```
> restart: with(plots): with(LinearAlgebra):
```

Prozeduren für Quaternionen und duale Quaternionen

Schreiben Quaternion als 4er Vektor

Prozedur für Quaternionenmultiplikation

```
> Qmult:=proc(a,b):
         simplify(<a[1]*b[1]-a[2]*b[2]-a[3]*b[3]-a[4]*b[4],
                   a[1]*b[2]+b[1]*a[2]+a[3]*b[4]-b[3]*a[4],
                   a[1]*b[3]+b[1]*a[3]+a[4]*b[2]-b[4]*a[2],
                   a[1]*b[4]+b[1]*a[4]+a[2]*b[3]-b[2]*a[3]>);
  end proc:
```

Prozedur für die konjugierte Quaternion

```
> Qkon:=proc(a):
        <a[1],-a[2],-a[3],-a[4]>;
  end proc:
```

Prozedur für die inverse Quaternion

```
> Qinv:=proc(a):
        1/Norm(a,2)^2*Qkon(a):
  end proc:
```

Duale Quaternion ist durch ein Quaternionenpaar gegeben

Prezedur für Produkt dualer Quaternionen

```
> DQmult:=proc(a,a_,b,b_):
> Qmult(a,b),VectorAdd(Qmult(a_,b),Qmult(a,b_));
> end proc:
```

Prozedur für die konjugierte duale Quaternion

```
> DQkon:=proc(a,a_):
        Qkon(a),Qkon(a_) ;
  end proc:
```

Prozedur für die dual konjugierte duale Quaternion

```
> DQdkon:=proc(a,a_):
        a,VectorScalarMultiply(a_,-1) ;
  end proc:
```

Prozedur für die sowohl konjugierte alsauch dual konjugierte duale Quaternion

```
> DQkdk:=proc(a,a_):
        Qkon(a),VectorScalarMultiply(Qkon(a_),-1) ;
  end proc:
```

Angabe des 6R Roboters mit Handgelenk in der Ausgangslage

Achsen (pi,pi_) in Ausgangslage

```
> a2:=3:b2:=1:
> a3:=3:b3:=4:x3:=3:y3:=4:z3:=7:
> a4:=2:b4:=1:x4:=7:y4:=-2:z4:=11:
> a5:=0:b5:=1:
> a6:=2:b6:=3:
```

```
> p1:=<1,0,0>:  p1:=VectorScalarMultiply(p1,1/Norm(p1,2)):P1:=<0,0,
  0>:p1_:=CrossProduct(P1,p1):
> p2:=<a2,b2,0>:p2:=VectorScalarMultiply(p2,1/Norm(p2,2)):P2:=<0,0,
  1>:p2_:=CrossProduct(P2,p2):
> p3:=<a3,b3,0>:p3:=VectorScalarMultiply(p3,1/Norm(p3,2)):P3:=<x3,
  y3,z3>:p3_:=CrossProduct(P3,p3):
> p4:=<a4,b4,0>:p4:=VectorScalarMultiply(p4,1/Norm(p4,2)):P4:=<x4,
  y4,z4>:p4_:=CrossProduct(P4,p4):
> p5:=<a5,b5,0>:p5:=VectorScalarMultiply(p5,1/Norm(p5,2)):P5:=<x4,
  y4,z4>:p5_:=CrossProduct(P5,p5):
> p6:=<a6,b6,0>:p6:=VectorScalarMultiply(p6,1/Norm(p6,2)):P6:=<x4,
  y4,z4>:p6_:=CrossProduct(P6,p6):
```

Bemerke, dass der Handgelenkspunkt die Rastkoordinaten (x4,y4,z4) in der Ausgangslage hat.

Drehungen um Achsen in dualen Quaternionen

```
> DQ1:=<cos(theta1/2),sin(theta1/2)*p1[1],sin(theta1/2)*p1[2],sin
  (theta1/2)*p1[3]>,<0,sin(theta1/2)*p1_[1],sin(theta1/2)*p1_[2],
  sin(theta1/2)*p1_[3]>:
> DQ2:=<cos(theta2/2),sin(theta2/2)*p2[1],sin(theta2/2)*p2[2],sin
  (theta2/2)*p2[3]>,<0,sin(theta2/2)*p2_[1],sin(theta2/2)*p2_[2],
  sin(theta2/2)*p2_[3]>:
> DQ3:=<cos(theta3/2),sin(theta3/2)*p3[1],sin(theta3/2)*p3[2],sin
  (theta3/2)*p3[3]>,<0,sin(theta3/2)*p3_[1],sin(theta3/2)*p3_[2],
  sin(theta3/2)*p3_[3]>:
> DQ4:=<cos(theta4/2),sin(theta4/2)*p4[1],sin(theta4/2)*p4[2],sin
  (theta4/2)*p4[3]>,<0,sin(theta4/2)*p4_[1],sin(theta4/2)*p4_[2],
  sin(theta4/2)*p4_[3]>:
> DQ5:=<cos(theta5/2),sin(theta5/2)*p5[1],sin(theta5/2)*p5[2],sin
  (theta5/2)*p5[3]>,<0,sin(theta5/2)*p5_[1],sin(theta5/2)*p5_[2],
  sin(theta5/2)*p5_[3]>:
> DQ6:=<cos(theta6/2),sin(theta6/2)*p6[1],sin(theta6/2)*p6[2],sin
  (theta6/2)*p6[3]>,<0,sin(theta6/2)*p6_[1],sin(theta6/2)*p6_[2],
```

```
sin(theta6/2)*p6_[3]>:
```

