ADVANCED COMPUTATIONAL DESIGN

From axial C-hedra to general P-hedra and their construction & deformation using control polylines

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Nawratil: From axial C-hedra to general P-nets. Advances in Robot Kinematics (2024)
Nawratil: Construction and deformation of P-hedra using control polylines. Advances in Robot Kinematics (2026)

Introduction
Quad-Surfaces

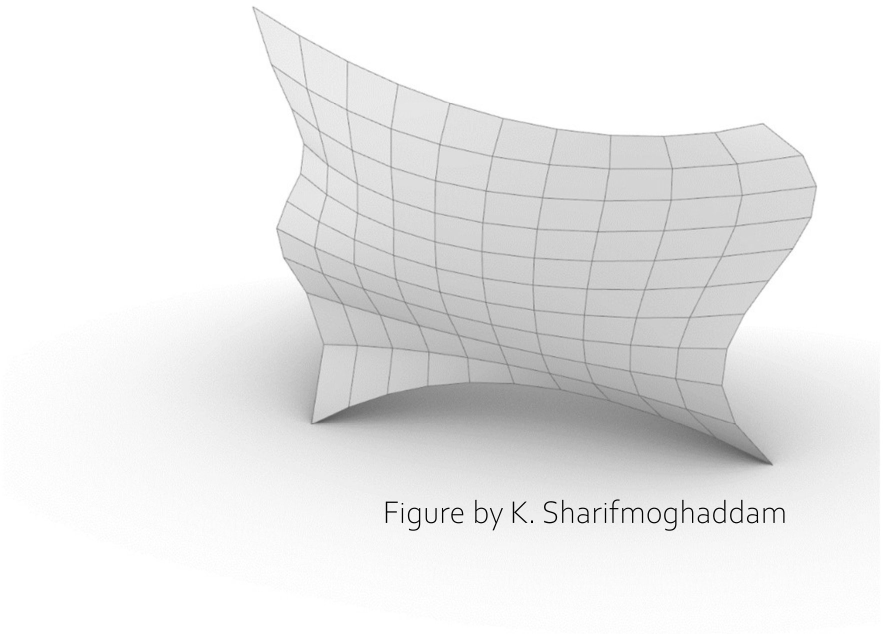
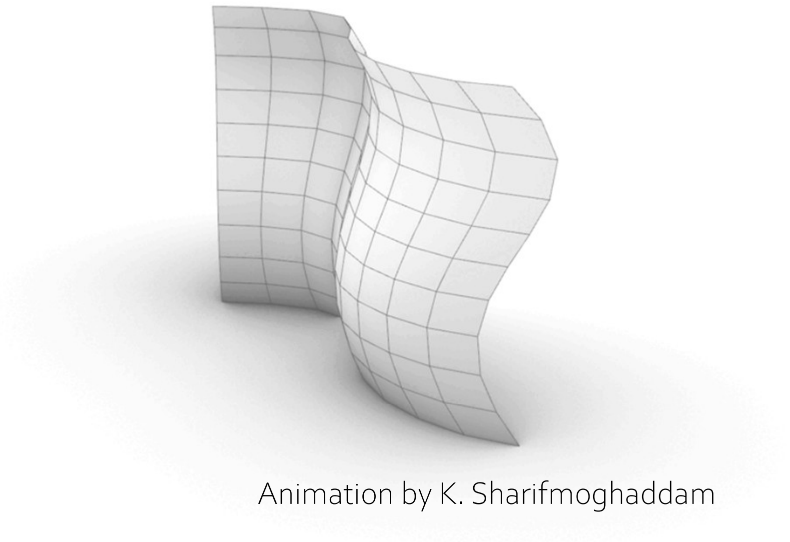


Figure by K. Sharifmoghaddam

Rigid



Animation by K. Sharifmoghaddam

Flexible

Introduction
Well-known classes

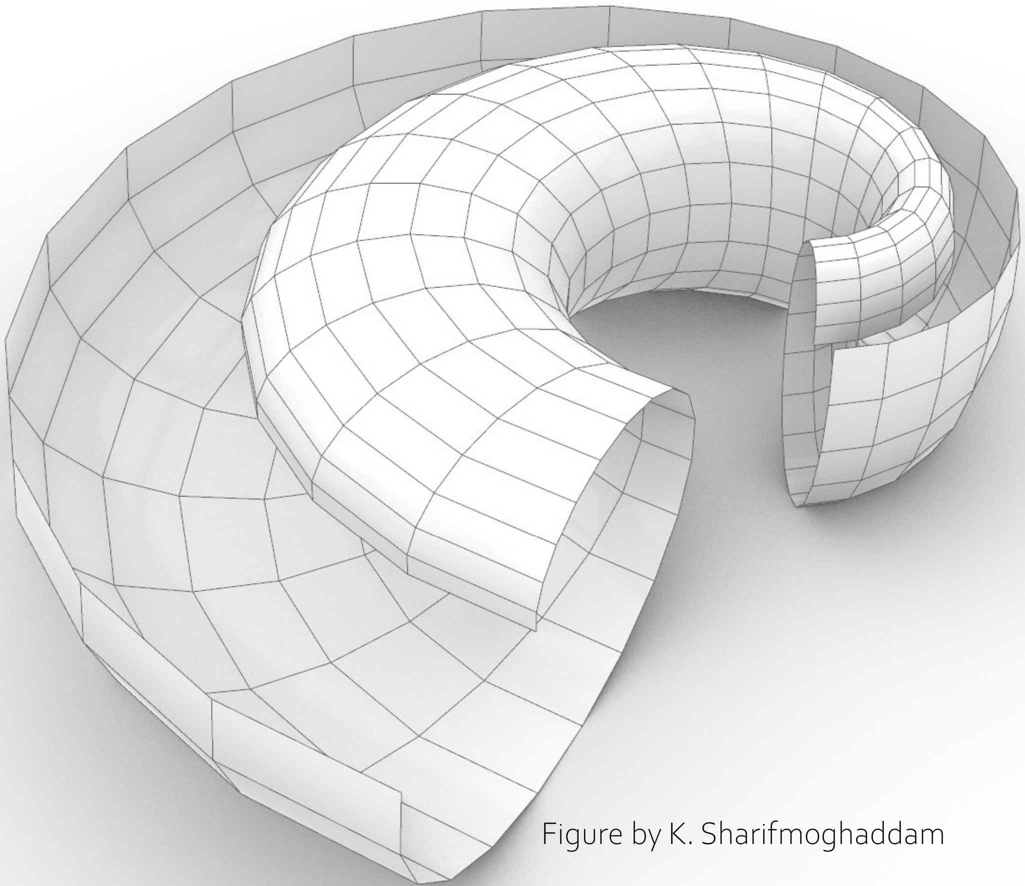


Figure by K. Sharifmoghaddam

T-hedra

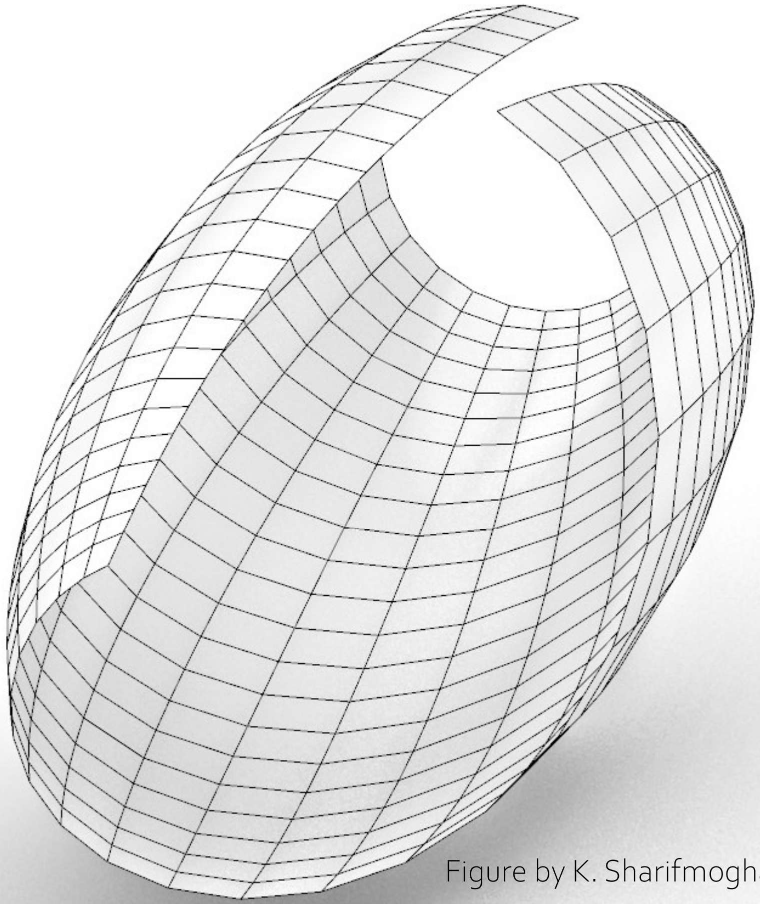


Figure by K. Sharifmoghaddam

V-hedra

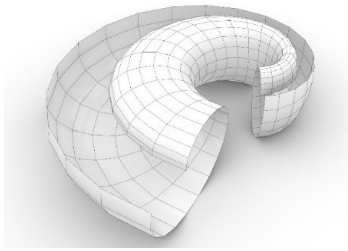
Sauer, Graf: Über Flächenverbiegung in Analogie zur Verknickung offener Facettenflache. Mathematische Annalen (1931)

