Self-motions of parallel manipulators associated with flexible octahedra

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1 Introduction

A Stewart Gough (SG) platform is a parallel manipulator consisting of a moving platform, which is connected via six Spherical-Prismatical-Spherical (SPS) legs with the base. Moreover a SG platform is called planar if the six platform anchor points m_i $(i = 1,\dots,6)$ are located in a plane and if the corresponding base anchor points M_i are coplanar as well (see Fig. 1a). Note that a SG platform is controlled only by a variation of the six leg lengths ($\Rightarrow$ P-joints are active, S-joints are passive). If all P-joints are locked, a SG platform is in general rigid in one of its 40 possible assembly modes (cf. HUSTY [1], DIETMAIER [2]). But, it can even be the case that the manipulator can perform an n-parametric motion $(n > 0)$, which is called self-motion. Moreover, all self-motions of SG platforms are solutions to the famous *Borel Bricard problem* (cf. BOREL [3], BRICARD [4], HUSTY [5], VOGLER [6]) which is still unsolved.

In this talk, which is subdivided into two parts (cf. Section 2 and 3), we discuss the connection between self-motions of parallel manipulators and flexible octahedra (= spatial shape of the octahedron can be changed continuously due to changes of its dihedral angles only). The latter are also associated with the name of BRICARD, as he firstly determined in [7] the three types of flexible octahedra in the Euclidean 3-space. These so-called *Bricard octahedra* are as follows:

Type 1 All three pairs of opposite vertices are symmetric with respect to a common line.

Type 2 Two pairs of opposite vertices are symmetric with respect to a common plane, which passes through the remaining two vertices.

Type 3 For a detailed discussion of this more complicated type we refer to STACHEL [8].

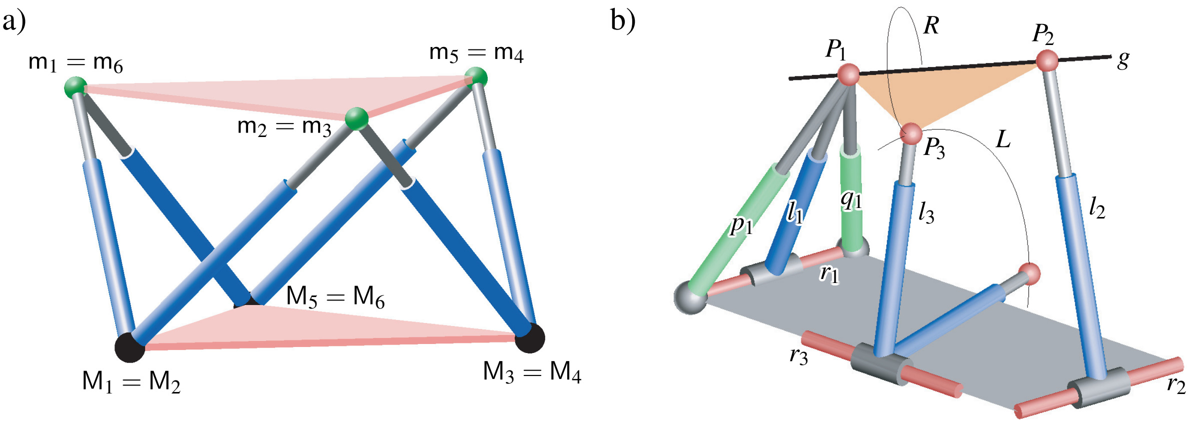

Figure 1: a) A special planar SG platform which is also known as the octahedral manipulator. b) The substitution of l_1 by p_1 and q_1 is illustrated for a TSSM with parallel rotary axes r_1, r_2.

2 Self-motions of parallel manipulators of TSSM type

In the first part of the talk, we discuss self-motions of parallel manipulators of TSSM type (Triangular Symmetric Simplified Manipulator), because they are probably of special interest for the audience as they are very often used in practice. A TSSM consists of a platform, which is connected via three Spherical-Prismatical-Rotational (SPR) legs l_j ($j = 1,2,3$) with the base (see Fig. 1b), where the axes r_j of the R-joints are coplanar. As we can replace each SPR leg l_j by two SPS legs p_j and q_j (as shown in Fig. 1b for $j = 1$) the determination of TSSM self-motions can be traced back to those of planar 6-3 parallel manipulators of SG type.

In spite of all the work done on singularities of these manipulators (e.g. MERLET [9], DI-GREGORIO [10], DOWNING ET. AL. [11], BEN-HORIN AND SHOHAM [12, 13]) a complete classification of TSSM designs with self-motions was missing up to recent. The author closed the gap in [14] and presents a listing of all TSSMs with self-motions within the talk. Moreover, we show that these TSSM designs are closely related with *Bricard octahedra* and flexible octahedra with one vertex in the plane at infinity (see also NAWRATIL [15]).

3 Self-motions of parallel manipulators of SG type

Beside the self-motions of 6-3 parallel manipulators (cf. Section 2), there are only a few self-motions of non-architecturally[1] singular SG platforms known, as their computation is a very complicated task. These self-motions are as follows: The first paper, which mentioned self-motions of SG platforms was written by HUSTY AND ZSOMBOR-MURRAY [24] in 1994. The reported self-motion was a *Schönflies motion*. Beside this self-motion, a two-parametric line-symmetric self-motion was given in ZSOMBOR-MURRAY ET AL. [25] (see also KRAMES [26]). Moreover, HUSTY AND KARGER [27] proved that the list of *Schönflies Borel Bricard motions*, given by BOREL [3], is complete. The classification of all self-motions of the original SG platform was done by KARGER AND HUSTY [28]. Based on this study KARGER [29, 30] presented a method for designing planar SG platforms with self-motions of the type $e_0 = 0$.

In a recent publication [31] the author showed that self-motions of general planar SG platforms can be classified into two so-called Darboux Mannheim (DM) types (type I and type II).[2] Based on this result, we present a surprising simple geometric method on how non-architecturally singular SG platforms with one-parametric self-motions of type II DM can be constructed from flexible octahedra of type 1.

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[1] It is already known that manipulators, which are singular in every possible configuration, possess self-motions in each pose. These manipulators are so-called architecturally singular SG platforms (cf. MA AND ANGELES [16]) and they are well studied (see KARGER [17], NAWRATIL [18], RÖSCHEL AND MICK [19] and WOHLHART [20] for the planar case and HUSTY AND KARGER [21], KARGER [22] and NAWRATIL [23] for the non-planar case). Therefore, we are only interested in the computation of self-motions of non-architecturally singular SG platforms.

[2] Moreover in [31] we presented a way on how the set of equations yielding a type II DM self-motion can be computed explicitly. Based on these equations, which are of great simplicity seen in the context of self-motions, we were already able to compute first results for this class of self-motions (cf. [32]).

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