Generierung einer Endeffektorstellung

```
> Pose:=DQmult(DQmult(DQmult(DQmult(DQmult(DQ1,DQ2),DQ3),DQ4),DQ5),
  DQ6):
> Given1:=(simplify(subs(theta1=Pi/2,theta2=Pi/3,theta3=-Pi/2,
  theta4=-Pi/3,theta5=Pi/2,theta6=-Pi/2,Pose[1])));
```

$$Given1 := \left[\begin{array}{c} \frac{((25\sqrt{15}+60)\sqrt{2}-10\sqrt{15}+180)\sqrt{13}}{2600}+\frac{9}{20}+\frac{(-52\sqrt{15}+195)\sqrt{2}}{2600} \\ \frac{((26\sqrt{2}-45)\sqrt{15}-30\sqrt{2}-225)\sqrt{13}}{2600}+\frac{9}{40}+\frac{(104\sqrt{2}-65)\sqrt{15}}{2600} \\ \frac{(-45\sqrt{2}+30\sqrt{15}-240)\sqrt{13}}{2600}+\frac{3}{20}+\frac{(39\sqrt{15}+260)\sqrt{2}}{2600} \\ \frac{((-18\sqrt{2}+15)\sqrt{15}+40\sqrt{2}-75)\sqrt{13}}{2600}-\frac{3}{40}+\frac{(78\sqrt{2}-195)\sqrt{15}}{2600} \end{array}\right] \quad (1)$$

```
> Given2:=(simplify(subs(theta1=Pi/2,theta2=Pi/3,theta3=-Pi/2,
  theta4=-Pi/3,theta5=Pi/2,theta6=-Pi/2,Pose[2])));
```

$$Given2 := \left[\left[\frac{((-134\sqrt{2}+63)\sqrt{15}+1075\sqrt{2}-5265)\sqrt{13}}{5200}+\frac{(1261\sqrt{2}-1833)\sqrt{15}}{5200}-\frac{69\sqrt{2}}{100}+\frac{129}{80}\right], \left[\frac{((-317\sqrt{2}-615)\sqrt{15}+344\sqrt{2}+3345)\sqrt{13}}{5200}+\frac{(806\sqrt{2}+3081)\sqrt{15}}{5200}-\frac{213\sqrt{2}}{400}-\frac{201}{80}\right], \left[\frac{((663\sqrt{2}-659)\sqrt{15}+300\sqrt{2}-5205)\sqrt{13}}{5200}+\frac{(-702\sqrt{2}+39)\sqrt{15}}{5200}+\frac{207\sqrt{2}}{400}-\frac{57}{80}\right], \left[\frac{((-644\sqrt{2}-485)\sqrt{15}-1017\sqrt{2}-6015)\sqrt{13}}{5200}+\frac{(117\sqrt{2}-1027)\sqrt{15}}{5200}-\frac{33\sqrt{2}}{200}+\frac{57}{80}\right]\right] \quad (2)$$

```
> Given:=Given1,Given2:
```

Rückwärtskinematik

```
> DQ1n:=<c1,s1*p1[1],s1*p1[2],s1*p1[3]>,<0,s1*p1_[1],s1*p1_[2],s1*
```

```
p1_[3]>:
```

```
> DQ2n:=<c2,s2*p2[1],s2*p2[2],s2*p2[3]>,<0,s2*p2_[1],s2*p2_[2],s2*
  p2_[3]>:
```

```
> DQ3n:=<c3,s3*p3[1],s3*p3[2],s3*p3[3]>,<0,s3*p3_[1],s3*p3_[2],s3*
  p3_[3]>:
```

```
> DQ4n:=<c4,s4*p4[1],s4*p4[2],s4*p4[3]>,<0,s4*p4_[1],s4*p4_[2],s4*
  p4_[3]>:
```

```
> DQ5n:=<c5,s5*p5[1],s5*p5[2],s5*p5[3]>,<0,s5*p5_[1],s5*p5_[2],s5*
  p5_[3]>:
```

```
> DQ6n:=<c6,s6*p6[1],s6*p6[2],s6*p6[3]>,<0,s6*p6_[1],s6*p6_[2],s6*
  p6_[3]>:
```

```
> HG:=<1,0,0,0>,<0,x4,y4,z4>;
```

$$HG := \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ -2 \\ 11 \end{bmatrix} \qquad \textbf{(3)}$$

Teil 1

```
> linksHG:=DQmult(DQkon(DQ1n),Given):
```

```
> rechtsHG:=DQmult(DQkdk(Given),DQdkon(DQ1n)):
```

```
> finaleHG:=DQmult(DQmult(linksHG,HG),rechtsHG);
```

$$finaleHG := \begin{bmatrix} c1^2 + s1^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \left[\left[\begin{bmatrix} 0 \end{bmatrix} \right], \right. \qquad \textbf{(4)}$$

$$\left[\frac{1}{2600} \left(53 \left(c1^2 + s1^2\right) \left(\left(\left(\left(\sqrt{2} + \frac{355}{53} \right) \sqrt{15} - \frac{495\sqrt{2}}{53} + \frac{5064}{265} \right) \sqrt{13} + \left(\frac{808\sqrt{2}}{53} + \frac{1056}{265} \right) \sqrt{15} + \frac{240\sqrt{2}}{53} + \frac{1889}{265} \right) \right) - \frac{1}{2600} \left(53 \left(\left(\left(\left(\sqrt{2} + \frac{355}{53} \right) \sqrt{15} - \frac{495\sqrt{2}}{53} + \frac{5064}{265} \right) \sqrt{13} + \left(-\frac{856\sqrt{2}}{53} + \frac{1056}{265} \right) \sqrt{15} + \frac{240\sqrt{2}}{53} + \frac{13407}{265} \right) \left(c1^2 + s1^2\right) \right) \right) \right],$$

$$\left[\frac{1}{13000} \left(\left(\left(\left(-2454\, c1^2 - 1590\, c1\, s1 + 2454\, s1^2 \right) \sqrt{2} + 1650\, c1^2 + 1350\, c1\, s1 \right. \right. \right. \right.$$

$$-1650\,s1^{2})\sqrt{15}+(-375\,c1^{2}+14850\,c1\,s1+375\,s1^{2})\sqrt{2}-2650\,c1^{2}+19736\,c1\,s1$$

$$+2650\,s1^{2})\sqrt{13})$$

$$+\frac{((-5029\,c1^{2}-24240\,c1\,s1+5029\,s1^{2})\sqrt{2}-800\,c1^{2}-1856\,c1\,s1+800\,s1^{2})\sqrt{15}}{13000}$$