Overview

T-hedra

Construction & Types

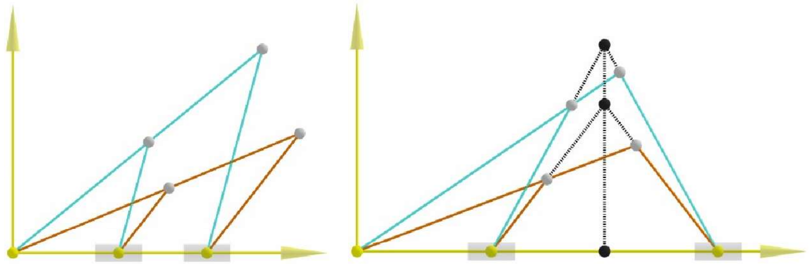
Projective Transformation



Axial C-hedra

Review

Full Classification

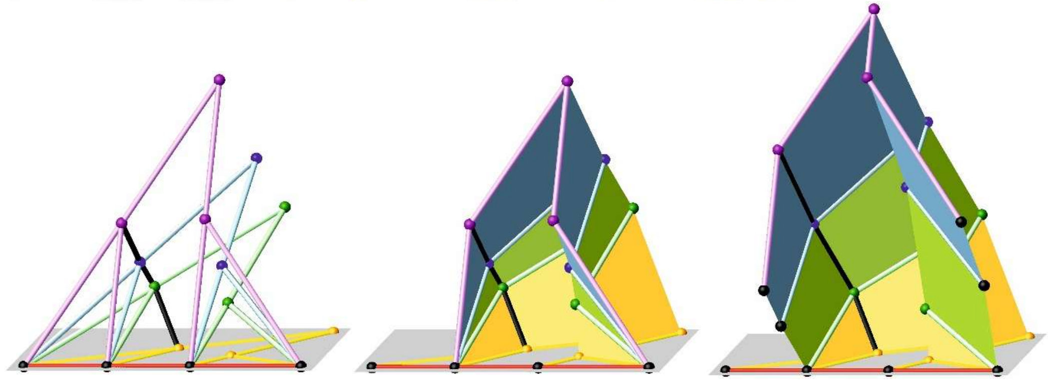


P-hedra

Construction

Deformation

Flexion Limit & Bifurcation



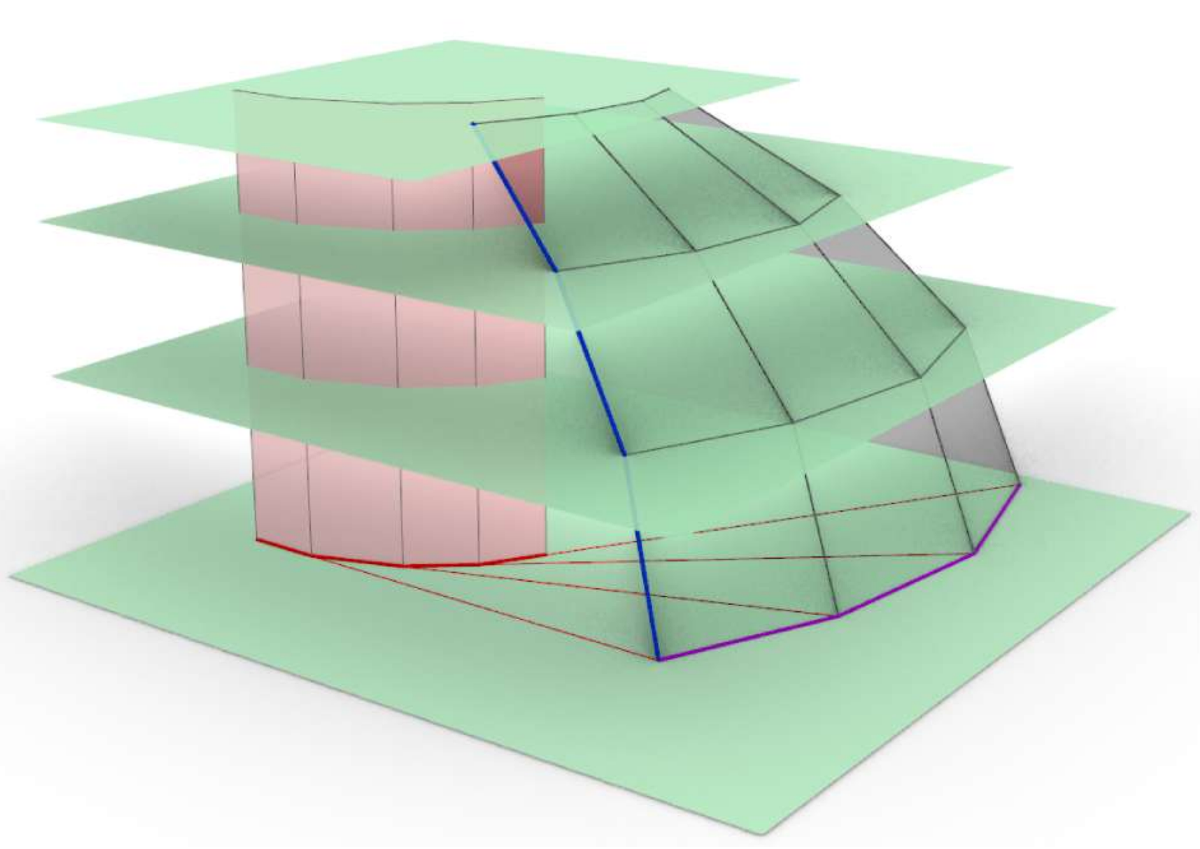
Final Remarks

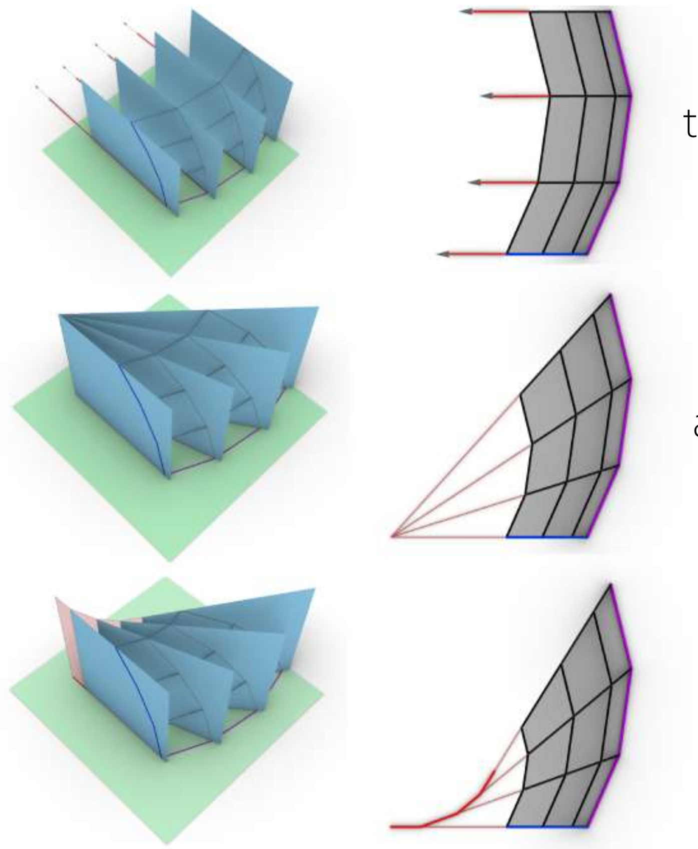
Nawratil: From axial C-hedra to general P-nets. Advances in Robot Kinematics (2024)

Nawratil: Construction and deformation of P-hedra using control polylines. Advances in Robot Kinematics (2026)

T-hedra

T-hedra
Construction and Types

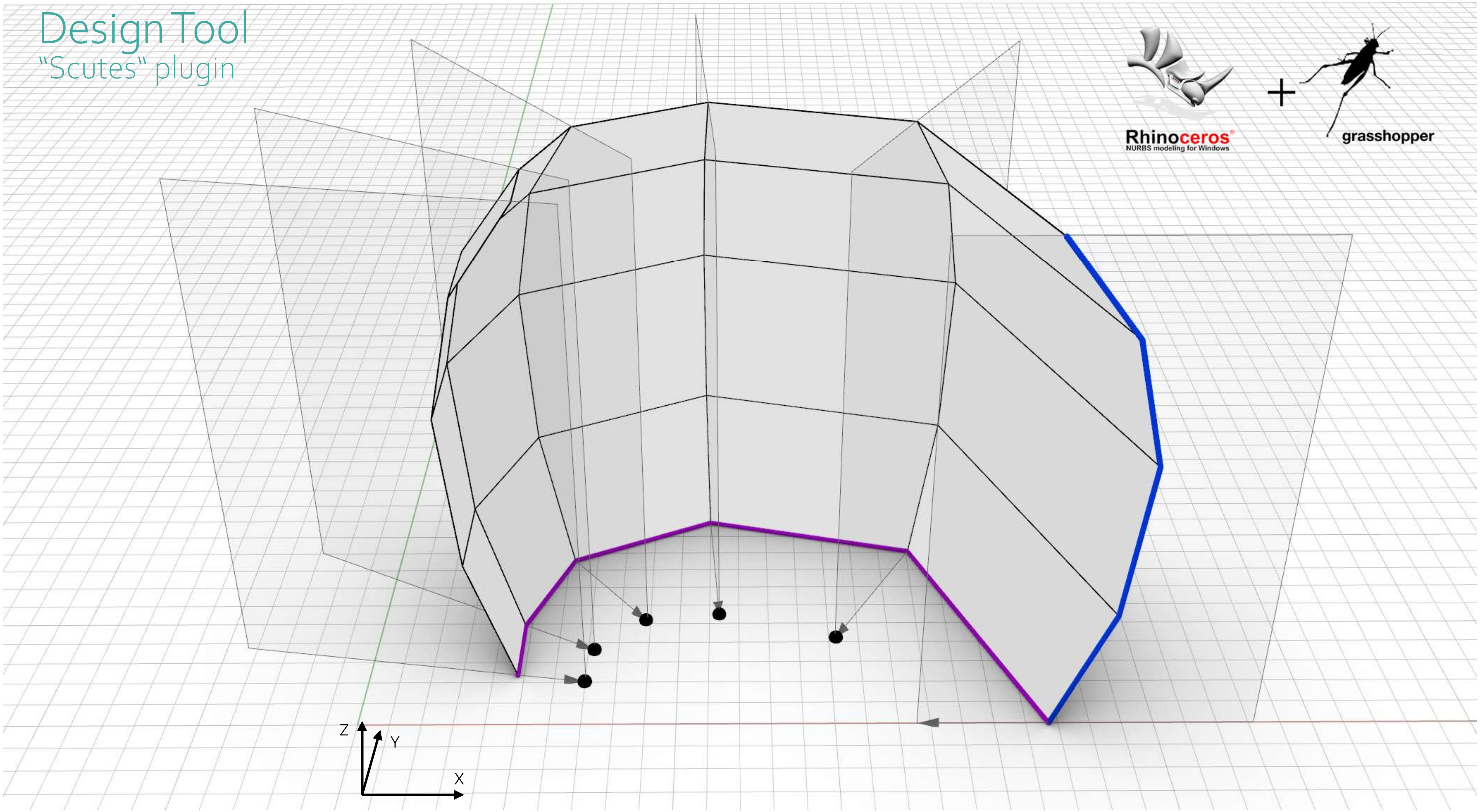




Sharifmoghaddam, Maleczek, Nawratil: Generalizing rigid-foldable tubular structures of T-hedral type.
Mechanics Research Communications (2023)

