$. Note, that for steering the translation of the end effector is the deciding quantity for turning the wheels, thus the orientation is omitted in the following. The manipulability is given by

$$m = \sqrt{\det(JJ^{\mathrm{T}})} \,, \tag{3}$$

see, [2]. Turning the steering wheel is achieved by moving the hand in the direction tangent to the steering wheel at the current contact point. This steering movement is actuated by joint velocities which can be mapped to the steering wheel using the Jacobian matrix. Thus, the *directional manipulability*

$$m_{\mathrm{dir}} = \| t^{\mathrm{T}} J \| \tag{4}$$

is introduced, which is the projection of the position part of the Jacobian matrix (and thus the manipulability ellipsoid) onto the unit tangent vector to the steering wheel t. This measure indicates the possibility to move the end effector in the direction t only.

These measures for a 90° turn using scenario 1 (s1) and 2 (s2) are depicted in Figure 2.

V. Shoulder Joint Position

In order to calculate the optimal position of the driver seat the optimization problem is solved for different shoulder joint positions. To design an indicator for the optimal seat position, the manipulability m is integrated along the steering angle φ, i.e.

$$M = \int_0^{90^\circ} m(\varphi) \mathrm{d}\varphi \,. \tag{5}$$

The classical manipulability has a distinct optimal shoulder position since both straight and lateral movements are considered, see Figure 3. In contrast, the directional manipulability suggests that the driver should be seated far away from the steering wheel. The z coordinate of the seat has only little influence on the manipulability. This can be reasoned with the longer distance causing higher velocities for a given joint angular velocity.

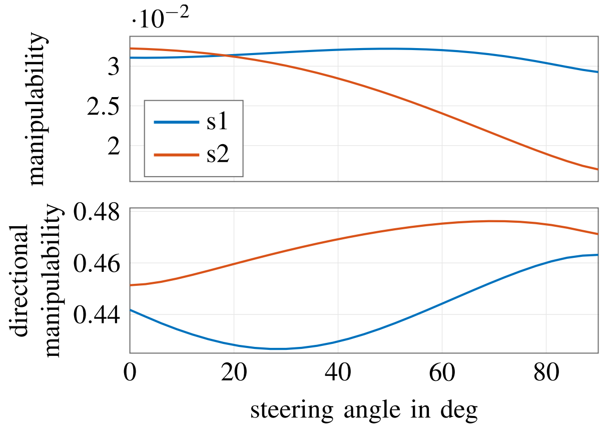

Fig. 2. Manipulability of scenario 1 and 2.

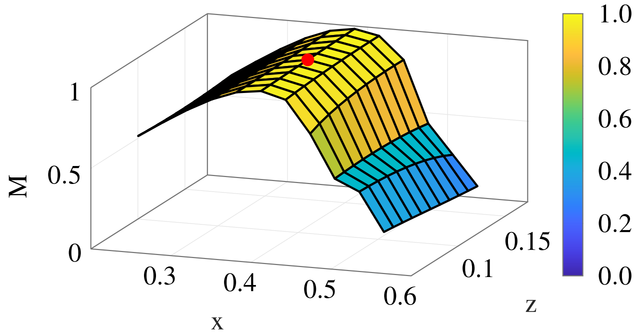

Fig. 3. Manipulability for scenario 1. The red dot marks the manipulability for the nominal values.

VI. Conclusions

A kinematic model of a human arm was used to analyze the steering of a car. In this context a specialized manipulability measure was introduced to evaluate the tangent velocity acting on the steering wheel. Two manipulability measures were used to optimize the position of the seat in the car. This spacially discretized optimization showed that the vertical position has only little influence on the manipulability. The presented algorithm proved to be a viable approach for optimizing the driver seat position in a car. Furthermore, it can be extended to satisfy specific requirements by using problem specific optimality criteria.

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