$$+\frac{(5760\,c1^{2}-7200\,c1\,s1-5760\,s1^{2})\sqrt{2}}{13000}-\frac{37\,c1^{2}}{5}+\frac{1109\,c1\,s1}{250}+\frac{37\,s1^{2}}{5}$$

$$+\frac{1}{13000}((((2454\,c1^{2}+1590\,c1\,s1-2454\,s1^{2})\sqrt{2}-1650\,c1^{2}-1350\,c1\,s1$$

$$+1650\,s1^{2})\sqrt{15}+(375\,c1^{2}-14850\,c1\,s1-375\,s1^{2})\sqrt{2}+2650\,c1^{2}-19736\,c1\,s1$$

$$-2650\,s1^{2})\sqrt{13})$$

$$+\frac{((-4773\,c1^{2}-25680\,c1\,s1+4773\,s1^{2})\sqrt{2}+800\,c1^{2}+1856\,c1\,s1-800\,s1^{2})\sqrt{15}}{13000}$$

$$+\frac{(-5760\,c1^{2}+7200\,c1\,s1+5760\,s1^{2})\sqrt{2}}{13000}\Bigg],$$

$$\Bigg[\frac{1}{13000}((((-795\,c1^{2}+4908\,c1\,s1+795\,s1^{2})\sqrt{2}+675\,c1^{2}-3300\,c1\,s1$$

$$-675\,s1^{2})\sqrt{15}+(7425\,c1^{2}+750\,c1\,s1-7425\,s1^{2})\sqrt{2}+9868\,c1^{2}+5300\,c1\,s1$$

$$-9868\,s1^{2})\sqrt{13})$$

$$+\frac{1}{13000}(((-12120\,c1^{2}+10058\,c1\,s1+12120\,s1^{2})\sqrt{2}-928\,c1^{2}+1600\,c1\,s1$$

$$+928\,s1^{2})\sqrt{15})+\frac{(-3600\,c1^{2}-11520\,c1\,s1+3600\,s1^{2})\sqrt{2}}{13000}+\frac{1109\,c1^{2}}{500}$$

$$+\frac{74\,c1\,s1}{5}-\frac{1109\,s1^{2}}{500}+\frac{1}{13000}((((795\,c1^{2}-4908\,c1\,s1-795\,s1^{2})\sqrt{2}$$

$$-675\,c1^{2}+3300\,c1\,s1+675\,s1^{2})\sqrt{15}+(-7425\,c1^{2}-750\,c1\,s1+7425\,s1^{2})\sqrt{2}$$

$$-9868\,c1^{2}-5300\,c1\,s1+9868\,s1^{2})\sqrt{13})$$

$$+\frac{1}{13000}(((-12840\,c1^{2}+9546\,c1\,s1+12840\,s1^{2})\sqrt{2}+928\,c1^{2}-1600\,c1\,s1$$

$$-928\,s1^{2})\sqrt{15})+\frac{(3600\,c1^{2}+11520\,c1\,s1-3600\,s1^{2})\sqrt{2}}{13000}\Bigg]\Bigg]$$

```
> links:=DQmult(DQ2n,DQ3n):
> rechts:=DQmult(DQkdk(DQ3n),DQkdk(DQ2n)):
> finale:=DQmult(DQmult(links,HG),rechts);
```

$$finale := \left[\begin{array}{c} (c3^2+s3^2)\,(c2^2+s2^2) \\ 0 \\ 0 \\ 0 \end{array}\right], \left[\left[0\right], \left[\frac{21\, c2\, s2 \left(c3^2 - \frac{136}{105}\, c3\, s3 - \frac{3}{7}\, s3^2\right)\sqrt{10}}{10} + \frac{(1750\, c3^2 + 3000\, c3\, s3 - 970\, s3^2)\, c2^2}{250} + \frac{22\, s2^2 \left(c3^2 + \frac{21}{22}\, c3\, s3 + \frac{37}{275}\, s3^2\right)}{5} - \frac{c2\, s2\,(c3^2 - 13\, s3^2)\sqrt{10}}{10} - \frac{28\left(c2^2 + \frac{7\, s2^2}{20}\right) c3\, s3}{5}\right], \left[-\frac{63\, c2\, s2\left(c3^2 - \frac{136}{105}\, c3\, s3 - \frac{3}{7}\, s3^2\right)\sqrt{10}}{10} + \frac{(-500\, c3^2 - 2250\, c3\, s3 + 1540\, s3^2)\, c2^2}{250} + \frac{29\, s2^2\left(c3^2 + \frac{72}{29}\, c3\, s3 - \frac{907}{725}\, s3^2\right)}{5} + \frac{3\, c2\, s2\,(c3^2 - 13\, s3^2)\sqrt{10}}{10} + \frac{21\, c3\, s3\left(c2^2 - \frac{8\, s2^2}{5}\right)}{5}\right], \left[-\frac{13\, c2\, s2\left(c3^2 + 3\, c3\, s3 - \frac{43}{25}\, s3^2\right)\sqrt{10}}{5} + \frac{(1375\, c3^2 - 1700\, c3\, s3 - 500\, s3^2)\, c2^2}{125} - 10\, s2^2\left(c3^2 - \frac{34}{25}\, c3\, s3 - \frac{1}{2}\, s3^2\right) + \frac{91\, c2\, c3\, s2\sqrt{10}\, s3}{25} - 6\, s2^2\, s3^2 + 7\, c2^2\, s3^2 + s2^2\, c3^2\right]\right] \quad (5)$$

```
> lang:=factor(Norm(finale[2],2,conjugate=false)^2);
```

