On the snappability and singularity-distance of frameworks with bars & triangular plates

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Outline

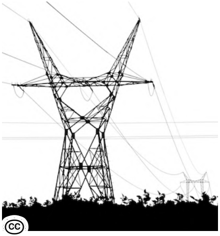
1. **Introduction**
2. **Theory**
3. **Examples**

Presentation of the publication:

GN: *On the snappability and singularity-distance of frameworks with bars and triangular plates*. Proc. of 2nd IMA Conference on Mathematics of Robotics (W. Holderbaum, J.M. Selig eds.), Springer (in press)

1. Introduction

Frameworks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 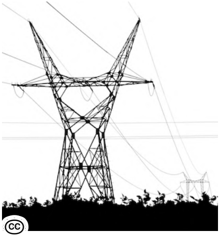 

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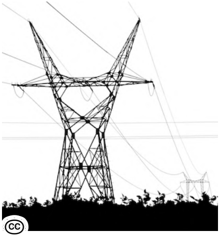

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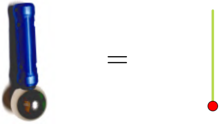
Geometric abstraction

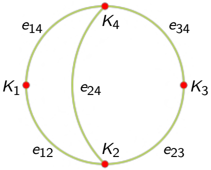
Knots are reduced to points and bars to straight line-segments.



Fundamentals

Graph G of a framework

consists of a knot-set $\mathcal{K} = \{K_1, \ldots, K_s\}$, where knots K_i and K_j are connected by edges e_{ij} ($\Rightarrow$ combinatorial structure).



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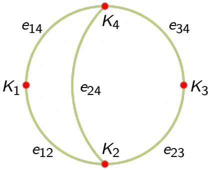
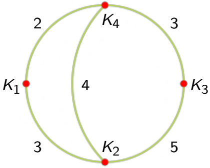
Inner geometry

is determined by assigning to each edge e_{ij} a length $L_{ij} > 0$ ($\Leftrightarrow$ fixing intrinsic metric).





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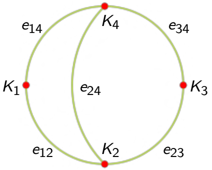

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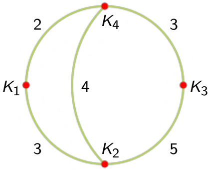
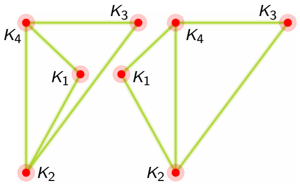


Realization $G(\mathbf{K})$

with $\mathbf{K} = (\mathbf{k}_1, \ldots, \mathbf{k}_s) \in \mathbb{R}^{sd}$ corresponds to the embedding of the framework with fixed inner geometry into the Euclidean d-space.



Algebraic approach to rigidity theory

The relation that two knots K_i and K_j are edge-connected can also be expressed algebraically as $\|\mathbf{k}_i - \mathbf{k}_j\|^2 = L_{ij}^2$.

In addition we can add 6 (for $d = 3$) or 3 (for $d = 2$) linear conditions to eliminate isometries. We end up with n algebraic conditions in $m = sd$ unknowns constituting an algebraic variety $A(c_1, \ldots, c_n)$.

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Def.: *A realization is* **rigid**

if it corresponds to an real isolated solution of $A(c_1, \ldots, c_n)$. If it is unique then we have a **global** rigidity; otherwise a **local** one.

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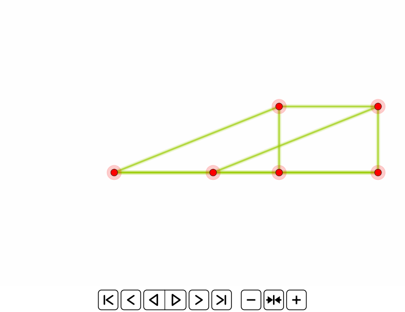
Algebraic approach to rigidity theory

Example: planar parallel mechanism



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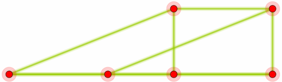

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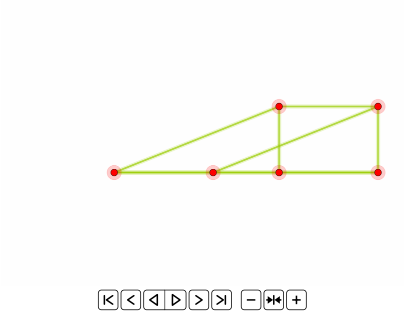

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Rigid realization

The realization is called **isostatic** (minimally rigid) if the removal of any edge constraint will make the realization flexible ($\Leftrightarrow m = n$).

