

Metriken zur Abstandsberechnung zwischen 3-RPR Konfigurationen

Metrics for distance computation between 3-RPR configurations

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Abstract

A 3-RPR manipulator is a three degree-of-freedom (dof) planar parallel manipulator (two translational and one rotational dof) consisting of a base and a moving platform linked by three legs, where each leg is composed of two passive revolute joints connected by an actuated prismatic joint (P). We present metrics for evaluating the distance between configurations \mathcal{H} and \mathcal{H}' of these planar manipulators, which can even differ in their geometry (shape of platform/base). An example for such a distance function is given [5] and reads as follows

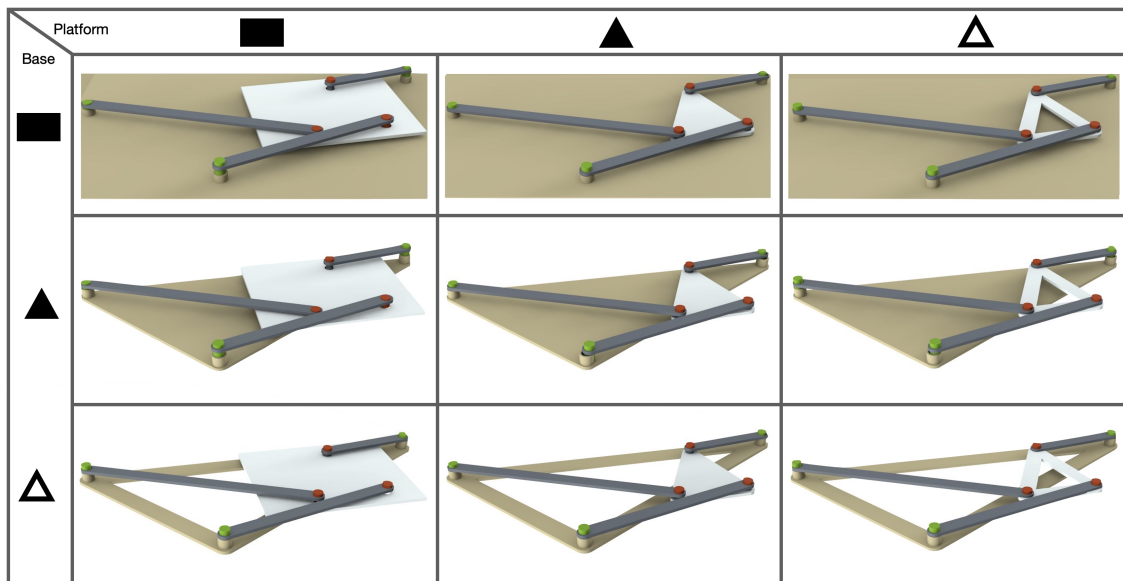
$$d(\mathcal{H}, \mathcal{H}')^2 = \frac{1}{6} \sum_{i=1}^6 \|\mathbf{k}'_i - \mathbf{k}_i\|^2, \quad (1)$$

where \mathbf{k}_i and \mathbf{k}'_i are the vectors of the six anchor points in the two configurations \mathcal{H} and \mathcal{H}' .

To do so, we consider 3-RPR manipulators as planar frameworks, which can be done in different ways, as the platform/base can be modeled as a triangular plate (\blacktriangle) or a pin-jointed triangular bar structure (\triangle). By defining the lengths of the bars and the shapes of the bodies, respectively, the inner metric of the framework is determined, which fixes, together with the combinatorial structure, the inner framework geometry. Note that there are multiple embeddings of the framework into the Euclidean plane.

Our motivation for dealing with these extrinsic metrics is twofold; they can be used to (a) compute singularity-free spheres in the embedding space (cf. paragraph B) and (b) quantify the change in shape implied by variations of the inner metric (e.g. [3]). The measurement of these variations can be done by an intrinsic metric based on the total elastic strain energy density of the framework [6], which also rely on the design of the platform/base as triangular plate or as pin-jointed triangular bar structure, respectively. Moreover, there is the additional possibility of pinning down a system (e.g. the base) thus it cannot be deformed. Therefore we ask for extrinsic metrics taking also these design options into account beside the combinatorial structure of the framework, which is for example not respected by $d(\mathcal{H}, \mathcal{H}')$ given in Eq. (1).