$$lang := -\frac{1}{125}\left(2\left(325\, c2\, c3^2\, s2\sqrt{10} + 520\, c2\, c3\, s2\sqrt{10}\, s3 - 559\, s2\sqrt{10}\, s3^2\, c2 - 10875\, c2^2\, c3^2 + 11900\, c2^2\, c3\, s3 - 3875\, c2^2\, s3^2 - 8375\, s2^2\, c3^2 + 8500\, s2^2\, s3\, c3 - 3375\, s2^2\, s3^2\right)(c2^2+s2^2)\,(c3^2+s3^2)\right) \quad (6)$$

```
> langHG:=factor(Norm(finaleHG[2],2,conjugate=false)^2);
```

$$langHG := \frac{\left(377\sqrt{2}\,\sqrt{15} + 50100\right)\left(c1^2+s1^2\right)^2}{250} \quad (7)$$

```
> eq1:=simplify(numer(finaleHG[2][2]-finale[2][2]));
```

$$eq1 := \left(\left(320\, c1^2 + 320\, s1^2\right)\sqrt{3} - 1000\, s2\left(c3^2 - \frac{34}{25}\, c3\, s3 + \frac{1}{5}\, s3^2\right) c2\right)\sqrt{10} + \left(-3500\, c3^2 - 3200\, c3\, s3 + 1940\, s3^2\right) c2^2 - 2200\, s2^2\, c3^2 - 1120\, s2^2\, s3\, c3 - 296\, s2^2\, s3^2 - 443\, c1^2 - 443\, s1^2 \quad \textbf{(8)}$$

```
> eq2:=simplify(numer(finaleHG[2][3]-finale[2][3]));
```

$$eq2 := \left(\left(-377\, c1^2 - 1920\, c1\, s1 + 377\, s1^2\right)\sqrt{3} + 3000\, s2\left(c3^2 - \frac{34}{25}\, c3\, s3 + \frac{1}{5}\, s3^2\right) c2\right)\sqrt{10} + \left(1000\, c3^2 + 2400\, c3\, s3 - 3080\, s3^2\right) c2^2 - 2900\, s2^2\, c3^2 - 3840\, s2^2\, s3\, c3 + 3628\, s2^2\, s3^2 - 3700\, c1^2 + 2218\, c1\, s1 + 3700\, s1^2 \quad \textbf{(9)}$$

```
> eq3:=simplify(numer(finaleHG[2][4]-finale[2][4]));
```

$$eq3 := \left(\left(-960\, c1^2 + 754\, c1\, s1 + 960\, s1^2\right)\sqrt{3} + 1300\, s2\, c2\left(c3^2 + \frac{8}{5}\, c3\, s3 - \frac{43}{25}\, s3^2\right)\right)\sqrt{10} + \left(-5500\, c3^2 + 6800\, c3\, s3 - 1500\, s3^2\right) c2^2 + 4500\, s2^2\, c3^2 - 6800\, s2^2\, s3\, c3 + 500\, s2^2\, s3^2 + 1109\, c1^2 + 7400\, c1\, s1 - 1109\, s1^2 \quad \textbf{(10)}$$

```
> eq4:=simplify(numer(langHG-lang));
```

$$eq4 := \left(377\left(c1^2 + s1^2\right)^2\sqrt{3} + 1300\, s2\left(c3^2 + s3^2\right) c2\left(c3^2 + \frac{8}{5}\, c3\, s3 - \frac{43}{25}\, s3^2\right)\left(c2^2 + s2^2\right)\right)\sqrt{10} - 43500\left(c3^2 - \frac{476}{435}\, c3\, s3 + \frac{31}{87}\, s3^2\right)\left(c3^2 + s3^2\right) c2^4 - 77000\, s2^2\left(c3^2 + s3^2\right)\left(c3^2 - \frac{408}{385}\, c3\, s3 + \frac{29}{77}\, s3^2\right) c2^2 - 33500\, s2^4\, c3^4 + 34000\, s2^4\, c3^3\, s3 - 47000\, s2^4\, c3^2\, s3^2 + 34000\, s2^4\, s3^3\, c3 - 13500\, s2^4\, s3^4 + 50100\left(c1^2 + s1^2\right)^2 \quad \textbf{(11)}$$

```
> s3:=2*t3/(1+t3^2):c3:=(1-t3^2)/(1+t3^2):
```

```
> yes1:=numer(simplify(eq1,{s2^2+c2^2=1,s1^2+c1^2=1}));
```

$$yes1 := -1000\sqrt{10}\, c2\, s2\, t3^4 + 320\sqrt{3}\sqrt{10}\, t3^4 - 2720\sqrt{10}\, c2\, s2\, t3^3 - 1300\, c2^2\, t3^4 + 1200\sqrt{10}\, c2\, s2\, t3^2 + 4160\, c2^2\, t3^3 + 640\sqrt{3}\sqrt{10}\, t3^2 + 2720\sqrt{10}\, c2\, s2\, t3 + 11544\, c2^2\, t3^2 - 2643\, t3^4 - 1000\, c2\, s2\sqrt{10} - 4160\, c2^2\, t3 + 2240\, t3^3 + 320\sqrt{10}\sqrt{3} - 1300\, c2^2 + 2330\, t3^2 - 2240\, t3 - 2643 \quad \textbf{(12)}$$

```
> yes2:=numer(simplify(eq2)):
> yes3:=numer(simplify(eq3)):
> yes4:=numer(simplify(eq4,{s2^2+c2^2=1,s1^2+c1^2=1}));
```

