On the set of oriented line-elements: point-models, metrics and applications

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For a large number of applications in robotics the *end-effector* has a rotational symmetry; e.g. milling, spot-welding, laser or water-jet engraving/cutting, etc. For the determination of these axial symmetric tasks the rotation axis a of the tool is of importance as well as the location of the *tool tip* A (cf. Fig. 1). In addition, the orientation of the line a has to be taken into account, thus we consider the oriented line $\overrightarrow{\mathsf{a}}$. The two geometric objects A and $\overrightarrow{\mathsf{a}}$ can be combined to a so-called oriented line-element $(\mathsf{A}, \overrightarrow{\mathsf{a}})$.

We study point-models for the set $\overrightarrow{\mathcal{L}}$ of oriented line-elements by reviewing existing ones and by constructing them from different approaches proposed in the literature. We distinguish between point-models resulting from transformations and those implied by representations, where we also pursue the strategy to represent oriented line-elements by oriented line-segments with a constant length. For path planning in robotics (e.g. Fig. 1) it is desirable to have a metric on $\overrightarrow{\mathcal{L}}$, thus we also study geometric meaningful distance measures. Based on the resulting metric spaces we finally discuss the problem of motion design (e.g. Fig. 1).

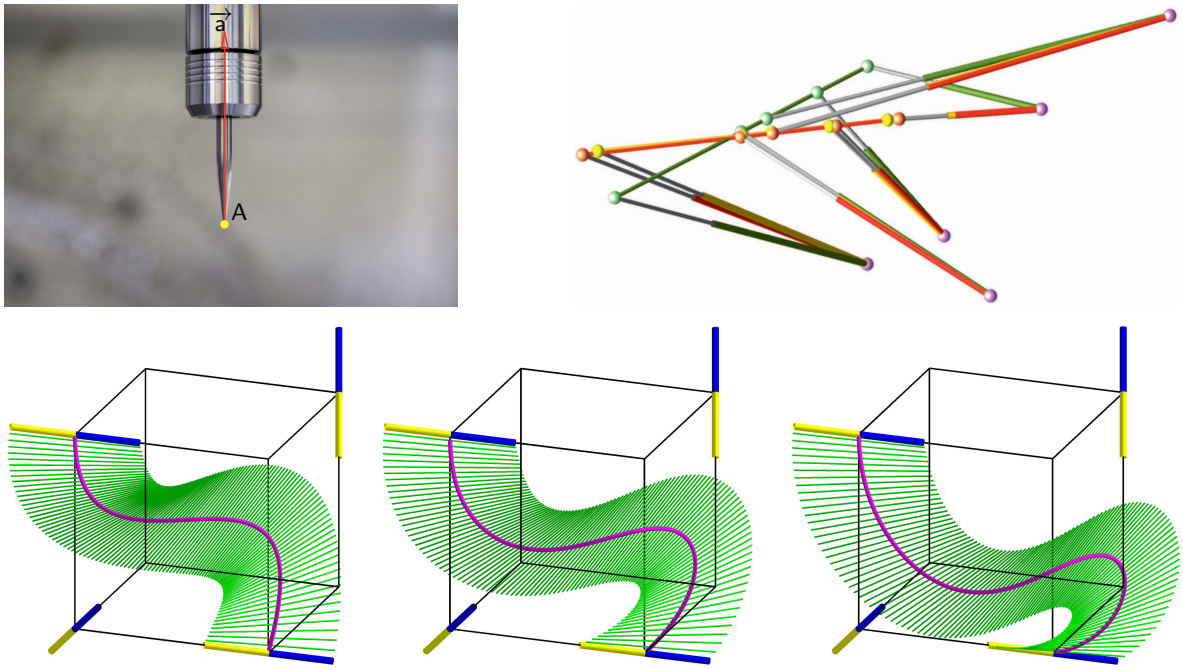

Figure 1: Upper Left: End-effector with rotational symmetry represented by the oriented axis $\overrightarrow{\mathsf{a}}$ and the tool tip A. Upper Right: A pentapod in the given (green) configuration and the closest singular configuration (red). The yellow configuration is the closest singularity under similarity transformations of the platform. Lower three pictures: For the motion design of oriented line-elements one can e.g. adapt the famous algorithm of De Casteljau for Bézier curves (the result depends on the underlying geodesic motions).

References

[1] Nawratil, G.: *Point-models for the set of oriented line-elements – a survey.* Mechanism and Machine Theory, Vol. 111, Elsevier, 2017, pp. 118-134

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