Computer Animation a Tool for the Design of Mechanisms

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1. Introduction

In the last years I was involved in the design of mechanisms in collaboration with a Viennese engineer, Ing. J. BROSOWITSCH, who even took out patents for each design. I personally played the role of a consulting engineer, in particular with regard to the implemented gearing. Additionally I developed PC-software that shows the mechanisms in action, that analyses the quality of transmission and that at the same time creates the DXF-files used for the production by NC-machines. One reason for this teamwork is that due to an Austrian tradition the courses for the *"Theory of Mechanisms"* at mechanical engineering faculties are held by geometers. This makes sense since plenty of geometry is included in kinematics.

In the first part of my paper I explain three of these mechanisms. The first has been developed in order to elude an existing patent. This mechanism (see Fig. 1) is used at garbidge trucks for lifting and emptying the garbage boxes. Here some new geometric ideas were necessary for the numerical design of gears for nonuniform transmission (compare [8] and [5]). The other two mechanisms (Fig. 3 and Fig. 6) provide uniform transmissions with high transmission ratios. The basic idea of J. BROSOWITSCH was the generation of a trochoid motion between two systems Σ_1 and Σ_2 by combining a pure rotation Σ_1/Σ_0 with a circular translating motion Σ_2/Σ_0. One of these mechanisms (Fig. 3) is implemented on garbage trucks in order to condense the garbage by revolving a drum on the truck.

In the second part I present the main ideas and the basic procedures which are used in the computer programs. The source code is written in Turbo-Pascal 7.0.

The animation was an essential part of this work as it offers a control for the correctness and the precision of the gearing. But the animation has also a commercial aspect: J. BROSOWITSCH, the owner of a small company dealing with the engineering components of "environmental technology", presents the software also as an advertising film when he is visiting his clients.

2. Three planar mechanisms and their geometry

All the following mechanisms have been invented with remarkable intuition by J. BROSOWITSCH. It was my job to deliver the theory behind and to solve particular geometric design problems.

2.1 An emptying device for garbage boxes

In order to bypass an existing patent a new mechanism was created for emptying garbage boxes into the garbage truck. The box is lifted by hydraulic power. This translation is combined with an accelerated rotation such that in the top position the garbage is thrown into the container. The user of the software can interactively modify the functional relation between the length l of translation and the angle φ of rotation by specifying the graph of the function $\varphi = \varphi(l)$.

How to generate this motion Σ_1/Σ_0 with one point tracing a straight line segment? J. BROSOWITSCH suggested a gearing that causes the rolling of the polodes p_0, p_1 (see Fig. 1). The following method for the computerized design of gears for a nonuniform transmission is based on an idea due to O. BAIER [1], p. 243 (compare [4], Abb. 446, p. 284):

Let a gear profile c be decomposed into its line elements (A_i, t_i). And suppose that each element is attached to the polode p at the corresponding pole P_i by a bar connecting P_i and A_i, perpendicular to t_i. This bar shall make a rigid angle α_i with p at P_i due to REULEAUX's theory. Now imagine that the polode is made from a flexing metal band with the attached line elements (A_i, t_i). Then each pair of flexes p_0, p_1 of this band defines the polodes of a motion where the attached line segments belong to a pair c_0, c_1 of mating tooth profiles[1]) (see Fig. 2).

The algorithm starts with the computation of the polodes p_0, p_1 of the given motion Σ_1/Σ_0. Then we stretch the polodes into a line segment p. We define a prototype of our tooth profile as a circular arc c with arbitrary center and radius or as a line segment in arbitrary position. c is of course located near p. After that the straight line segment p together with the attached prototype c of line elements is re-bent into its initial shapes p_0 and p_1 respectively. This gives one pair of corresponding profiles c_0, c_1 for the given motion Σ_1/Σ_0. By equidistant spacing of p_0 and p_1 the points of intersection between the polodes and the profiles of the different gear teeth are obtained.

Now a careful analysis of these profiles has to be carried out: The user has to look for global interferences between c_0 und c_1 and for cusps. The latter can be done automatically since the differential-geometric condition for cusps of c_0 or c_1 can be stated explicitely due to the wellknown EULER-SAVARY formula (see e.g. [7], p. 180). The offered zooming function assists at the optical analysis. If cusps or other interferences are detected, the user can react by modifying the addendum and dedendum of c_0 and c_1 or the radius ρ of c and the angle α made by c and p.

[1]) The evolutes of each pair of tooth profiles form locally a pair of mating chain-wheel profiles (see [6], Theorem 2).