$$yes4 := 1300\sqrt{10}\, c2\, s2\, t3^4 + 377\sqrt{3}\sqrt{10}\, t3^4 - 4160\sqrt{10}\, c2\, s2\, t3^3 - 10000\, c2^2\, t3^4 - 11544\sqrt{10}\, c2\, s2\, t3^2 - 27200\, c2^2\, t3^3 + 754\sqrt{3}\sqrt{10}\, t3^2 + 4160\sqrt{10}\, c2\, s2\, t3 + 12000\, c2^2\, t3^2 + 16600\, t3^4 + 1300\, c2\, s2\sqrt{10} + 27200\, c2^2\, t3 - 68000\, t3^3 + 377\sqrt{10}\sqrt{3} - 10000\, c2^2 + 113200\, t3^2 + 68000\, t3 + 16600 \quad \textbf{(13)}$$

```
> degree(yes1,t1);degree(yes4,t1);
```
$$0$$
$$0 \quad (14)$$

```
> factor(simplify(resultant(yes1,yes4,t3)));
```
$$63468627558400 \left(3151200 \sqrt{3} \sqrt{10}\, c2^6 + 3151200 \sqrt{3} \sqrt{10}\, c2^4 s2^2 + 777400000\, c2^8 + 1554800000\, c2^6 s2^2 + 777400000\, c2^4 s2^4 + 1158799200 \sqrt{10}\, c2^5 s2 + 1158799200 \sqrt{10}\, c2^3 s2^3 + 1474356000 \sqrt{3}\, c2^5 s2 + 1474356000 \sqrt{3}\, c2^3 s2^3 - 720912720 \sqrt{3} \sqrt{10}\, c2^4 - 1440073440 \sqrt{3} \sqrt{10}\, c2^2 s2^2 + 22506484000\, c2^6 + 22506484000\, c2^4 s2^2 + 1019375760 \sqrt{10}\, c2^3 s2 + 8555528436 \sqrt{3}\, c2^3 s2 + 1083882240 \sqrt{3} \sqrt{10}\, c2^2 + 12599014694\, c2^4 - 75603813662\, s2^2 c2^2 - 14932048560\, c2\, s2 \sqrt{10} + 28854498504 \sqrt{3}\, c2\, s2 + 3185447280 \sqrt{10} \sqrt{3} - 15455396368\, c2^2 - 21699552051\right)^2 \quad (15)$$

```
> T:=-25616084000*s2^6+777400000*s2^8-1272049725-80371685020*
  s2^2+3551568000*2^(1/2)*3^(1/2)*5^(1/2)+1158799200*c2*s2^5*5^
  (1/2)*2^(1/2)-711459120*s2^4*2^(1/2)*3^(1/2)*5^(1/2)+1474356000*
  c2*s2^5*3^(1/2)+1474356000*c2^3*s2^3*3^(1/2)-11504240436*c2*s2^3*
  3^(1/2)+38884382940*c2*s2*3^(1/2)-12753873600*c2*s2*5^(1/2)*2^
  (1/2)-51542529662*c2^2*s2^2+84782866694*s2^4+1158799200*c2^3*
  s2^3*5^(1/2)*2^(1/2)-1436922240*c2^2*s2^2*5^(1/2)*2^(1/2)*3^(1/2)
  -3151200*c2^2*s2^4*5^(1/2)*2^(1/2)*3^(1/2)-3336974160*c2*s2^3*5^
  (1/2)*2^(1/2)+348489600*2^(1/2)*3^(1/2)*5^(1/2)*s2^2-3151200*
  s2^6*2^(1/2)*3^(1/2)*5^(1/2)+777400000*c2^4*s2^4-25616084000*
  c2^2*s2^4+1554800000*c2^2*s2^6:
```

```
> degree(T,c2);degree(T,s2);
```
$$4$$
$$8 \quad (16)$$

```
> hilf2:=c2^2+s2^2-1;
```
$$hilf2 := c2^2 + s2^2 - 1 \quad (17)$$

```
> weg:=numer(factor(resultant(T,hilf2,s2)));
```
$$weg := -9 \left(246801297727191511 + 39496537186164740 \sqrt{30}\right) \left(4\, c2^2 - 3\right) \left(-870377581802546524522262810187710657424\, c2^6 + 789133920246289856263684218383558280 0\, c2^4 \sqrt{30} + 6756103268379993365326874580392106351 6\, c2^4 + 100250161596696741730397215170942144 00\, c2^2 \sqrt{30} + 124313624406328887767692460150077966 40\, c2^2 - 137708450756800706910047248006662347 00 \sqrt{30} + 769338745420152837349190322981600728 93\right) \quad (18)$$

```
> # explizit loesbar, da nur gerade exponenten
```

```
> Digits:=20:
```

```
> los:=solve(evalf(weg),c2);
```

$los := 0.8660254038, -0.8660254038, -0.9553704535, 0.9553704535, -0.008467518621 - 0.2085542404\,I, 0.008467518621 + 0.2085542404\,I, -0.008467518621 + 0.2085542404\,I, 0.008467518621 - 0.2085542404\,I$ **(19)**

```
> c2:=solve(-3^(1/2)+2*c2,c2);
```

$$c2 := \frac{\sqrt{3}}{2}$$ **(20)**

```
> evalf(2*arccos(c2)*180/Pi);
```

$$60.$$ **(21)**

```
> factor(T);
```

$$-\frac{1}{2}\left((-1+2\,s2)\left(-777400000\,s2^7 + 3151200\,s2^5\sqrt{30} - 388700000\,s2^6 - 577824000\,s2^4\sqrt{30} + 24255634000\,s2^5 + 424910520\,s2^3\sqrt{30} + 9916283000\,s2^4 + 1446392640\,s2^2\sqrt{30} - 61049949694\,s2^3 + 1452398400\,s2\sqrt{30} - 14927264693\,s2^2 + 7103136000\sqrt{30} + 111564949920\,s2 - 2544099450\right)\right)$$ **(22)**

```
> factor(hilf2);
```

$$\frac{(-1+2\,s2)\,(2\,s2+1)}{4}$$ **(23)**

```
> s2:=solve(-1 + 2*s2,s2);
```

$$s2 := \frac{1}{2}$$ **(24)**

```
> factor(yes1);
```

$$\frac{1}{3235731}\left(2\left(35\sqrt{30} - 1809\right)\left(521260\,t3\sqrt{30} + 3235731\,t3^2 + 1980342\,t3 - 3235731\right)\left(t3^2 - 2\,t3 - 1\right)\right)$$ **(25)**

```
> factor(yes4);
```

$$\frac{26\left(27\sqrt{30} + 350\right)\left(7780\,t3\sqrt{30} + 10063\,t3^2 - 95634\,t3 - 10063\right)\left(t3^2 - 2\,t3 - 1\right)}{10063}$$ **(26)**

```
> t3:=solve(-t3+1+2^(1/2),t3);
```