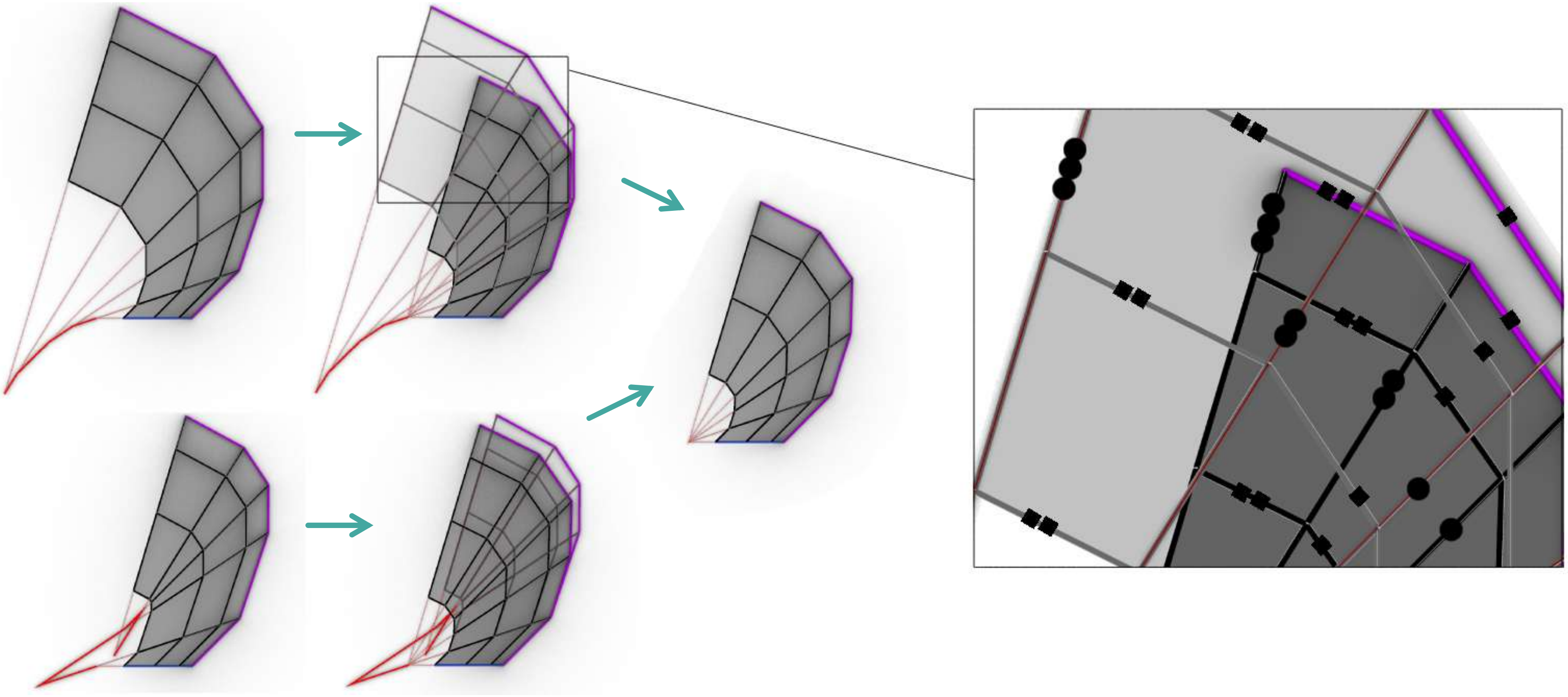
Design Tool

“Scutes“ plugin



Sharifmoghaddam, Nawratil, Rasoulzadeh, Tervooren: Using Flexible Trapezoidal Quad-Surfaces for Transformable Design. IASS (2021)

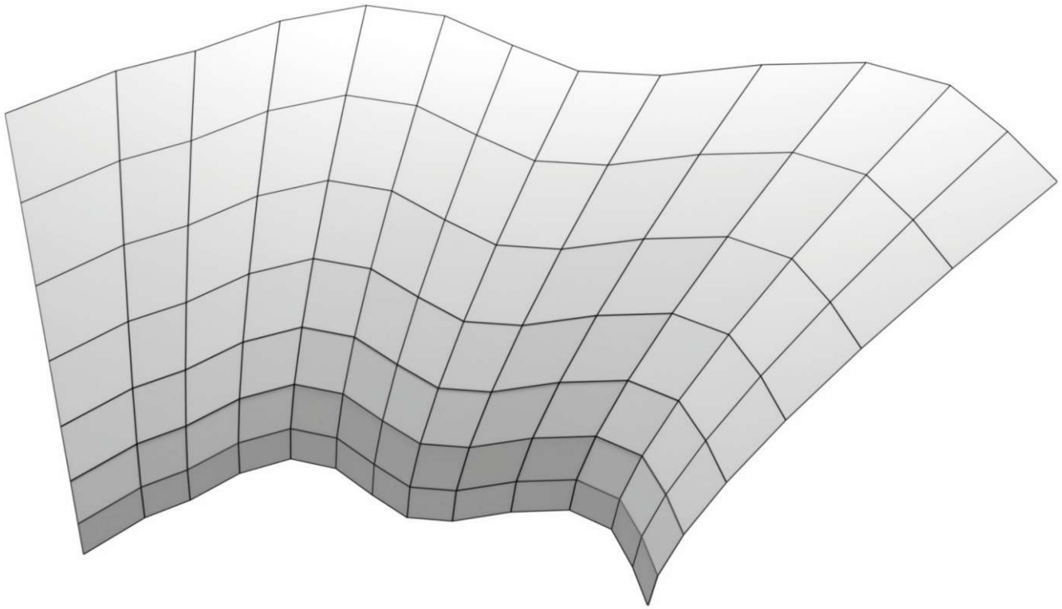
T-hedra
Parallel operation



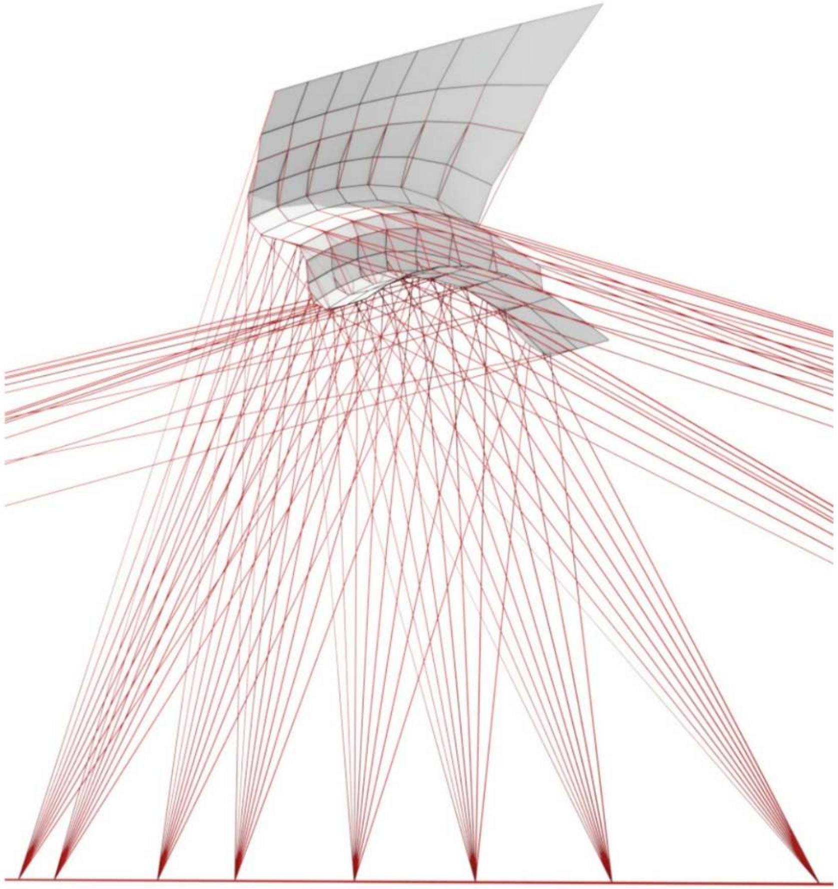
Sharifmoghaddam, Maleczek, Nawratil: Generalizing rigid-foldable tubular structures of T-hedral type.
Mechanics Research Communications (2023)