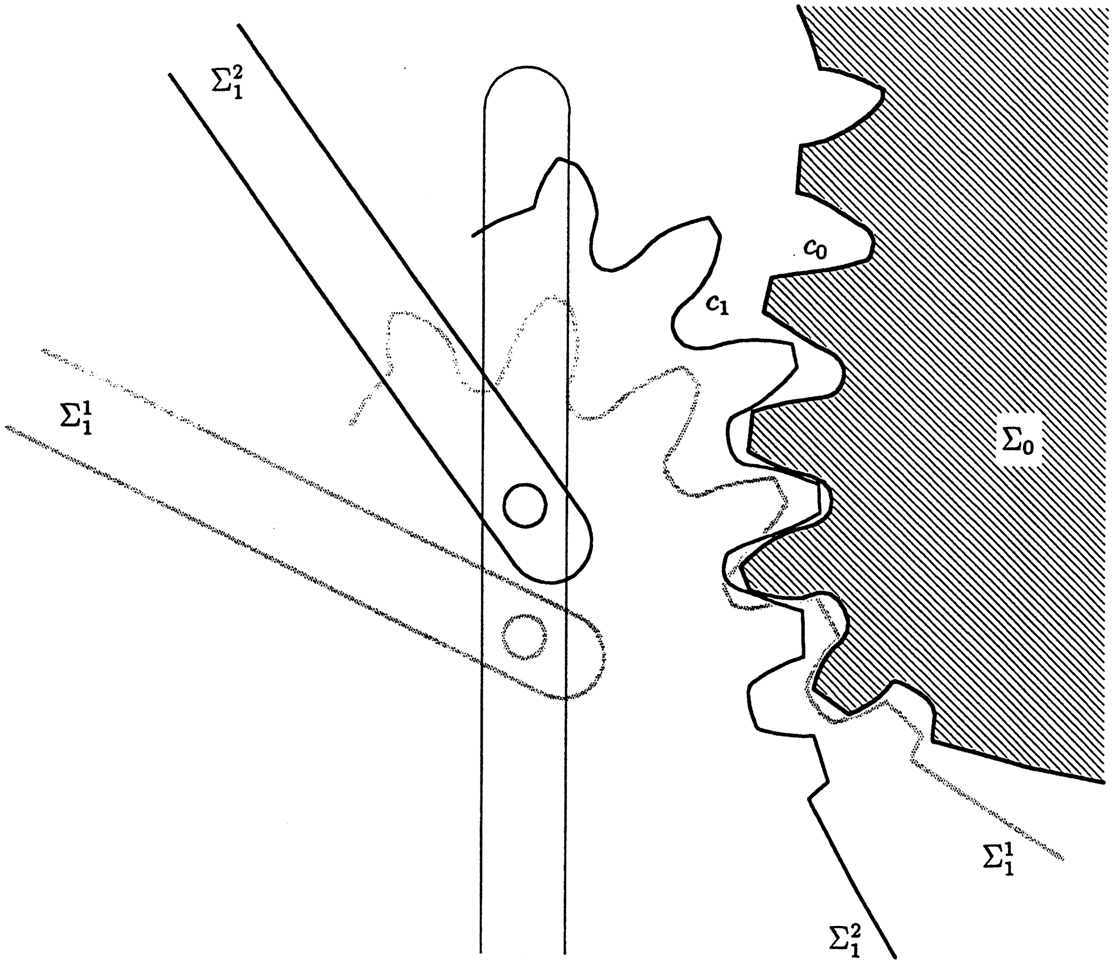

Figure 1: Mechanism Σ_1/Σ_0 for lifting and emptying garbage boxes. Two positions Σ_1^1 and Σ_1^2 of the lifting arm are displayed.

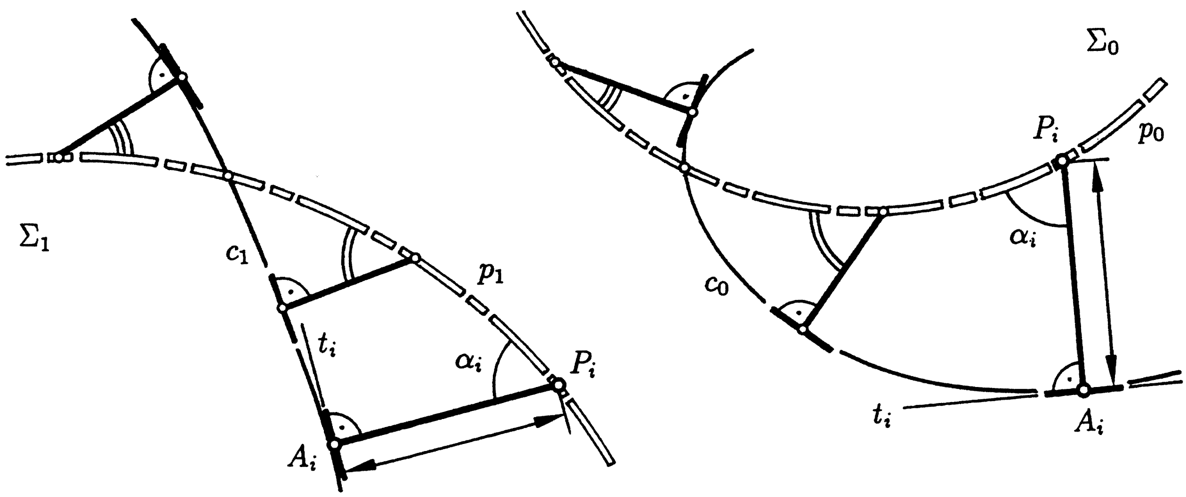

Figure 2: Mating tooth profiles c_0, c_1 decomposed into line elements, each attached to the polodes p_0, p_1 resp., which are seen as flexings of a metal band.

