On the spin surface of RSSR mechanisms with parallel rotary axes

Georg Nawratil[1]

[1] *Vienna University of Technology, Institute of Discrete Mathematics and Geometry, Wiedner Hauptstrasse 8-10/104, Vienna, A-1040, Austria*

Due to Cayley's theorem, given by Fichter and Hunt [1], the line $s \in \Sigma$ (= moving system) spanned by the centers of the spherical joints of an RSSR linkage generates a surface of degree 8. In the special case of parallel rotary axes of the R-joints the corresponding ruled surface is only of degree 6.

Hunt [2] proved by means of algebraic connection theory that the spin surface Φ generated by the rotation of a point $X \in \Sigma \setminus \{s\}$ about s is of order $2 \times 8 = 16$ (general case) and $2 \times 6 = 12$ (special case), respectively. Moreover Hunt [3] suggested that the circularity of the spin surface is 8 which was later proved by Merlet [5].

We demonstrate that the circularity of the spin-surface for the special case is 4 instead of 6 as given in the literature (cf. Lazard and Merlet [4], Merlet [7]). As a consequence generalized TSSM manipulators (rotary axes must not be coplanar) with two parallel rotary joints can have up to 16 solutions (as the general case; cf. Merlet [6]) instead of 12. We show that this upper bound cannot be improved by constructing an example for which the maximal number of assembly modes is reached.

Moreover we list all parallel manipulators of this type where more than $4 \times 2 = 8$ points are located on the imaginary spherical circle.

References

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[†]E-mail address: nawratil@geometrie.tuwien.ac.at