T-hedra

Projective Transformation



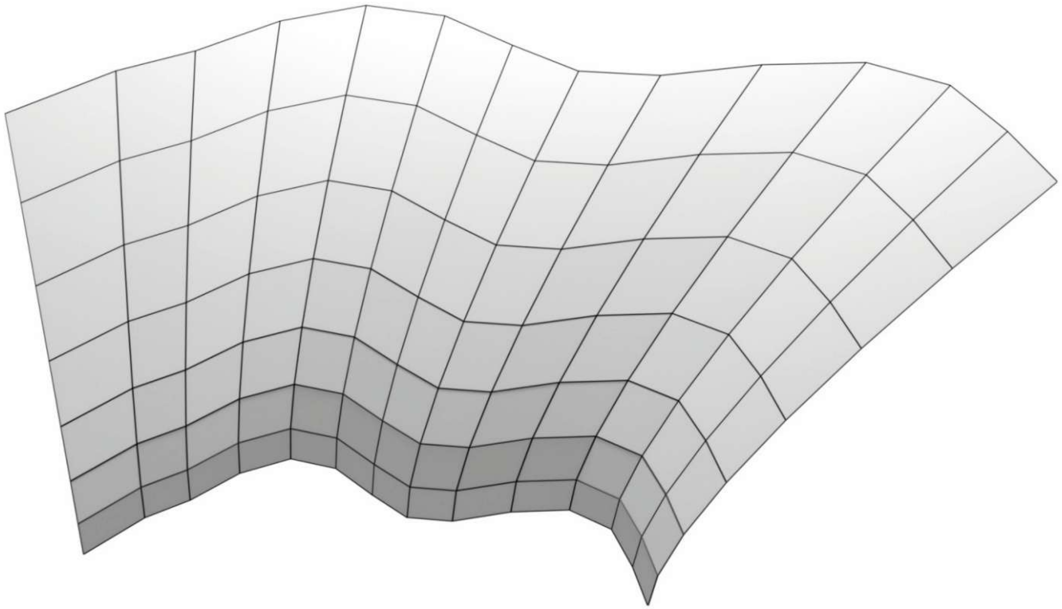
T-hedron



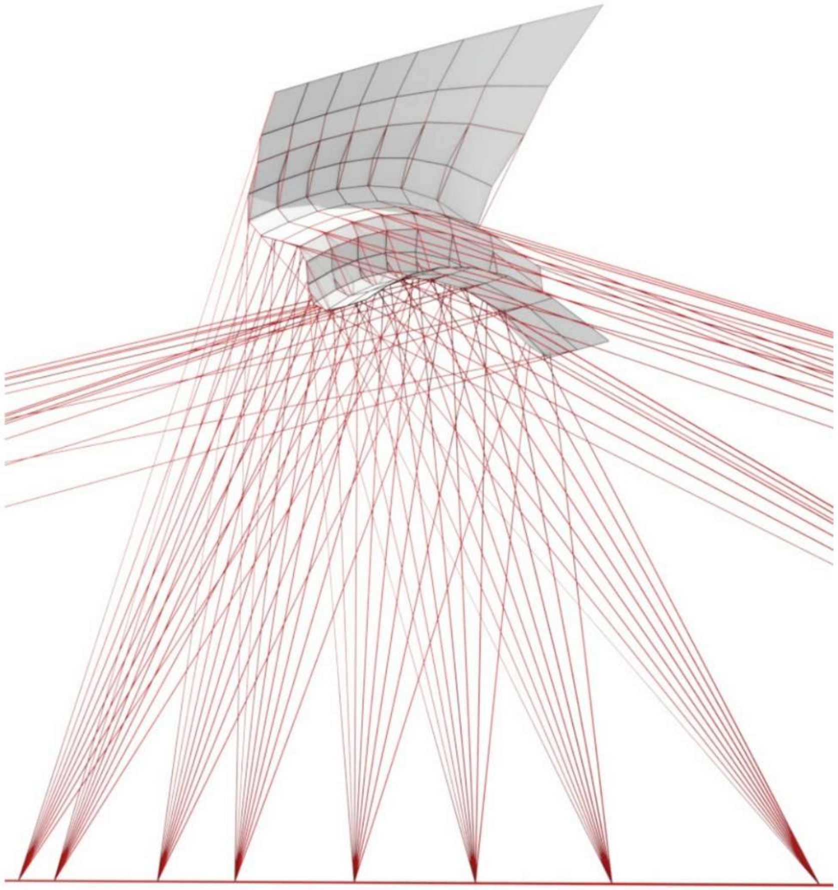
projectively transformed T-hedron

Kilian, Müller, Tervooren: Smooth and Discrete cone-Nets. Results in Mathematics (2023)

T-hedra
Projective Transformation



T-hedron
flexible



projectively transformed T-hedron
only infinitesimal flexible

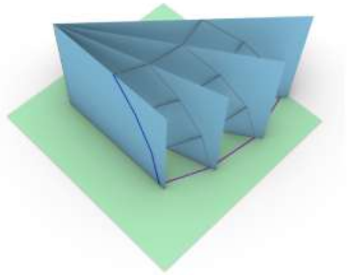
Kilian, Müller, Tervooren: Smooth and Discrete cone-Nets. Results in Mathematics (2023)

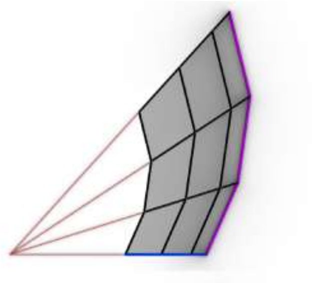
Axial C-hedra

Axial C-hedra
Review

T-hedra

Axial T-hedra can be seen as axial C-hedra





Axial C-hedra

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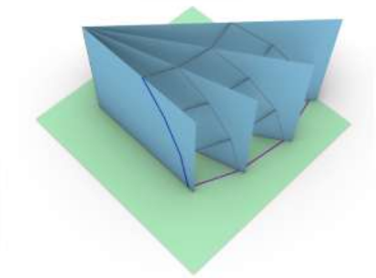
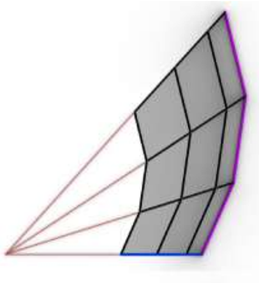
T-hedra

Axial T-hedra can be seen as axial C-hedra





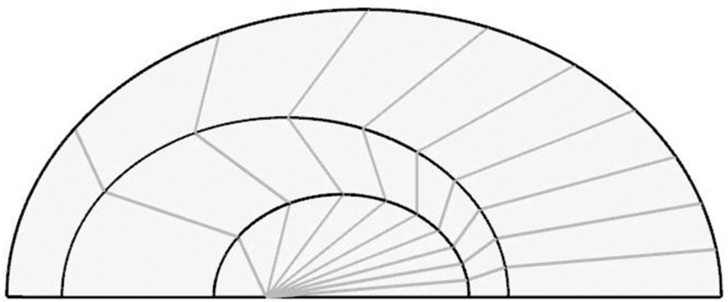
Flexible Conic Crease Patterns

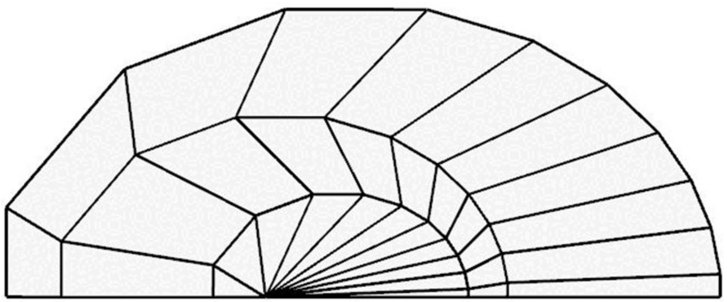
Semi-discrete axial cone-nets with rigid-ruling folding

Demaine et al: Conic crease patterns with reflecing rule lines. Origami7 (2018)

Discretization implies a family of axial C-hedra

Demaine, Mundilova, Tachi: Locally flat and rigidly foldable discretizations of conic crease patterns with reflecting rule lines. ICGG (2022)

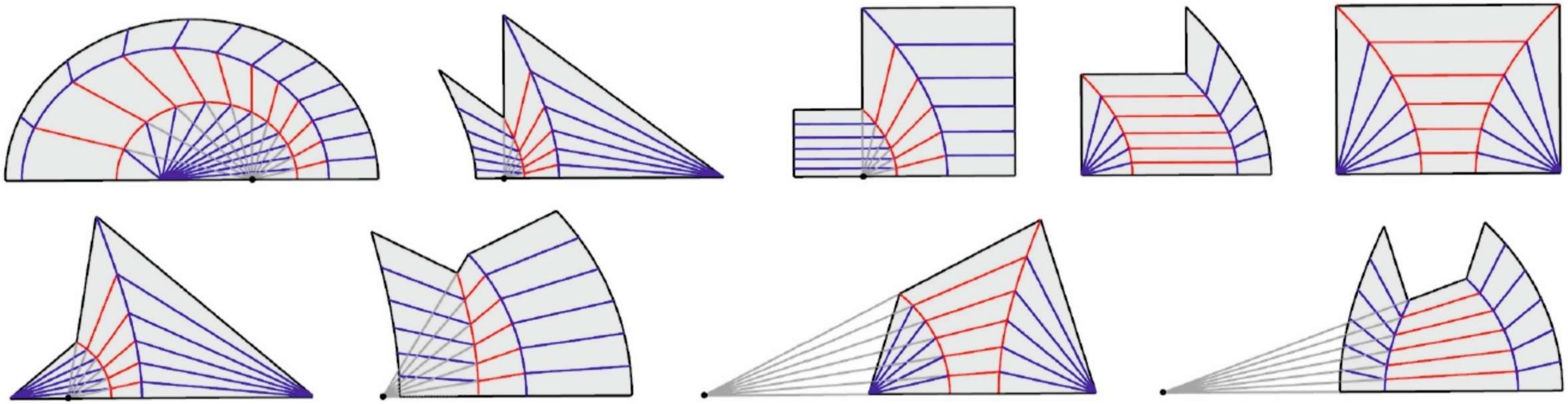




Animations by K. Mundilova

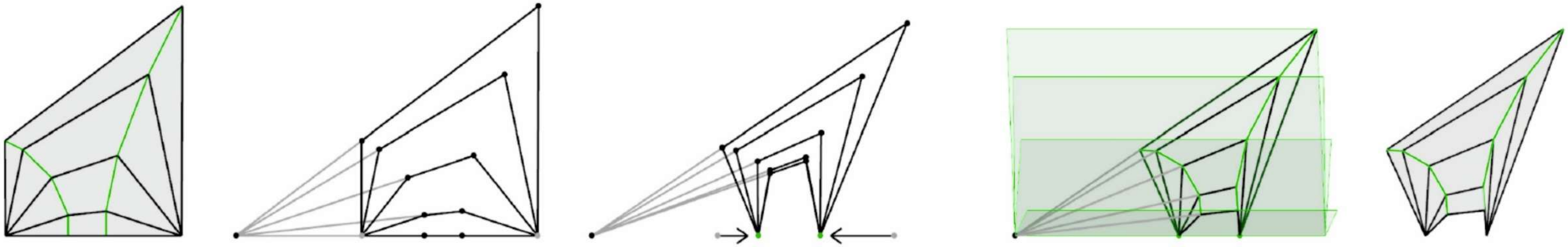
Axial C-hedra

Discrete Conic Crease Patterns



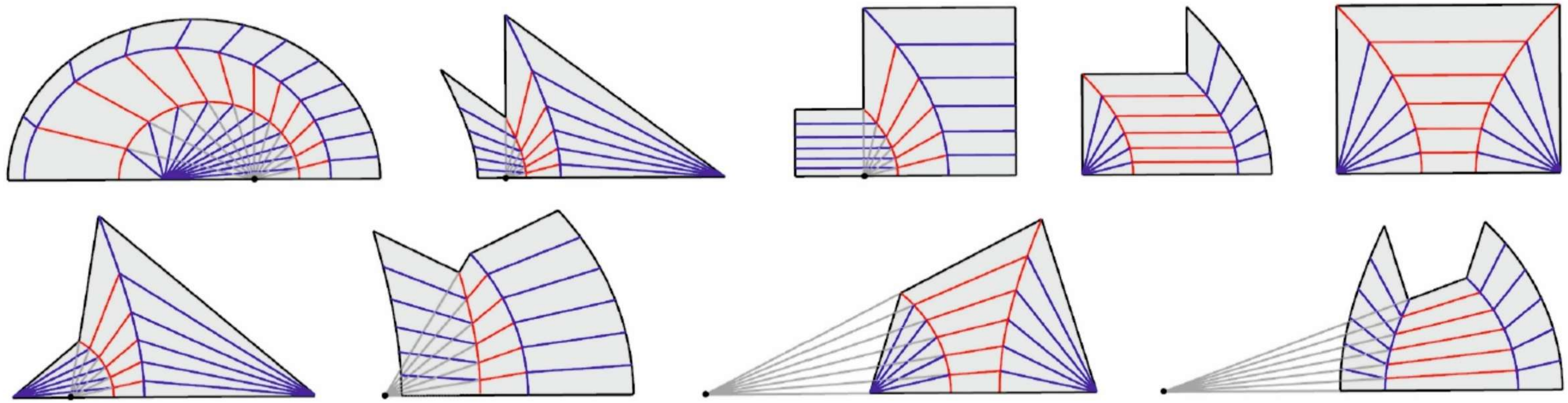
Demaine, Mundilova, Tachi: Locally flat and rigidly foldable discretizations of conic crease patterns with reflecting rule lines. ICGG (2022)