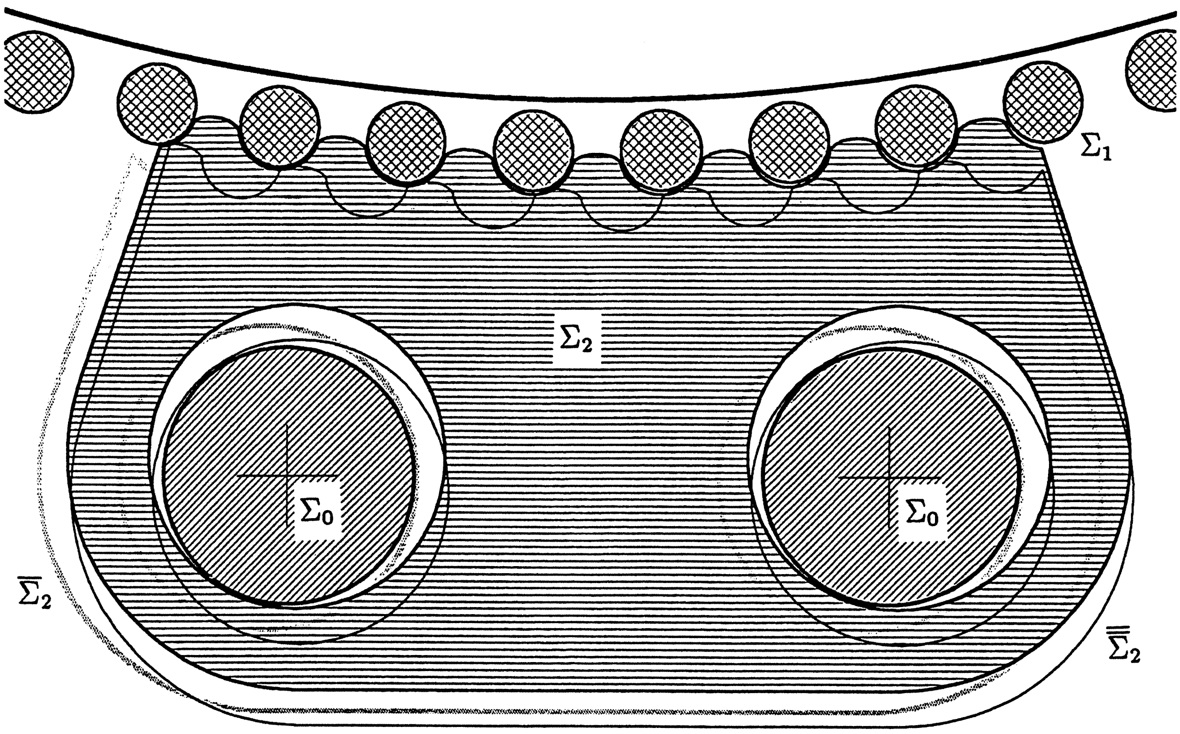

Figure 3: Driving mechanism for the garbage drum: Three driving elements in Σ_2, $\overline{\Sigma}_2$ and $\overline{\overline{\Sigma}}_2$ resp. perform a circular translating motion against Σ_0 while they are in action with the chain in Σ_1. At the displayed example the transmission angle μ varies between $39,8°$ and $76,6°$.

The proposed gearing contains the wellknown involute gearing as the limiting case $\rho = \infty$. This sort of gearing, which can also be applied in spherical kinematics, has the advantage that the shape of each profiles is controlled by the two parameters ρ and α only. But this method could of course be generalized by replacing the circle c by a rational B-spline curve.

2.2 A driving device for a drum on a garbage truck

Garbage trucks are usually equipped with a huge revolving drum, in which the garbage is condensed. J. Brosowitsch proposed a new driving mechanism for this drum. He enclosed the drum with a standard chain and attached it to the drum. The corresponding drive consists of three or more geared driving elements that perform a circular translating motion[2]) (see Fig. 3). Each driving element meshes with the chain. Actually, the beginning of our teamwork was the following question asked by J. Brosowitsch at the Institute of Geometry: How long should the driving elements be in order to guarantee a good transmission?

In the following the geometry behind this mechanism is briefly explained: Let Σ_1

[2]) In a variant the circular translating motion is replaced by a slider crank motion. In this case only one driving shaft with excenters is necessary instead of two in the previous case.