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We can compute in a realization the tangent-hyperplane to each of the hypersurfaces $c_i = 0$ in $\mathbb{R}^m$ for $i = 1, \ldots, n$. The normal vectors of these tangent-hyperplanes constitute the columns of the $m \times n$ **rigidity matrix** $\mathbf{R}_{G(\mathbf{K})}$ of the realization $G(\mathbf{K})$.

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Remark: For isostatic frameworks the infinitesimal flexibility is characterized by $\det(\mathbf{R}_{G(\mathbf{K})}) = 0$, which is also known as *pure condition*.

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Open problem: The meaning of closeness!

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2. Theory

In the presentation we restrict to pin-jointed frameworks composed of bars and triangular panels but the theory can be generalized to polygonal panels and polyhedra as well; cf.

GN: *Snappability and singularity-distance of pin-jointed body-bar frameworks*. Mechanism and Machine Theory **167**:104520 (2022)

Physical Model of Deformation

Due to the fact that the elastic deformation during the process of snapping are expected to be small, we can apply **Hooke's law**; i.e.

$$\underbrace{\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}}_{\mathbf{e}} = \underbrace{\frac{1}{E} \begin{pmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{pmatrix}}_{=:\mathbf{D}(\nu)} \begin{pmatrix} \delta_x \\ \delta_y \\ \tau_{xy} \end{pmatrix}. \qquad (1)$$

- normal stress $\delta_{x/y}$ and normal strain $\varepsilon_{x/y}$ in x/y-direction
- shear stress τ_{xy} and shear strain γ_{xy} in the xy-plane
- Poisson ration ν and Young modulus E

For a bar (in x-direction) the relation reduces to $\varepsilon_x = \frac{\delta_x}{E}$.

Assumptions

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(II) all bars have the same cross-sectional area A,

(III) triangular bar structure and triangular panel are made of the same amount of material.

Strain energy according to Green-Lagrange

The deformation of the triangular panel K_i, K_j, K_k into K_i', K_j', K_k' can be represented by a 2×2 matrix $\mathbf{F}$ in terms of edge lengths.

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Based on $\mathbf{F}$ the Green-Lagrange (GL) strains can be computed as

$$\begin{pmatrix} \varepsilon_x & \gamma_{xy} \\ \gamma_{xy} & \varepsilon_y \end{pmatrix} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right).$$

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The elastic GL strain energy of the deformation is calculated as

$$U_{ijk} = V_{ijk}\tfrac{1}{2}\mathbf{e}^T\mathbf{D}^{-1}\mathbf{e} \quad \text{where } V_{ijk} \text{ denotes the panel volume.}$$

Strain energy according to Green-Lagrange

The deformation of the triangular panel K_i, K_j, K_k into K'_i, K'_j, K'_k can be represented by a 2×2 matrix $\mathbf{F}$ in terms of edge lengths.

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Analogue: GL strain energy of a deformed bar can be computed as

$$U_{ij} = \frac{EA}{8L_{ij}^3}({L'_{ij}}^2 - L_{ij}^2)^2 \quad \text{where } L'_{ij} \text{ denotes the deformed length.}$$

Total elastic strain energy density

$$u(\mathbf{L}') = \frac{\sum U_{ij} + \sum U_{ijk}}{\sum AL_{ij} + \sum A(L_{ij} + L_{ik} + L_{jk})} \quad \text{with } \mathbf{L}' = (\ldots, L'_{ij}, \ldots)$$

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Lemma 1.

$u(\mathbf{L}')$ is a fourth order polynomial with respect to the variables L'_{ij} which only appear with even powers, but it does not depend on A. Moreover, $u(\mathbf{L}')$ is positive semi-definite.

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Remark. Therefore $u(\mathbf{L}')$ can be written in matrix formulation as $u(\mathbf{Q}') = \mathbf{Q}'^T\mathbf{M}\mathbf{Q}'$ where $\mathbf{M}$ is a symmetric $(b+1)$-matrix and $\mathbf{Q}' := (1, \dots, Q'_{ij}, \dots)^T$ is composed of the b squared edge lengths $Q'_{ij} := L'^2_{ij}$ and the number 1.

Pseudometric on the space $\mathbb{R}^b$

Lemma 2.