$$t3 := 1 + \sqrt{2}$$ **(27)**

```
> evalf(2*arctan(t3)*180/Pi);
```

$$135.$$ **(28)**

```
> factor(yes2);
```

$$-\frac{1}{4713065}\left(4\left(1131\sqrt{2}\sqrt{15} + 1508\sqrt{15} + 7400\sqrt{2} + 11100\right)\left(7940186\,c1\sqrt{2}\sqrt{15}\,s1 - 3970093\sqrt{2}\sqrt{15} + 9426130\,c1^2 - 29921800\,c1\,s1 - 9426130\,s1^2 + 14960900\right)\right)$$ **(29)**

```
> factor(yes3);
```

$$\frac{1}{26418119}\left(8\left(2880\sqrt{2}\sqrt{15} + 3840\sqrt{15} - 2218\sqrt{2} - 3327\right)\left(7940186\,c1\sqrt{2}\sqrt{15}\,s1 - 3970093\sqrt{2}\sqrt{15} - 26418119\,c1^2 + 29921800\,c1\,s1 + 26418119\,s1^2 - 14960900\right)\right)$$ **(30)**

```
> yo2:=7940186*c1*sqrt(2)*sqrt(15)*s1 - 3970093*sqrt(2)*sqrt(15) +
  9426130*c1^2 - 29921800*c1*s1 - 9426130*s1^2 + 14960900;
```

$yo2 := 7940186\,c1\sqrt{2}\sqrt{15}\,s1 - 3970093\sqrt{2}\sqrt{15} + 9426130\,c1^2 - 29921800\,c1\,s1$ **(31)**

$- 9426130\, s1^2 + 14960900$

```
> yo3:=7940186*c1*sqrt(2)*sqrt(15)*s1 - 3970093*sqrt(2)*sqrt(15) -
  26418119*c1^2 + 29921800*c1*s1 + 26418119*s1^2 - 14960900;
```

$$yo3 := 7940186\, c1 \sqrt{2} \sqrt{15}\, s1 - 3970093 \sqrt{2} \sqrt{15} - 26418119\, c1^2 + 29921800\, c1\, s1 + 26418119\, s1^2 - 14960900 \quad (32)$$

```
> hilf1:=c1^2+s1^2-1;
```

$$hilf1 := c1^2 + s1^2 - 1 \quad (33)$$

```
> factor(resultant(yo2,hilf1,c1));
```

$$-\frac{1}{2180124301614001} \left(10 \left(-78552960844637 + 11879232872740 \sqrt{30}\right) \left(4360248603228002\, s1^2 + 284610004090314 \sqrt{30} - 1671693405153801\right) \left(2\, s1^2 - 1\right)\right) \quad (34)$$

```
> factor(resultant(yo3,hilf1,c1));
```

$$-\frac{1}{2180124301614001} \left(\left(1394594693167631 + 118792328727400 \sqrt{30}\right) \left(-4360248603228002\, s1^2 + 284610004090314 \sqrt{30} + 2688555198074201\right) \left(2\, s1^2 - 1\right)\right) \quad (35)$$

```
> s1:=solve(2*s1-2^(1/2),s1);
```

$$s1 := \frac{\sqrt{2}}{2} \quad (36)$$

```
> factor(yo2);
```

$$-\frac{\left(9426130\, c1 + 7940186 \sqrt{15} - 10247835 \sqrt{2}\right) \left(-2\, c1 + \sqrt{2}\right)}{2} \quad (37)$$

```
> factor(yo3);
```

$$-\frac{\left(-2\, c1 + \sqrt{2}\right) \left(-52836238\, c1 + 15880372 \sqrt{15} + 3503681 \sqrt{2}\right)}{4} \quad (38)$$

```
> c1:=solve(2*c1-2^(1/2),c1);
```

$$c1 := \frac{\sqrt{2}}{2} \quad (39)$$

```
> #Probe
> #theta1=Pi/2,theta2=Pi/3,theta3=-Pi/2,
> simplify(arccos(c1)*180/Pi); #theta1/2
```

$$45 \quad (40)$$

```
> simplify(arccos(c2)*180/Pi*2); #theta2/2
```

$$60 \quad (41)$$

```
> simplify(arccos(c3)*180/Pi); #theta3/2
```

$$135 \quad (42)$$

Teil 2

```
> end4:=Qmult(Qkon(DQ4n[1]),Qmult(Qkon(DQ3n[1]),Qmult(Qkon(DQ2n[1]
  ),Qmult(Qkon(DQ1n[1]),Given[1]))));
```

$$end4 := \left[\left[\frac{\left(\left(7\,s4\,\sqrt{5}-15\,c4\right)\sqrt{3}+7\,\sqrt{5}\,c4+15\,s4\right)\sqrt{13}}{260} + \frac{\left(-13\,s4\,\sqrt{5}-65\,c4\right)\sqrt{3}}{260} - \frac{\sqrt{5}\,c4}{20} + \frac{s4}{4}\right],\right.$$
$$\left[\frac{\left(\left(4\,\sqrt{3}\,s4+4\,c4\right)\sqrt{5}+5\,\sqrt{3}\,c4-5\,s4\right)\sqrt{13}}{130} + \frac{\sqrt{5}\,\left(\sqrt{3}\,s4+c4\right)}{10}\right],$$
$$\left[\frac{\left(\left(-s4\,\sqrt{5}+15\,c4\right)\sqrt{3}-\sqrt{5}\,c4-15\,s4\right)\sqrt{13}}{260} + \frac{\left(13\,s4\,\sqrt{5}-65\,c4\right)\sqrt{3}}{260} + \frac{\sqrt{5}\,c4}{20} + \frac{s4}{4}\right],$$
$$\left.\left[\frac{\left(\left(-2\,\sqrt{3}\,s4-2\,c4\right)\sqrt{5}-5\,\sqrt{3}\,c4+5\,s4\right)\sqrt{13}}{130} + \frac{\sqrt{5}\,\left(\sqrt{3}\,s4+c4\right)}{10}\right]\right] \quad (43)$$

```
> end56:=Qmult(DQ5n[1],DQ6n[1]);
```