Proof of rigid-foldability was boiled down to the mobility of an overconstrained planar linkage



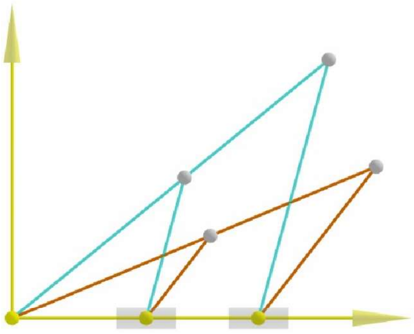
Demaine, Mundilova, Tachi: Locally flat and rigidly foldable discretizations of conic crease patterns with reflecting rule lines. ICGG (2022)

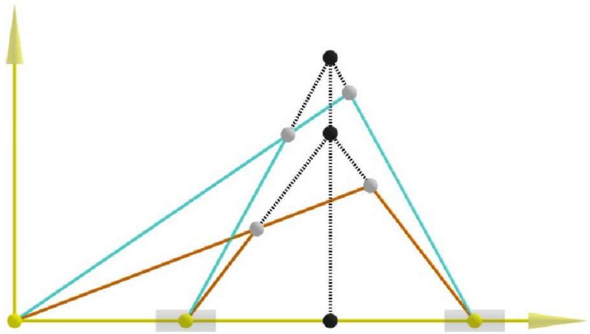
Axial C-hedra
Full Classification

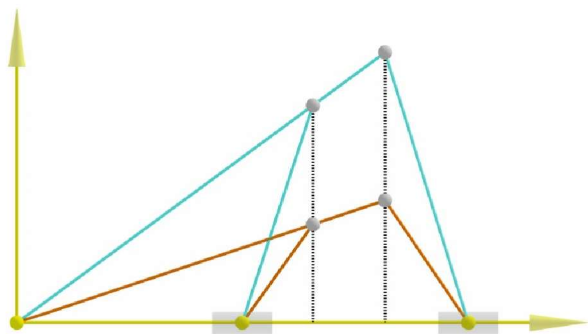


Demaine, Mundilova, Tachi: Locally flat and rigidly foldable discretizations of conic crease patterns with reflecting rule lines. ICGG (2022)

A detailed study of these planar linkages revealed following three cases:







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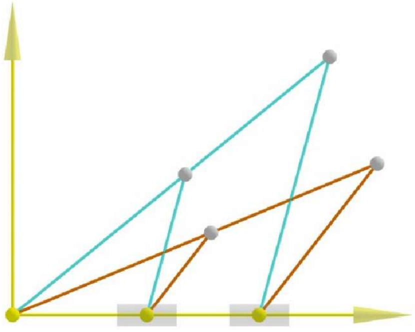
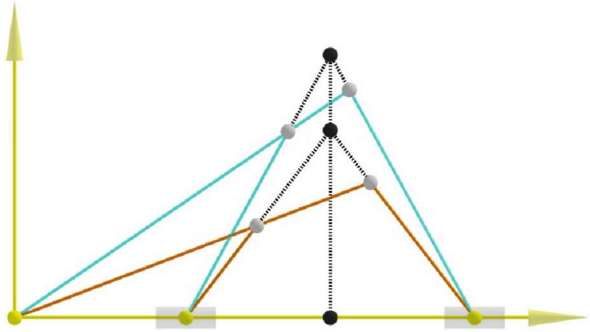
Axial C-hedra

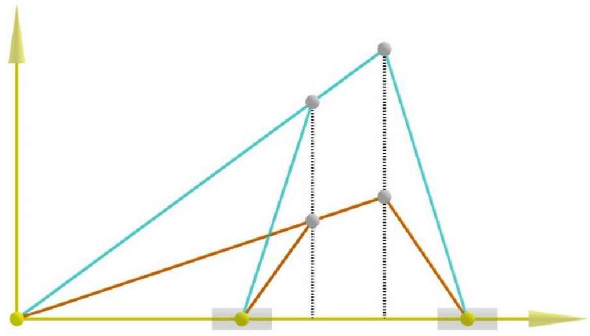
Full Classification



Central scaling

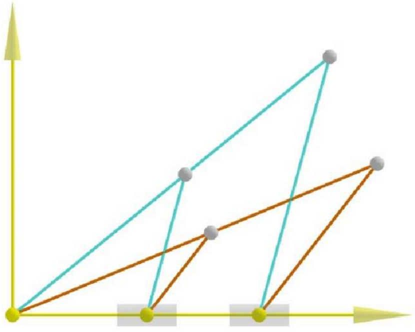


Perspective collineation

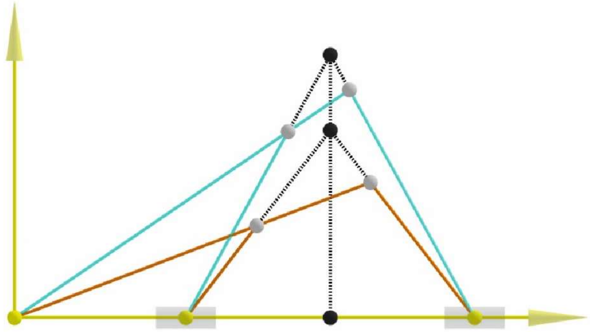


Central perspectivity

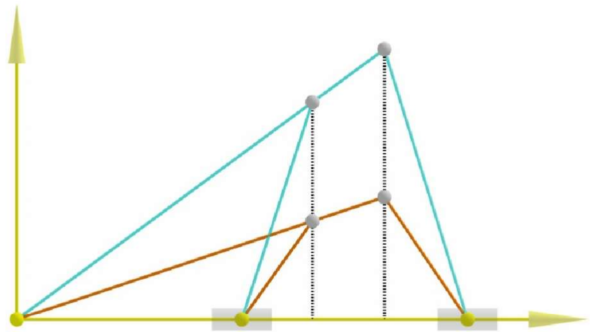
Axial C-hedra
Full Classification



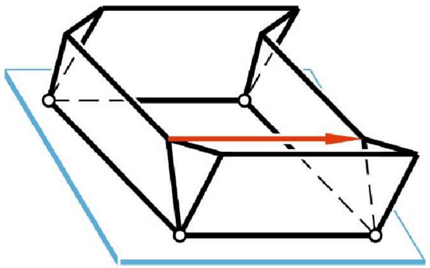
Central scaling

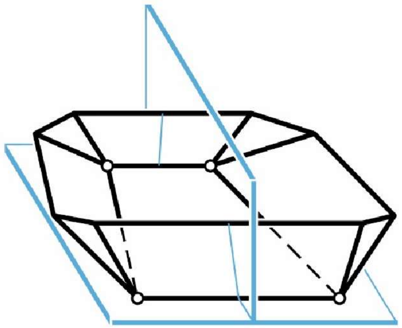


Perspective collineation



Central perspectivity

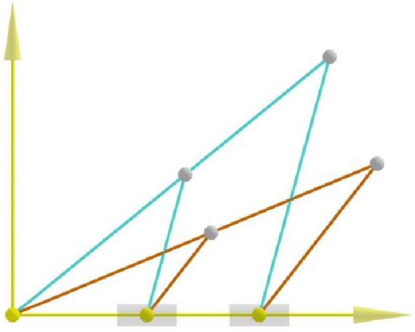




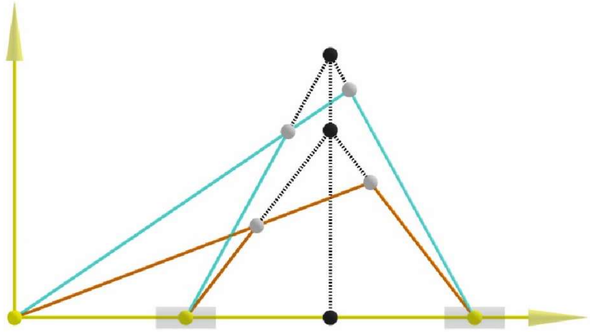
Stachel: A kinematic approach to Kokotsakis meshes. CAGD (2010)

Axial C-hedra

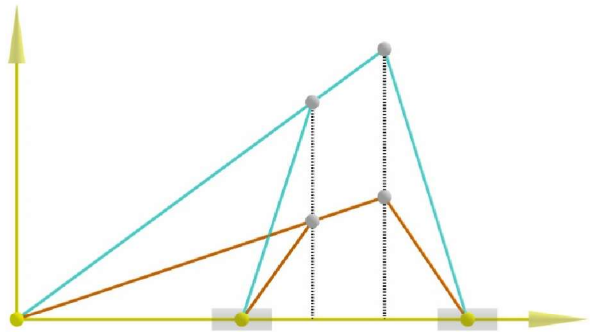
Full Classification



Central scaling



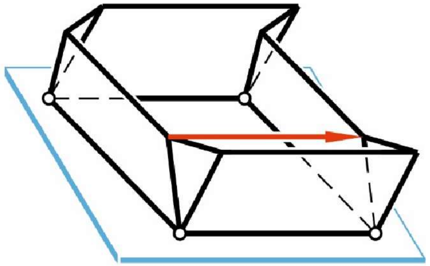
Perspective collineation

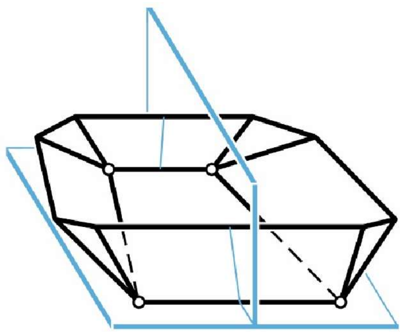


Central perspectivity

The lengths of the links fulfill the proportion (P) of the intercept theorem => axial P-hedra