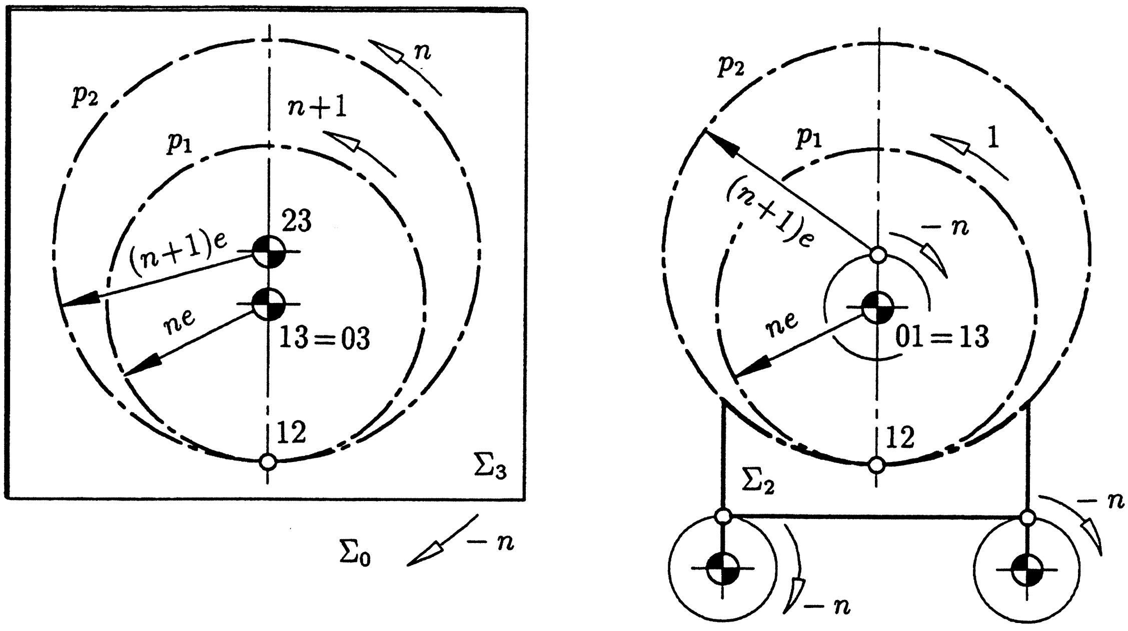

Figure 4: Trochoid motion Σ_1/Σ_2 as the composition of a pure rotation Σ_1/Σ_0 and a circular translating motion Σ_2/Σ_0.

and Σ_2 be two systems rotating against Σ_3 with constant angular velocities ω_{13}, ω_{23} about the fixed pivots 13, 23 respectively. Then the relative motion Σ_1/Σ_2 is a trochoid motion. The radii r_1, r_2 of the polodes p_1, p_2 match the equation

$$r_1\,\omega_{13} = r_2\,\omega_{23}\,.$$

These rolling circles p_1, p_2 serve as pitch circles when the relative motion Σ_1/Σ_2 is achieved by geared wheels. We now assume

$$\omega_{13} := n+1\,, \quad \omega_{23} := n \quad \text{for} \quad n \in \mathbb{N}, \text{ hence } \; r_1 : r_2 = \omega_{23} : \omega_{13} = n : (n+1)\,.$$

Then the radii in terms of the *excentricity* $e := \overline{13\,23}$ are

$$r_1 = n\,e\,, \quad r_2 = (n+1)\,e\,.$$

Now the following ideas are essential:

1. We superpose a rotation Σ_3/Σ_0 about $03 = 13$ with the constant angular velocity $\omega_{30} := -\omega_{23} = -n$ (see Fig. 4, left). Then Σ_2 performs a translating motion against Σ_0, as
$$\omega_{20} = \omega_{23} + \omega_{30} = -n + n = 0\,.$$

2. All points of Σ_2 share with the center 23 of p_2 the property of tracing a circle with radius e and with angular velocity $-n$ in the fixed plane Σ_0. The circle $p_2 \subset \Sigma_2$ is still rolling on $p_1 \subset \Sigma_1$, but the center 23 looses its exceptional position against Σ_0.

