CONGRUENT STEWART GOUGH PLATFORMS WITH NON-TRANSLATIONAL SELF-MOTIONS

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A parallel manipulator of STEWART GOUGH (SG) type consists of a platform and a base, which are connected via six spherical-prismatic-spherical legs, where all prismatic joint are active (actuated) and all spherical joints are passive (non-actuated). If we fix the prismatic joints, the SG platform is generically rigid, but under particular geometric conditions the manipulator can perform a n-dimensional motion ($n > 0$), which is called self-motion.

It is well known that each SG manipulator, where the platform is congruent with the base (= congruent SG manipulator), has a 2-dimensional translational self-motion $\mathscr{T}$ if all legs have equal (non-zero) length. As congruent SG manipulators with planar platform and planar base are only special cases of so-called planar affine/projective SG platforms, which were already studied by the author in foregoing publications, we focus on the non-planar case.

Within the lecture we first show that non-planar congruent SG manipulators cannot have further translational self-motions beside $\mathscr{T}$. In a second step we modify the well known WREN platform in a way that we obtain a non-translational self-motion of a non-planar congruent SG platform, which is not architecturally singular[1] (cf. Figure 1 and Figure 2). Note that this existence is not self-evident, as congruent SG manipulators with planar platform and planar base can only possess translational self-motions if they are not architecturally singular. Based on this example, we prove the following main theorem by means of bond theory:

Main Theorem: *A non-planar congruent SG manipulator can have a real non-translational self-motion only if the six base (resp. platform) anchor points have equal distance to a finite line* s*, i.e. they are located on at least one of the following cylinders of revolution* Φ *with axis* s*:*

- $\star$ s *is real and* Φ *is not reducible:* Φ *is a cylinder of revolution over* $\mathbb{R}$*.*
- $\star$ s *is imaginary and* Φ *is not reducible:* Φ *is a cylinder of revolution over* $\mathbb{C}$*. The real points of* Φ *are located on the 4th order intersection curve of* Φ *and its conjugate* $\overline{\Phi}$*.*
- $\star$ s *is imaginary and* Φ *is reducible: In this case* Φ *equals a pair of isotropic planes* γ_1 *and* γ_2*, which are not conjugate complex. Moreover* Φ *contains two real lines* g_i *(*$i = 1,2$*), which are the intersections of* γ_i *and its isotropic conjugate* $\overline{\gamma}_i$*.*

Moreover this condition is also sufficient for the existence of self-motions over $\mathbb{C}$*.*

Although this result is known, a complete list of all self-motions is still missing. Because from the example of the modified WREN platform it cannot be concluded that the SCHÖNFLIES self-motions with

[1]The set of non-planar congruent SG platforms, which are architecturally singular, consists of all manipulators with four collinear anchor points.

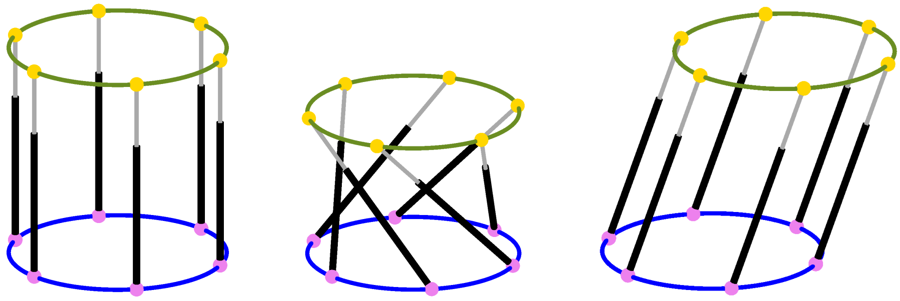

Figure 1: WREN platform: branching singularity (left) of the 2-dimensional self-motion $\mathscr{T}$ (right) and the 1-dimensional SCHÖNFLIES self-motion (center). Therefore this architecturally singular congruent SG manipulator with planar platform and planar base is kinematotropic.

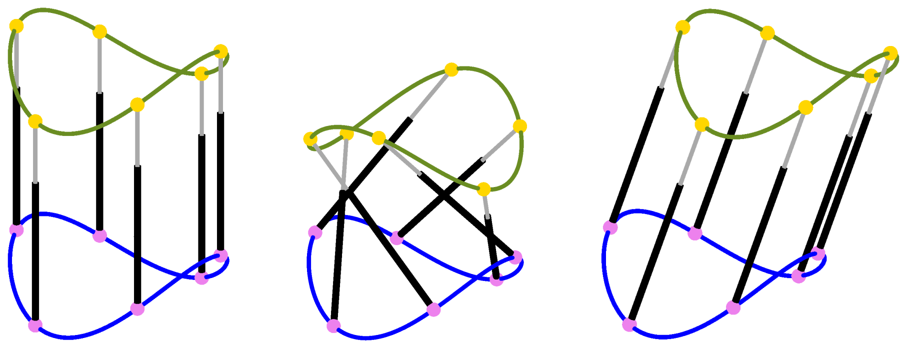

Figure 2: Modified WREN platform: branching singularity (left) of the 2-dimensional self-motion $\mathscr{T}$ (right) and the 1-dimensional SCHÖNFLIES self-motion (center). This example also shows that the property of kinematotropy is not restricted to architecturally singular manipulators.

equal leg lengths (cf. Figure 2) are the only non-translational self-motions, which can be performed by the manipulators characterized in the main theorem. A trivial counter example is the architecturally singular case (cf. Footnote 1), as the self-motions are the motions of the 5-legged manipulator, which results from the removal of one of the four legs, whose anchor points are collinear. But also the following two counter examples of non-architecturally singular manipulators are given within the talk:

- If the six points are located on two skew lines, where each line carries three pairwise distinct anchor points, then the manipulator can also perform so-called butterfly self-motions.
- If the manipulator is plane-symmetric, then it possesses a 4-parametric set of self-motions in addition, which is not known until now to the best knowledge of the author. We close the presentation by showing animations of exemplary self-motions of this new set.

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