$$end56 := \left[\begin{array}{c} c5\,c6-\frac{3\,s5\,s6\,\sqrt{13}}{13} \\ \frac{2\,c5\,s6\,\sqrt{13}}{13} \\ \frac{3\,c5\,s6\,\sqrt{13}}{13}+s5\,c6 \\ -\frac{2\,s5\,s6\,\sqrt{13}}{13} \end{array}\right] \quad (44)$$

```
> factor(simplify(Norm(end4,2,conjugate=false)^2));
```

$$c4^2+s4^2 \quad (45)$$

```
> factor(simplify(Norm(end56,2,conjugate=false)^2));
```

$$\left(c6^2+s6^2\right)\left(c5^2+s5^2\right) \quad (46)$$

```
> s4:=2*t4/(1+t4^2):c4:=(1-t4^2)/(1+t4^2):
> s5:=2*t5/(1+t5^2):c5:=(1-t5^2)/(1+t5^2):
> s6:=2*t6/(1+t6^2):c6:=(1-t6^2)/(1+t6^2):
```

```
> bed1:=factor(numer(simplify(end4[1]-end56[1]))):
> bed2:=factor(numer(simplify(end4[2]-end56[2]))):
> bed3:=factor(numer(simplify(end4[3]-end56[3]))):
> bed4:=factor(numer(simplify(end4[4]-end56[4]))):
```

```
> aha2:=factor(simplify(resultant(bed1,bed2,t4)/((1+t5^2)^2*(1+
  t6^2)^2)));
```

$$aha2 := \frac{1}{3929}\left(5408000\left(8\,\sqrt{13}+69\right)\left(3929+3929\,t5^4\,t6^4-87360\,t5^3\,t6^3+87360\,t5^3\,t6\right.\right. \quad (47)$$

$$+87360\,t5\,t6^3-30054\,t5^4\,t6^2-70870\,t5^2\,t6^4+36336\sqrt{13}\,t5^3\,t6+36336\sqrt{13}\,t5\,t6^3-7176\,t6^3+3929\,t5^4+3929\,t6^4-6624\sqrt{13}\,t5\,t6^2-7176\,t5^4\,t6^3+832\sqrt{13}\,t6^3-15648\sqrt{13}\,t5^4\,t6^2-36336\sqrt{13}\,t5^3\,t6^3-14560\sqrt{13}\,t5^2\,t6^4+832\sqrt{13}\,t5^4\,t6^3-832\sqrt{13}\,t5^4\,t6+6624\sqrt{13}\,t5^3\,t6^2+42496\sqrt{13}\,t5^2\,t6^2-14560\sqrt{13}\,t5^2-15648\sqrt{13}\,t6^2+152100\,t5^2\,t6^2-1664\sqrt{13}\,t5^2\,t6^3+7176\,t6+7176\,t5^4\,t6-9984\,t5^3\,t6^2-87360\,t5\,t6+14352\,t5^2\,t6^3-14352\,t5^2\,t6+9984\,t5\,t6^2-36336\,t5\,t6\sqrt{13}-70870\,t5^2-30054\,t6^2-832\,t6\sqrt{13}+1664\sqrt{13}\,t5^2\,t6)\big)$$

```
> aha3:=factor(simplify(resultant(bed1,bed3,t4)/((1+t5^2)^2*(1+t6^2)^2)));
```

$$aha3 := -\frac{1}{61}\Big(43264000\,(6\sqrt{13}-23)\,\big(305+305\,t5^4\,t6^4+54288\,t5^3\,t6^3-54288\,t5^3\,t6-54288\,t5\,t6^3+21714\,t5^4\,t6^2+32122\,t5^2\,t6^4-14544\sqrt{13}\,t5^3\,t6-14544\sqrt{13}\,t5\,t6^3-4056\,t5+3588\,t6^3+305\,t5^4+305\,t6^4+5704\sqrt{13}\,t5\,t6^2+3588\,t5^4\,t6^3+4056\,t5^3\,t6^4+936\sqrt{13}\,t6^3+6248\sqrt{13}\,t5^4\,t6^2+14544\sqrt{13}\,t5^3\,t6^3+9048\sqrt{13}\,t5^2\,t6^4+936\sqrt{13}\,t5^4\,t6^3+1196\sqrt{13}\,t5^3\,t6^4-936\sqrt{13}\,t5^4\,t6-5704\sqrt{13}\,t5^3\,t6^2-1196\sqrt{13}\,t5\,t6^4+4056\,t5^3-30656\sqrt{13}\,t5^2\,t6^2+9048\sqrt{13}\,t5^2+6248\sqrt{13}\,t6^2-106860\,t5^2\,t6^2-5616\sqrt{13}\,t5^2\,t6^3-3588\,t6-3588\,t5^4\,t6-19344\,t5^3\,t6^2-4056\,t5\,t6^4+54288\,t5\,t6-21528\,t5^2\,t6^3+21528\,t5^2\,t6+1196\sqrt{13}\,t5^3-1196\,t5\sqrt{13}+19344\,t5\,t6^2+14544\,t5\,t6\sqrt{13}+32122\,t5^2+21714\,t6^2-936\,t6\sqrt{13}+5616\sqrt{13}\,t5^2\,t6\big)\Big) \quad \textbf{(48)}$$

```
> aha4:=factor(simplify(resultant(bed1,bed4,t4)/((1+t5^2)^2*(1+t6^2)^2)));
```