=> axial T-hedra



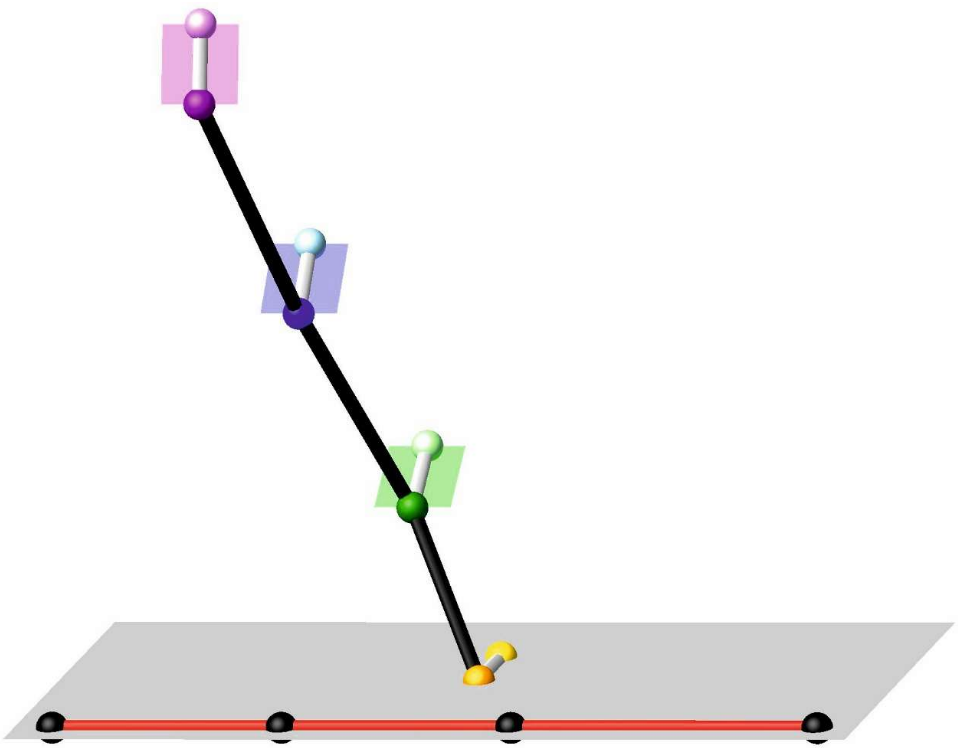


Stachel: A kinematic approach to Kokotsakis meshes. CAGD (2010)

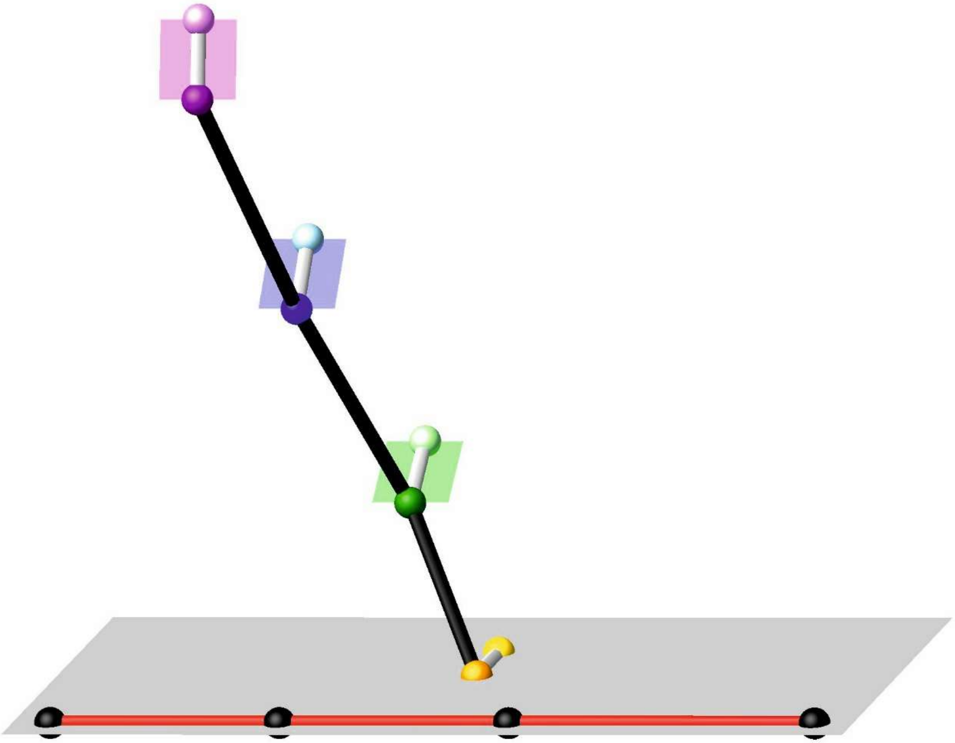
P-hedra

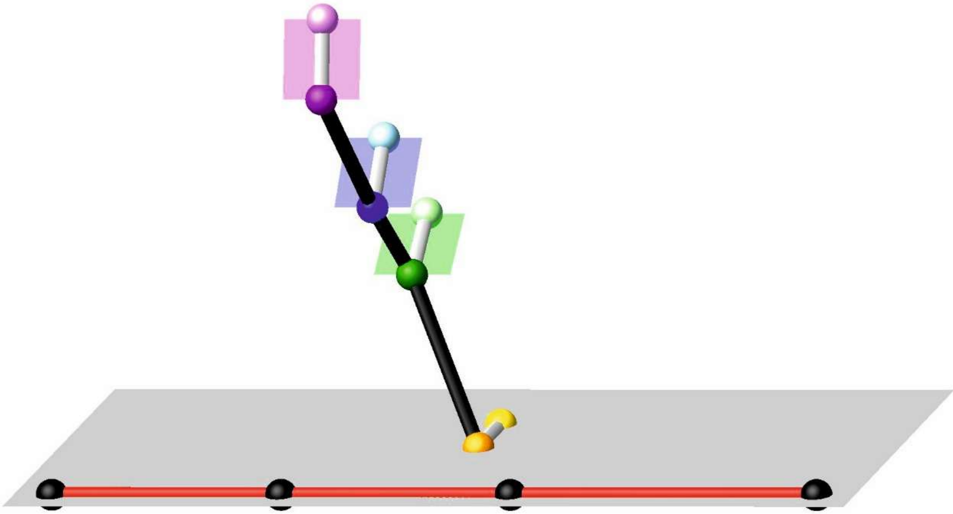
P-hedra
Construction

This class allows the construction based on three input polylines (like the T-hedral class)