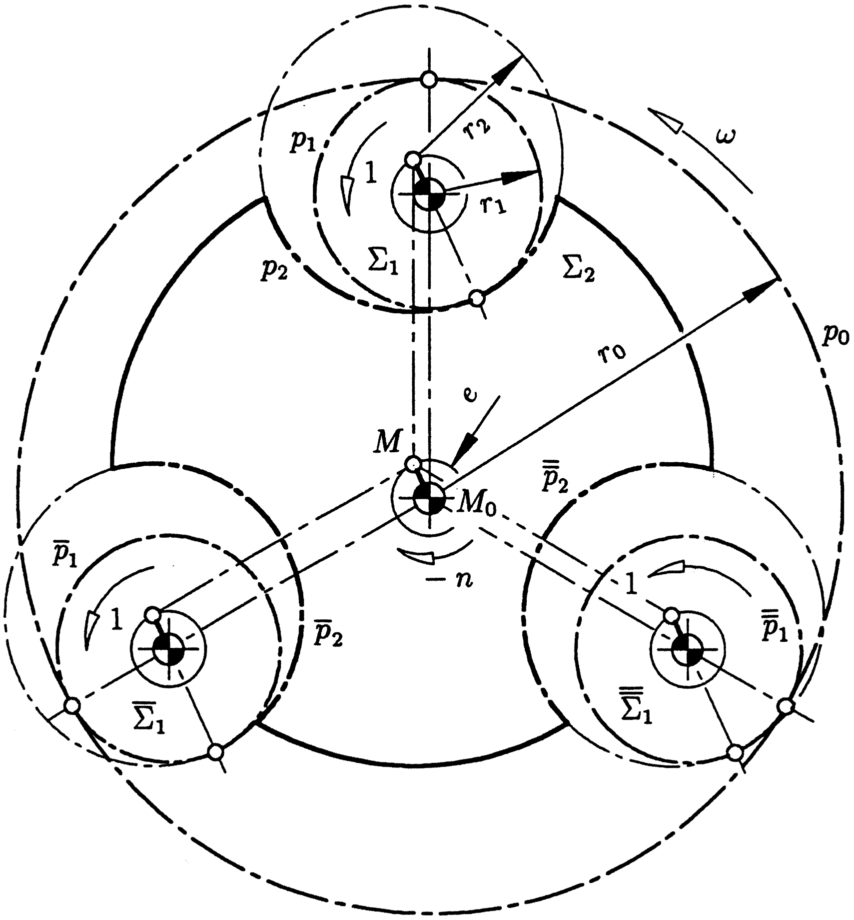

Figure 5: The translating motion Σ_2/Σ_0 with circles $p_2, \bar{p}_2, \bar{\bar{p}}_2, \ldots \subset \Sigma_2$ rolling on $p_1, \bar{p}_1, \bar{\bar{p}}_1, \ldots \subset \Sigma_1$ and on $p_0 \subset \Sigma_0$ too.

3. Two or more other circles $\bar{p}_2 \in \bar{\Sigma}_2$, $\bar{\bar{p}}_2 \in \bar{\bar{\Sigma}}_2, \ldots$ can be added which at the same time are rolling on p_1 while performing circular translating motions against Σ_0. Now instead of the complete geared wheels $p_2, \bar{p}_2, \bar{\bar{p}}_2, \ldots$ only portions can be used (Fig. 4, right side), provided one takes care that the next driving element comes in action before the forerunner stops meshing.

The product $2\pi e$ with the excentricity e must be an integer multiple of the circular pitch of the gearing. At BROSOWITSCH's version a pin gearing is used where $2\pi e$ equals the arc length on p_1 between two consecutive pin centers. Therefore, while each point of the driving elements is tracing a full circle, Σ_1 rotates from one tooth to the next.

The quality of the transmission depends on the transmission angle. At the pin gearing this angle varies. During the animation the extreme values and the instantanous value of the pitch angle are displayed on the screen. So the user can immediately check whether the specified length of the driving elements is sufficient (see Fig. 3). Obviously it is much more effective to increase the number of driving elements than to lengthen the driving elements by adding a few teeth.