$$aha4 := \frac{1}{233}\Big(70304000\,(4\sqrt{13}-21)\,\big(-233\,t5^4\,t6^4+16\sqrt{13}\,t5^4\,t6^2+176\sqrt{13}\,t5^3\,t6^3+16\sqrt{13}\,t5^2\,t6^4+550\,t5^4\,t6^2-1872\,t5^3\,t6^3+550\,t5^2\,t6^4-176\sqrt{13}\,t5^3\,t6+512\sqrt{13}\,t5^2\,t6^2-176\sqrt{13}\,t5\,t6^3-233\,t5^4+1872\,t5^3\,t6-5700\,t5^2\,t6^2+1872\,t5\,t6^3-233\,t6^4+16\sqrt{13}\,t5^2+176\,t5\,t6\sqrt{13}+16\sqrt{13}\,t6^2+550\,t5^2-1872\,t5\,t6+550\,t6^2-233\big)\Big) \quad \textbf{(49)}$$

```
> fin34:=factor(simplify(resultant(aha3,aha4,t6)));
```

$$fin34 := 1\,/\,278578035568083225382 81\,\Big(350579702885347096087692204425646142652416000000000000000000000000\,(260338167157+55413243156\sqrt{13})\,(t5^2+2\,t5-1)^2\,(28\sqrt{13}\,t5^3-419\,t5^4+312\sqrt{13}\,t5^2+312\,t5^3-28\,t5\sqrt{13}+1202\,t5^2-312\,t5-419)^2\,(t5^2-2\,t5-1)^2\,(622913636\sqrt{13}\,t5^7-398345039\,t5^8-695354504\sqrt{13}\,t5^6-1804640968\,t5^7+775284100\sqrt{13}\,t5^5+7252955380\,t5^6-2238907216\sqrt{13}\,t5^4-5623219784\,t5^5-775284100\sqrt{13}\,t5^3-6694649274\,t5^4-695354504\sqrt{13}\,t5^2+5623219784\,t5^3-622913636\,t5\sqrt{13}+7252955380\,t5^2+1804640968\,t5-398345039)^2\Big) \quad \textbf{(50)}$$

```
> fin24:=factor(simplify(resultant(aha2,aha4,t6)));
```

$$fin24 := -1 / 535945902914586778849\,(2191123143033419350548076277660288391577600000000000000000000000\,(-23150565361+14606688\sqrt{13})\,(t5^2+2\,t5-1)^2\,(19665584\sqrt{13}\,t5^7-55251803\,t5^8+43977284\sqrt{13}\,t5^6+31416866\,t5^7-77167336\sqrt{13}\,t5^5+558400304\,t5^6-302690024\sqrt{13}\,t5^4-294937318\,t5^5+77167336\sqrt{13}\,t5^3-1242005722\,t5^4+43977284\sqrt{13}\,t5^2+294937318\,t5^3-19665584\,t5\sqrt{13}+558400304\,t5^2-31416866\,t5-55251803)^2\,(t5^2-2\,t5-1)^2\,(28\sqrt{13}\,t5^3+419\,t5^4-312\sqrt{13}\,t5^2+312\,t5^3-28\,t5\sqrt{13}-1202\,t5^2-312\,t5+419)^2) \quad (51)$$

```
> fin23:=factor(simplify(resultant(aha2,aha3,t6)));
```

$$fin23 := 1 / 4234521942610370800998 01\,(20744361117476159531816106770748292464640000000000000000000000\,(655477567501+21836185980\sqrt{13})\,(t5^2+2\,t5-1)^2\,(t5^2-2\,t5-1)^2\,(4\sqrt{13}\,t5^3+13\,t5^4+24\sqrt{13}\,t5^2-4\,t5\sqrt{13}+50\,t5^2+13)^2\,(191178446884\sqrt{13}\,t5^7+650732044901\,t5^8+408246663656\sqrt{13}\,t5^6+291411154216\,t5^7-475230994684\sqrt{13}\,t5^5+505721922836\,t5^6-3806495434352\sqrt{13}\,t5^4-1061741755672\,t5^5+475230994684\sqrt{13}\,t5^3-12407394352866\,t5^4+408246663656\sqrt{13}\,t5^2+1061741755672\,t5^3-191178446884\,t5\sqrt{13}+505721922836\,t5^2-291411154216\,t5+650732044901)^2) \quad (52)$$

```
> gcd(gcd(fin34,fin24),fin23);
```

$$(t5^2+2\,t5-1)^2\,(t5^2-2\,t5-1)^2 \quad (53)$$

```
> solve(t5^2+2*t5-1,t5);
```

$$\sqrt{2}-1,\ -1-\sqrt{2} \quad (54)$$

```
> arctan(2^(1/2)-1)*180/Pi*2; arctan(-2^(1/2)-1)*180/Pi*2;
  #theta5/2
```

$$45$$
$$-135 \quad (55)$$

```
> solve(t5^2-2*t5-1,t5);
```

$$1+\sqrt{2},\ 1-\sqrt{2} \quad (56)$$

```
> arctan(2^(1/2)+1)*180/Pi*2; arctan(-2^(1/2)+1)*180/Pi*2;
  #theta5/2
```

$$135$$
$$-45 \quad (57)$$

```
> # somit zwei Loesungen
```

```
> t5:=2^(1/2)-1;
```

$$t5 := \sqrt{2} - 1 \quad (58)$$

```
> factor(gcd(gcd(aha2,aha3),aha4));
```

$$-1 - 2\,t6 + t6^2 \quad (59)$$

```
> solve(-1-2*t6+t6^2,t6);
```

$$\sqrt{2} + 1, 1 - \sqrt{2} \quad (60)$$

```
> t6:=1-2^(1/2);
```

$$t6 := 1 - \sqrt{2} \quad (61)$$

```
> factor(gcd(gcd(bed1,bed2),gcd(bed3,bed4)));
```

$$t4 - 2 - \sqrt{3} \quad (62)$$

```
> t4:=solve(t4-2-3^(1/2),t4);
```

$$t4 := 2 + \sqrt{3} \quad (63)$$

```
> arctan(t4)*180/Pi*2;
```

$$150 \quad (64)$$

```
> arctan(t5)*180/Pi*2;
```

$$45 \quad (65)$$

```
> arctan(t6)*180/Pi*2;
```

$$-45 \quad (66)$$

```
> # somit erhalten wir genau die Drehwinkel, mit welchen die Pose
  "Given" generiert wurde.
```