Flexible arrangement of two Bennett tubes

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In 1897 Bricard [2] proved that there are three types of flexible octahedra in the Euclidean 3-space. These so-called *Bricard octahedra* can also be seen as flexible bipyramids, where each quadrilateral pyramid corresponds to a spherical 4R-loop. As also planar 4R-loops are flexible, one can replace one or both pyramids by quadrilateral prisms. The full classification of flexible arrangements of quadrilateral pyramids and prisms was given in [3] and of two quadrilateral prisms in [4], respectively.

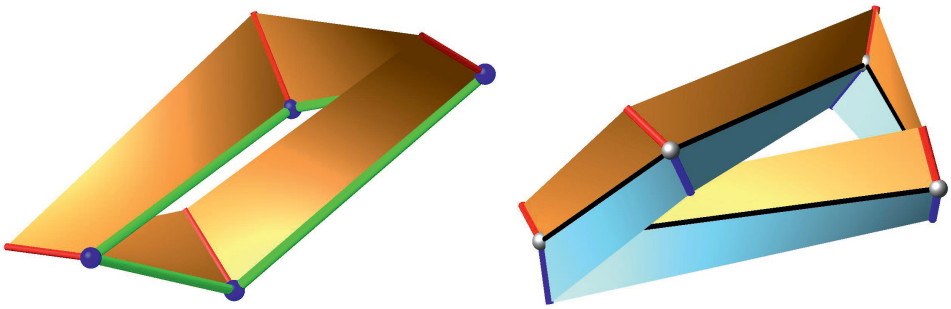

Figure 1: Left: Bennett mechanism consisting of four rotation axes (red) where adjacent ones are connected by common normals (green). By adding skew faces (e.g. orange hyperbolic paraboloid patches) one obtaines a Bennet tube. Right: Flexible arrangement of two Bennett tubes in a self-collision-free configuration. The intersection polygon of the blue Bennett tube and the orange one is illustrated in black.

But beside planar and spherical 4R-loops there also exist spatial ones, known as Bennett mechanisms [1], which can be realized as so-called Bennett tubes by using skew faces (see Figure 1-left). Therefore one can ask for flexible arrangements of a Bennett tube with a quadrilateral pyramid/prism and of two Bennett tubes, respectively. In this talk we present results on the latter case, where an example is illustrated in Figure 1-right.

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References

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