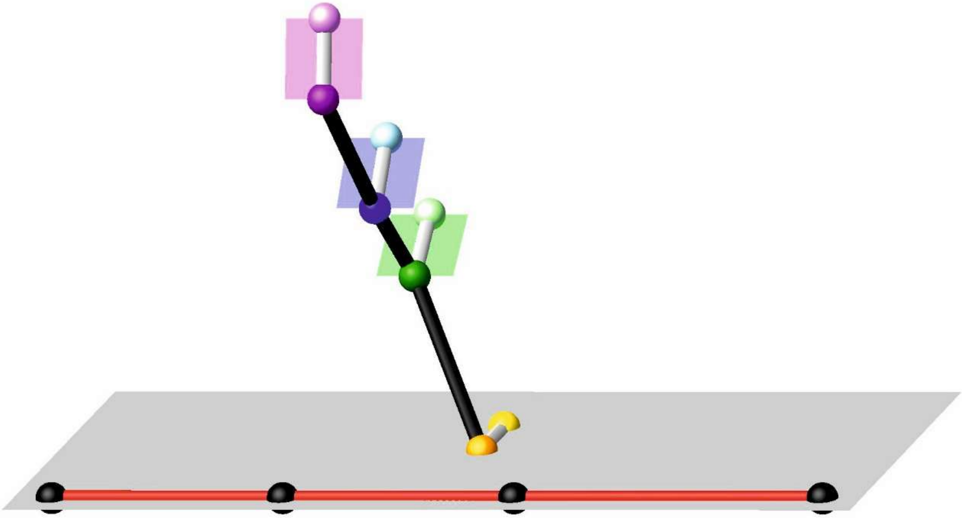


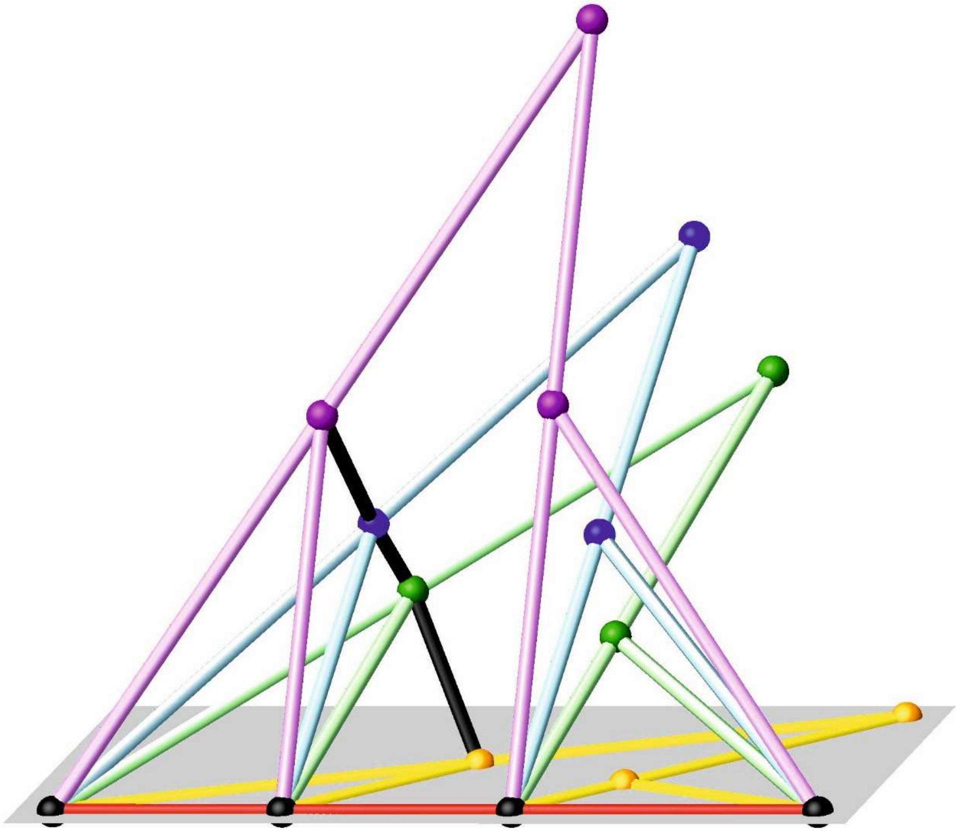
P-hedra
Construction





P-hedra
Construction





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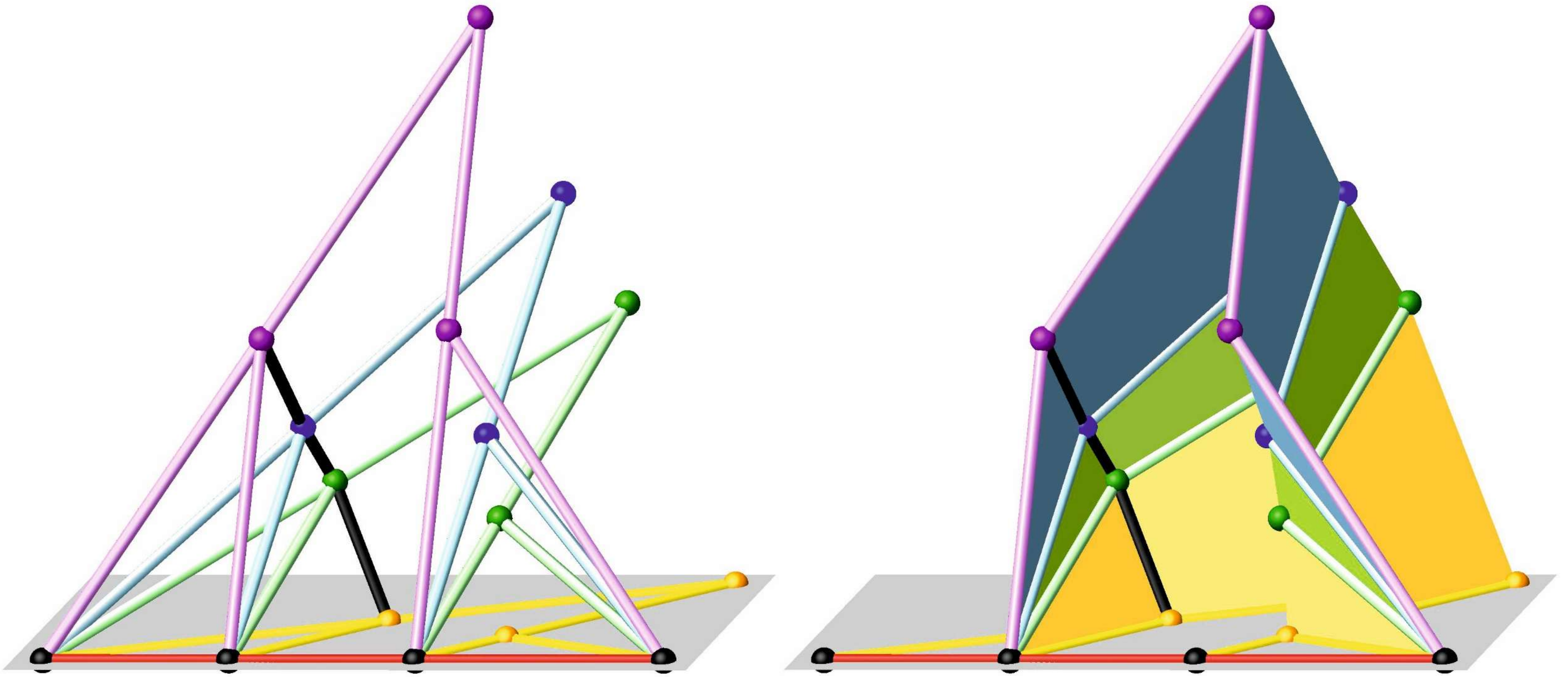
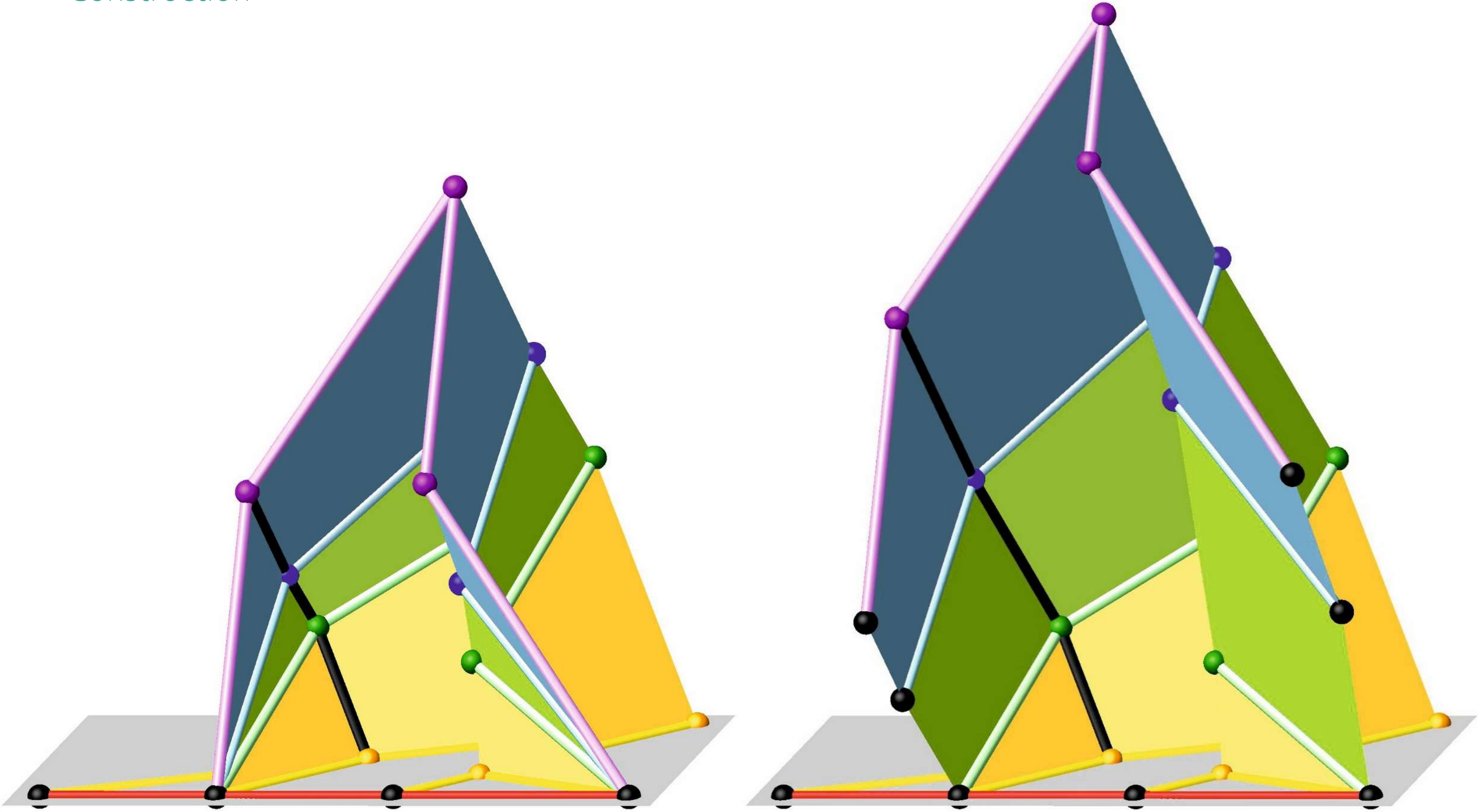
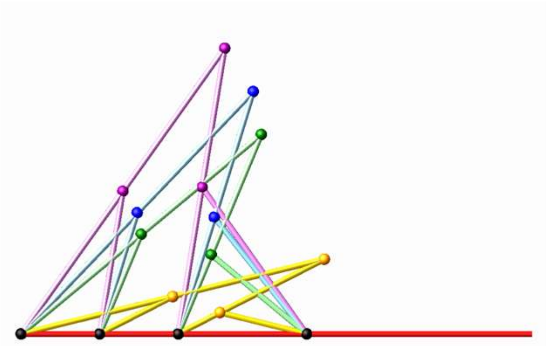
P-hedra
Construction