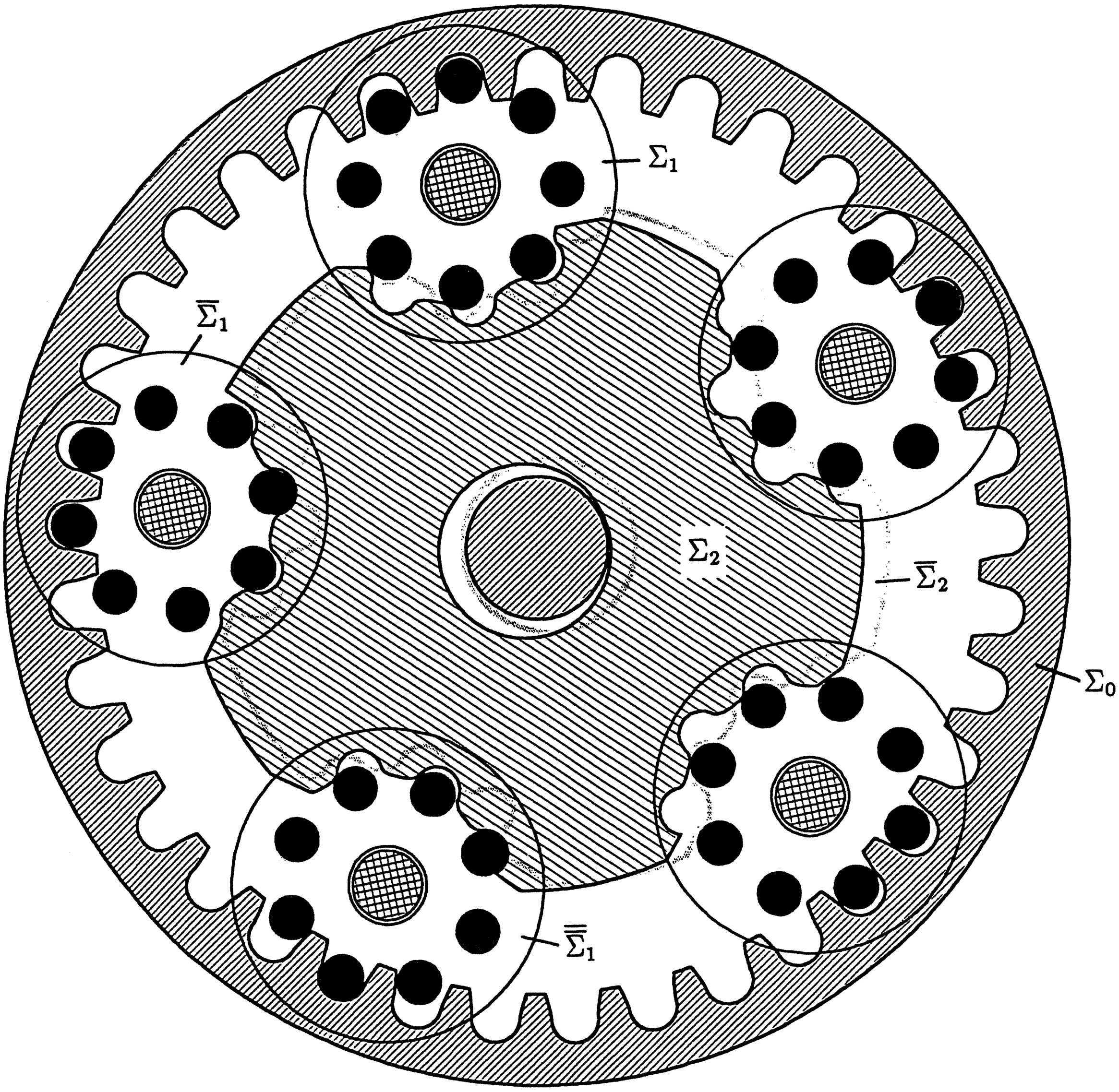

Figure 6: J. BROSOWITSCH's variant of the CYCLO-mechanism: Input: The centers of the disks Σ_2 and $\overline{\Sigma}_2$ rotate with the angular velocity $z_0 + 1 = 38$. Output: The centers of the five planetary wheels Σ_1, $\overline{\Sigma}_1, \ldots$ rotate with the angular velocity 1.

2.3 A mechanism with high transmission ratio

The idea presented above gives also rise to a new mechanism (Fig. 6) with high transmission ratio and with coinciding input and output shaft, similar to the CYCLO-mechanism: Here not only the translated circle p_2 but also the rotating circle p_1 appears in more exemplares $\overline{p}_1, \overline{\overline{p}}_1, \ldots$. Each of these circles $p_1, \overline{p}_1, \overline{\overline{p}}_1, \ldots$ is rolling on a circle $p_2, \overline{p}_2, \overline{\overline{p}}_2, \ldots$ resp. in Σ_2 (see Fig. 5).

Let $M \in \Sigma_2$ denote the center of symmetry for these translated circles $p_2, \overline{p}_2, \overline{\overline{p}}_2, \ldots$. Point M traces a circle centered at M_0 with radius e and angular velocity $-n$ in Σ_0. The angular velocity of Σ_2 with respect to Σ_0 is 0, that of Σ_1, $\overline{\Sigma}_1$, $\overline{\overline{\Sigma}}_1, \ldots$ is 1.

There is a circle $p_0 \subset \Sigma_0$ with center M_0 and radius r_0 which encloses and at the same time is rolling on $p_1, \overline{p}_1, \overline{\overline{p}}_1, \ldots$. Its angular velocity ω matches the equation

$$r_0 \cdot \omega = r_1 \cdot 1\,, \quad \text{hence} \quad \omega = \frac{r_1}{r_0} = \frac{ne}{r_0}\,.$$

This situation is shown in Fig. 5.

When we additionally superpose a rotation of $\Sigma_0, \Sigma_1, \ldots, \Sigma_2$ about M_0 with the angular velocity $-\omega$, then p_0 remains fixed. The circles $p_1, \overline{p}_1, \overline{\overline{p}}_1, \ldots$ are rolling on the inside of p_0; the angular velocity of the centers equals $-\omega$. The angular velocity of M with respect to M_0 becomes $-n-\omega$, and this results in the transmission ratio

$$(-n-\omega) : (-\omega) = (n+\omega) : \omega = \left(\frac{n}{\omega} + 1\right) : 1 = \left(\frac{r_0}{e} + 1\right) : 1$$

for the following mechanism (see Fig. 6):

Excenters on the input shaft (center M_0) are used for the drive of two or more disks which are meshing with three or more pin geared planetary wheels in Σ_1, $\overline{\Sigma}_1, \overline{\overline{\Sigma}}_1, \ldots$. At the same time these wheels perform a trochoid motion inside the geared frame with the pitch circle p_0. The rotation of the centers of the planetary wheels around the center M_0 of the frame is the output motion. If z_0 denotes the number of teeth at the frame wheel in Σ_0 and if $2\pi e$ is the specified circular pitch, then the transmission ratio of this mechanism reads

$$\left(\frac{r_0}{e} + 1\right) : 1 = (z_0 + 1) : 1\,.$$

The boundary of each driving element contains geared sections according to the number a of planet wheels (Fig. 6). What is the relation between the geared sections corresponding to the consecutive planets in Σ_1 and $\overline{\Sigma}_1$?

