ALTERNATIVE INTERPRETATION OF THE PLÜCKER QUADRIC'S AMBIENT SPACE AND ITS APPLICATION

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It is well-known that there exists a bijection between the set of lines of the projective 3-dimensional space P^3 and all real points of the so-called Plücker quadric Ψ. Moreover one can identify each point of the Plücker quadric's ambient space with a linear complex of lines in P^3. Within this paper we give an alternative interpretation for the points of P^5 as lines of an Euclidean 4-space E^4, which are orthogonal to a fixed direction. By using the quaternionic notation for lines, we study straight lines in P^5 which correspond in the general case to cubic 2-surfaces in E^4. We show that these surfaces are geometrically connected with circular Darboux 2-motions in E^4, as they are basic surfaces of the underlying line-symmetric motions.

Moreover we extend the obtained results to line-elements of the Euclidean 3-space E^3, which can be represented as points of a cone over Ψ sliced along the 2-dimensional generator space of ideal lines. We also study straight lines of its ambient space P^6 and show that they correspond to ruled surface strips composed of the mentioned 2-surfaces with circles on it.

Finally we present an application of this interpretation in the context of interactive design of ruled surfaces and ruled surface strips/patches based on the algorithm of De Casteljau. The more detailed procedure is as follows:

For the design of a ruled surface (strip/patch) we work in P^5 (P^6 resp. P^7). We perform a projective De Casteljau algorithm (using Farin points) in this projective space of dimension 5 (6 resp. 7). The resulting curve can be interpreted as a conoidal ruled 2-surface (strip/patch) in E^4 with respect to the director hyperplane $x_0 = 0$. By applying the orthogonal projection π in x_0-direction we obtain the desired ruled surface (strip/patch) in E^3. Moreover we label the projected lines (line-elements/line-segments) by the x_0-coordinate. In German such a map is known as "*kotierte Projektion*". In this way the user can modify very intuitively the control structure; i.e. the Farin and control lines (line-elements/line-segments) can be changed by *mouse action* and their x_0-heights by the *scroll wheel*. This user-friendly method for interactive design is illustrated in Figure 1 for line-elements.

Keywords: Plücker Quadric, Line-Element, Euclidean 4-space, Circular Darboux 2-Motion, De Casteljau Algorithm

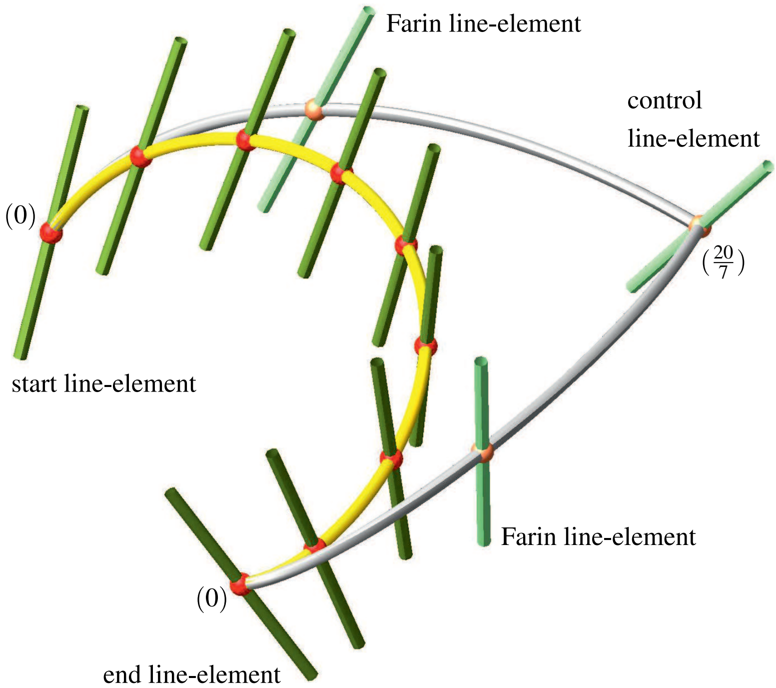

Figure 1: The illustrated quartic rational ruled surface strip corresponds to a quadratic Bezier curve in P^6. Each *Farin line-element* can only be modified within the ruled surface strip (composed of a Plücker conoid and an ellipse on it) determined by the *control line-element* and *start/end line-element*, respectively. In contrast the *control line-element* has 6 degrees of freedom. The x_0-coordinates of the control, start and end line-element are given in parentheses.