These goals can be achieved by constructing extrinsic metrics, which rely on the distance computation between corresponding structural components (bars or plates) of \mathcal{H} and \mathcal{H}' , which is outlined in paragraph A. In the general case, these structural components are assumed to be made of deformable material allowing not only a variation of the leg lengths but also a change of the platform/base geometry (affine deformation of platform/base). By assuming that the platform/base consists of undeformable material (indicated by \blacksquare) we obtain a generalization of the above mentioned grounding of a system, as it restricts the affine transformation to an Euclidean motion. In total we end up with nine interpretations of a 3-RPR manipulator as a framework, which are illustrated below.



A) Distance between line-segments and triangles plates

According to [4], the squared distance between two line-segments $\ell_{ij} = (\mathbf{k}_i, \mathbf{k}_j)$ and $\ell'_{ij} = (\mathbf{k}'_i, \mathbf{k}'_j)$ can be written as:

$$d(\ell_{ij}, \ell'_{ij})^2 = \frac{1}{3} \left[\|\mathbf{k}_i - \mathbf{k}'_i\|^2 + \|\mathbf{k}_j - \mathbf{k}'_j\|^2 + (\mathbf{k}_i - \mathbf{k}'_i)^T (\mathbf{k}_j - \mathbf{k}'_j) \right], \quad (2)$$

thus the distance metric equals the square root of the mean of squared distances of corresponding points over the entire line-segment. One can extend this idea to compute the squared distance between two triangles $\blacktriangle_{ijk} = (\mathbf{k}_i, \mathbf{k}_j, \mathbf{k}_k)$ and $\blacktriangle'_{ijk} = (\mathbf{k}'_i, \mathbf{k}'_j, \mathbf{k}'_k)$, which yields the following formula:

$$d(\blacktriangle_{ijk}, \blacktriangle'_{ijk})^2 = \frac{1}{6} \left[\sum_{x=i,j,k} \|\mathbf{k}_x - \mathbf{k}'_x\|^2 + (\mathbf{k}_i - \mathbf{k}'_i)^T (\mathbf{k}_k - \mathbf{k}'_k) + (\mathbf{k}_i - \mathbf{k}'_i)^T (\mathbf{k}_j - \mathbf{k}'_j) + (\mathbf{k}_k - \mathbf{k}'_k)^T (\mathbf{k}_j - \mathbf{k}'_j) \right]. \quad (3)$$

B) Application in the computation of singularity-distances

Given is a non-singular configuration of a 3-RPR manipulator and we are interested in the closest singular configuration with respect to the extrinsic metric associated with the chosen framework interpretation. From line geometry it is well-known that a 3-RPR manipulator is singular if and only if the carrier lines of the three legs intersect in a common point or are parallel. Furthermore, if the platform/base is interpreted as a pin-jointed triangular bar structure, then there exist additional singular configurations, namely those where three platform/base anchor points are collinear. Taking this into account, we compute the closest singular configuration as the global minimizer of a Lagrange optimization problem. Based on this formulation, we set up an algorithm for computing the singularity-distance along a 1-parametric motion of the manipulator, whose pipeline is as follows:

Step 0: By using a suitable homotopy continuation algorithm implemented in BERTINI [1] we compute all critical points of the Lagrangian for a randomly selected set of six complex anchor points, which is the so-called *source configuration*. Note that these generic solutions only have to be computed once (for each of the different extrinsic metrics).

Step 1: Based on the input of the manipulator's geometry and its 1-parametric motion, we get the so-called *seed configuration* by setting the motion parameter equal to a randomly chosen complex number. Now we define a user-defined homotopy from the source configuration to the seed configuration by considering the simplest possible path in \mathbb{C}^{12} (i.e. the straight line-segment spanned by these two complex configurations) and track the solutions of Step 0.

Step 2: Using PARAMOTOPY [2] and the solutions obtained from Step 1, we compute the critical points in each pose (number can be defined by the user) of the discretized 1-parametric motion. The obtained solution sets are post-processed in MAPLE, where the global minimizers can easily be filtered out.

To operate this algorithm, we developed an open-source software interface between MAPLE and BERTINI as well as PARAMOTOPY, thus that all calls can be made within MAPLE rather than switching between the systems. Using this implementation of the singularity-distance computation, we present a comparison between the nine different extrinsic metrics based on the example taken from [5, Section 3].

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