The polodes of the trochoid motion Σ_1/Σ_2 are p_1, p_2, that of $\overline{\Sigma}_1/\Sigma_2$ are $\overline{p}_1, \overline{p}_2$. The geared section of Σ_2 that meshes with Σ_1 is transformed into that meshing with $\overline{\Sigma}_1$ by the product of two rotations (see Fig. 7): The first is performed about the symmetry center M of Σ_2 through the angle $\varphi := \dfrac{2\pi}{a}$. The center of the second rotation coincides with the center of $\overline{p}_2$. In the following we derive the angle ψ of this rotation. If z_2 denotes the number of teeth of the full circle p_2, then of course only the residue class of ψ modulo the center angle $\zeta := \dfrac{2\pi}{z_2} = \dfrac{2\pi e}{r_2}$ between two adjacent teeth on $\overline{p}_2$ is essential:

We assume an initial position (see Fig. 7) where the points of contact between p_0, p_1 and p_2 are coinciding. This point of contact is supposed the center C of a pin on the planetary wheel in Σ_1. While p_1 is rolling along p_0 into the initial position of $\overline{p}_1$, the point of contact covers the arc length $r_0\varphi$ on p_0. The corresponding center angle on $\overline{p}_1$ reads $\dfrac{r_0\varphi}{r_1}$ and it defines the position of a pin center $\overline{C}$ on the following planet in $\overline{\Sigma}_1$. The point of $\overline{p}_2$ which meshes with $\overline{C}$ determines the angle

$$\psi = \left(\frac{-r_0\varphi}{r_1} + \varphi\right)\frac{r_1}{r_2} - \varphi = \frac{-r_0 + r_1 - r_2}{r_2}\varphi = \frac{-r_0 - e}{r_2}\cdot\frac{2\pi}{a} = -\frac{z_0+1}{a}\cdot\frac{2\pi}{z_2}\,.$$

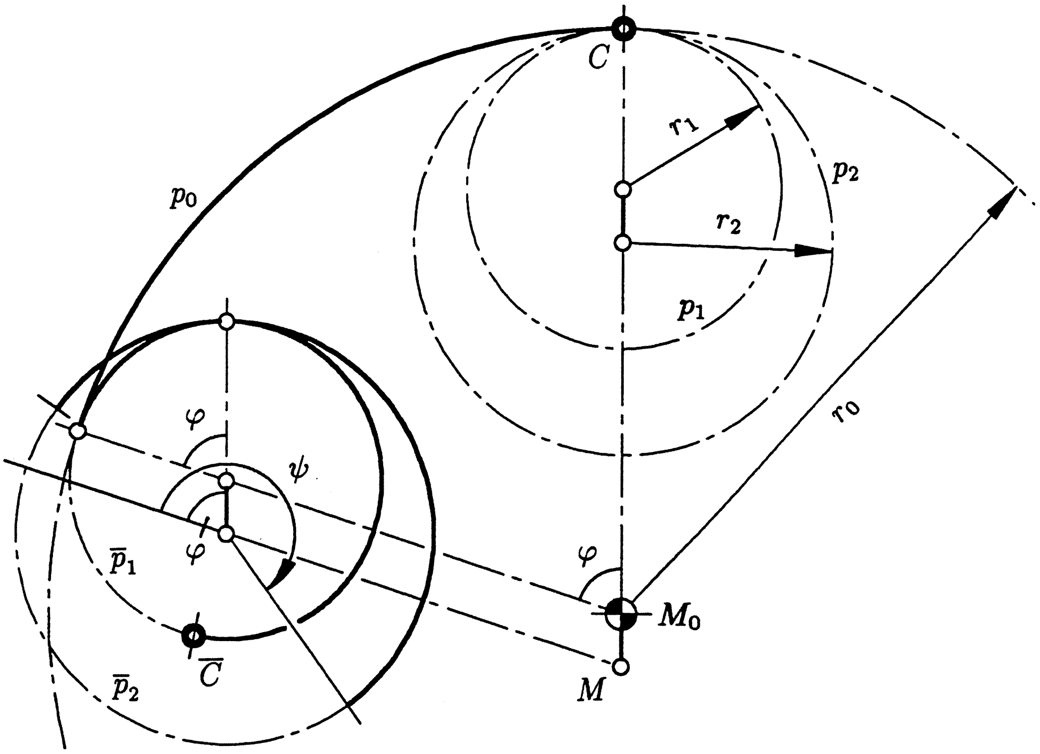

Figure 7: Phase difference between consecutive geared sections of Σ_2.

The example in Fig. 6 shows the case $z_0 = 37$ and $a = 5$. From $-(z_0 + 1) \equiv 2 \pmod a$ we obtain the following sequence of phase differences for the five geared sections of Σ_2 meshing with Σ_1, $\overline{\Sigma}_1$, $\overline{\overline{\Sigma}}_1, \ldots$ respectively:

$$0,\ \frac{2\zeta}{5},\ \frac{-\zeta}{5},\ \frac{\zeta}{5},\ \frac{-2\zeta}{5} \quad \text{with } \zeta = \frac{2\pi}{z_2} = \frac{2\pi e}{r_2}.$$

3. Programming techniques for the animation

There are several reasons for creating a computer animation for these mechanisms:

1. The animation shows each mechanism in action. This eases the understanding of the mechanism.

2. The computer program is a design tool. It offers the variation of parameters. E.g. at the example in chapter 2.2 the number of driving elements and the number of teeth at each driving element can be specified at the beginning of each run. Such variations make sense only when the animation runs in real time.

