Multistable Design of Triangulated Cones

Georg Nawratil (TU Wien, Austria)

It is well known that a flat strip of congruent triangles, which is also known as Kresling pattern, can be arranged in two different ways on a cylinder:

(1) Circular arrangement to a closed strip: The strip of finite length is folded up such that its ends are identified, where the points of the upper and lower rim form regular polygons. This discretized cylindrical strip has a bi-stable behavior (cf. [1]) and can be composed repetitively to cylindrical towers finding practical application as energy/shock absorber or vibration isolator.

(2) Helical arrangement according to C.R. Calladine: The strip can be assumed of infinite length and is wrapped up in a way that the points on the lower rim are identified with those of the upper rim after one winding around the cylinder. The resulting cylindrical structure is multistable and its vertices are located on a helix (cf. [2, 3])

Within this talk we generalize these two constructions for conical structures, i.e. vertices are located on a cone of revolution. There is only one journal article written in English language (cf. [4]) dealing with these conical structures, where their crease patterns are generated from those of cylindrical structures by applying planar conformal maps. The drawback of this method is that it does not allow direct access to the spatial conical shape with exception of the flat foldable state, which can be added as an additional condition (cf. [4, Eq. (14)]).

Based on a kinematical generation of these structures, we compute directly the triangulation on the cone. In this way we construct structures, which can snap between conical shapes whose apex angles serve as design parameters. In this context, we also derive shaky realizations and intervals of self-intersection free designs. Finally, we analyze these structures with respect to their capability to snap, where we focus on the cases with a flat-foldability (apex angle equals π), as deployable structures are of great practical interest.

Acknowledgement: This research is supported by project P 30855-N32 of the Austrian Science Fund (FWF).

References

[1] Walter Wunderlich. Snapping and Shaky Antiprisms. *Math. Mag.*, 52(4): 235–236, 1979.

[2] Simon D. Guest and Sergio Pellegrino. The folding of triangulated cylinders, Part I: Geometric considerations. *J. Appl. Mech.*, 61(4):773–777, 1994.

[3] Jens Wittenburg. Foldable and Self-Intersecting Polyhedral Cylinders Based on Triangles. *J. Geom. Graph.*, 23(2):245–258, 2019.

[4] Sachiko Ishida et al. Mathematical approach to model foldable conical structures using conformal mapping. *J. Mech. Des.*, 136(9):091007, 2014.