The following function

$$d : \mathbb{R}^b \times \mathbb{R}^b \to \mathbb{R}_{\geq 0} \quad \text{with} \quad (\mathbf{L}', \mathbf{L}'') \mapsto d(\mathbf{L}', \mathbf{L}'') := \frac{|u(\mathbf{L}') - u(\mathbf{L}'')|}{E}$$

is a pseudometric on the b-dimensional space of intrinsic framework metrics given by $\mathbf{L}'$ and $\mathbf{L}''$, respectively. Moreover, the pseudometric does not depend on the choice of E.

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Proof: One can easily check the axioms for a pseudometric:

(1) $d(\mathbf{L}', \mathbf{L}'') \geq 0$

(2) $d(\mathbf{L}', \mathbf{L}') = 0$

(3) $d(\mathbf{L}', \mathbf{L}'') = d(\mathbf{L}'', \mathbf{L}')$

(4) $d(\mathbf{L}', \mathbf{L}''') \leq d(\mathbf{L}', \mathbf{L}'') + d(\mathbf{L}'', \mathbf{L}''')$

Due to Assumption I, Young's modulus E factors out of $u(\mathbf{L}')$. $\square$

$u(\mathbf{L}') \Longrightarrow u(\mathbf{K}')$ and its critical points

Theorem 1.

The critical points of the total elastic stain energy density $u(\mathbf{K}')$ of an isostatic framework correspond to realizations $G(\mathbf{K}')$ that are either undeformed or deformed and shaky.

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Proof: It is based on the characterization of shakiness in terms of self-stress. For details please see the presented paper. $\square$

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The formulation of the next theorem requires the notion of stability:

Def.: *A realization is* **stable**

if it corresponds to a local minimum of the total elastic strain energy (density) of the framework.

Connection between shakiness and snapping

Theorem 2.

If an isostatic framework snaps out of a stable realization $G(\mathbf{K})$ by applying the minimum GL strain energy needed to it, then the corresponding deformation has to pass a shaky realization $G(\mathbf{K}')$ at the maximum state of deformation.

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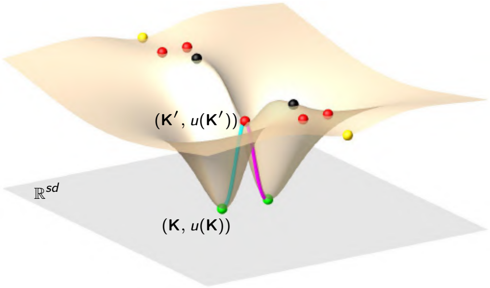
Proof: We think of u as a graph function over $\mathbb{R}^{sd}$. To get out of the valley of the local minimum $(\mathbf{K}, u(\mathbf{K}))$ with a minimum of energy needed, one has to pass a *saddle point* $(\mathbf{K}', u(\mathbf{K}'))$. $\square$



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(1) We compute the set $\mathcal{S}$ of saddle points. Let us assume that $G(\mathbf{K}') \in \mathcal{S}$ yields the minimal value for $d(\mathbf{L}, \mathbf{L}')$.

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(2) $\mathbf{Q}_t := \mathbf{Q} + t(\mathbf{Q}' - \mathbf{Q})$ with $t \in [0, 1]$ implies a path $\mathbf{L}_t$ in $\mathbb{R}^b$. Along this path the deformation energy of each bar and triangular plate is *monotonic increasing* ensuring that the minimum mechanical work needed is applied to reach $G(\mathbf{K}')$.
This results from Lemma 1, as $U_{ijk}(\mathbf{L}_t)$ as well as $U_{ij}(\mathbf{L}_t)$ are quadratic functions in t, which are at their minima for $t = 0$.

Snappability

(3) The path $\mathbf{L}_t$ corresponds to different 1-parametric deformations of realizations in $\mathbb{R}^d$. If among these a deformation $G(\mathbf{K}_t)$ with

$$G(\mathbf{K}_t)\big|_{t=0} = G(\mathbf{K}), \quad G(\mathbf{K}_t)\big|_{t=1} = G(\mathbf{K}')$$

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(4) Otherwise we redefine $\mathcal{S}$ as $\mathcal{S} \setminus \{G(\mathbf{K}')\}$ and run again the procedure. If we end up with $\mathcal{S} = \varnothing$ then we set $s(\mathbf{K}) = \infty$.

Singularity-distance

Theorem 3.

For a non-shaky realization $G(\mathbf{K})$ of an isostatic framework the singularity-distance $\varsigma(\mathbf{K})$ equals the snappability $s(\mathbf{K})$.