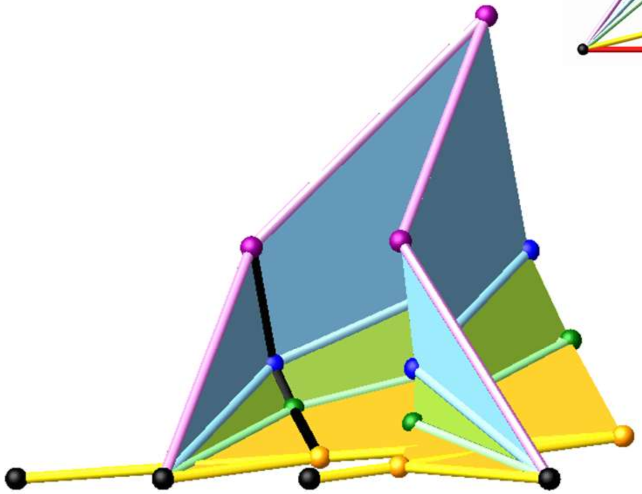


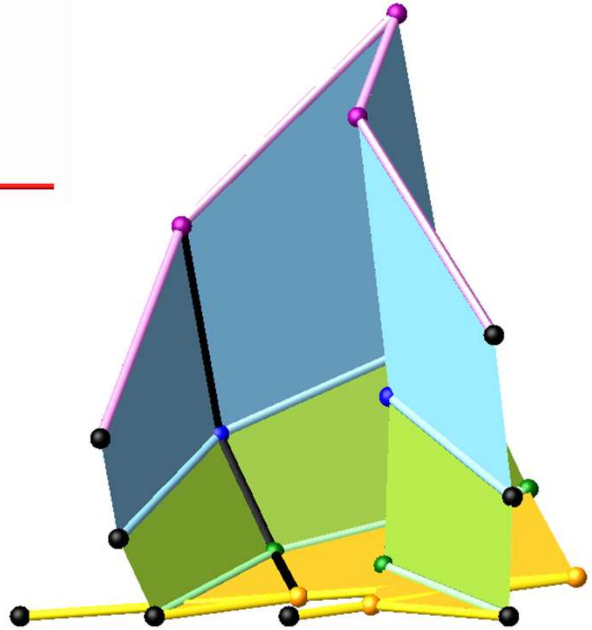
P-hedra
Construction



P-hedra
Deformation





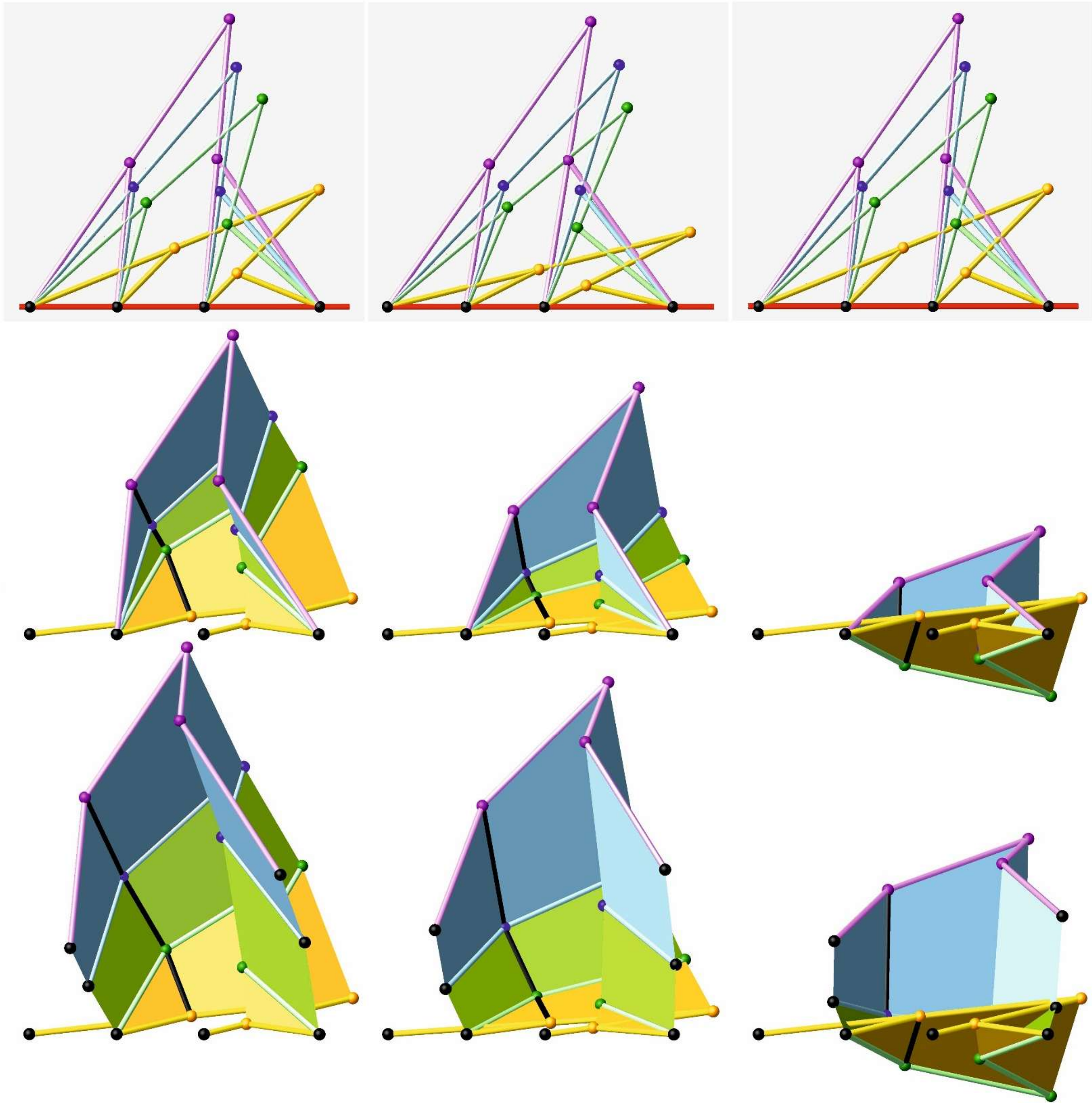


P-hedra
Flexion Limits & Bifurcation

Planar linkage

Axial P-hedron

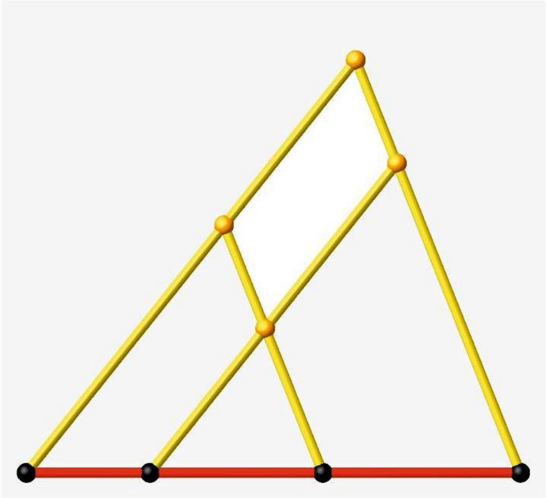
General P-hedron



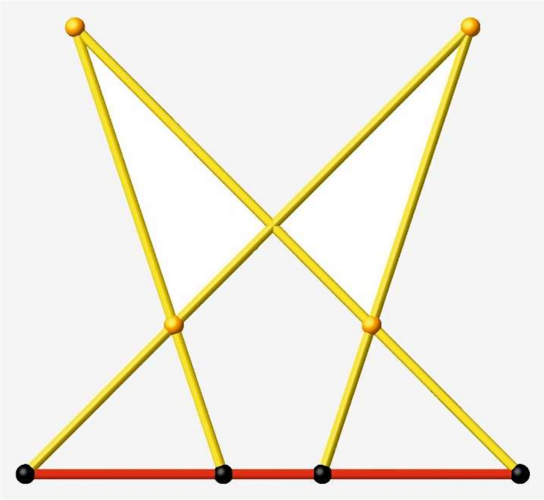
Final Remarks

Final Remarks
Extensions

P-hedral tubes with quadrilateral cross-sections



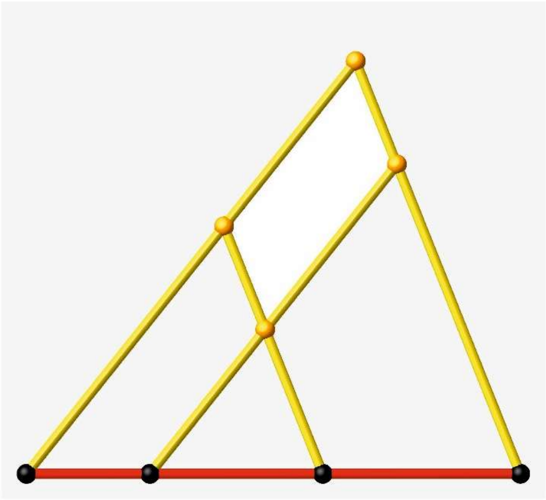
Prismatic tube with parallel-ogrammic cross-section



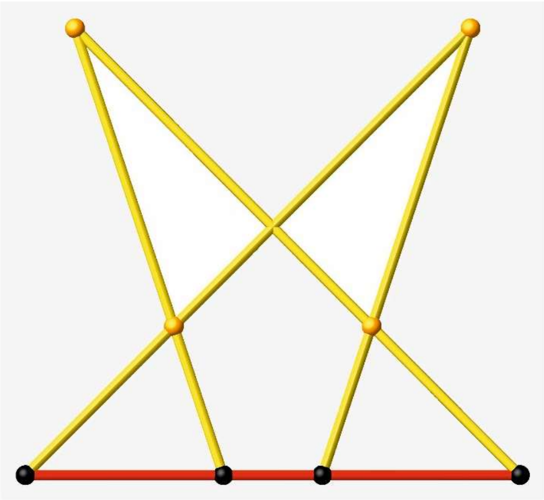
Composition of plane-symmetric Bricard octahedra

Final Remarks
Extensions

P-hedral tubes with quadrilateral cross-sections

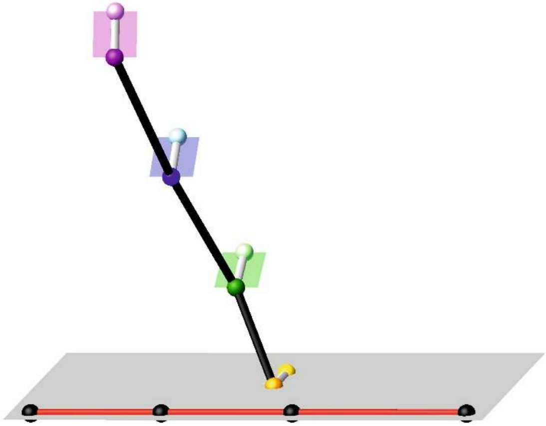


Prismatic tube with parallel-ogrammic cross-section

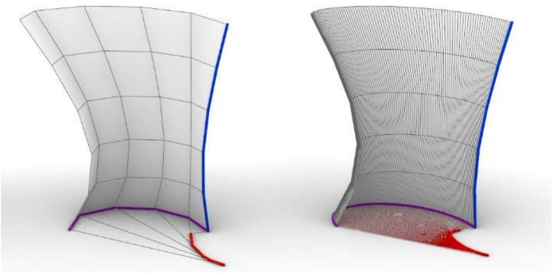


Composition of plane-symmetric Bricard octahedra

Semi-discrete P-surfaces



T-hedral analogy



Replace the trajectory polyline and direction points by a smooth curve plus vector field

Acknowledgements

This research was funded in whole or in part by the Austrian Science Fund (FWF) [grant F77: SFB "Advanced Computational Design", SP7].

Further thanks to Christian Müller, Klara Mundilova, Kiumars Sharifmoghaddam and Hellmuth Stachel for providing some of the figures and animations contained in this presentation.

Thank you!