3. The animation offers a control for the correctness of the gearing. Especially in the nonuniform case it provides a control of the contact ratio and it helps to detect interferences. In case of interferences in the example of 2.1 the user can modify the

parameters ρ and α of each profile as well as the addendum and the dedendum. A zooming function assists at this visual analysis.

4. The base of the animation is a numerical representation of the motion. This allows also to observe the transmission angle, which is an important parameter for the quality of transmission. This angle should range between 40° and 140°.

5. The data needed for displaying the motion are also the base for DXF-files which are necessary for the NC-manufacturing of all components with a curved shape.

6. Finally the animation has also an important advertising aspect. For selling the patent it is necessary to contact various industries and to convince the staff of the effectiveness of the mechanism.

For animations the use of a two-sided screen is essential (see e.g. [2] or [3]). Here the frame-buffer is split into two pages, the *visual page*, which is displayed on the screen, and the *active page*, where the pixels are set. By use of Turbo-Pascal the exchange of the two buffers with internal numbers 0 and 1 can be performed with the following procedure:

```
uses
    Graph;
var
    visual_page: integer;

procedure swap_pages;
begin
  SetActivePage (visual_page);
  visual_page:= 1 - visual_page;
  SetVisualPage (visual_page);
end; {swap_pages}
```

Here the number of the current visual page is stored under the global variable *visual_page.*

When the animation shows moving figures in front of a rather complex background, then the figures should be drawn in the XOR-mode. In this mode the color of each pixel in the buffer is the bitwise XOR-composition of the original color and the new one. The XOR-operation, which is equivalent to "$\neq$", has the important property

$$(A \,\mathrm{XOR}\, B) \,\mathrm{XOR}\, B = A .$$

Drawing a line twice in this mode means therefore that the new line is erased and the original background restored.

Plane figures are seen as a union of vertices and lines. We store each plane figure as an array of *figure-elements.* Each figure-element contains the coordinates of the vertex (called *point*) and a boolean variable (=*draw*) that specifies whether this point has to be connected with its forerunner or not. A *plane figure* contains a pointer p to this array and the number n of its figure-elements. This explains the following definitions in Turbo-Pascal notation:

```
type
    coord2 = record x, y: real
             end;
    matrix22 = array [1..2,0..2] of real;
    figure_element_type = record draw: boolean;
                                 point: coord2
                          end;
    figure_element_pool = array [1..4000] of figure_element_type;
    ptr_figure_element_pool = ^figure_element_pool;
    plane_figure_type = record n: integer;
                               p: ptr_figure_element_pool
                        end;
```

In the definition of the *figure_element_pool* the maximal index 4000 is just an upper bound. The following procedure shows how the required memory for a figure with given number of figure-elements can be allocated dynamically:

```
procedure allocate_memory (var fig: plane_figure_type);
var
    size: integer;
begin
  with fig do begin
    size:= n*SizeOf (figure_element_type);
    GetMem (p, size)
  end
end; {allocate_memory}
```

When all figure-elements of a plane figure named *fig* are stored, then *fig.p^[i].point* gives the coordinates its i-th vertex.

Each moving system Σ_i has its own coordinate system. For each position Σ_i^j of Σ_i we store two 2×3-matrices, *in_wc^[j]* and *in_dc^[j]*. The latter is used for the transformation into the screen-coordinates in order to obtain a fast display of all figures belonging to Σ_i. It turns out that in this way the animation in real time on a PC works quite satisfactory for the mechanisms presented above. The first matrix *in_wc^[j]* transforms the Σ_i-coordinates into the current coordinates of Σ_0. This is used for updating *in_dc^[j]* after zooming.

Each plane figure belongs to a system and has to be drawn in a specified *display_mode* and color. Each plane mechanism consists of a number of plane figures that are moved. This results in the following type definitions:

```
type
    matrix22_pool = array [0..1000] of matrix22;
    ptr_matrix22_pool = ^matrix22_pool;
    plane_motion_type = record  n: integer;
                                in_wc, in_dc: ptr_matrix22_pool
                        end; {world coordinates and device coordinates}
```

```
display_type = (normal, thick, solid_fill, hatched);
plane_system_type = record figure: plane_figure_type;
                           motion: plane_motion_type;
                           color: word;
                           display: display_type
                    end;
plane_system_pool = array[1..1000] of plane_system_type;
ptr_plane_system_pool = ^plane_system_pool;
plane_mechanism_type = record n: integer;
                              system: ptr_plane_system_pool
                       end;
```

There is just one procedure necessary that shows the animation of a given mechanism of type *plane_mechanism_type.*

Turbo Pascal offers a procedure for filling simply connected polygons in the normal writing mode. For all other polygons and for the XOR-mode a subroutine had to be created for filling or hatching.

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