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For a non-shaky realization $G(\mathbf{K})$ of an isostatic framework the singularity-distance $\varsigma(\mathbf{K})$ equals the snappability $s(\mathbf{K})$.

Proof: $\varsigma(\mathbf{K}) \leq s(\mathbf{K})$ has to hold, as $G(\mathbf{K}')$ of Theorem 2 is shaky. We show that the assumption $\varsigma(\mathbf{K}) < s(\mathbf{K})$ implies a contradiction.

We denote by $G(\mathbf{K}'')$ shaky configuration implying $\varsigma(\mathbf{K}) = d(\mathbf{L}, \mathbf{L}'')$.

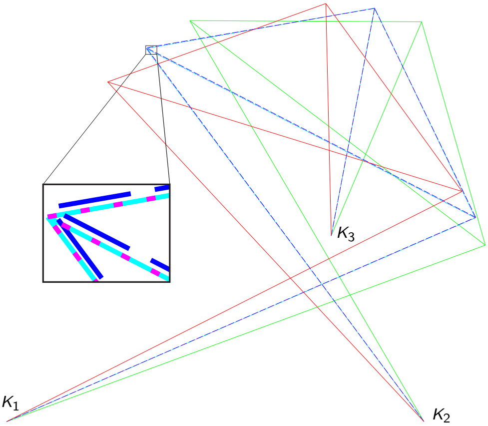
Then $\mathbf{Q}_t := \mathbf{Q} + t(\mathbf{Q}'' - \mathbf{Q})$ with $t \in [0, 1]$ corresponds to a set of 1-parametric deformations $\{G(\mathbf{K}_t^1), G(\mathbf{K}_t^2), \ldots\}$.

A subset $\mathcal{D}$ of this set has the property $G(\mathbf{K}_t^i)|_{t=1} = G(\mathbf{K}'')$ where $\#\mathcal{D} > 1$ holds as $G(\mathbf{K}'')$ is shaky. Hence the framework can snap out of $G(\mathbf{K})$ over $G(\mathbf{K}'')$ which contradicts $\varsigma(\mathbf{K}) < s(\mathbf{K})$. $\square$

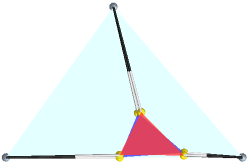
3. Examples

Planar parallel manipulator with pinned base

The snap between the green and red undeformed realization passes the shaky deformed configuration $G(\mathbf{K}')$.



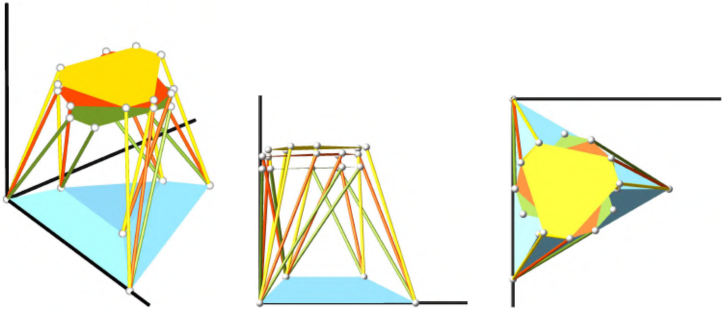
	structure	strain	tracked paths	#solutions $\in \mathbb{C}$	$\#\mathcal{S}$	$\varsigma(\mathbf{K}) = s(\mathbf{K})$
$G(\mathbf{K}')$	3 bars + 1 plate	Green-Lagrange	729	285	62	$3.2531/10^6$
$G(\mathbf{K}')$	6 bars	Green-Lagrange	729	219	58	$1.8271/10^6$
$G(\mathbf{K}')$	6 bars	Cauchy/Engineering	59 163	758	142	$1.8285/10^6$



One-parametric motion of the platform in blue and the closest singular configuration in red (6 bars, GL strain)

A. Kapilavai, GN: Comparison of extrinsic and intrinsic singularity distance measures for planar 3-RPR manipulators (in preparation)

Hexapod with undeformable base and platform



The snap between the green and yellow undeformed realization passes the shaky deformed configuration in red implying $\varsigma(\mathbf{K}) = s(\mathbf{K}) = 3.3241/10^5$.

GN: *Snappability and singularity-distance of pin-jointed body-bar frameworks*. Mechanism and Machine Theory **167**:104520 (2022)

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Thank you for your attention!